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AN INTUITIVE PICTURE OF THE MATRIX ELEMENTS OF COMPOSITE PARTICLES AND THE RATIOS OF ψ AND ψ' DECAYS*

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ABSTRACT

In this paper we discuss the intuitive picture of the matrix element of composite particles, which has been represented in the field theory of composite particles.[1] In addition, we discuss the ratio of ψ and ψ' particle decays under a reasonable assumption and obtain some interesting results.

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1. Introduction

It is well known that hadrons are composite particles and that they are composed of quarks. Because hadrons have the phenomena of creation and annihilation, the composite particle model of hadrons should be discussed in the field theory of composite particles.

We proposed previously a quantized field theory of composite particles.[1] In this field theory hadrons are regarded as the excited quanta of a composite field, composed of quark fields. For example, a composite field operator of a meson can be defined by:

$$B(x_1,x_2) = T\Big(\psi(x_1)\ \bar{\psi}(x_2)\Big) \qquad (1)$$

where $\psi(x)$ is the quark field. The method used in establishing the composite fields theory essentially follows the formulation of the LSZ quantum field theory. Since we hope that many energy levels are involved, we have introduced some different assumptions: an asymptotic condition and completeness. Under these assumptions we have obtained many kinds of representations of S-matrix elements. Here we will discuss one of them; it shows that there is an equivalence relation between composite particle field and quark currents for any physical matrix element. We call it the field-current relation.

In this paper we will discuss the field-current relation from an intuitive picture. It will be seen that it is natural and is a kind of projection process. The quantized field theory of composite particles puts this intuitive picture in the framework of the field theory. Therefore we can discuss any physical matrix element of hadrons. When it is combined with current algebra, we can discuss many concrete processes.

The ψ' particle is the radially excited state of ψ; they have the same intrinsic quantum number. We will see that according to the field-current relation, there is a similar form in the matrix element. If we calculate their decay rate to the same final state, then the same Green function should be calculated. With a reasonable assumption about analytic continuation, we may suggest that their ratio does not depend on the final products. Using this assumption, we have analyzed a series of radially excited states and obtain some interesting results.

2. A Brief Review of the Field Theory of Composite Particles

At first, the wave function of mesons may be defined as

$$\begin{aligned}\chi_{\vec{k},\zeta}(x_1,x_2) &= \langle 0 \mid T\big(\psi(x_1)\ \bar{\psi}(x_2)\big) \mid \vec{k},\zeta \rangle \\ &= \frac{1}{\sqrt{2\omega}}\, e^{ikX}\, \chi_{\vec{k},\zeta}(x) \qquad (2)\end{aligned}$$

where $|\vec{k},\zeta\rangle$ is the bound state of the meson, ζ labels the quantum numbers of the meson including spin and isospin indices, and

$$\begin{aligned}X &= \frac{1}{2}(x_1+x_2) \\ x &= x_1 - x_2 \qquad . \qquad (3)\end{aligned}$$

$\chi_{\vec{k},\zeta}(x_1,x_2)$ satisfies the Bethe-Salpeter equation.[2]

The method essentially follows the formulation of the LSZ quantum field theory. Here we do not restate those fundamental principles discussed elsewhere. In the following we shall give two different assumptions:

(a) We introduce an asymptotic condition by means of the orthogonal and normalized condition

$$i\mathrm{Sp}\int d^3X\, d^4x \;\; \bar{\chi}_{\vec{k}',\zeta'}(X,x)\,\hat{Q}\frac{\overleftrightarrow{\partial}}{\partial X_0}\,\chi_{\vec{k},\zeta}(X,x) = \delta_{\vec{k},\vec{k}'}\,\delta_{\zeta,\zeta'} \qquad (4)$$

where $\hat{Q}$ is the weight operator of the orthogonal and normalized condition, which is given by Ref. 1 from the Bethe-Salpeter equation.

(b) We suggest the concept of completeness so as to cover all the bound states including the higher excited states, but not including scattering states of quarks.

Under these assumptions the reduction formula and S-matrix of hadrons may be obtained. For example, the form factor of the π-meson may be written as[1]

$$\langle k',\pi|J_\mu(z)|\vec{k},\pi\rangle = -\int d^4X\, d^4Y\, d^4x\, d^4y\;\frac{e^{ikX - ik'Y}}{\sqrt{4\omega\omega'}}\,\bar{\chi}_{\vec{k}',\pi}(X;x)\,\hat{Q}\left(m_\pi^2 - \Box_X\right)$$

$$\times\left(m_\pi^2 - \Box_Y\right)\left\langle 0\left|T\left(\psi\left(X+\frac{x}{2}\right)\bar{\psi}\left(X-\frac{x}{2}\right)J_\mu(z)\,\psi\left(Y-\frac{y}{2}\right)\bar{\psi}\left(Y+\frac{y}{2}\right)\right)\right|0\right\rangle\hat{Q}\chi_{\vec{k},\pi}(Y,y) \qquad (5)$$

where $J_\mu(z)$ is the electromagnetic current and the wave function may be written as

$$\chi_{\vec{k},\pi}(x_1,x_2) = \langle 0|T\left(\psi(x_1)\;\bar{\psi}(x_2)\right)|\vec{k},\pi\rangle = i\int d^4Y\, d^4y\;\frac{e^{ikY}}{\sqrt{2\omega}}\,\bar{\chi}_{\vec{k},\pi}(Y,y)\hat{Q}\left(m_\pi^2 - \Box_Y\right)$$

$$\times\left\langle 0\left|T\left(\psi\left(X+\frac{x}{2}\right)\bar{\psi}\left(X-\frac{x}{2}\right)\psi\left(Y-\frac{y}{2}\right)\bar{\psi}\left(Y+\frac{y}{2}\right)\right)\right|0\right\rangle \quad . \qquad (6)$$

According to the definition of Mandelstam:[3]

$$\langle 0|T\left(\psi(x_1)\;\bar{\psi}(x_2)\;J_\mu(z)\;\psi(y_2)\;\bar{\psi}(y_1)\right)|0\rangle \qquad (7)$$

$$= \int d^4u_1\, d^4u_2\, d^4v_1\, d^4v_2\;\; K\left(x_1,x_2;u_2,u_1\right)G_\mu\left(u_1,u_2;z;v_2,v_1\right)K\left(v_1,v_2;y_2,y_1\right)$$

where the four point Green's function is

$$\tilde{K}(x_1, x_2 \,;\, y_2, y_1) = \langle 0|T\Big(\psi(x_1)\ \bar{\psi}(x_2)\ \psi(y_2)\ \bar{\psi}(y_1)\Big)|0\rangle \quad . \tag{8}$$

From Eqs. (5)-(8) it follows that

$$\langle \vec{k}', \pi | J_\mu(z) | \vec{k}, \pi\rangle$$
$$= \int \bar{\chi}_{\vec{k}', \pi}(x_2, x_1)\ G_\mu(x_1, x_2 \,;\, z \,;\, y_2, y_1)\chi_{\vec{k}, \pi}(y_1, y_2)\ d^4x_1\, d^4x_2\, d^4y_1\, d^4y_2 \tag{9}$$

which is the Mandelstam representation[3] of the transition matrix element for the bound state. However in the field theory of composite particles any matrix element of composite particles may be discussed; the transition matrix element is given here as an example.

Notice that the Green's function $K(x_1, x_2; y_2, y_1)$ has the following representation (near the pole $p^2 + m^2 = 0$)

$$K(x_1, x_2 \,;\, y_2, y_1) = \frac{i}{(2\pi)^4}\int d^4P\ \frac{e^{iP(X-Y)}}{2P_0}\left[\frac{\chi_P(x)\ \bar{\chi}_P(y)}{P_0 - \omega + i\varepsilon}\right.$$
$$\left. + \text{ terms regular at } P_0 = \omega\right] \quad . \tag{10}$$

From Eqs. (7), (9) and (10) it may be proved easily that

$$\chi_{\vec{k}', \pi}(x)\langle \vec{k}'\pi \mid J_\mu(z) \mid \vec{k}\pi\rangle\bar{\chi}_{\vec{k}, \pi}(y)$$
$$= -\int d^4X\ d^4y\ \frac{e^{ikX - ik'Y}}{\sqrt{4\omega\omega'}}\ \left(m_\pi^2 - \Box_X\right)\left(m_\pi^2 - \Box_Y\right)$$
$$\left\langle 0\middle|T\Big(\psi\big(X + \tfrac{x}{2}\big)\bar{\psi}\big(X - \tfrac{x}{2}\big)J_\mu(z)\psi\big(Y - \tfrac{y}{2}\big)\bar{\psi}\big(Y + \tfrac{y}{2}\big)\Big)\middle|0\right\rangle \tag{11}$$

or

$$\langle \vec{k}',\pi \mid J_\mu(z) \mid \vec{k},\pi\rangle$$

$$= \frac{-1}{f_{\vec{k}',\pi}(x)\, f^*_{\vec{k},\pi}(y)} \int d^4X\; d^4Y\; \frac{e^{ikX - ik'Y}}{\sqrt{4\omega\omega'}} \left(m_\pi^2 - \Box_X\right)\left(m_\pi^2 - \Box_Y\right)$$

$$\times \quad \left\langle 0 \left| T\left[Sp\left(\Gamma\psi\left(X + \frac{x}{2}\right)\bar{\psi}\left(X - \frac{x}{2}\right)\right) J_\mu(z)\;\; Sp\left(\Gamma\psi\left(Y - \frac{y}{2}\right)\bar{\psi}\left(Y + \frac{y}{2}\right)\right)\right] \right| 0 \right\rangle \qquad (12)$$

where

$$\begin{aligned} f_{\vec{k},\pi}(x) &= Sp\left(\Gamma\chi_{\vec{k},\pi}(x)\right) \\ f^*_{\vec{k},\pi}(y) &= Sp\left(\Gamma\bar{\chi}_{\vec{k},\pi}(y)\right) \end{aligned} \qquad (13)$$

and Γ is the specific Dirac γ matrix. It may be seen that the left hand side of Eq. (12) does not depend on the variables x and y; formally, we may take the limit $x, y \to 0$. The singularity of the composite operator $\psi(Y)\ \bar{\psi}(Y)$ will be cancelled by the singularity of π-meson wave function at the origin $f_{\vec{k},\pi}(0)$. Therefore we have

$$\langle \vec{k}',\pi \mid J_\mu(z) \mid \vec{k},\pi\rangle$$

$$= \lim_{\substack{x \to 0 \\ y \to 0 \\ \text{spacelike}}} \left\{ \frac{-1}{f_{\vec{k}',\pi}(x)\, f^*_{\vec{k},\pi}(y)} \int d^4X\; d^4Y\; \frac{e^{ikX - ik'X}}{\sqrt{4\omega\omega'}} \left(m_\pi^2 - \Box_X\right)\left(m_\pi^2 - \Box_Y\right) \right.$$

$$\left. \times \quad \left\langle 0 \left| T\left[Sp\left(\Gamma\psi\left(X + \frac{x}{2}\right)\bar{\psi}\left(X - \frac{x}{2}\right)\right) J_\mu(z)\;\; Sp\left(\Gamma\psi\left(Y - \frac{y}{2}\right)\bar{\psi}\left(Y + \frac{y}{2}\right)\right)\right] \right| 0 \right\rangle \right\} \quad . \qquad (14)$$

Comparing Eq. (14) with the LSZ field theory, it may be seen that there is an equivalence relation:

$$\varphi_\pi(Y) \sim \lim_{\substack{y \to 0 \\ \text{spacelike}}} \left\{ \frac{1}{f_{\vec{k},\pi}(y)} \bar{\psi}\left(Y + \frac{y}{2}\right)\ \Gamma\psi\left(Y - \frac{y}{2}\right) \right\} \qquad (15)$$

where Γ may be selected as γ_5, $\not{k}\gamma_5$, ... Here we call the equivalence relation in the physical matrix element the "field-current relation." It does not depend on the Lagrangian model[4] and the above discussion has been extended to any type of meson and has been extended to any baryon. Therefore for any composite particle there is an equivalence relation, and it is correct for any physical matrix element of composite particles.

3. An Intuitive Picture

It should be noted that from Eq. (14) any process, in which hadrons participate, is calculable by the corresponding Green's function of quarks, using an equivalence relation of this type. Of course, the Green's function depends on the model for dynamics. Here we don't discuss how to calculate the Green's function, it is a difficult problem; however the procedure can be understood by an intuitive picture.

At first, as an example of the physical matrix element, we consider the wave function $\chi_{\vec{k},\pi}(x_1,x_2)$ (see Fig. 1). From Eq. (10) it may be seen that the wave function may be represented by a Green's function $K(x_1,x_2,y_2,y_1)$ given in Eq. (8) and

$$\int d^4Y \, \frac{e^{ikY}}{\sqrt{2\omega}} \left(m_\pi^2 - \Box_Y\right) \langle 0 \mid T\left(\psi(x_1)\bar{\psi}(x_2)\psi(y_2)\bar{\psi}(y_1)\right) \mid 0\rangle$$

$$= \langle 0 \mid T\left(\psi(\underline{x}_1)\ \bar{\psi}(x_2)\right) \mid \vec{k},\pi\rangle \ \bar{\chi}_{\vec{k},\pi}(y) \tag{16}$$

where k is on mass shell, i.e., $k^2 + m^2 = 0$. This equation may be represented by the intuitive picture shown in Fig. 2, where

$$\mathscr{P}_{\vec{k},\pi} = \int d^4Y \, \frac{e^{ikY}}{\sqrt{2\omega}} \left(m_\pi^2 - \Box_Y\right)$$

is an (unnormalized) projection operator. It projects the Green's function $K(x_1,x_2,y_2,y_1)$ into the product of two wave functions. In general, $K(x_1,x_2,y_2,y_1)$ satisfies the following equation (see Fig. 3),

$$K = V + VSV + \ldots \tag{17}$$

where $V(x_1,x_2;y_2,y_1)$ is the sum of all the irreducible parts, S is the propagator of quark and antiquark. If $\bar{\chi}_{\vec{k},\pi}(y)$ were a scalar, it could be transferred to the left-hand side, but it is a spinor. However by multiplying on both sides of Eq. (16) by a Γ matrix and taking the trace we can obtain a representation of the wave function:

$$\langle 0 \mid T\big(\psi(x_1)\ \bar{\psi}(x_2)\big) \mid \vec{k},\pi\rangle$$

$$= \frac{1}{f^*_{\vec{k},\pi}(y)} \int d^4Y\, \frac{e^{ikY}}{\sqrt{2\omega}} \left(m_\pi^2 - \Box_Y\right) \left\langle 0 \middle| T\left[\psi(x_1)\bar{\psi}(x_2)\,\mathrm{Sp}\left(\Gamma\psi\left(Y-\frac{y}{2}\right)\bar{\psi}\left(Y+\frac{y}{2}\right)\right)\right] \middle| 0 \right\rangle \tag{18}$$

$$= \lim_{\substack{y \to 0 \\ \text{spacelike}}} \left\{ \frac{1}{f^*_{\vec{k},\pi}(y)} \int d^4Y\, \frac{e^{ikY}}{\sqrt{2\omega}} \left(m_\pi^2 - \Box_Y\right) \right.$$

$$\left. \times \left\langle 0 \middle| T\left[\psi(x_1)\bar{\psi}(x_2)\,\mathrm{Sp}\left(\Gamma\psi\left(Y-\frac{y}{2}\right)\bar{\psi}\left(Y+\frac{y}{2}\right)\right)\right] \middle| 0 \right\rangle \right\} \tag{19}$$

Equations (18) and (19) may be represented diagramatically as shown in Figs. 4a and 4b, respectively. In particular, for the pseudoscalar wave function we have the following form

$$\chi_{\vec{k},\pi}(x) = \gamma_5 f_1^\pi(x) + \frac{i\not{k}}{m_\pi}\gamma_5 f_2^\pi(x) + \frac{i(k\cdot x)}{m_\pi}\not{x}\gamma_5 f_3^\pi(x) + \frac{ik_\mu x_\nu}{m_\pi}\gamma_5\sigma_{\mu\nu} f_4^\pi(x) \tag{20}$$

Obviously, we have

$$f_{\vec{k},\pi}(0) = 4f_1^\pi(0) \quad \text{if} \quad \Gamma = \gamma_5$$
$$f_{\vec{k},\pi}(0) = -4if_2^\pi(0) \quad \text{if} \quad \Gamma = \not{k}\gamma_5 \tag{21}$$

Thus the field-current relation can be understood through this intuitive picture.

Similarly, we may apply the projection method to any physical matrix element. As an example, we consider the electromagnetic form factor of the π-meson which has been studied in Eq. (11) and (14) by using two projection operators; the diagram is shown in Fig. 5.

From the above discussion it may be seen that there are three general features:

(a) In order to get the physical matrix element of hadronic processes, we may transform it into the above mentioned representation in which the corresponding Green's function (off mass shell) should be calculated. For example, for the form factor of the π-meson we may discuss the following Green's function

$$\left[(\not{k}'\gamma_5)_{\alpha\beta}\langle 0|T\ \psi_\beta(x_1)\bar{\psi}_\alpha(x_2)J_\mu(z)\psi_\delta(y_2)\psi_\gamma(y_1)|0\rangle(\not{k}\gamma_5)_{\gamma\delta}\right] \quad . \tag{22}$$

Then taking the projection, we may obtain the form factors of the π-meson. Of course, it is as difficult to calculate as the matrix element.

(b) Comparing these formulae with the LSZ field theory, it may be seen that there is a correspondence between the quark current $\left(1/f_{\vec{k},\pi}(0)\right)\bar{\psi}(Y)\Gamma\psi(Y)$ and the field quantity of the π-meson, $\varphi_\pi(Y)$, i.e., there is an equivalence relation (15). It is correct only on the mass shell of the π-meson. In other words, it is correct only for physical matrix elements of the π-meson after taking the projection. In this relation Y is the coordinate of the center of mass. The equivalent field quantity $\varphi_\pi(Y)$ may be regarded as a description for the motion of the center of mass of the π-meson.

(c) If the π-meson is replaced by any other meson, the above mentioned projection method can still be used. For example we may consider the vector meson, ρ. Similarly after using projection operator for the Green's function we may write its wave function as

$$\langle 0|T\left(\psi(x_1)\bar{\psi}(x_2)\right)|\vec{k},\rho\rangle = \frac{1}{Sp\left(\Gamma_\nu \chi_{\vec{k},\rho}(0)_\nu\right)} \int d^4Y \, \frac{e^{ikY}}{\sqrt{2\omega}} \left(m_\rho^2 - \Box_Y\right)$$

$$\times \langle 0|T\left(\psi(x_1)\bar{\psi}(x_2)\, Sp\left(\Gamma_\mu \psi(Y)\bar{\psi}(Y)\right)\right)|0\rangle\, e_\mu^\lambda \tag{23}$$

where

$$\chi_{\vec{k},\rho}(x) = \left(\chi_{\vec{k},\rho}(x)\right)_\mu e_\mu^\lambda \tag{24}$$

Notice that the general form of the vector meson wave function can be written as

$$\chi_{\vec{k},\rho}(x)_\mu = \Big[\gamma_\mu g_1^\rho(x) + \frac{i\not{k}}{m_\rho}\gamma_\mu g_2^\rho(x) + ix_\mu g_3^\rho(x) + x_\mu \not{x} g_4^\rho(x)$$

$$+ x_\mu x_\nu \frac{k_\lambda}{m_\rho}\sigma_{\nu\lambda} g_5^\rho(x) + \frac{i}{m_\rho}\varepsilon_{\mu\nu\rho\sigma} x_\nu k_\rho \gamma_\sigma \gamma_5 g_6^\rho(x)$$

$$+ \sigma_{\mu\nu} x_\nu \frac{k\cdot x}{m_\rho} g_7^\rho(x) + x_\mu \frac{\not{k}}{m_\rho}\frac{k\cdot x}{m_\rho} g_8^\rho(x)\Big] \tag{25}$$

By taking $\Gamma_\mu = \gamma_\mu$, then

$$\langle 0|T\left(\psi(x_1)\bar{\psi}(x_2)\right)|\vec{k},\rho\rangle \tag{26}$$

$$= \frac{1}{4g_1^\rho(0)} \int d^4Y \, \frac{e^{ikY}}{\sqrt{2\omega}} \left(m_\rho^2 - \Box_Y\right) \langle 0|T\left(\psi(x_1)\bar{\psi}(x_2) Sp\left(\gamma_\mu \psi(Y)\bar{\psi}(Y)\right)\right)|0\rangle\, e_\mu^\lambda$$

which means that on the mass shell of the ρ-meson there is an equivalence relation

$$\varphi_\mu(Y) \sim \frac{1}{4g_1^\rho(0)} \bar{\psi}(Y)\,\gamma_\mu\,\psi(Y) \tag{27}$$

Equations (15) and (27) are satisfied by any physical matrix element. It may be seen that PCAC and VDM are particular cases of the field-current relation. There are field-current relations not only for meson but also for baryons.[5] By similar arguments we may obtain the following equality

$$\int d^4X\, e^{iPX}\sqrt{\frac{m_B}{E}}\left(m_B^2 - \Box_X\right)\langle\alpha|T\Big(\psi(x_1)\psi(x_2)\psi(x_3)\Big)|\beta\rangle = i\psi^\lambda_{\vec{p},B}(x,x')\langle\alpha;\vec{p},B,\lambda|\beta\rangle \tag{28}$$

where $\psi^\lambda_{\vec{p},B}(x,x')$ is the wave function for baryon B with the momentum p and spin λ

$$\psi^\lambda_{\vec{p},B}(x_1,x_2,x_3) = \langle 0|T\Big(\psi(x_1)\psi(x_2)\psi(x_3)\Big)|\vec{p},B,\lambda\rangle$$

$$= \sqrt{\frac{m_B}{E}}\, e^{iPX}\, \psi^\lambda_{\vec{p},B}(x,x') \tag{29}$$

$$X = \frac{1}{3}(x_1 + x_2 + x_3)$$

$$x = x_1 - x_2$$

$$x' = \frac{1}{2}(x_1 + x_2) - x_3 \tag{30}$$

The problem that remains is how to project the wave functions. In other words, how to get the inverse form of the wave function, since the wave function of baryons is more complicated than that of a meson. Following the method suggested by A. B. Hemigues *et al.*,[6] we can deduce the following general form of the baryon wave function:

$$\left(\psi^{\lambda,j}_{p,k}(x,x')\right)^{abc}_{\alpha\beta\gamma} = \frac{1}{\sqrt{2}}\left[\left(\phi^j_k\right)^{abc}\chi^{\lambda}_{\alpha\beta\gamma} + \left(\varphi^j_k\right)^{abc}\xi^{\lambda}_{\alpha\beta\gamma}\right] \tag{31}$$

where ϕ^j_k and φ^j_k are the bases of the symmetry operators O_1 and O_2 separately in SU(3) space,

$$\left(\phi^j_k\right)^{abc} = -\frac{1}{\sqrt{6}}\left[\epsilon^{dba}\left(\lambda^j_k\right)^c_d + \epsilon^{dca}\left(\lambda^j_k\right)^b_a\right]$$

$$\left(\varphi^j_k\right)^{abc} = \frac{1}{\sqrt{2}}\,\epsilon^{dbc}\left(\lambda^j_k\right)^a_b \tag{32}$$

χ^{λ} and ξ^{λ} are the wave functions in spin space and the bases of the symmetry operators O_1 and O_2,

$$\begin{aligned}\left(\chi^{\lambda}\right)_{\alpha\beta\gamma} = \Big\{&(h-f)(\gamma_5 C)_{\beta\gamma}U^{\lambda}_a(\vec{P}) + \left[\left(h - f\frac{i\not{P}}{m_B}\right)C\right]_{\beta\gamma}\left(\gamma_5 U^{\lambda}(\vec{P})\right)_{\alpha} \\ &+ (h-f)\left(\frac{i\not{P}}{m_B}\gamma_\mu\gamma_5 C\right)_{\beta\gamma}\left(\gamma_\mu U^{\lambda}(\vec{P})\right)_{\alpha} \\ &+ \left[\left(g+f-h\frac{i\not{P}}{m_B}\right)\gamma_\mu C\right]_{\beta\gamma}\left(\gamma_\mu\gamma_5 U^{\lambda}(\vec{P})\right)_{\alpha} \\ &+ \text{other term including } x,x'\Big\}\end{aligned} \tag{33}$$

$$\begin{aligned}\left(\xi^{\lambda}\right)_{\alpha\beta\gamma} = \frac{1}{\sqrt{3}}\Big\{&\left[\left(-2g-3h+3f\frac{i\not{P}}{m_B}\right)\gamma_5 C\right]_{\beta\gamma}U^{\lambda}_{\alpha}(\vec{P}) \\ &+ (2g-3h+3f)(C)_{\beta\gamma}\left(\gamma_5 U^{\lambda}(\vec{P})\right)_{\alpha} \\ &- \frac{1}{\sqrt{3}}\,g\left(\gamma_\mu\gamma_5 C\right)_{\beta\gamma}\left(\gamma_\mu U^{\lambda}(\vec{P})\right)_{\alpha} \\ &+ \text{other terms including } x,x'\Big\}\ .\end{aligned} \tag{34}$$

where C is the charge conjugation operator. $U(\vec{P})$ is the spinor wave function, f,g,h are the invariant functions of P,x,x'. Substituting Eqs. (32)-(34) into Eq. (31) and noticing that the proton wave function in SU(3) space is λ_2^1, we may obtain an equivalent representation

$$\langle\alpha;\vec{P},\lambda|\beta\rangle = \frac{i}{B(0)}\,\varepsilon_{dbc}(\lambda_2^1)_a^d(C\gamma_5)_{\gamma\beta}\int d^4X\; e^{-iPX}\sqrt{\frac{m_P}{E}}\,\bar{U}_{\alpha'}(\vec{P})\Big(m_P+i\not{P}\Big)_{\alpha'\alpha}$$

$$\times\;\langle\alpha|T\Big(\psi_\alpha^a(X)\,\psi_\beta^b(X)\,\psi_\gamma^c(X)\Big)|\beta\rangle \qquad (35)$$

From Eq. (35) it follows that if one introduces an equivalent proton field quantity $\Psi_P(X)$, then we have

$$\Psi_P(X) \sim \frac{1}{B(0)}\,\varepsilon_{dbc}(\lambda_2^1)_a^d \lim_{\substack{x\to 0\\ x'\to 0}}\Big\{(C\gamma_5)_{\alpha\beta}T\Big(\psi_\alpha^a(x_1)\psi_\beta^b(x_2)\psi_\gamma^c(x_3)\Big)\Big\}$$

$$B(0) = \frac{16}{\sqrt{3}}\Big(g(0)+\frac{3}{2}\,h(0)\Big) \qquad (36)$$

It may be seen that there are different field-current relations for particles with different quantum numbers, and even for the same particle there are different field-current relations, because one may take different spinor operators, e.g., $\gamma_\mu\gamma_5$, γ_5, $C\gamma_5$, C, ...

Since there are many kinds of field-current relations, one may select what one needs according to the actual problem. By combining the above discussion with current algebra techniques, we may discuss some concrete processes. For example, using Eqs. (15) and (36) we can discuss the vertex $NN\pi$. Under the soft π approximation we obtain a relation which is very much like the Goldberger-Treiman relation. From the process $\pi\to\mu\nu$, we may obtain $g^2/4\pi \simeq 15.4$, which is in agreement with the experimental value.[5]

4. The Decay Ratio of ψ and ψ' Particles

Now we will discuss some phenomenological relationships between the ground state and the radially excited states with the same intrinsic quantum number. For example, we can consider ψ and ψ', if they have the same decay products, i.e.,

$$\psi \to f$$

$$\psi' \to f$$

Then according to the field-current relation for vector mesons we have ($k^2+m^2 = 0$, $k'^2+m'^2 = 0$)

$$\langle f|\psi,\lambda\rangle = \frac{1}{4g_1^{\psi}(0)}\int d^4Y\, \frac{e^{ikY}}{\sqrt{2\omega}}\left(m_\psi^2-\Box_Y\right)\langle f|\bar{\psi}_c(Y)\gamma_\mu\psi_c(Y)|0\rangle\, e_\mu^\lambda \tag{37}$$

$$\langle f|\psi',\lambda'\rangle = \frac{1}{4g_1^{\psi'}(0)}\int d^4Y\, \frac{e^{ik'Y}}{\sqrt{2\omega}}\left(m_{\psi'}^2-\Box_Y\right)\langle f|\bar{\psi}_c(Y)\gamma_\mu\psi_c(Y)|0\rangle\, e_\mu^{\lambda'} \tag{38}$$

where $\psi_c(Y)$ is the field quantity of the charm quark. It may be seen that the Green's function which one wants to calculate has the same form for the two kinds of processes, the only difference being projection onto different mass states. In general, there may be many poles in the matrix element $\langle f|\bar{\psi}_c(Y)\gamma_\mu\psi_c(Y)|0\rangle$, and certainly there are at least two poles, ψ and ψ'. As an example, in the case of the two poles we may change the above matrix element into another form

$$\langle f|k,\lambda,\psi\rangle = \frac{1}{4g_1^{\psi}(0)}\int d^4Y\, \frac{e^{ikY}}{\sqrt{2\omega}}\left(m_\psi^2-\Box_Y\right)\left(m_{\psi'}^2-\Box_Y\right)\frac{\langle f|\bar{\psi}_c(Y)\gamma_\mu\psi_c(Y)|0\rangle e_\mu^\lambda}{m_{\psi'}^2 - m_\psi^2} \tag{39}$$

and

$$\langle f|k',\lambda',\psi'\rangle = \frac{1}{4g_1^{\psi'}(0)}\int d^4Y\,\frac{e^{ik'Y}}{\sqrt{2\omega'}}\left(m_{\psi'}^2-\Box_Y\right)\left(m_\psi^2-\Box_Y\right)$$

$$\times\,\frac{\langle f|\bar{\psi}_c(Y)\gamma_\mu\psi_c(Y)|0\rangle e_\mu^{\lambda'}}{m_\psi^2-m_{\psi'}^2}. \qquad (40)$$

It may be seen that the function,

$$\int d^4Y\,\frac{e^{ikY}}{\sqrt{2\omega}}\left(m_\psi^2-\Box_Y\right)\left(m_{\psi'}^2-\Box_Y\right)\langle f|\bar{\psi}_c(Y)\gamma_\mu\psi_c(Y)|0\rangle \quad ,$$

does not have any pole. Hence we might assume that it is a smooth function of k^2. Under this assumption we may obtain an approximate equality for their amplitude

$$\frac{T_{\psi'\to f}}{T_{\psi\to f}} = c(\psi,\psi') \qquad (41)$$

where $c(\psi,\psi')$ only depends on ψ and ψ', but not on f. Once the constant $c(\psi,\psi')$ is determined from $\Gamma_{\psi\to ee}$ and $\Gamma_{\psi'\to ee}$, one may calculate $\Gamma_{\psi'\to f}$ from $\Gamma_{\psi\to f}$. Theoretical values are listed in Table I (we have considered the phase space correction).

For four particle decay, ignoring the phase space difference, it may be expected that

$$\frac{\Gamma_{\psi'\to 2\pi^+2\pi^-}}{\Gamma_{\psi\to 2\pi^+2\pi^-}} \simeq \frac{\Gamma_{\psi'\to K^+K^-\pi^+\pi^-}}{\Gamma_{\psi\to K^+K^-\pi^+\pi^-}}$$

The experiment result is

$$\frac{\Gamma_{\psi' \to 2\pi^+ 2\pi^-}}{\Gamma_{\psi \to 2\pi^+ 2\pi^-}} \simeq 0.68 \pm 0.32$$

$$\frac{\Gamma_{\psi \to K^+ K^- \pi^+ \pi^-}}{\Gamma_{\psi \to K^+ K^- \pi^+ \pi^-}} \simeq 0.66 \pm 0.34$$

Similarly, it may be expected that for T, we have

$$\frac{T_{T' \to f}}{T_{T \to f}} \simeq c'(T,T') \tag{42}$$

where $c'(T,T')$ only depends on T and T', but not on f. This prediction will be checked by experiments.

Following a similar method we have discussed a series of the radially excited states, for example, $N^*(1470)$ and $N^*(1780)$, $N^*(1535)$ and $N^*(1700)$, $\Lambda(1520)$ and $\Lambda(1690)$, $\Sigma(1670)$ and $\Sigma(1940)$, ... At present, the experimental measurements for these decay processes are not accurate. But it seems that the theoretical results agree with the experiments.[8]

All the above analyses show that the amplitude ratios for the radically excited states only depend on the different radially excited states themselves and do not depend on final products (or the ratio is only slightly dependent on final states). The reason is that they have the same intrinsic quantum numbers, except for the radial quantum number. Probably they have the same decay mechanism. It seems that these decay processes can be factorized into two parts. The problem about the decay mechanism may be investigated in the future.

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TABLE I

Two Particles Decay of ψ'

Process	Cal (%)	Exp. (%) [a]
$B(\psi' \to ee)$	0.88 (input)	0.88 ± 0.13
$B(\psi' \to \mu\mu)$	0.88	0.88 ± 0.13
$B(\psi' \to p\bar{p})$	0.027	0.023 ± 0.007
$B(\psi' \to \rho\pi)$	0.13	< 0.1
$B(\psi' \to \pi\pi)$	0.0013	< 0.005
$B(\psi' \to KK)$	0.002	< 0.005
$B(\psi' \to \Lambda\bar{\Lambda})$	0.022	< 0.04
$B(\psi' \to \Xi\bar{\Xi})$	0.0064	< 0.02
$B(\psi' \to \pi^{o}\gamma)$	0.01	< 0.7
$B(\psi' \to \eta\gamma)$	0.011	< 0.042
$B(\psi' \to \eta'\gamma)$	0.03	< 0.11
$B(\psi' \to K^{o}\bar{K}^{o*})$	0.042	
$B(\psi' \to K^{+}K^{-*})$	0.034	
$B(\psi' \to \omega f)$	0.04	
$B(\psi' \to \rho A_2)$	0.12	
$B(\psi' \to B\pi)$	0.048	
$B(\psi' \to \phi\eta)$	0.013	
$B(\psi' \to \phi f')$	0.011	

[a] Ref. 7.

FIGURE CAPTIONS

1. The wave function of the π meson.

2. The diagrammatical representation of Eq. (16).

3. The Bethe-Salpeter equation of $K(x_1,x_2;y_2,y_1)$.

4. (a) The diagrammatical representation of Eq. (18).

   (b) The diagrammatical representation of Eq. (19).

5. The diagrammatical representation of the form factor of the π meson.

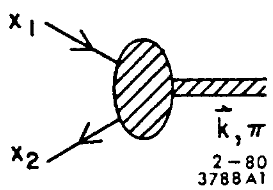

Fig. 1

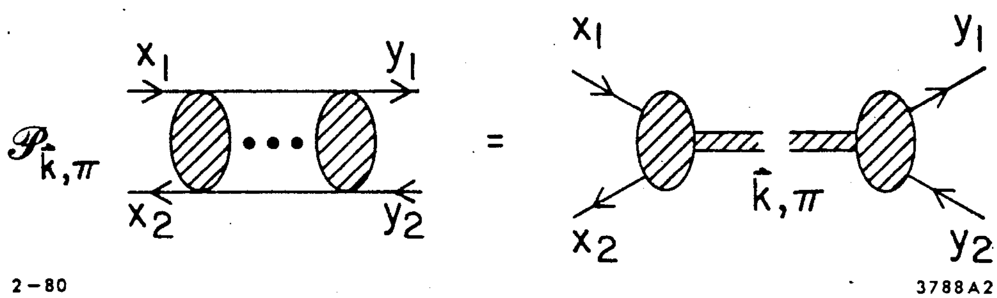

Fig. 2

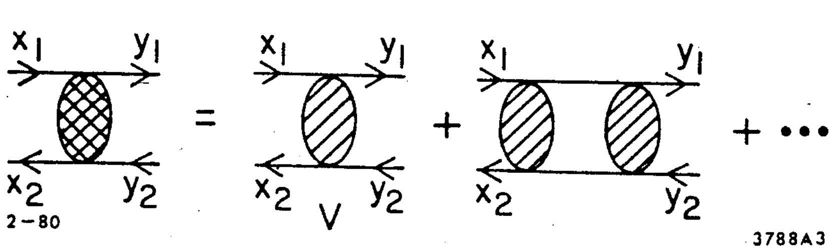

Fig. 3

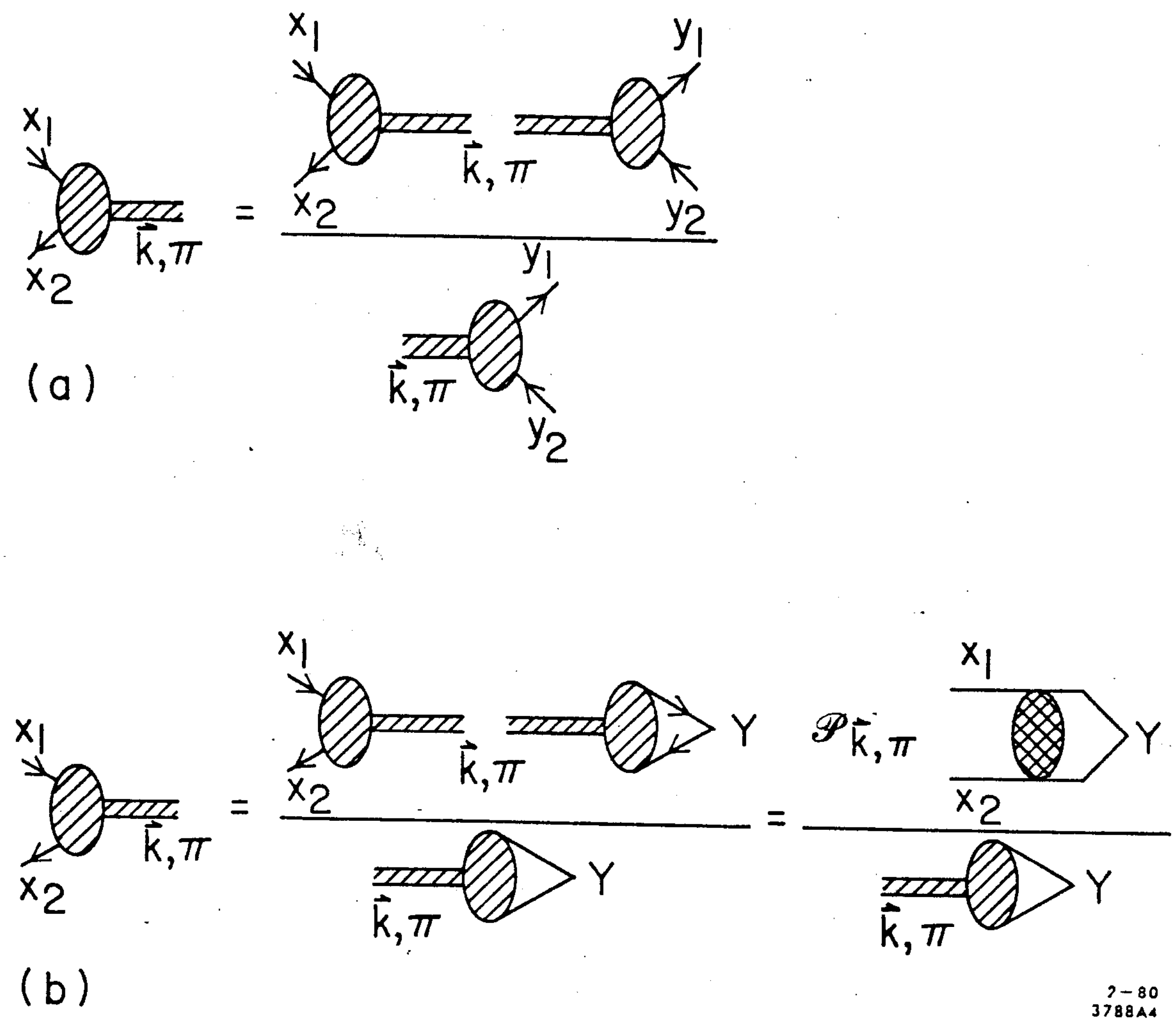

Fig. 4

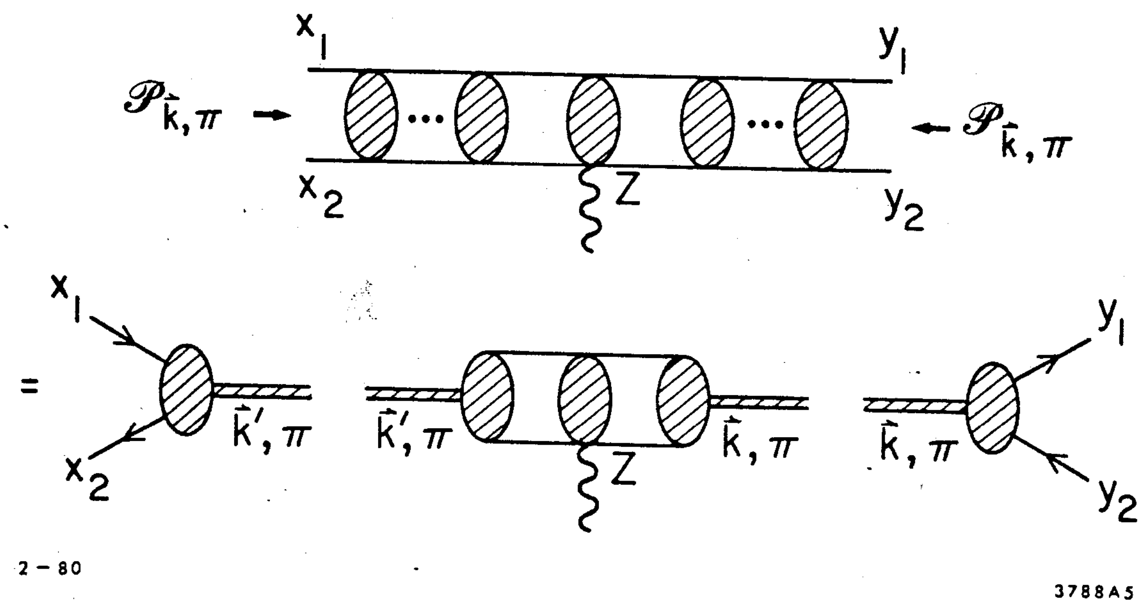
x_1
y_1
$\mathscr{P}_{\vec{k},\pi}$
$\mathscr{P}_{\vec{k},\pi}$
x_2
Z
y_2
x_1
y_1
=
$\vec{k}', \pi$
$\vec{k}', \pi$
$\vec{k}, \pi$
$\vec{k}, \pi$
x_2
Z
y_2
2 − 80
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Fig. 5