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NEW ESTIMATE OF QUARK MASS PARAMETERS*

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ABSTRACT

An estimate of quark mass parameters suitable for the study of chiral symmetry breaking effects is made using exact Goldberger-Treiman relations for the Goldstone bosons which appear in the Weinberg-Salam theory when the gauge coupling is set to zero. The numbers obtained are $m_u = 13.7 \pm 5.9$ MeV, $m_d = 24.7 \pm 12.9$ MeV, and $m_s = 497 \pm 299$ MeV. Consistency of the various Goldberger-Treiman relations with current algebra mass ratios is discussed. The relationship (or otherwise) of these parameters to those obtained in previous evaluations is discussed.

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1. Introduction

In this paper, I relate the current quark masses to the deviations in the usual Goldberger-Treiman relations in the context of the standard QCD/Weinberg-Salam-Higgs model. A connection between quark masses and the Goldberger-Treiman relation is hardly surprising in light of the fact that Goldberger-Treiman would be exact in the absence of chiral symmetry breaking; the mass terms in the Lagrangian are primarily responsible for breaking the chiral flavor symmetry. I reestablish this connection but, now, from a different and, hopefully, more insightful viewpoint. The point of view to be presented here was first stressed several years ago by M. Weinstein.[1]

The treatment of quark masses to be presented below is based on the observation that the Goldstone states eaten by the electroweak gauge bosons are not pure Higgs states: they have quark content as well. To see this, let us first imagine a world where Higgs and quarks are decoupled from each other. This means that the quarks interact only through color gauge interactions whereas the Higgs interact strictly through the usual ϕ^4 couplings. Any interactions that link the two sectors in the standard strong interaction/electroweak model -- electroweak gauge couplings and Yukawa couplings -- are turned off. Each sector possesses a global chiral symmetry: the quarks are invariant under flavor chiral $SU_L(N) \times SU_R(N)$ transformations (N standing for the number of quark flavors), and the Higgs fall into representations of the chiral $SU_L(2) \times SU_R(2)$ group (the Higgs sector has the same structure as a pion-sigma system without fermionic couplings). Both global symmetries are spontaneously broken. In the Higgs case, a negative mass term drives the spontaneous symmetry breaking while, in the quark sector, the breaking is assumed to be dynamical and leads to the formation of a quark-antiquark condensate. The chiral $SU_L(N) \times SU_R(N)$ symmetry of the strongly interacting sector spontaneously breaks to SU(N) and is accompanied by the existence of N pion-like Goldstone particles. At the same time, the Higgs sector breaks to SU(2), which implies the existence of three Higgs Goldstone states.

In actuality, the quark and Higgs sectors are locked together through Yukawa and gauge boson couplings. The coupled world does not possess as much symmetry as the unlocked world: it is no longer possible to rotate the Higgs degrees of freedom independently of the quark degrees of freedom. The conserved currents in the combined theory now have both quark and Higgs content. The standard model is $SU(2) \times U(1)$ symmetric. When this symmetry is spontaneously broken down to U(1), the three resulting Goldstone states will, to zeroth order in the Yukawa and gauge boson coupling constants, be admixtures of the Higgs and pion-like Goldstone states of the uncoupled worlds. This follows from the fact the Goldstone content of the theory is uniquely fixed by the form of the conserved currents associated with the spontaneously broken symmetries. It needs to be emphasized that the degree of mixing between quark and Higgs in the formation of the Goldstone states is not determined by the Yukawa or electroweak gauge boson couplings; rather, it is the relative sizes of the Higgs vacuum expectation value and the pion decay constant that determines the magnitude of the mixing. The truth of this last comment will become apparent in the next section. Those Goldstone degrees of freedom present in the uncoupled worlds but, up to this point, unaccounted for in the coupled world gain mass and appear as low-mass pseudoscalar states -- pions, kaons, etc.

The quark-Higgs Yukawa couplings lead to fermion masses. The values of these couplings at $q^2 = 0$ are estimated in this paper using exact Goldberger-Treiman relations: exact relations can be written down for the true Goldstone bosons that exist in the coupled theory of scalars and quarks. The introduction of gauge bosons and gauge couplings are assumed to affect the results only by finite higher order corrections. The Yukawa couplings at $q^2 = 0$ multiplied by the scalar vacuum expectation value, f_χ, define the quantities referred to in this paper as quark masses. These parameters are of interest because they set the scale of chiral symmetry breaking effects; however, they cannot readily be compared with other definitions of quark mass since perturbative renormalization group arguments cannot be used to relate a $q^2 = 0$ coupling to couplings defined at some higher momentum scale.

2. Formalism

In light of the above scenario, the Goldstone bosons of Weinberg-Salam theory -- i.e., those states eventually eaten by the W and Z bosons -- are seen to have both quark and Higgs content. As an example, let us consider the following charged weak current:

$$J^{\mu L}_{1+i2} = \bar{u}\gamma^\mu\left(\frac{1-\gamma_5}{2}\right) d\cos\theta_c + \bar{u}\gamma^\mu\left(\frac{1-\gamma_5}{2}\right) s\sin\theta_c$$
$$+ \frac{i}{2}\partial^\mu\left(\sigma' + i\phi_3\right)\left(\phi_1 - i\phi_2\right) - \frac{i}{2}\left(\sigma' + f_{\tilde{\chi}} + i\phi_3\right)\partial^\mu\left(\phi_1 - i\phi_2\right) \tag{1}$$

I have dropped any terms involving the charmed quarks in the interest of staying within the realm of flavor SU(3). The ϕ_i's are to be identified as the real parts of the complex SU(2) Higgs doublet:

$$\begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix} = \begin{pmatrix} (\phi_1 + i\phi_2)/\sqrt{2} \\ (\sigma + i\phi_3)/\sqrt{2} \end{pmatrix}$$

This current couples to one of the three Goldstone bosons in the theory. The magnitude of the coupling of the charged Goldstone state, $|\chi^-\rangle$, to the current is measured by the matrix element of the conserved current taken between $|\chi^-\rangle$ and the vacuum:

$$\langle 0 \mid J^{\mu L}_{1+i2} \mid \chi^-\rangle = iq^\mu f_\chi \frac{\sqrt{2}}{2} = iq^\mu \sqrt{f^2_{\tilde{\chi}} + f'^2}\,\frac{\sqrt{2}}{2} \tag{2}$$

To interpret Eq. (2), a few definitions are required. The constant, f', is related to the pion (π) and kaon (k) decay constants, $f_{\tilde{\pi}}$ and $f_{\tilde{k}}$, which, in turn, are defined by the matrix elements:

$$\begin{aligned} \langle 0 \mid \bar{u}\gamma^\mu\gamma^5 d \mid \tilde{\pi}^-\rangle &= iq^\mu f'\sqrt{2} = iq^\mu f_{\tilde{\pi}}\sqrt{2}\big/\cos\theta_c \\ \langle 0 \mid \bar{u}\gamma^\mu\gamma^5 s \mid \tilde{k}^-\rangle &= iq^\mu f'\sqrt{2} = iq^\mu f_{\tilde{k}}\sqrt{2}\big/\sin\theta_c \end{aligned} \tag{3}$$

Tilda's indicate quantities defined in the absence of Yukawa and electroweak gauge couplings. Furthermore, the Higgs vacuum expectation value, $\langle\sigma\rangle = f_{\tilde{\chi}}$, can also be defined by the matrix element:

$$\langle 0 \mid i\left(\sigma' + f_{\tilde{\chi}}\right)\partial^\mu\left(\phi_1 - i\phi_2\right) - i\partial^\mu\sigma'\left(\phi_1 - i\phi_2\right) \mid \tilde{\chi}^-\rangle = iq^\mu f_{\tilde{\chi}}\sqrt{2} \tag{4}$$

The true Goldstone state, consistent with Eq. (2), is given by:

$$|\chi^-\rangle = \frac{f_{\tilde{\chi}}\,|\tilde{\chi}^-\rangle}{\sqrt{f^2_{\tilde{\chi}} + f'^2}} + \frac{f'}{\sqrt{f^2_{\tilde{\chi}} + f'^2}}\left(\cos\theta_c|\tilde{\pi}\rangle + \sin\theta_c|\tilde{k}\rangle\right) \tag{5}$$

This relation is really only correct up to higher order corrections: in the coupled theory the field operators and decay constants are renormalized by the interactions. It is of interest to note that the degree of mixing between the Higgs and quark sectors is controlled by the ration of f' to $f_{\tilde{\chi}}$ and not by the size of the Yukawa or gauge coupling constants. For the sake of completeness, I point out that the pseudoscalar meson states, $|\pi^-\rangle$ and $|k^-\rangle$, are linear combinations of $|\tilde{\chi}^-\rangle$, $|\tilde{\pi}^-\rangle$, and $|\tilde{k}^-\rangle$ orthogonal to $|\chi^-\rangle$ and totally decoupled from the conserved currents.

The Goldstone particle $|\chi^-\rangle$ satisfies an exact Goldberger-Treiman relation. I will outline the derivation of this relation; the final result will be an expression for the quark mass parameters in terms of physically known quantities. The derivation begins with the insertion of the conserved current between baryon states; the divergence of this quantity is zero.

$$0 = \partial_\mu \langle B'(p')|\, 2J^{\mu L}_{1+i2}\,|\, B(p)\rangle = iq_\mu \sum_{\ell} \langle B'(p')\,|\, 2J^{\mu L}_{1+i2}\,|\,\ell\rangle\langle\ell\,|\,B(p)\rangle \tag{6}$$

where, in the second part of the equality, a complete set of physical states, $|\ell\rangle$, has been inserted. The current operator can be separated into a piece that contains pole contributions and one that does not:

$$iq_\mu \left\{ \sum_{m,B'} \langle B'(p')\,|2J^{\mu L}_{1+i2}|B''(p''),m\rangle\langle m,B''(p'')\,|B(p)\rangle + \langle B'(p')\,|2\bar{J}^{\mu L}_{1+i2}|B(p)\rangle \right\} = 0 \tag{7}$$

Here I have used the fact that the only intermediate states contributing involve either pseudoscalar meson poles or a Goldstone pole; any member of this set of states is represented by the symbol, m. $\bar{J}^{\mu L}_{1+i2}$ stands for the nonpole part of the current operator. Since $J^{\mu L}_{1+i2}$ couples only to the physical state, $|\chi^-\rangle$, Eq. (7) becomes

$$0 = iq_\mu\left\{\langle 0|2J^{\mu L}_{1+i2}|\chi^-(q)\rangle\langle\chi^-(q),B'(p')|B(p)\rangle + \langle B'(p')|2\tilde{J}^{\mu L}_{1+i2}|B(p)\rangle\right\} \tag{8}$$

where q = p' − p. The axial vector and vector parts of (8) can be separated. The axial part of the nonpole term on the rhs of (8) is the usual axial-vector form factor piece:

$$\left[\langle B'(p')\mid 2\tilde{J}^{\mu L}_{1+i2}\mid B(p)\rangle\right]_{axial}$$

$$= \begin{cases} \langle B'(p')|\widetilde{\bar{u}\gamma^\mu\gamma^5}\, d\cos\theta_c|B(p)\rangle & \text{if } \begin{array}{l} B' = \text{proton (p)} \\ B = \text{neutron (n)} \end{array} \\ \langle B'(p')|\widetilde{\bar{u}\gamma^\mu}\,\widetilde{\gamma^5}\, s\sin\theta_c|B(p)\rangle & \text{if } \begin{array}{l} B' = \text{neutron} \\ B = \Sigma^- \end{array} \\ & \text{or } \begin{array}{l} B' = \text{proton} \\ B = \Lambda \end{array} \end{cases} \tag{9a}$$

$$= \begin{cases} \bar{u}_p\, g_{A\tilde{\pi}pn}(q^2)\,\gamma^\mu\gamma^5 u_n\cos\theta_c \\ \bar{u}_{\substack{n\\p}}\, g_{A\tilde{k}\left(\substack{n\ \Sigma^-\\ p\ \Lambda}\right)}(q^2)\,\gamma^\mu\gamma^5 u_{\substack{\Sigma^-\\ \Lambda}}\sin\theta_c \end{cases} \tag{9b}$$

Line (9a) follows from the fact that the neutron and proton, for all practical purposes, contain no valence s-quark as well as no Higgs component; hence, only $\bar{u}\gamma^\mu\gamma^5 d\cos\theta_c$ can make a nonpole contribution if B = neutron and B' = proton. Similarly, there is no nonpole overlap between the neutron state and $\bar{u}\gamma^\mu\gamma^5 d\cos\theta_c|\Sigma^-\rangle$ or between the proton state and the state, $\bar{u}\gamma^\mu\gamma^5 d\cos\theta_c|\Lambda\rangle$.

The evaluation of the axial part of the pole term on the rhs of (8) yields:

$$\left[\langle 0 \mid 2J^{\mu L}_{1+i2} \mid \chi^-(q)\rangle\langle\chi^-(q), B'(p')\,|B(p)\rangle\right]_{axial}$$

$$= \; iq^\mu f_\chi \sqrt{2}\, \frac{i}{q^2} \sqrt{2}\; g_{r\chi B'B}(q^2)\, \bar{u}_{B'}\, \gamma_5\, u_B \qquad (10)$$

The second factor of $\sqrt{2}$ is convention (it can be absorbed into the definition of $g_{r\chi B'B}(q^2)$). Using Eq. (5), the form factor, $g_{r\chi B'B}(q^2)$ can be rewritten (to $o(g^2)$, where g is the Yukawa coupling or the electroweak gauge coupling) as follows:

$$g_{r\chi B'B}(q^2) = \frac{f_{\tilde{\chi}}}{\left(f^2_{\tilde{\chi}} + f'^2\right)^{\frac{1}{2}}}\, g_{r\tilde{\chi}B'B}(q^2) + \frac{f'\cos\theta_c}{\left(f^2_{\tilde{\chi}} + f'^2\right)^{\frac{1}{2}}}\left(\cos\theta_c\; g_{r\tilde{\pi}B'B}(q^2)\right)$$

$$+ \frac{f'\sin\theta_c}{\left(f^2_{\tilde{\chi}} + f'^2\right)^{\frac{1}{2}}}\left(\sin\theta_c\; g_{r\tilde{K}B'B}(q^2)\right) \qquad (11)$$

Combining Eqs. (8)-(11) the results are:

$$f_{\tilde{\chi}}\left[2g_{r\tilde{\chi}pn}(q^2)\right] = \cos\theta_c\left(m_p + m_n\right) g_{A\tilde{\pi}pn}(q^2) - f'\cos^2\theta_c\, 2g_{r\tilde{\pi}pn}(q^2)$$

$$\equiv \cos\theta_c \Delta_{pn} \qquad (12a)$$

$$f_{\tilde{\chi}}\left[2g_{r\tilde{\chi}n\Sigma^-}(q^2)\right] = \sin\theta_c\left(m_n + m_{\Sigma^-}\right) g_{A\tilde{K}n\Sigma^-}(q^2) - f'\sin^2\theta_c\, 2g_{r\tilde{K}n\Sigma^-}(q^2)$$

$$\equiv \sin\theta_c \Delta_{n\Sigma^-} \qquad (12b)$$

$$f_{\tilde{\chi}}\left[2g_{r\tilde{\chi}p\Lambda}(q^2)\right] = \sin\theta_c\left(m_p + m_\Lambda\right)\; g_{A\tilde{K}p\Lambda}(q^2) - f'\sin^2\theta_c\, 2g_{r\tilde{K}p\Lambda}(q^2)$$

$$\equiv \sin\theta_c \Delta_{p\Lambda} \qquad (12c)$$

These equations have been simplified by recognizing that $\not{q}\gamma_5$ sandwiched between spinors $\bar{u}_{B'}$ and u_B gives $\bar{u}_{B'}\gamma_5 u_B\left(m_{B'} + m_B\right)$; moreover, in the standard version of the quark model, $g_{r\tilde{K}pn}(q^2) = g_{r\tilde{\pi}n\Sigma^-}(q^2) = g_{r\tilde{\pi}p\Lambda}(q^2) = 0$. The axial vector form factors, $g_{A\tilde{\pi}pn}(q^2)$, $g_{A\tilde{K}n\Sigma^-}(q^2)$, and $g_{A\tilde{K}p\Lambda}(q^2)$, are the same, to within corrections of order $(f'/f_{\tilde{\chi}})^2$ and g^2, as the form factors derived using the physical pion or kaon currents. There is a Higgs component in the physical pion and kaon current that is down by only a factor of $(f'/f_{\tilde{\chi}})$; however, as was seen earlier, the Higgs current makes no nonpole contributions. The other form factors, $g_{r\tilde{\pi}pn}(q^2)$, $g_{r\tilde{K}n\Sigma^-}(q^2)$, and $g_{r\tilde{K}p\Lambda}(q^2)$, are the same as their physical counterparts up to factors of $(f'/f_{\tilde{\chi}})^2$. This can be seen from a decomposition of the physical currents in terms of the unmixed currents.

Finally, the form factors, $g_{\tilde{\chi}B'B}(q^2)$, are related to the Higgs-quark Yukawa couplings (see Weinberg[2] for Lagrangian). The relevant interactions in the Lagrangian are:

$$\frac{\phi^+ m_d \cos\theta_c}{\left(f_{\tilde{\chi}}/\sqrt{2}\right)}\, \bar{u}\left(\frac{1+\gamma_5}{2}\right) d - \frac{\phi^+ m_u \cos\theta_c}{\left(f_{\tilde{\chi}}/\sqrt{2}\right)}\, \bar{u}\left(\frac{1-\gamma_5}{2}\right) d \left(\text{for} \quad g_{r\tilde{\chi}pn}(q^2)\right)$$

or (13)

$$\frac{\phi^+ m_s \sin\theta_c}{\left(f_{\tilde{\chi}}/\sqrt{2}\right)}\, \bar{u}\left(\frac{1+\gamma_5}{2}\right) s - \frac{\phi^+ m_u \sin\theta_c}{\left(f_{\tilde{\chi}}/\sqrt{2}\right)}\, \bar{u}\left(\frac{1-\gamma_5}{2}\right) s \left(\begin{array}{l}\text{for} \quad g_{r\tilde{\chi}n\Sigma^-}(q^2) \\ \text{or} \quad g_{r\tilde{\chi}p\Lambda}(q^2)\end{array}\right)$$

and where $\phi^+ = (\phi_1 + i\phi_2)/\sqrt{2}$ and $\langle\phi^o\rangle = f_{\tilde{\chi}}/\sqrt{2}$. The matrix element that enters into the definition of the form factor involves the source current for the fields, ϕ_1 and ϕ_2, sandwiched between the appropriate baryon states:

$$\langle B'(p')|j^5_\phi(0)|B(p)\rangle = \bar{u}(B')\gamma_5 u(B)\, 2G_{\phi B'B}(q^2)\, (\text{vol})^{-1} \tag{14}$$

The $(\text{volume})^{-1}$ factor is present because the quantization procedure is carried out with box normalization and periodic boundary conditions. The current $j^5_\phi(x)$ is determined from the equations of motion for ϕ_1 and ϕ_2. Using translational invariance of the operator, $j^5_\phi(x)$, and integrating over the appropriate space-time box that encloses the baryon, it follows that:

$$\int_{-\infty}^{\infty} dt\ \bar{u}(B')\,\gamma_5\,u(B)\,2G_{\tilde{\chi}B'B}(q^2)$$

$$= \int_{-\infty}^{\infty} dt \int_{\text{box}} d^3x\ e^{iqx} \begin{pmatrix} \dfrac{m_u+m_d}{f_{\tilde{\chi}}}\cos\theta_c \\ \dfrac{m_u+m_s}{f_{\tilde{\chi}}}\sin\theta_c \end{pmatrix} \left\langle B'(p')\left|\begin{pmatrix}\bar{u}\gamma_5 d\\ \bar{u}\gamma_5 s\end{pmatrix}\right|B(p)\right\rangle \quad (15)$$

The matrix elements $\langle B'(p')|\bar{u}\gamma_5 d|B(p)\rangle$ and $\langle B'(p')|\bar{u}\gamma_5 s|B(p)\rangle$ can only be calculated in the context of a specific model. Since the MIT bag model has met with considerable phenomenological success, I choose it. Hence, "bag" should be substituted for "box" in the integral. I will assume that the quarks inside the baryons are in the lowest eigenmodes. The lowest mode solution of the massive Dirac equation with the MIT bag model boundary conditions is given in Ref. 3. It should be emphasized that the mass parameter occurring in the wave function is the current quark mass. This identification is justified in work done recently by Donoghue and Johnson.[4] In the $q_\mu \to 0$ limit (I am assuming $m_{B'} \sim m_B$), the exponential can be expanded. The lone surviving term in this limit is the one linear in $|\vec{q}|$. For simplicity, I will assume that the vector $\vec{q}$ points in the z-direction:

$$\lim_{q_\mu \to 0} \bar{u}(B')\gamma_5 u(B)\, 2G_{\tilde{\chi}B'B}(q^2)$$

$$= \lim_{q_\mu \to 0} -i\int d^3\vec{x}\; q_z z \begin{pmatrix} \dfrac{m_u + m_d}{f_{\tilde{\chi}}} \cos\theta_c \\ \dfrac{m_u + m_s}{f_{\tilde{\chi}}} \sin\theta_c \end{pmatrix} \left\langle B'(p') \middle| \begin{pmatrix} \bar{u}\gamma_5 d \\ \bar{u}\gamma_5 s \end{pmatrix} \middle| B(p) \right\rangle \quad (16)$$

Next, the pseudoscalar operator $\bar{u}\gamma_5 d$ (or $\bar{u}\gamma_5 s$) is inserted between the wave functions of reference 3, and the integral is carried out. The resulting relation (for $x_{B'} \neq x_B$) is:

$$\lim_{q_\mu \to 0} \bar{u}(B')\gamma_5 u(B)\, 2G_{\tilde{\chi}B'B}(q^2) = \lim_{q_\mu \to 0} q_z \begin{pmatrix} \dfrac{m_u + m_d}{f_{\tilde{\chi}}} \cos\theta_c \\ \dfrac{m_u + m_s}{f_{\tilde{\chi}}} \sin\theta_c \end{pmatrix} \frac{N(x_{B'})N(x_B)}{3}$$

$$\times \quad (\text{Clebsch factor}) \left\{ \left[\frac{R^4}{x_{B'} x_B^2} \left(\frac{\omega_{B'} + m_{B'}}{\omega_{B'}} \right)^{\frac{1}{2}} \left(\frac{\omega_B - m_B}{\omega_B} \right)^{\frac{1}{2}} + \frac{R^4}{x_{B'}^2 x_B} \left(\frac{\omega_B + m_B}{\omega_B} \right)^{\frac{1}{2}} \right. \right.$$

$$\times \left. \left(\frac{\omega_{B'} - m_{B'}}{\omega_{B'}} \right)^{\frac{1}{2}} \right] \left(\frac{\sin(x_{B'} - x_B)}{2(x_{B'} - x_B)} - \frac{\sin(x_{B'} + x_B)}{2(x_{B'} + x_B)} \right) + \left[\frac{R^4}{x_{B'} x_B} \left(\frac{\cos(x_{B'} + x_B)}{2(x_{B'} + x_B)} \right. \right.$$

$$+ \left. \left. \frac{\cos(x_{B'} - x_B)}{2(x_{B'} - x_B)} \right) - \frac{R^4 \sin(x_{B'} + x_B)}{2x_{B'} x_B (x_{B'} + x_B)^2} - \frac{R^4 \sin(x_{B'} - x_B)}{2x_{B'} x_B (x_{B'} - x_B)^2} \right]$$

$$\times \left(\frac{\omega_{B'} + m_{B'}}{\omega_{B'}} \right)^{\frac{1}{2}} \left(\frac{\omega_B - m_B}{\omega_B} \right)^{\frac{1}{2}} + \left[\frac{R^4}{x_{B'} x_B} \left(\frac{\cos(x_{B'} + x_B)}{2(x_{B'} + x_B)} + \frac{\cos(x_B - x_{B'})}{2(x_B - x_{B'})} \right) \right.$$

$$- \left. \left. \frac{R^4 \sin(x_{B'} + x_B)}{2x_{B'} x_B (x_{B'} + x_B)^2} - \frac{R^4 \sin(x_B - x_{B'})}{2x_{B'} x_B (x_{B'} - x_B)^2} \right] \left(\frac{\omega_B + m_B}{\omega_B} \right)^{\frac{1}{2}} \left(\frac{\omega_{B'} - m_{B'}}{\omega_{B'}} \right)^{\frac{1}{2}} \right\} \quad (17)$$

($N(x_B)$, ω_B, and x_B are all defined in Ref. 3). The subscript B' (B) labels the quark state in baryon B' (B). The Clebsch term is given by $\langle B'|B_0^{f\dagger} U^\dagger \sigma_z U B_0^f|B\rangle$, where σ_z is the zth component of the spin operator; U is a 2-component Pauli spinor; and B_0^f destroys a quark of flavor f in the lowest mode state. R is the dimension of the bag.

Equation (16) reduces to a considerably simpler expression if $f' = u$ and $f = d$; then, to good approximation, $m_u = m_d$, and the integral can be written as:

$$\lim_{q_\mu \to 0} \bar{u}(B') \gamma_5 u(B) 2G_{\tilde{\chi}B'B}(q^2)$$

$$= \lim_{q_\mu \to 0} q_z \frac{R}{6} \frac{(4\alpha + 2\lambda - 3)}{2\alpha(\alpha-1) + \lambda} \text{(Clebsch factor)} \begin{pmatrix} \dfrac{m_u + m_d}{f_{\tilde{\chi}}} \cos\theta_c \\ \dfrac{m_u + m_s}{f_{\tilde{\chi}}} \sin\theta_c \end{pmatrix} \quad (18)$$

with $\alpha^2 = \lambda^2 + x^2$ and $\lambda = mR$. However, if $f' = u$ and $f = s$, the two quark masses can no longer be considered equal, and the integral does not reduce to such a simple form.

The numbers arising from relations (17) and (18) are relevant for discussions of chiral symmetry breaking. The masses appearing in these equations as well as in the Lagrangian are products of the Higgs vacuum expectation value and the appropriate Yukawa coupling constants. The quark mass, therefore, is a hidden coupling parameter in the quark-quark-Higgs Goldstone three-point vertex function and is renormalized just like a Yukawa coupling constant. Since the Yukawa coupling will "run" with the momentum, it is necessary to choose an appropriate momentum scale for defining the masses. The form factors in Eq. (12) and, hence, masses in Eqs. (17) and (18) are all defined at $q_\mu = 0$. This is an appropriate choice, since the low momentum region is the domain of chiral symmetry breaking. Moreover, on the basis of the PCAC of $K_{\ell 3}$ decays it can be argued that the results at $q^2 = 0$ carry over to q^2 values as high as m_k^2.

A final question remains: what effect does the introduction of gauge bosons -- the Higgs phenomenon -- have on these arguments? It is assumed that the Higgs phenomenon does not alter any of the above results in a non-smooth way: the Goldstone particle is absorbed by the vector mesons, but, otherwise, all results following from the Goldstone structure of the theory are left intact.

3. Discussion of Results

To extract actual numbers for the quark mass parameters defined by Eqs. (17) and (18), I use the Goldberger-Treiman relation for the π-N system and the current algebra mass ratios. The current algebra mass ratios are obtained from the pseudoscalar meson mass formula. Though the individual mass parameters are functions of the momentum -- as pointed out above, they run like Yukawa couplings -- the ratios are taken to be renormalization invariants. This is true as long as flavor breaking weak and electromagnetic interactions are negligible; numerical estimates indicate that this is indeed the case. Following Weinberg,[5] I set

$$m_d/m_u = 1.8 \qquad \text{and} \qquad m_s/m_d = 20.1$$

I choose to study the π-N system since, for this case, the result of the bag model calculation assumes a relatively simple form (see Eq. (18)): all the mass dependence, to a good approximation, lies in the coefficient term, $(m_u + m_d)$. The numerical results and relevant numbers are either presented in the accompanying tables or can be found in Ref. 6. The result is:

$$m_u + m_d = 38.46 \pm 14.63 \tag{19}$$

The error factor includes only the experimental error in Δ_{pn} (see Eq.(12a)). Combining this result with the current algebra mass ratios yields:

$$m_u \cong 13.7 \pm 5.9 \quad , \quad m_d \cong 24.7 \pm 12.9 \tag{20}$$

and the prediction:

$$m_s \cong 497 \pm 299 \tag{21}$$

The errors include both the uncertainty in Δ_{pn} and a 30% uncertainty in the current algebra mass ratios: the 30% uncertainty represents the unreliability of kaon PCAC.

It is interesting to test whether the value for m_s is consistent with the Goldberger-Treiman relations for the p-Λ (see Eq. (12c)) and the n-Σ^- (Eq. (12b)) systems. With $m_s = 497$ MeV and $m_u = 13.7$ MeV, the calculated value of the lhs of Eq. (12c) is 924 MeV, which is within the error range for the Goldberger-Treiman deviation, $\Delta_{\Lambda p}$, given in Table II. There is considerably more uncertainty associated with $\Delta_{n\Sigma^-}$. The main uncertainty lies with the axial vector form factor, $g_{Ak\Sigma^- n}$. The most recent results, from the Yale-NAL-BNL neutron spectrum experiment, disagree by roughly three standard deviations with the value obtained from the Orsay-Ecole Polytechnique neutron-spectrum data. The Orsay value for $g_{Ak\Sigma^- n}$ leads to consistent quark mass predictions whereas the Yale value does not. It should be pointed out that the Yale value is actually closer to the SU(3) prediction for the axial vector form factor: using the n-p and Λ-p axial vector form factors to establish the D and F coupling constants, I find:

$$\left(g_{Ak\Sigma^- n}\right)_{\text{predicted}} = 0.65 \pm 0.15$$

There are several approximations that enter into the definition of quark masses proposed in this section and the previous section. It is important to understand these approximations and how they differ from those used in other definitions of the quark mass. I am defining a quark mass to be the product of the Higgs vacuum expectation value and the Yukawa coupling constant that appears in the Lagrangian. This defines the current quark mass; for it is this combination of parameters that enters into the pseudoscalar meson mass formula and any other current algebra formula involving Lagrangian mass terms. The numerical estimates of these quark masses (cf. Eqs. (20)-(21)) should be qualitatively correct: they are considerably larger than one might expect on the basis of baryon mass splittings alone.

There are three major ingredients that play a role in the estimates. The first ingredient is the "error" -- Δ_{pn} -- in the proton-neutron Goldberger-Treiman relation at $q_\mu = 0$. This "error" is known to only fair accuracy: the experimental errors in the physical measurements of the relevant form factors are quite small; however, Δ_{pn} involves the difference of two large numbers. The second ingredient is the quark mass ratios deduced from the pseudoscalar mass formula. The relevant derivation of the ratios has been carried out by Weinberg,[5] who is careful to take into account the virtual photon contributions. The largest errors in the derivation stem from SU(3) symmetry considerations: the accuracy of kaon PCAC and the assumption that $\langle 0|\bar{q}_f q_f|0\rangle$ is the same for $f = u$, d, or s. The resulting errors are probably no worse than 30% -- as suggested by the kaon Goldberger-Treiman relations. The final ingredient is the model-dependent calculation of the overlap factor: $\langle p(p')|\bar{u}\gamma_5 d|n(p)\rangle$ at $q_\mu = 0$. The bag model wave functions used in this calculation are not sensitive to the small up and down quark masses; this implies that $\langle p|\bar{u}\gamma_5 d|n\rangle$ is independent of the quark masses, a fact that greatly simplifies the calculation. Moreover, the integral in Eq. (18) is proportional to the axial-vector form factor, which turns out to be 1.09 in the MIT bag model. This number is within 1.09/1.24 of the observed number; so one expects the calculation of the overlap factor to be good to this accuracy. All in all, the errors in the estimates of the quark masses seem under control (or, at least, known).

It is of interest to compare the mass estimates summarized by Eqs. (20) and (21) with numbers obtained in other schemes. In a scheme proposed by Weinberg,[5] the renormalized quark masses (m_i^*) are given by $m_i^* = Z_m m_i$, where Z_m is a universal renormalization factor, defined by the equation:

$$\langle H(p) \mid \bar{q}_i q_i \mid H(p)\rangle = Z_m N_{Hi} \qquad (22)$$

N_{Hi} is the number of quarks of flavor i in the hadron, H. Attributing the mass splittings in various unitary multiplets to the terms $m_s\bar{s}s$ and next applying the mass ratio quoted earlier, Weinberg finds:

$$m_s^* \cong 150 \text{ MeV} \quad ; \quad m_d^* \cong 7.5 \text{ MeV} \quad ; \quad m_u^* \cong 4.2 \text{ MeV} \qquad (23)$$

These numbers are unreliable because of the uncertainty involved in estimating m_s^* from mass splittings. Not only is there considerable variation in mass splittings from multiplet to multiplet (110 MeV to

190 MeV), but it is not clear at what momentum scale m_s^* is being defined. An even more telling argument is provided by the kaon vector Goldberger-Treiman relation. Using arguments similar to those used in the axial case, one finds:

$$\left(m_u - m_s\right) \lim_{q_\mu \to 0} \int_{\text{bag}} d^3x\; e^{-i\vec{q}\cdot\vec{x}} \langle p|\bar{u}s|\Lambda\rangle \left(\bar{u}(p)u(\Lambda)\right)^{-1}$$

$$= \left(m_\Lambda - m_p\right) g_{V\tilde{K}p\Lambda}(0) \qquad \left(g_{V\tilde{K}p\Lambda}(0) \sim -1\right)$$

Doing the bag integral, I find: $m_s \cong \left(m_\Lambda - m_p\right)/.61$. Hence, the mass values recorded in Eq. (23) probably are not the relevant parameters to use in estimates of chiral symmetry breaking. Finally, the Weinberg mass values cannot be compared directly with those presented in Eqs. (20)-(21), since the $g_{r\tilde{\chi}B'B}(0)$ form factors, on which the estimates in this paper are based, involve the quantity $\langle B'|\bar{q}_{f'}\gamma_5 q_f|B\rangle$, not $\langle \bar{B}|\bar{q}q|B\rangle$. Hence, for the sake of comparison, I need to know Z_m, but, for now, Z_m can only be calculated in the context of a model.

In the MIT model,[3,4] Z_m has been calculated and is roughly equal to 0.5. In addition, the MIT group finds:

$$m_q^* = \frac{1}{2}\left(m_u^* + m_d^*\right) = 17 \text{ MeV} \quad \text{(to be compared with Eq. (23))}$$

and

$$m_q = \frac{1}{2}\left(m_u + m_d\right) = 34 \text{ MeV} \quad \text{(to be compared with Eq. (19))}$$

On the other hand, m_s^* is taken to be one of the parameters fitted to the light hadron mass spectrum. The MIT result is $m_s^* = 165$ MeV, which is close to the Weinberg value. It should be noted that the mass ratios in the MIT scheme are very different from the current algebra ratios.

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TABLE I

Form Factors

Axial Vector Form Factor	Numerical Value
$g_{A\pi np}(0)$	1.253 ± 0.007
$g_{Ak\Sigma^- n}(0)$	0.435 ± 0.035 (1a) $0.17 \; {}^{+0.07}_{-0.09}$ (1b)
$g_{Ak\Lambda p}(0)$	0.76 ± 0.06 (2)

Vertex Functions	Numerical Value
$g_{r\pi NN}$	13.50 ± 0.25 (3)
$g_{rkn\Sigma^-}$	4.89 ± 4.11 (4)
$g_{rk\Lambda p}$	16.01 ± 1.45 (5)

(1a) Yale-NAL-BNL neutron spectrum data.
W. Tanebaum et al., Phys. Rev. D12, 1871 (1975).

(1b) Orsay-Ecole Polytechnique neutron spectrum data.
D. Decamp et al., Orsay preprint.

(2) $g_{Ak\Lambda p} = (g_1/f_1)f_1$ where $f_1 = (-3/\sqrt{6})$
See S. Wojcicki, "Proceedings of Summer Institute on Particle Physics," SLAC Report No. 215.

(3) G. E. Hite, R. J. Jacob and D. C. Moir, Phys. Rev. D12, 2677 (1975). The number quoted represents the best estimator for the two experimental numbers presented in the paper.

(4) P. Baillon et al., Phys. Lett. 61B, 171 (1978).

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TABLE II

Errors ($\Delta_{B'B}$) in Goldberger-Treiman Relations.[1]

$\Delta_{B'B}$	Numerical Value
Δ_{pn}	-125.38 ± 47.70
$\Delta_{\Lambda p}$	867.08 ± 138.33
$\Delta_{n\Sigma^-}$	717.62 ± 193.19 $\left(\text{for } g_{Ak\Sigma^- n} = 0.435 \pm 0.035\right)^{(2)}$ 151.34 ± 247.14 $\left(\text{for } g_{Ak\Sigma^- n} = 0.17 \pm 0.08\right)^{(2)}$

(1) In the computation of $\Delta_{B'B}$, I set:

$$f' = \frac{0.93 m_{\pi^\pm}}{\sqrt{2}\cos\theta_c} = \frac{91.79 \text{ MeV}}{\cos\theta_c} ,$$

with $\theta_c = 0.232 \pm 0.003$.

(2) cf. footnotes (1a) and (1b) in Table I.

TABLE III

System	$\bar{u}(B')\gamma_5 u(B)$ (1)	Clebsch Factor (see text)	$f_{\tilde{\chi}}\left[2G_{\tilde{\chi}B'B}(0)\right]$ (2)
p-n	$\frac{1}{2m_p} q_z$	-5/3	$(m_u + m_d)(-3.26)$
Λ-p	$q_z \frac{1.16}{2m_\Lambda}$	$\sqrt{3/2}$	$(m_u + m_s)(1.81)$
$n-\Sigma^-$	$q_z \frac{1.22}{2m_{\Sigma^-}}$	1/3	$(m_u + m_s)(1.50)$

(1) This factor is derived under the assumption that $p_z = 0$ and $p_{z'} \to 0$, where p_z is the initial baryon momentum, $p_{z'}$ is the final baryon momentum, and $q_z = p_z - p_{z'}$.

(2) The bag radius for each system is taken to be 1/200 $(\text{MeV})^{-1}$.