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STRONG INTERACTION CORRECTIONS TO $K^0 - \bar{K}^0$ MIXING IN THE SIX QUARK MODEL*

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ABSTRACT

Strong interaction corrections to $K^0 - \bar{K}^0$ mixing in the six quark model are computed in quantum chromodynamics using the leading logarithm approximation. The corrections to the real and imaginary parts of the off diagonal $K^0 - \bar{K}^0$ mass matrix elements can be large.

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The $K^o - \overline{K}^o$ mass matrix has played an important role in particle physics over the past decade. The small value of the real part of the off diagonal elements found an explanation in the GIM mechanism[1] which invoked a fourth, charmed quark. Later calculations[2] of the magnitude of these mass matrix elements led to a quantitative estimate of the charmed quark mass. While these calculations were originally done without strong interaction corrections, with the development of Quantum Chromodynamics (QCD) the short distance effects due to strong interactions were soon computed[3,4] and found to change the answer rather little.

With the standard phase conventions an imaginary part of the off diagonal mass matrix elements is an expression of CP non-invariance and leads to the neutral kaon eigenstates, K_L^o and K_S^o, not being CP eigenstates. With four quark flavors there is no imaginary part,[5] but in a six quark model a phase in the heavy quark couplings to the weak vector bosons leads to CP violation and an imaginary part in the mass matrix. The phenomenology of CP violation in the six quark model has been discussed[6] without account of QCD corrections and found to be consistent with experiment and in particular with its observation in the $K_L - K_S$ system.

In this paper we calculate QCD corrections to the $K^o - \overline{K}^o$ mass matrix in the six quark model with its attendant CP violation. We find that these strong interaction corrections, calculated in the leading logarithmic approximation, are generally not small.

We work within the standard model[7] where the gauge group of electroweak interactions is $SU(2) \otimes U(1)$ and the six quarks, u,c,t with charge 2/3 and d,s,b with charge -1/3 are assigned to left-handed doublets and right-handed singlets:

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L ; \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L ; \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L \quad ;$$

$$(u)_R; \quad (d)_R; \quad (c)_R; \quad (s)_R; \quad (t)_R; \quad (b)_R \quad .$$

The choice of quark fields is such that[8]

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}_L = \begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \tag{1}$$

where $c_i = \cos\theta_i$, $s_i = \sin\theta_i$, $i\epsilon\{1,2,3\}$. Equation (1) defines three real Cabibbo-type mixing angles, θ_i, and the CP-violating phase δ.

The portion (with $\Delta S=2$) of the effective weak Hamiltonian density which contributes to the matrix element between a $\bar{K}^o$ and K^o may be written uniquely as

$$\begin{aligned} \mathcal{H} = \; & s_1^2c_2^2(c_1c_2c_3-s_2s_3e^{i\delta})^2\mathcal{H}_1 \\ & + \; s_1^2s_2^2(c_1s_2c_3 + c_2s_3e^{i\delta})^2\mathcal{H}_2 \\ & + \; 2s_1^2s_2c_2(c_1c_2c_3 - s_2s_3e^{i\delta})(c_1s_2c_3 + c_2s_3e^{i\delta})\mathcal{H}_3 \quad . \end{aligned} \tag{2}$$

The components $\mathcal{H}_1$, $\mathcal{H}_2$, and $\mathcal{H}_3$ of the complete Hamiltonian have relatively complicated expressions in terms of time ordered products of four weak charged currents contracted with W boson fields corresponding in the free quark model to forming a "box diagram" with virtual quarks and W bosons in the loop.

In the free quark model, successively treating the W boson, t, and c quarks as heavy results in the following expressions:

$$\mathcal{H}_1 = -\frac{G_F^2 m_c^2}{16\pi^2}\left(\bar{d}_\alpha \gamma_\mu (1-\gamma_5) s_\alpha\right)\left(\bar{d}_\beta \gamma^\mu (1-\gamma_5) s_\beta\right) \tag{3a}$$

$$\mathcal{H}_2 = -\frac{G_F^2 m_t^2}{16\pi^2}\left(\bar{d}_\alpha \gamma_\mu (1-\gamma_5) s_\alpha\right)\left(\bar{d}_\beta \gamma^\mu (1-\gamma_5) s_\beta\right) \tag{3b}$$

$$\mathcal{H}_3 = -\frac{G_F^2 m_c^2 \ln\left(m_t^2/m_c^2\right)}{16\pi^2}\left(\bar{d}_\alpha \gamma_\mu (1-\gamma_5) s_\alpha\right)\left(\bar{d}_\beta \gamma^\mu (1-\gamma_5) s_\beta\right) \tag{3c}$$

where G_F is the Fermi constant, and m_c and m_t the c and t quark masses. The color indices α and β are summed when repeated. Terms which are higher order in m_t^2/m_W^2, m_c^2/m_t^2, etc. have been dropped.

In the presence of strong interactions as described by QCD the results in Eqs. (3) will be modified. We shall derive in leading logarithmic approximation the form of the effective Hamiltonian when the W bosons, t, b, and c quarks are treated as heavy and their fields removed from explicitly appearing in the theory. For this purpose it is convenient to separate the Hamiltonian into pieces that will not mix under renormalization by taking two-currents (each a quark bilinear) which are to be joined by a W boson propagator and writing it as half the sum of a color-index symmetrized (superscript +) and a color-index antisymmetrized (superscript -) piece. We obtain in this way the division

$$\mathcal{H}_1 = \mathcal{H}_1^{(++)} + \mathcal{H}_1^{(+-)} + \mathcal{H}_1^{(-+)} + \mathcal{H}_1^{(--)} \quad ,$$

and similarly for $\mathscr{H}_2$ and $\mathscr{H}_3$.

The first step is to treat the W boson as heavy and remove it from explicitly appearing in the Hamiltonian. This is done in a manner similar to the analogous step in the derivation of the effective Hamiltonian for $\Delta S = 1$ weak nonleptonic decays.[9] In the leading logarithmic approximation each of the Hamiltonians $\mathscr{H}_j$, defined by Eq. (2), can be written as:

$$\mathscr{H}_j = \left[\frac{\alpha_s(M_W^2)}{\alpha_s(\mu^2)}\right]^{2a^{(+)}} \mathscr{H}_j^{(++)} + \left[\frac{\alpha_s(M_W^2)}{\alpha_s(\mu^2)}\right]^{a^{(+)}+a^{(-)}} \mathscr{H}_j^{(+-)}$$

$$+ \left[\frac{\alpha_s(M_W^2)}{\alpha_s(\mu\ )}\right]^{a^{(-)}+a^{(+)}} \mathscr{H}_j^{(-+)} + \left[\frac{\alpha_s(M_W^2)}{\alpha_s(\mu^2)}\right]^{2a^{(-)}} \mathscr{H}_j^{(--)} \quad , \qquad (4)$$

where $a^{(+)} = 6/21$ and $a^{(-)} = -12/21$, and μ is the renormalization point mass.

The next step is to successively consider the t quark, b quark, and c quark as heavy fields and remove them from appearing explicitly in $\mathscr{H}$. For $\mathscr{H}_1$, this is particularly simple since the t and b quark fields do not occur in it. The effect of removing the t quark and b quark fields from the theory of strong interactions on passing from a mass scale [10] given m_t to m_b' and thence to m_c'', is to change $a^{(+)}$ ($a^{(-)}$) from 6/21 (-12/21) to 6/23 (-12/23) and thence to 6/25 (-12/25). These factors are familiar from the calculation of the ordinary four-fermion ΔS=1 weak Hamiltonian.[9] On removing the c quark, each of the terms $\mathscr{H}_1^{(++)}$, $\mathscr{H}_1^{(+-)}$, $\mathscr{H}_1^{(-+)}$, and $\mathscr{H}_1^{(--)}$ yields to leading order in the c quark mass the (color index symmetric) operator[10] $m_c''^2(\bar{d}_\alpha\gamma_\mu(1-\gamma_5)s_\alpha)(\bar{d}_\beta\gamma^\mu(1-\gamma_5)s_\beta)$ with a Wilson coefficient that includes a factor from contraction of color indices (3/2, -1/2, -1/2, and + 1/2, respectively and a factor from renormalization (including mass) of this latter color index symmetric four-fermion operator. In the leading logarithm approximation

$$\mathscr{H}_1 = -\frac{G_F^2 m_c^{*2}}{16\pi^2}\left(\bar{d}_\alpha\gamma_\mu(1-\gamma_5)s_\alpha\right)\left(\bar{d}_\beta\gamma^\mu(1-\gamma_5)s_\beta\right)$$

$$\times\left\{K_c^{-6/27}\left[\frac{3}{2}K_b^{-12/25}K_t^{-12/23}K_W^{-12/21} - K_b^{6/25}K_t^{6/23}K_W^{6/21} + \frac{1}{2}K_b^{24/25}K_t^{24/23}K_W^{24/21}\right]\right\}, \qquad (5)$$

with $K_W = \alpha_s(m_t^2)/\alpha_s(M_W^2)$, $K_t = \alpha_s'(m_b'^2)/\alpha_s(m_t^2)$, $K_b = \alpha_s''(m_c''^2)/\alpha_s'(m_b'^2)$, and $K_c = \alpha_s'''(\mu^2)/\alpha_s''(m_c''^2)$. The quantities α_s, α_s', α_s'', and α_s''' are running strong interaction fine structure constants in field theories with 6, 5, 4, and 3 quark flavors, respectively. m_c^* is the running charm quark mass evaluated at $m_c''^2$, i.e., $m_c^* = m_c''[\alpha_s''(m_c''^2)/\alpha_s''(\mu^2)]^{12/25}$. The Hamiltonian $\mathscr{H}_1$ already occurs in the four-quark model, and with appropriate simplifications Eq. (5) agrees with some of the previous results[3] for the QCD corrected $\mathscr{H}_1$.

The analysis of QCD corrections to $\mathscr{H}_2$ proceeds along similar lines except that already at the step of removing the t quark field from explicitly appearing each of the $\mathscr{H}_2^{(++)}$, $\mathscr{H}_2^{(+-)}$, $\mathscr{H}_2^{(-+)}$, and $\mathscr{H}_2^{(--)}$ collapses to a Wilson coefficient times $m_t^2(\bar{d}_\alpha\gamma_\mu(1-\gamma_5)s_\alpha)(\bar{d}_\beta\gamma^\mu(1-\gamma_5)s_\beta)$ to leading order in the t quark mass. From that point on the successive steps are marked by renormalization of this latter color-index-symmetric four fermion operator. The result is

$$\mathcal{H}_2 = -\frac{G_F^2 m_t^{*2}}{16\pi^2}\left(\bar{d}_\alpha \gamma_\mu (1-\gamma_5) s_\alpha\right)\left(\bar{d}_\beta \gamma^\mu (1-\gamma_5) s_\beta\right)$$

$$\times \left\{ K_c^{-6/27} K_b^{-6/25} K_t^{-6/23} \left[\frac{3}{2} K_W^{-12/21} - K_W^{6/21} + \frac{1}{2} K_W^{24/21} \right] \right\} \quad (6)$$

where m_t^* is the running t quark mass evaluated at m_t^2, i.e., $m_t^* = m_t \left[\alpha(m_t^2)/\alpha(\mu^2)\right]^{12/21}$.

The calculation of the QCD corrections to $\mathcal{H}_3$ is considerably more complicated. At the step of removing the t quark eight operators are generated even with the condition of keeping only those which will yield a final contribution of the same order in heavy quark masses as the free quark results in Eq. (3). Six of these operators are easily understood as arising from a portion of $\mathcal{H}_3$ which is expressible as an integral of a time ordered product of two pieces of the four-quark local $\Delta S=1$ Hamiltonian,[9] one containing a charmed quark and the other a t quark. As is already known from the calculation of the $\Delta S=1$ Hamiltonian, the latter piece generates six operators on removing the t quark.[11] The additional two operators needed for $\mathcal{H}_3^{(\pm\pm)}$ are

$$O_7^{(\pm\pm)} = i\int d^4x T\left\{\left[\left(\bar{d}_\alpha(x)\gamma_\mu(1-\gamma_5)c_\alpha(x)\right)\left(\bar{u}_\beta(x)\gamma^\mu(1-\gamma_5)s_\beta(x)\right) \pm \left(\bar{d}_\alpha(x)\gamma_\mu(1-\gamma_5)s_{\alpha}(x)\right)\right.\right.$$

$$\left.\left(\bar{u}_\beta(x)\gamma^\mu(1-\gamma_5)c_\beta(x)\right)\right]\left[\left(\bar{d}_\gamma(o)\gamma_\nu(1-\gamma_5)u_\gamma(o)\right)\left(\bar{c}_\delta(o)\gamma^\nu(1-\gamma_5)s_\delta(o)\right)\right.$$

$$\left.\left.\pm\left(\bar{d}_\gamma(o)\gamma_\nu(1-\gamma_5)s_\gamma(o)\right)\left(\bar{c}_\delta(o)\gamma^\nu(1-\gamma_5)u_\delta(o)\right)\right]\right\} , \quad (7)$$

$$0_8 = \frac{m_c'^2}{g'^2}\left(\bar{d}_\alpha \gamma_\mu (1-\gamma_5) s_\alpha\right)\left(\bar{d}_\beta \gamma^\mu (1-\gamma_5) s_\beta\right) \quad . \tag{8}$$

Fortunately it is only those operators induced by the QCD corrections among the first six which mix with the last two. To a good approximation the contribution of the first six operators can be neglected (this has been checked by doing the full calculation[12]). From here on we retain only the last two. With our definition of 0_8 (with $1/g'^2$) the anomalous dimension matrix has in lowest order all its entries proportional to g'^2, the square of the strong interaction coupling in the effective five quark theory:[13]

$$\gamma'(g') = \frac{g'^2}{8\pi^2}\begin{pmatrix} 4 & -24 \\ 0 & 7/3 \end{pmatrix}, \ \frac{g'^2}{8\pi^2}\begin{pmatrix} -2 & 8 \\ 0 & 7/3 \end{pmatrix}, \ \frac{g'^2}{8\pi^2}\begin{pmatrix} -8 & -8 \\ 0 & 7/3 \end{pmatrix} \tag{9}$$

for $\mathscr{H}_3^{(++)}$, $\mathscr{H}_3^{(+-)}$ or $\mathscr{H}_3^{(-+)}$, and $\mathscr{H}_3^{(--)}$ respectively. Here renormalization of the coupling constant and mass in 0_8 as well as the usual anomalous dimension for a color symmetric four-fermion operator are taken into account. On removing the b quark we are left an effective four quark theory with the matrices in Eq. (9) all replaced by those with the 8-8 entry being 5/3. Finally, on removing the c quark, only an operator proportional to 0_8 is left. Carrying through the steps of successive renormalization using the anomalous dimension matrices given above, diagonalizing to eigenoperators at each stage, and using a QCD beta function appropriate to the operative number of quark flavors at that stage, results in

$$\mathcal{H}_3 = \frac{G_F^2 m_c^{*2}}{64\pi\alpha_s(m_t^2)} \left(\bar{d}\alpha\gamma_\mu(1-\gamma_5)s_\alpha\right)\left(\bar{d}_\beta\gamma^\mu(1-\gamma_5)s_\beta\right)K_t^{-1}K_b^{-1}K_c^{-6/27}$$

$$\times \left\{\frac{72}{35}\left[5K_b^{-12/25}K_t^{-12/23} + 2K_b^{-5/25}K_t^{-12/23} - 7K_b^{-5/25}K_t^{-7/23}\right]K_W^{-12/21}\right.$$

$$+ \frac{48}{143}\left[13K_b^{6/25}K_t^{6/23} - 2K_b^{-5/25}K_t^{6/23} - 11K_b^{-5/25}K_t^{-7/23}\right]K_W^{6/21} \tag{10}$$

$$\left. + \frac{24}{899}\left[-31K_b^{24/25}K_t^{24/23} + 2K_b^{-5/25}K_t^{24/23} + 29K_b^{-5/25}K_t^{-7/23}\right]K_W^{24/21}\right\}$$

with m_c^* defined as previously.[14] The matrix elements of the three parts of the effective Hamiltonian for $K^o - \bar{K}^o$ mixing in Eqs. (5), (6), and (10) are to be evaluated using the mass independent $\overline{MS}$ regularization scheme in an effective theory of strong interactions with three light quark flavors.

The effects of QCD may now be examined by comparing $\mathcal{H}_1$, $\mathcal{H}_2$, and $\mathcal{H}_3$ in Eqs. (5), (6), and (10), respectively, with their free quark counterparts in Eqs. (3a), (3b), and (3c). In Table I we give the ratio of QCD corrected to free quark values for these three components of the full $\Delta S=2$ Hamiltonian in some typical cases: $M_W = 78$ GeV, $m_c^* = 1.5$ GeV, $m_b' = 4.5$ GeV, $m_t^* = 15$ and 30 GeV, and $\Lambda^2 = 0.1$ and 0.01 GeV^2 in[15]

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2N_f)\ln(Q^2/\Lambda^2)} \tag{11}$$

with N_f the number of quark flavors. The renormalization point mass μ is chosen so that $\alpha_s(\mu^2) = 1$ in each case. The numerical results in the Table for $\mathcal{H}_3$ stem from the approximate Eq. (10), although the full calculation with all eight operators involved at the intermediate stages gives essentially the same result.[12]

Within the six quark model the effects of QCD with typical choices of parameters are fairly large. They decrease the coefficients of each component of $\mathcal{H}$, particularly that of $\mathcal{H}_3$.

We recall that the real part of the matrix element of $\mathcal{H}$ between K^o and $\overline{K}^o$ states is proportional to the K_L^o-K_S^o mass difference, while the ratio of imaginary to real parts is proportional to the magnitude of the CP violation parameter ε, when CP violation in decay amplitudes is neglected. From Eqs. (2) and (3) we have in the free quark model the usual result

$$\varepsilon_m = \frac{\mathrm{Im}\langle K^o|\mathcal{H}|\overline{K}^o\rangle}{\mathrm{Re}\langle K^o|\mathcal{H}|\overline{K}^o\rangle} = 2s_2c_2s_3\sin\delta\left\{\frac{-c_2^2m_c^2+s_2^2m_t^2+(c_2^2-s_2^2)m_c^2\ln\left(\frac{m_t^2}{m_c^2}\right)}{c_2^4m_c^2+s_2^4m_t^2+2s_2^2c_2^2m_c^2\ln\left(\frac{m_t^2}{m_c^2}\right)}\right\} \tag{12}$$

when s_1 and s_3 are treated as small. In both numerator and denominator the order of the terms is that of contributions from $\mathcal{H}_1$, $\mathcal{H}_2$, and $\mathcal{H}_3$. Note that all μ dependence will cancel in the QCD corrected version of Eq. (12). If s_2 is very small, then the last term in the numerator (from $\mathcal{H}_3$) and the first term in the denominator (from $\mathcal{H}_1$) are the most important. Then the QCD corrections, which diminish the magnitude of $\mathcal{H}_3$ much more than $\mathcal{H}_1$ will cause the right hand side of Eq. (12) to decrease by a factor of two or more. The ratio of CP violation coming from K^o decay amplitudes to that from the mass matrix, measured experimentally by the ratio ε'/ε, is proportionally increased, all other parameters being held fixed. When s_2 is not so small (indeed for $s_2^2 \approx s_1^2 \approx 1/20$) or for m_t large the other terms (particularly $\mathscr{H}_2$) make important contributions to both the numerator and denominator of Eq. (12) and the effect of QCD corrections is less dramatic. However, then the real part of $\langle K^o|\mathscr{H}|\overline{K}^o\rangle$ has substantial QCD corrections. In either case there will be a change in the constraints placed by $\mathrm{Re}\langle K^o|\mathscr{H}|\overline{K}^o\rangle$ and $\mathrm{Im}\langle K^o|\mathscr{H}|\overline{K}^o\rangle$ through comparison with $m_{K_L} - m_{K_S}$ and ε on the allowed domain[16] of the angles Θ_2, Θ_3, and δ. Work on this is in progress.

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10. m_t, m_b, and m_c are the t, b, and c quark mass parameters in the six quark theory of strong interactions, m_b' and m_c' and the b are c quark mass parameters in an effective five quark theory, and m_c'' is the c quark mass parameter in an effective theory with four quark flavors.

11. In the notation of Ref. 9 the six operators,

$$i\int d^4x \quad T\left\{O_c^{(\pm)}(x)O_j(0)\right\} \quad , \quad j = 1,\ldots,6$$

are generated when the t quark is removed from the portion of $\mathcal{H}_3^{(\pm\pm)}$ which has the form

$$i\int d^4x \quad T\left\{O_c^{(\pm)}(x)O_t^{(\pm)}(0)\right\} \quad .$$

The operators, $O_c^{(\pm)}$, $O_t^{(\pm)}$, and O_j are defined in Eqs. (7) and (20) of Ref. 9.

12. F. J. Gilman and M. B. Wise, to be published.

13. The advantages of using this definition were discussed in detail by F. J. Gilman and M. B. Wise, SLAC-PUB-2437 (1979) (unpublished).

14. M. I. Vysotsky, ITEP preprint, ITEP-121, 1979 (unpublished) has also performed a calculation of the QCD corrections to $K^o - \overline{K}^o$ mixing. Our separation into $\mathcal{H}_1$, $\mathcal{H}_2$, and $\mathcal{H}_3$ follows his and our results for $\mathcal{H}_1$ and $\mathcal{H}_2$, with appropriate simplifications, agree with his. Our results for $\mathcal{H}_3$ do not.

15. m_c^*, which is independent of the renormalization point, is a physical quantity most appropriately associated with charmonium spectroscopy. In the leading logarithmic approximation the difference between m_c^* and m_c'' can be neglected in the argument of α_s. Similar remarks hold for b and t quark masses. In comparing the QCD corrected Hamiltonian with the free quark Hamiltonian we have used $m_c = 1.5$ GeV and $m_t = 15$ and 30 GeV in the free quark case.

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TABLE I

Ratio of QCD corrected to free quark values of the components $\mathcal{H}_1$, $\mathcal{H}_2$, and $\mathcal{H}_3$ of the $\Delta S = 2$ effective Hamiltonian.

Parameters	$\mathcal{H}_1^{QCD}/\mathcal{H}_1^{free}$	$\mathcal{H}_2^{QCD}/\mathcal{H}_2^{free}$	$\mathcal{H}_3^{QCD}/\mathcal{H}_3^{free}$
$m_t = 15$ GeV $\Lambda^2 = 0.1$ GeV^2	0.92	0.62	0.37
$m_t = 30$ GeV $\Lambda^2 = 0.1$ GeV^2	0.91	0.62	0.34
$m_t = 15$ GeV $\Lambda^2 = 0.01$ GeV^2	0.67	0.60	0.33
$m_t = 30$ GeV $\Lambda^2 = 0.01$ GeV^2	0.66	0.60	0.33