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FERMION MASSES AND HIERARCHY OF SYMMETRY BREAKING*

G. C. Branco
Carnegie-Mellon University
Pittsburgh, PA 15213

H. P. Nilles†
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

and

V. Rittenberg
Physikalisches Institut der Universitaet Bonn
D53 Bonn, W. Germany

ABSTRACT

We suggest that the breaking of a symmetry unifying the families of fermions occurs in stages. We consider the total Lagrangian to be invariant under the group $SU(2) \times U(1) \times G$, where G is a discrete group. The Higgs potential is, however, invariant under $SU(2) \times U(1) \times \tilde{G}$, where $\tilde{G} \supset G$. In a first stage $\tilde{G}$ is broken to a subgroup $H \subset \tilde{G}$, but H is not contained in G. The u and d quarks are naturally massless at the tree level, and we discuss how they could acquire mass through radiative corrections.

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† Max Kade Fellow.

1. Introduction

One of the outstanding problems in the unified gauge theories of weak and electromagnetic interactions is the understanding of the fermion mass spectrum and the flavor mixing angles. It has been shown that the unification of the various fermion families through a discrete[1] or continuous[2] symmetry can lead to calculable Cabibbo-like angles. Motivated by the smallness of some of the fermion mass ratios, we suggest that the spontaneous breakdown of the family-symmetry occurs in stages.[3,4] More precisely, we are envisaging a situation such that in a first stage the discrete symmetry is broken to one of its subgroups, with the heavy fermions acquiring mass and the light ones remaining massless. At a second stage, the symmetry would be further broken and as a result the light fermions would acquire mass. The hope is that in the correct unification, these steps of symmetry breaking would occur in a natural way.[5] In general, in order to obtain a natural hierarchy of symmetry breaking, some rather strict conditions have to be satisfied. For definiteness, consider a gauge theory invariant under $G \times SU(2) \times U(1)$, where G is a finite discrete group. The constraints of renormalizability and gauge invariance may lead to a classical Higgs potential with a higher symmetry $\tilde{G} \supset G$. Consider now the exact effective potential written as:

$$V(\phi) = V^{(0)}(\phi) + \Delta V(\phi) \tag{1.1}$$

where $V^{(0)}$ is the tree approximation to the effective potential and ΔV are radiative corrections. Assume that the minimum of $V^{(0)}(\phi)$ leaves a subgroup $H \subset \tilde{G}$ invariant:

$$\mathscr{H}(v^{o}) = v^{o} \tag{1.2}$$

where v^{o} are the vacuum expectation values of ϕ in the tree approximation and $\mathscr{H}$ stands for the matrices of group H. We seek a situation such that

$$\mathscr{H}(\Delta v) \neq \Delta v \tag{1.3}$$

where Δv is the change in the vacuum expectation values produced by ΔV. In order to find under what conditions (1.2) and (1.3) may be satisfied, it is convenient to consider the following two cases:

(a) H is a subgroup of G. In this case the Georgi and Pais theorem[6] gives a necessary condition to achieve a hierarchy of symmetry breaking. The theorem states that if a Lagrangian is invariant under a (discrete or continuous) symmetry and if the vacuum expectation values respect this symmetry in the tree approximation, then the symmetry will still hold in higher orders, provided that at the tree level there are no massless scalar meson fields which are not Goldstone bosons. A well known situation where this necessary condition can be satisfied is when the Higgs potential has a larger (continuous) symmetry than the total Lagrangian, in which case pseudo-Goldstone bosons[7] occur at the tree level. It is clear that this necessary condition is unlikely to be satisfied if the unification of fermion families is done through a discrete group.

(b) H is not a subgroup of G. This is probably the most interesting case. Since H is not a symmetry of the total Lagrangian, quantum corrections to the effective potential will in general produce a breaking of H, even if there are no massless non-Goldstone bosons at tree level. In the sequel, we will consider in detail an example in which G in chosen to be the tetrahedral group[8] T and we show that the first step of symmetry breaking naturally leads to a subgroup D_2 (a dihedral group) which is not a subgroup of T.

This paper is organized as follows: In section 2 we present some general results about patterns of symmetry breaking and their implications for the fermion mass spectrum and the Cabibbo-like angles. In Section 3 we work out in detail a specific example where the total Lagrangian is T-invariant while the Higgs potential has an O_h symmetry. We analyse the various patterns of symmetry breaking of O_h and consider in detail the case in which the tree approximation minimum has a D_2 symmetry. It turns out that the correspondent mass matrices for the up and down quarks contain each a zero eigenvalue. The masslessness of the light quarks in the tree approximation is thus obtained in a natural way, being the result of a particular pattern of symmetry breaking. We further show how a small perturbation about the D_2 symmetric minimum (and along a particular irreducible representation) leads to a realistic mass spectrum and calculable Cabibbo-like angles. Finally we discuss the possibility of obtaining the breaking of D_2 through radiative corrections.

2. Patterns of Symmetry Breaking

Consider the standard $SU(2) \times U(1)$ gauge theory,[9] with the families of fermions unified through a finite discrete group G. We will assume that G commutes with $SU(2) \times U(1)$, thus insuring that all members of irreducible representations of G are identical representations of the gauge group. The fermions are assumed to appear in n families, where the left handed components

$$\ell_i = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}_L \quad ; \quad L_i = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L \quad ; \quad i = 1, \ldots, n$$

behave as SU(2) doublets whereas u_{iR}, d_{iR} and e_{iR} are singlets. We further assume that L_i, ℓ_i as well as the right handed components form n-dimensional irreducible representations of the horizontal symmetry G. Given the observed fermions, this assumption (which is primarily based on asthetical reasons) restricts the choice of G to groups with 3-dimensional (or higher) irreducible representations. The Yukawa interaction can be expressed as:

$$\begin{aligned} \mathscr{L}_y &= \mathscr{L}_y^u + \mathscr{L}_y^d + \mathscr{L}_y^e \\ \mathscr{L}_y^u &= g_\alpha^u \bar{L}_i C_{i,k}^{\alpha,\ell} u_{kR} \phi_\ell + \text{h.c.} \end{aligned} \tag{2.1}$$

with similar expressions for the down quarks and leptons. In expression (2.1), ϕ denotes the Higgs doublets which form an irreducible multiplet under G and $C_{i,k}^{\alpha,\ell}$ are Clebsch-Gordan coefficients. The label α takes as many values as the number of times the representation of ϕ is contained in the irreducible expansion of the product $\bar{L} \times u_R$. After spontaneous symmetry breaking (SSB) the mass matrix reads:

$$M_{ik}^u = \sum_{\alpha,\ell} g_\alpha^u C_{ik}^{\alpha,\ell} v_\ell \tag{2.2}$$

where $v_\ell = \langle 0|\phi_\ell|0\rangle$. The quark mass matrices are diagonalized through biunitary transformations $U_L^{u\dagger} M^u U_R^u$, $U_L^{d\dagger} M^d U_R^d$, and the generalized Cabibbo matrix is given by $C = U_L^{u\dagger} U_L^d$. The calculability of flavor mixing angles crucially depends on the number of independent parameters in the mass matrix. The sources of independent parameters are: (i) the number of independent Yukawa couplings, which equals the number of times the representation ϕ appears in the Clebsch-Gordan expansion of $\bar{L}\times u_R$ and $\bar{L}\times d_R$; (ii) the number of independent vacuum expectation values (v.e.v.) which depends on the pattern of symmetry breaking. The Higgs potential is constructed to be G-invariant, but since it only contains terms that are quadratic and quartic in ϕ, if often has a higher symmetry $\tilde{G}\supset G$. The SSB will not, in general, break $\tilde{G}$ completely and the minimum of $V(\phi)$ will still be invariant with respect to a non-trivial subgroup $H\subset\tilde{G}$ (not necessarily a subgroup of G). In order that a given subgroup H may be an invariance of the minimum of the potential for a given ϕ, it is necessary that the Frobenius decomposition of the induced representation of ϕ (with respect to the subgroup H) contains at least one singlet scalar representation. In most cases, one has only one independent vacuum expectation value (v.e.v.) and further steps of symmetry breaking (obtained, e.g., through the introduction of additional Higgs multiplets) are required to have more than one independent v.e.v. Some of the features of the fermion mass spectrum and Cabibbo-like matrix can be deduced on group theoretical grounds, without explicit calculation. For simplicity, we will assume that L_i, u_{iR}, d_{iR} form equivalent irreducible n-dimensional representations Γ of $\tilde{G}$, and consider H to be the largest subgroup of $\tilde{G}$ which is left invariant after SSB. Assume further that the reducible representation $\Gamma^{(s)}$ of H induced by Γ, decomposes with respect to H in the following way:

$$\Gamma^{(s)} = \left(d^{(1)},\ldots,d^{(k)}\right) \qquad (2.3)$$

where $d^{(i)}$ denotes the dimension of the representation and $\sum_{i=1}^{k} d^{(i)} = n$. Then the structure of the mass matrix will depend on the properties of the $d^{(i)}$. For example, if Γ contains ℓ scalars, i.e., $d^{(1)} = d^{(2)} = \ldots = d^{(\ell)} = 1$, while $d^{(i)} \neq 1$ for $i > \ell$, the mass matrix will be block diagonal:

$$M = \left(\begin{array}{c|c} A & \\ \hline & B \end{array}\right) \tag{2.4}$$

where A and B are $\ell \times \ell$ and $(n-\ell) \times (n-\ell)$ matrices, respectively. The two blocks completely decouple, i.e., the generalized Cabibbo angles between the two sectors vanish. The matrix B can have a special substructure depending on the properties of the representations $d^{(i)}$, $\ell \leq i \leq k$. If for example $d^{(\ell+1)}$ has dimension $(n-\ell)$, then B will be diagonal and the corresponding $(n-\ell)$ masses will be degenerate. It often happens that in the breakdown of $\tilde{G}$ with one Higgs multiplet, there is only one independent v.e.v. In the case of simply reducible groups all fermion mass ratios will then be given by Clebsch-Gordan coefficients, which is likely to be unrealistic, given the known fermion spectrum. In order to obtain more independent parameters, without proliferation of Higgs bosons one has to look for not simply reducible groups.

3. A Special Example

In order to illustrate the statements of the previous sections, we consider the tetrahedral group T which has been recently proposed by Wyler.[10] We restrict ourselves to the case of three generations $(n = 3)$, and put L, u_R, d_R in triplets. Since $\bar{3} \times 3$ contains the 3 twice, ϕ is chosen to be a T-triplet as well. The most general Higgs potential with a $SU(2)\times U(1)\times T$ invariance is given by:

$$V(\phi) = \mu^2\left(\sum_{i=1}^{3} \phi_i^\dagger\phi_i\right) + \lambda_1\left(\sum_{i=1}^{3} \phi_i^\dagger\phi_i\right)^2$$
$$+ \lambda_2\left[\left(2\phi_3^\dagger\phi_3 - \phi_1^\dagger\phi_1 - \phi_2^\dagger\phi_2\right)^2 + 3\left(\phi_1^\dagger\phi_1 - \phi_2^\dagger\phi_2\right)^2\right]$$
$$+ \lambda_3\left[\left(\phi_2^\dagger\phi_3 + \phi_3^\dagger\phi_2\right)^2 + \left(\phi_3^\dagger\phi_1 + \phi_1^\dagger\phi_3\right)^2 + \left(\phi_1^\dagger\phi_2 + \phi_2^\dagger\phi_1\right)^2\right]$$
$$+ \lambda_4\left[\left(\phi_2^\dagger\phi_3 - \phi_3^\dagger\phi_2\right)^2 + \left(\phi_3^\dagger\phi_1 - \phi_1^\dagger\phi_3\right)^2 + \left(\phi_1^\dagger\phi_2 - \phi_2^\dagger\phi_1\right)^2\right] \quad (3.1)$$

The Yukawa interaction is given by:

$$\mathscr{L}_y^d = g_1 \bar{L}_i C_{i,k}^{1,\ell} d_{kR}\phi_\ell + g_2 \bar{L}_i C_{i,k}^{2,\ell} d_{kR}\phi_\ell + \text{h.c.} \quad (3.2)$$

where $C_{i,k}^{1,\ell} = (1/\sqrt{2})|\varepsilon_{ik\ell}|$ and $C_{i,k}^{2,\ell} = (1/\sqrt{2})\varepsilon_{ik\ell}$ with similar expressions for the up quarks. The actual symmetry of $V(\phi)$ turns out to be much larger and it can be shown to be $SU(2)\times U(1)\times O_h$. The group O_h [8] is a 48 element subgroup of $O(3)$: $O_h = O\times C_i$, where O denotes the octahedral group which is isomorphic to S_4.

We first investigate the possible minima in a group theoretical language. According to our strategy, this corresponds to looking at the maximal subgroups that contain a scalar in the induced representations in one of the four O_h triplets (there is an ambiguity that these four triplets coincide with the T-triplet on T). These maximal subgroups and the corresponding Frobenius decomposition turn out to be (in the notation of Hamermesh[8]):

(i) $S_3 = D_3$ corresponding to $3_1 \rightarrow (1,2)$

(ii) $D_2 \subset 0$; $D_2 \not\subset T$ for $3_1 \rightarrow (1 , 1' , 1'')$

(iii) C_4 for $3_2 \rightarrow (1 , 1'' , 1''')$

(iv) $D_2 \not\subset 0$ for $3_{2i} \rightarrow (1 , 1'' , 1')$

(v) $C_3 \not\subset 0$ for $3_{1i} \rightarrow (1 , 1' , 1'')$

From these results one can read of, that in all possible patterns of symmetry breaking corresponding to a ϕ-triplet, there is only one scalar in the decomposition of the subduced representations. This in turn implies that there is only one independent v.e.v. and it is then possible, for each case, to choose a basis such that the minima of the classical potential correspond to $\langle 0|\phi|0\rangle = (v,0,0)$. In a different basis, the non-vanishing v.e.v. will be related. The expression for $V(\phi)$ in (3.1) is written in the T-triplet basis which coincides with the natural basis for the SO(3) triplet. In this basis the minima of the potential are:

(a) $(v,0,0)$; $(0,v,0)$; $(0,0,v)$

These solutions have a remaining $D_2(C_4)$ invariance for $3_1(3_2)$ breaking. There are three $D_2(C_4)$ subgroups of 0 which correspond to these three solutions. The fourth D_2 subgroup of 0, which is simultaneously a subgroup of T, cannot be reached by triplet breaking.

(b) $(v,v,0)$; $(v,0,v)$; $(0,v,v)$

These solutions correspond to D_2 symmetries which are not subgroups of 0. If the Higgs potential would have had only an 0-symmetry these minima would correspond to a C_2 subgroup. The decomposition of 3_1 with respect to C_2 contains two scalars and therefore if it were not for the accidental O_h symmetry, this C_2 subgroup would be a maximal subgroup and allow for two independent v.e.v.

(c) (v,v,v); (-v,v,v) and permutations. These minima have a remaining S_3 symmetry and correspond to pattern (i).

(d) $(v, ve^{i\pi/3}, ve^{i2\pi/3})$ and permutations correspond to (v) and have a C_3 symmetry.

<u>The first stage of symmetry breaking</u>: Among the patterns of symmetry breaking previously considered, we are specially interested in the solution that corresponds to a D_2 invariance. In particular we will show that it is a good candidate for a first stage of symmetry breaking. First of all, one has to make sure that this first stage can be achieved in a natural way, i.e., it has to be shown that for a range of values of the free parameters of the Higgs potential the D_2 solution corresponds indeed to an absolute minimum of the classical potential. From (3.1) it is straightforward to check that $\langle 0|\phi|0\rangle = (v,0,0)$ is a stationary point of the potential. In order to find the conditions for being a minimum, we have to examine the corresponding Higgs scalars mass matrix. It turns out that after we make the replacement $\langle 0|\phi|0\rangle = (v,0,0)$ the Higgs-scalar mass matrix is diagonal, with diagonal elements given by (we denote $\phi_j = R_j + iI_j$):

$$\frac{\partial^2 V}{\partial R_1^2} = 2\mu^2 + 12(\lambda_1 + 4\lambda_2)v^2 = 8(\lambda_1 + 4\lambda_2)v^2$$

$$\frac{\partial^2 V}{\partial I_1^2} = 2\mu^2 + 4(\lambda_1 + 4\lambda_2)v^2 = 0$$

$$\frac{\partial^2 V}{\partial R_2^2} = \frac{\partial^2 V}{\partial R_3^2} = 4(2\lambda_3 - 6\lambda_2)v^2$$

$$\frac{\partial^2 V}{\partial I_2^2} = \frac{\partial^2 V}{\partial I_3^2} = -4(6\lambda_2 + 2\lambda_4)v^2 \qquad (3.4)$$

where we have used that $2v^2(\lambda_1 + 4\lambda_2) = -\mu^2$. In order for (v,0,0) to be a minimum the following conditions must be satisfied:

$$\mu^2 < 0; \quad \lambda_1 > -4\lambda_2; \quad \lambda_2 < \lambda_3/3; \quad \lambda_2 < -\lambda_4/3 \tag{3.5}$$

Next we will analyze the fermion mass matrix. Using (3.2) and $\langle 0|\phi|0\rangle$ = (v,0,0) one obtains:

$$M^d M^{d\dagger} = \frac{v^2}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & (g_1+g_2)^2 & 0 \\ 0 & 0 & (g_1-g_2)^2 \end{pmatrix} \tag{3.6}$$

and similarly for the up quarks. The mass matrix is diagonal and all Cabibbo angles vanish. This is to be expected from our analysis in Section 2, since the decomposition of 3_1 with respect to D_2 is (1,1',1"). Furthermore, the u and d quarks are naturally massless at this stage. We now consider the next stage of symmetry breaking, in which D_2 is further borken. This could in principle be achieved by either taking into account quantum corrections to the Higgs potential (if they lead to a further breaking) or more modestly by adding extra Higgs particles. But independently of the mechanism which produces the breaking of D_2, we will consider an example in which a "perturbation" along a given irreducible representation of D_2 leads to a calculable Cabibbo mixing matrix. We recall that the nontrivial elements of O_h which leave the vacuum (v,0,0) invariant are:

$$\begin{pmatrix} 1 & & 0 \\ & -1 & \\ 0 & & -1 \end{pmatrix}; \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}; \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \tag{3.7}$$

We exchange now to a basis allowing for a direct decomposition into D_2 representations. This is achieved by performing a $\pi/4$ rotation in the 2-3 plane. In the new basis the matrices (3.7) are given by:

$$\begin{pmatrix} 1 & & 0 \\ & -1 & \\ 0 & & -1 \end{pmatrix} \;;\; \begin{pmatrix} 1 & & 0 \\ & -1 & \\ 0 & & +1 \end{pmatrix} \;;\; \begin{pmatrix} 1 & & 0 \\ & 1 & \\ 0 & & -1 \end{pmatrix} \tag{3.8}$$

The decomposition $3_1 \to (1,1',1'')$ is transparent. Now we assume that the breaking of D_2 is along a given irreducible representation, say 1', i.e., the new minimum is given by

$$\langle 0|\phi|0\rangle = (v\,,\sqrt{2}\,\varepsilon\,v\,,0) \quad ; \quad \varepsilon \ll 1 \tag{3.9}$$

or, in the previous basis:

$$\langle 0|\phi|0\rangle = (v\,,\varepsilon v\,,-\varepsilon v) \tag{3.10}$$

Using (3.2), (3.10) one obtains for the fermion mass matrix:

$$M^d M^{d\dagger} = \frac{v^2}{2}\begin{pmatrix} (g_+^2+g_-^2)\varepsilon^2 & g_-g_+\varepsilon & -g_-g_+\varepsilon \\ g_-g_+\varepsilon & g_+^2+\varepsilon^2 g_-^2 & -g_-g_+\varepsilon^2 \\ -g_-g_+\varepsilon & -g_-g_+\varepsilon^2 & g_-^2+\varepsilon^2 g_+^2 \end{pmatrix} \tag{3.11}$$

where we have introduced $g_\pm = g_1 \pm g_2$. The eigenvalue equation can now be used to relate the quark masses to the parameters in the mass matrix.

$$r_1 = \frac{m_b^2 m_s^2 + m_b^2 m_d^2 + m_d^2 m_s^2}{\left(m_b^2 + m_s^2 + m_d^2\right)^2} = \frac{\delta^4(1-4\varepsilon^2) + 4\varepsilon^2(2+\varepsilon^2)}{4\left(1+2\varepsilon^2\right)^2} \tag{3.12}$$

$$r_2 = \frac{m_b^2\, m_d^2\, m_s^2}{\left(m_b^2 + m_s^2 + m_d^2\right)^3} = \frac{\varepsilon^4(1+\delta^2)(2-\delta^2)^2}{4\left(1+2\varepsilon^2\right)^3} \tag{3.13}$$

where $\delta^2 = \left(g_1^2 - g_2^2\right)/\left(g_1^2 + g_2^2\right)$. Assuming now that $\varepsilon << 1$ one obtains:

$$\delta^4 \approx 4r_1 \approx \frac{m_s^2}{m_b^2}$$
$$\varepsilon^2 \approx \frac{m_d m_s}{m_b^2} \qquad (3.14)$$

Using the Kobayashi-Maskawa[11] parametrization one obtains for the angles:

$$s_1^d \cong \sqrt{\frac{m_d}{m_s}} \quad ; \quad s_2^d \cong -s_3^d \cong -\frac{m_s^2}{m_b^2} \qquad (3.15)$$

with similar expressions for s_i^u.

The purpose of this example was to illustrate how a small perturbation around the D_2 symmetric vacuum leads to physically interesting results.

We now address ourselves to the following question: Can quantum corrections generate this second stage of symmetry breaking, thus making the smallness of the parameter ε (or equivalently the smallness of m_u, m_d) a natural feature of the model? We first note that the D_2 symmetry of the minimum (v,0,0) is not a symmetry of the total Lagrangian. In particular, it can be easily verified that the Yukawa interactions $\mathcal{L}_y$ are not invariant under (3.7) transformations. This is due to the fact that $\mathcal{L}_y$ is only T invariant and the particular subgroup of O_h considered here is not a subgroup of T. Since D_2 is not a symmetry of the total Lagrangian, the Georgi-Pais theorem[6] does not apply and this raises the hope of generating light quark masses through radiative corrections. Unfortunately it turns out that in the present example that is not possible. This can been seen in the following way: among the D_2 matrices in (3.7) only the first belongs to T. Together with the identity it forms a C_2 subgroup of T, to which the Georgi-Pais theorem applies. From (3.4) it is apparent that there are no non-Goldstone massless scalars (I_1 is the Goldstone boson needed to give mass to the neutral vector meson Z) as long as one doesn't choose nonnatural values of the λ_i, and therefore one concludes that the C_2 symmetry will be respected by higher order corrections. On the other hand this C_2 invariance is sufficient to guarantee the form (v,0,0) for the minimum of the Higgs potential.

In view of previous analysis, we examine now the other tree approximation minima listed in (3.3). Among these, only the one corresponding to

$$\langle 0|\phi|0\rangle = \frac{1}{\sqrt{2}}(v,v,0) \tag{3.16}$$

leads to massless u, d quarks in the tree approximation. In this case, O_h is broken to a D_2 subgroup whose nontrivial elements are given by:

$$\begin{pmatrix} 1 & & 0 \\ & 1 & \\ 0 & & -1 \end{pmatrix} \; ; \; \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \; ; \; \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{3.17}$$

This case has the interesting feature that none of these matrices belongs to T. Before pursuing, we have to investigate if (3.16) can be obtained in a natural way. The Higgs scalar mass matrix corresponding to (3.16) is given by:

$$\frac{\partial^2 V}{\partial R_1^2} = \frac{\partial^2 V}{\partial R_2^2} = 4v^2(\lambda_1 + 4\lambda_2)$$

$$\frac{\partial^2 V}{\partial R_1 \partial R_2} = 4v^2(\lambda_1 - 2\lambda_2 + 2\lambda_3)$$

$$\frac{\partial^2 V}{\partial C_1^2} = \frac{\partial^2 V}{\partial C_2^2} = -4v^2(\lambda_3 + \lambda_4)$$

$$\frac{\partial^2 V}{\partial C_1 \partial C_2} = 4v^2(\lambda_3 + \lambda_4)$$

$$\frac{\partial^2 V}{\partial R_3^2} = 4v^2(\lambda_3 - 3\lambda_2)$$

$$\frac{\partial^2 V}{\partial C_3^2} = -4v^2(3\lambda_2 + \lambda_3 + 2\lambda_4) \qquad (3.18)$$

where $2v^2(\lambda_1 + \lambda_2 + \lambda_3) = -\mu^2$, and the other nondiagonal elements vanish. Diagonalizing (3.18) gives the following scalar masses:

$$M_1 = 8v^2(\lambda_1 + \lambda_2 + \lambda_3) \quad ; \quad M_2 = 8v^2(3\lambda_2 - \lambda_3)$$

$$M_3 = -8v^2(\lambda_3 + \lambda_4) \quad ; \quad M_4 = 0$$

$$M_5 = 4v^2(\lambda_3 - 3\lambda_2) \quad ; \quad M_6 = -4v^2(3\lambda_2 + \lambda_3 + 2\lambda_4) \qquad (3.19)$$

As in the previous case, there is one massless scalar which is a Goldstone boson. In order that (3.16) is a minimum all the other eigenvalues in (3.19) should be positive. It happens here by accident, that this cannot be achieved in a natural way (i.e., for a finite range of the free parameters of the theory) since $M_2 \geq 0$ and $M_5 \geq 0$ lead to:[12]

$$3\lambda_2 = \lambda_3 \qquad (3.20)$$

From the previous discussion, we conclude that in the present example it is not possible to generate a hierarchy of symmetry breaking through radiative corrections in a natural way. However we feel that the possibility of avoiding the Georgi-Pais theorem through a first stage breaking into a group which is not a symmetry of the total Lagrangian is sufficiently attractive to deserve further attention.

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