

RF ENERGY COMPRESSOR\*

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Abstract

The RF Energy Compressor, REC described here, transforms cw rf into periodic pulses using an energy storage cavity, ESC, whose charging and discharging is controlled by 180° bi-phase modulation, PSK, and external Q switching,  $\beta_s$ . Compression efficiency,  $C_e$ , of 100% can be approached at any compression factor  $C_f$ .

Introduction

There are many applications where we need a train of rf pulses, each pulse containing a given energy over a given time such as for charged particle accelerators, for radar and for communication. Usually the energy is built up during the time between pulses in a capacitor which discharges into an rf tube that converts the dc energy into rf. The REC performs a similar task, but it stores rf instead of dc energy. Just like a capacitor in a hard tube modulator, the ESC acts as a fly-wheel. It takes energy in between pulses and delivers it during the pulse. The control that tells the ESC when to accept and when to deliver energy is a low power 180° bi-phase modulator. The energy is delivered at a rate determined by a switch that controls the effective size of the cavity opening, i.e., its external Q. We arrange the duration and rate of ESC charging and discharging such that average energy level in the cavity is such that during charging it accepts all the generator energy available and during discharging, delivers it, together with the generator energy available during discharge, into a load.

Theory

The differential equation that governs the emitted field amplitude of a tuned cavity with an incident field  $E_i$  is<sup>1</sup>

$$T_c (dE_e/dt) + E_e = \alpha E_i \quad (1)$$

$\beta = Q_0/Q_e$ ,  $\alpha = 2\beta/(1+\beta)$ ,  $T_{CO} = Q_0/\pi f$ ,  $T_c = T_{CO}/(1+\beta)$  where  $\beta$  is the coupling factor, the ratio of emitted to dissipated power,  $\alpha$  is the steady state emitted field,  $T_c$  is the cavity time constant,  $Q_0$  is the unloaded Q factor,  $T_{CO}$  is the unloaded cavity time constant. Its solution during a time interval  $t_n$ , during which  $T_c$  and  $E_i$  are constant is

$$E_{enl} = E_{enf} + [E_{eni} - E_{enf}] e^{-t_n/T_c} \quad (2)$$

where

$E_{enl}$  = the emitted field at the end of time interval  $t_n$ ,

$E_{enf}$  = the steady state emitted field, the emitted field that would have been reached if  $t_n$  were infinite =  $\alpha_n E_{i1}$ .

$E_{eni}$  = the emitted field at the beginning of interval  $t_n$ .

$T_n$  = the normalized duration of the nth interval =  $t_n/T_{cn}$ .

If we have a given  $E_{ia}$  and  $Q_{ca}$  during interval 'a' and  $E_{ib}$  and  $Q_{cb}$  during the next interval 'b' and this interval pair repeats indefinitely, then the initial and final emitted fields during the two intervals are:

$$E_{eal} = E_{eaf} + [E_{eai} - E_{eaf}] e^{-\tau_a} \quad (3)$$

$$E_{eb1} = E_{ebf} + [E_{ebi} - E_{ebf}] e^{-\tau_b} \quad (4)$$

Using  $E_{eai} = \beta_a^{1/2} E_{cai}$  and  $E_{cbl} = \beta_b^{1/2} E_{cbl}$  in Eqs. (3) and (4) and noting that  $E_c$  does not change during transition and hence  $E_{cai} = E_{cbl}/\beta_b^{1/2}$  and  $E_{cbl} = E_{cai} = E_{eal}/\beta_a^{1/2}$ , we obtain

$$E_{eal} = E_{eaf} + [(1/k)E_{eb1} - E_{eaf}] e^{-\tau_a} \quad (5)$$

$$E_{eb1} = E_{ebf} + [kE_{eal} - E_{ebf}] e^{-\tau_b} \quad (6)$$

where  $k = (\beta_b/\beta_a)^{1/2} = (Q_{ea}/Q_{eb})^{1/2}$ . Solving Eqs. (5) and (6) simultaneously and using  $\tau_2 = \tau_a + \tau_b$  we obtain

$$E_{eaf} = \frac{E_{eaf}(1 - e^{-\tau_a}) + (1/k)E_{ebf}e^{-\tau_a}(1 - e^{-\tau_b})}{1 - e^{-\tau_2}} \quad (7)$$

$$E_{eb1} = \frac{E_{ebf}(1 - e^{-\tau_b}) + kE_{eaf}e^{-\tau_b}(1 - e^{-\tau_a})}{1 - e^{-\tau_2}} \quad (8)$$

Using  $E_{cai} = E_{eal}/\beta_a^{1/2}$  and  $E_{cbl} = E_{eb1}/\beta_b^{1/2}$ , we obtain

$E_{cai} = (1/k)E_{eb1}$ ;  $E_{eb1} = kE_{eal}$ ;  $E_i = 1$  during 'a' and  $-1$  during 'b'; hence,  $E_{eaf} = \alpha_a$ ,  $E_{ebf} = -\alpha_b$ . The field traveling away from the cavity  $E_r$ , is the superposition of the field reflected from the cavity wave guide interface,  $-E_i$  and  $E_e$ . Hence,  $E_r = E_e - E_i$  and  $E_{ra} = E_{ea} - 1$ , and  $E_{rb} = E_{eb} + 1$ . Thus, by imposing the periodicity condition, we have solved for all fields. Two periods of  $E_c$ ,  $E_e$ , and  $E_r$  are shown in Figs. 1a, 1b, and 1c respectively. An oscillogram of  $|E_r|$  vs. time is shown in Fig. 2.

Conversion Efficiency

Define the modulation period,  $t_2 = t_a + t_b$ , the compression factor,  $C_f = t_2/t_b$ , and the conversion efficiency, which is the normalized pulse power,  $P_p$ , divided

by the period,  $C_e = 1/t_2 \int_0^{t_b} E_{rb}^2 dt$ . Since  $E_{rb} = E_{ebf} + (E_{eb1} - E_{ebf})e^{-t/T_{cb}}$  we can obtain  $C_e$  as a function of  $\beta_a$ ,  $t_2$ ,  $C_f$ ,  $k$ .

To gain insight and obtain an approximate solution for  $C_f$  as a function of  $k$  at maximum efficiency, we will consider the limiting case of  $\tau_a, \tau_b \ll 1$ ,  $\beta_a, \beta_b \gg 1$ . Then,  $E_{eaf} = \alpha_a = 2$ ,  $E_{ebf} = -\alpha_b = -2$ ,  $1 - e^{-\tau} = \tau$ ,  $e^{-\tau} = 1 - \tau$ ,  $E_{eal} = 2(\tau_a - (1/k)\tau_b)/(\tau_a + \tau_b)$ . Imposing the zero reflection condition we set  $E_{eal} = 1$  and using  $T_{ca}/T_{cb} = Q_{ea}/Q_{eb} = k^2$ , and  $C_f = (t_a + t_b)/t_b$ , we obtain  $C_f + (1+k)^2$  or  $k = \sqrt{C_f} - 1$ . In the limit  $E_{ea} = 1$ ,  $E_{eb} = k$ ,  $E_{rb}^2 = (1+k)^2$ . Thus  $E_{rb}^2 = C_f$ . Hence, in the limit of  $\beta_a, \beta_b \gg 1$  and  $t_2/T_{CO} \ll 1$ , for any  $C_f$  from 1 to  $\infty$  we can choose the proper  $k$  and obtain 100% conversion efficiency. It turns out that not only does the above relationship between  $C_f$  and  $k$  yield  $C_e = 100\%$  at ideal

\*Work supported by the Department of Energy, contract DE-AC-3-76SF00515.

conditions but also yields maximum efficiency when  $C_f$  is considerably less than 100%.

The maximum efficiency for the nonideal case is obtained as follows. For a given  $t_2$ ,  $C_f$ ,  $T_{co}$  there is a unique value of  $\beta_a$  that maximizes  $C_e$ . If  $\beta_a$  is too large, then  $T_{ca}$  is too small and a large fraction of the charging energy is reflected; if it is too small then a large fraction of the charging energy is dissipated in the cavity and  $\alpha_a$  is too small. Plots of  $C_e$  vs.  $\beta_a$  for several values of  $\tau_{20}$  and for  $C_f = 10$  are shown in Fig. 3. The curves are nearly independent of  $C_f$ . Figure 4 is a plot of  $C_e$  vs.  $\tau_{20}$  with  $\beta_a$  optimized for maximum conversion efficiency. As  $C_f$  increases above its optimum value,  $P_p$  increases but at a slower rate than  $C_f$  to a limiting value of  $k\alpha_a + 1$ . Thus,  $C_e$  decreases first slowly and in the limit of large  $C_f$ , inversely as  $C_f$ . The curves of Fig. 5 are plots of  $C_e$  and  $P_p$  vs.  $C_f$  normalized to  $C_f = (1+k)^2 = C_{f0}$ , for which  $C_e = 1$  under ideal conditions. The points shown were experimentally obtained.

If, during  $t_b$ , the incident field is turned off instead of shifted  $180^\circ$  we have  $\beta_s$  only and  $E_{ebf} = 0$  instead of  $-\alpha_b$ . Then, the ideal gain is  $C_f - 1$  which can be approached if  $C_f = 1 + k^2$ . The bottom two curves of Fig. 5 are plots of  $C_e$  and  $P_p$  for  $\beta_s$  only. The points shown were experimentally obtained.

#### Practical Considerations

The period is given by:

$$t_2 = \tau_{20} T_{co} = \tau_{20} Q_0 / \pi f = \tau_{20} \Gamma / R_s \pi f$$

$\Gamma$  is the cavity geometric factor which depends only on cavity geometry and mode.  $R_s = 2.61 \times 10^{-4} f^2$ ,  $f$  is the operating frequency in MHz. The surface resistance of a niobium cavity operating at liquid helium temperature, 4.2°K,  $R_{ssc} = R_s 10^{-9} f^{3/2}$ .<sup>2</sup> If we use a TE<sub>011</sub> cavity for the ESC and we accept an efficiency of 86%, then  $\Gamma = 780$  and from Fig. 4,  $\tau_{20} = 90 \times 10^{-3}$ . Thus, with a copper ESC the maximum  $t_2$  is 0.56  $\mu$ s and with a superconducting ESC it is 3.7 ms. Using TE<sub>023</sub> spherical cavities, such as used by Ray Alvarez at Lawrence Livermore Laboratories,  $\Gamma$ , and hence  $t_2$  is increased by a factor of five.

The minimum pulse width,  $t_b$ , which determines the maximum  $C_f$  for a given period, is determined by the speed of PSK and  $\beta_s$  switches. The maximum power output depends on the power handling ability and the maximum period on the minimum dissipation of the  $\beta_s$ . Birx and Scallopino have reported a high intensity electron beam switch which can function in an evacuated x-band superconducting waveguide.<sup>3</sup> They obtained  $Q_e$  ratios of  $4 \times 10^3$  and  $T_{ca}$  of 500  $\mu$ s. Using an electron beam is analogous to a hard tube modulator. However, it can be much simpler because it does not require to hold off the high power amplifier voltage, and to carry the amplifier current.

The PSK switch does not limit output power because the power amplifier is interposed between the PSK switch and the ESC as shown in Fig. 6, so that the PSK switch operates at low power. Thus, for  $C_f$  approximately four, only low power switches are required. If  $C_f = 4$  then  $k = 1$  and, therefore, for this special case we approach 100% compression efficiency without  $\beta$ -switching, using PSK only. At the Stanford Linear Accelerator Center (SLAC) using single pulse PSK, and TE<sub>015</sub> ESC we have obtained output powers in excess of 125 MW. To illustrate the versatility of the REC I'll list in the table below the parameters of the present and several REC modes of SLAC operation, if we keep the 114 kW AC power input but replace the 50% efficient 38.5 MW, pulsed klystrons with 65% efficient 60kW cw klystrons (assume 80% AC to DC efficiency), and install REC with niobium

ESC at 4.2 K. The values are calculated as follows.  $T_{co} = 75$  ms,  $\tau_{20} = t_2 / T_{co} = 13.3 / \text{PRF}$ . Obtain  $C_e$  from plot of  $C_e$  vs.  $\tau_{20}$ , Fig. 4.  $P_p = 10^3 P_{cw} C_e / (\text{PRF}) (\text{PW})$ ,  $V = 30 (P_p / 38.5)^2$ .  $P_{cw}$  is the cw klystron power in kW, PRF is the pulse repetition frequency in  $s^{-1}$ , PW (or  $t_b$ ) is the pulse width in  $\mu$ s, and  $V$  is the SLAC beam voltage. The peak current  $I_p$ , and average current  $I_a$  are limited either by 10% loading,  $I = 2.86V$ , or by cw beam breakup,  $I = .4V$ , or by 1.5  $\mu$ s pulse beam breakup,  $I = 70V/24$ .  $dc$  is the duty cycle.

PRF 1/s	PW $\mu$ s	$C_e$	$P_p$ MW	V GeV	$I_p$ ma	$I_a$ $\mu$ a	dc %
360	2.5	~	38.5	30	87	50	.06
360	2.5	.92	61	38	87	50	.06
180	1	.86	287	82	200	11	.005
133	.9	.85	427	100	200	8	.004
cw	cw	~	.06	1.2	.400	400	100
$10^5$	2.5	.99	.24	2.4	7.0	1120	16
$10^4$	2.5	.98	2.35	7.4	21.6	345	1.6
$10^3$	2.5	.96	23	23	67	107	.16

With REC we can attain higher peak power, duty cycle, and average current than without REC. REC can be installed where cw klystrons already exist, as in electron-positron storage rings, and increase the beam energy as the number of bunches in the ring decrease. It is possible to have relatively closely spaced two or more pulses, and consider  $t_b$  as the sum of their widths, as long as in a super period the energy into the cavity is equal to the energy lost by it. It is also possible to have different length consecutive periods as long as for each period  $C_f = (1+k)^2$ .

A means has to be provided to separate the reverse and forward powers. The simplest way is to connect the generator, cavity, and load, each to one of the arms of a three-port circulator. A second method, which does not have the power limitations of the circulator is to connect the generator, two identical cavities and the load to consecutive ports of a 3-dB hybrid as shown in Fig. 6. A third method is to use a resonant ring.

#### Conclusion

It was shown that rf energy compression with a gain of four at compression efficiencies nearly 100% and up to a gain of eight with compression efficiencies above 60%, at power outputs in excess of 100 MW are possible, using a low power microwave switch whose insertion loss is of no consequence. At low power, high efficiencies and high compression ratios can be attained if we limit the period. To attain high compression factor at high power levels depends on further development of a low loss, fast turn-on turn-off high power microwave switch.

#### Acknowledgment

Thanks to D. L. Birx of Lawrence Livermore Laboratories for enlightening discussions.

#### References

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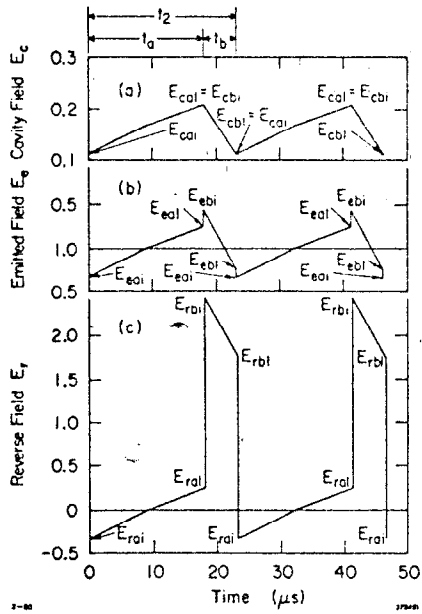


Fig. 1. REC fields vs time.  
 (a) Cavity field,  $E_c$ .  
 (b) Emitted field,  $E_e$ .  
 (c) Reverse field,  $E_r$ .

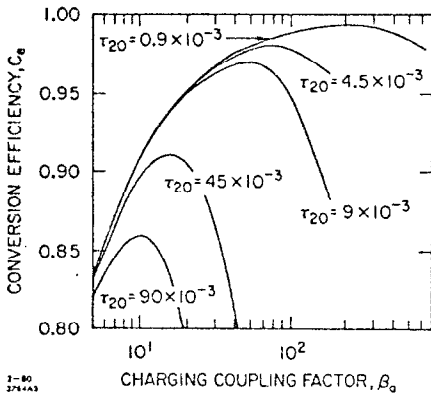


Fig. 3. Conversion efficiency,  $C_e$ , vs charging coupling factor,  $\beta_a$  for several normalized period,  $\tau_{20}$ .

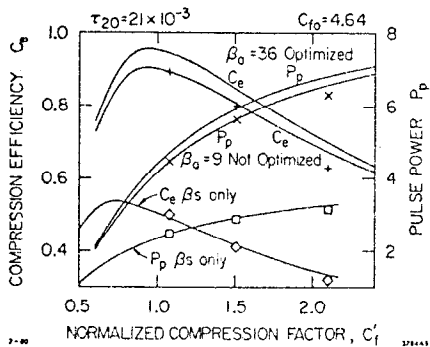


Fig. 5. Conversion efficiency  $C_e$  and normalized pulse power vs normalized compression factor,  $C_f$ .

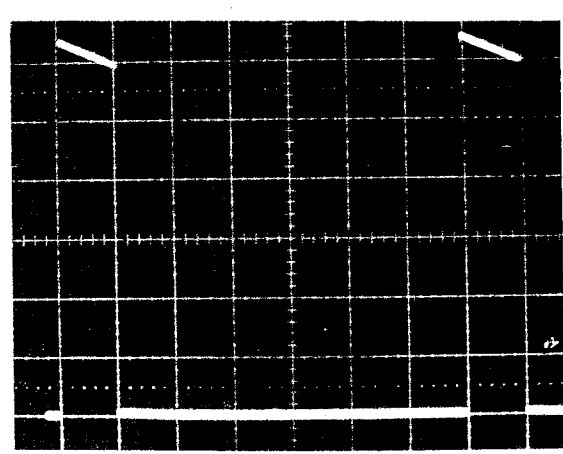


Fig. 2. Oscillogram of  $|E_r|$  vs time (10 μs/cm).

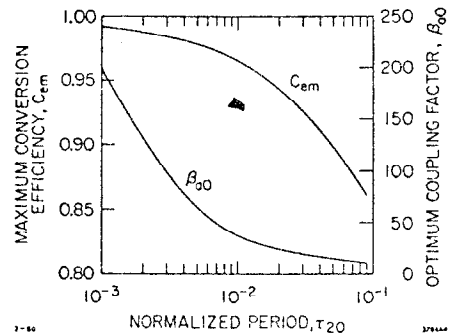


Fig. 4. Maximum  $C_e$ ,  $C_{em}$  and optimum  $\beta_a$ ,  $\beta_{00}$  vs normalized period,  $\tau_{20}$ .

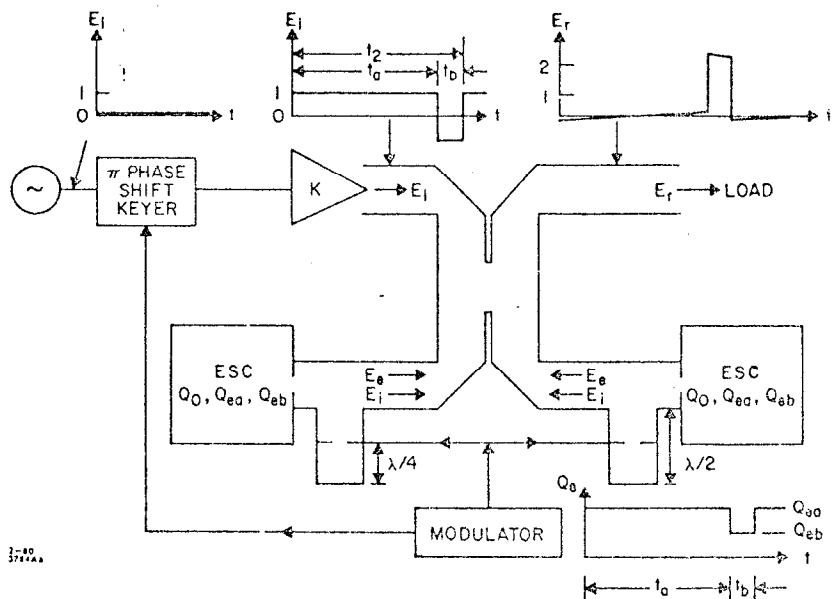


Fig. 6. RF energy compressor system.