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HADRONIC WAVE FUNCTIONS AT SHORT DISTANCES AND THE OPERATOR PRODUCT EXPANSION†

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ABSTRACT

The operator product expansion, of appropriate products of quark fields, is used to find the anomalous dimensions which control the short distance behavior of hadronic wave functions. This behavior in turn controls the high Q^2 limit of hadronic form factors. In particular, we relate each anomalous dimension of the non-singlet structure functions to a corresponding logarithmic correction factor to the nominal $\alpha_s(Q^2)/Q^2$ fall off of meson form factors. Unlike the case of deep inelastic lepton-hadron scattering, the operator product necessary here involves extra terms which do not contribute to forward matrix elements.

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In this paper we shall show how the operator product expansion links together two critical testing grounds of quantum chromodynamics (QCD): the evolution of the moments of the deep inelastic structure functions and the short distance structure of the hadronic wave functions which appear in large momentum transfer exclusive reactions. In particular, we shall be able to relate the anomalous dimension γ_n of the n-th non-singlet operator that appears in the description of deep inelastic lepton-hadron scattering to a corresponding logarithmic correction factor $(\ln Q^2/\Lambda^2)^{-\gamma_n/2\beta_1}$ that multiplies the nominal $\alpha_s(Q^2)/Q^2$ fall off of (helicity zero) meson form factors. In fact, each anomalous dimension is associated with a specific Gegenbauer moment of the lowest $q\bar{q}$ Fock state component of the meson wave function (q stands for quark, $\bar{q}$ for anti-quark). Here β_1 is defined by $\beta(g) = -\beta_1 g^3 + O(g^5)$, and $\gamma_n g^2$ is the anomalous dimension of the n-th non-singlet operator appearing in the expansion of two currents (in the leading contribution to forward matrix elements). Λ is the renormalization group invariant scale, and $\alpha_s(Q^2) = 1/[4\pi\beta_1 \log(Q^2/\Lambda^2)]$ with $\beta_1 = (1/16\pi^2)(11 - 2/3n_f)$ (n_f is the number of quark flavors).

The general result in QCD for the electromagnetic form factors of hadrons at large momentum transfer is ($Q^2 = -q^2 = -t$) [1,2,3,4]

$$F(Q^2) = \left[\frac{\alpha_s(Q^2)}{Q^2}\right]^{(n-1)} \left|\sum_{k=0}^{\infty} a_k \left(\ln \frac{Q^2}{\Lambda^2}\right)^{-\tilde{\gamma}_k/2\beta_1}\right|^2 \cdot \left[1 + O\left(\alpha_s(Q^2), \frac{m^2}{Q^2}\right)\right] \tag{1}$$

where $n = 2$ and 3 for mesons and baryons, respectively. For mesons, the $\tilde{\gamma}_k$ are equal to the γ_k that appear in the moments of the non-singlet structure functions. The quantity m^2/Q^2 represents mass effects (target mass, transverse motion, etc.). This result holds for all elastic and transition form factors where the constituents have zero orbital angular momentum along the hadrons' direction of motion, and the hadronic helicity is conserved and less than one (zero for mesons, one half for baryons); otherwise the form factor is suppressed by additional powers of Q^2 (with new powers of $\ln Q^2/\Lambda^2$). For mesons, the leading $k=0$ term in eq. (1) has $\tilde{\gamma}_0 = \gamma_0 = 0$ and is normalized to the $M \to \ell\bar{\ell}$ electromagnetic or weak decay amplitudes ($\pi^+ \to \mu^+\nu$ for π^+ form factor [5] and $\rho^0 \to e^+e^-$ for ρ^+ form factor). In the case of baryons the leading $k=0$ terms have $\tilde{\gamma}_0 =$ $(1/16\pi^2)(4/3)$, and the ratios of the asymptotic form factors are given by the SU(3) of color and the flavor group symmetry.

Following Ref. [1], we can write the hadronic form factor to leading order in $\alpha_s(Q^2)$ as (see fig. 1):

$$F(Q^2) = \int_0^1 [dx] \int_0^1 [dy]\ \phi^+(x_i,Q^2)\ T_H(x_i,y_i,Q^2)\ \phi(y_i,Q^2) \tag{2}$$

where $x_i = (k^0+k^3)_i/(p^0+p^3)$ is the longitudinal (light-cone) momentum fraction carried by the i-th constituent and $[dx] = dx_1 \ldots dx_n \delta\left(1 - \sum_{i=1}^{n} x_i\right)$. Equation (2) is obtained in the standard light-cone frame where the incident hadron and virtual photon momenta are $(p^\pm = p^0 \pm p^3)$

$$p^\mu = (p^+, p^-, \vec{p}_\perp) = \left(p^+, \frac{m^2}{p^+}, \vec{0}_\perp\right)$$

$$q^\mu = (q^+, q^-, \vec{q}_\perp) = \left(0, \frac{2p\cdot q}{p^+}, \vec{q}_\perp\right)$$

with $q^2 = -\vec{q}_\perp^{\,2} = -Q^2$.

The hard scattering amplitude T_H is defined as the amplitude for the form factor where each hadron is replaced by collinear valence quarks. The dominant momentum transfer occurs in T_H, and to leading order in $\alpha_s(Q^2)$ it has the form [1,2] (fig. 1)

$$T_H = \left[\frac{\alpha_s(Q^2)}{Q^2}\right]^{n-1} f(x_i,y_i) \tag{3}$$

The "quark distribution amplitude" $\phi(x_i,Q^2)$ in eq. (2) is the amplitude for finding n-valence quarks which are collinear up to the scale Q^2:

$$\phi(x_i,Q^2) = \left(\log \frac{Q^2}{\Lambda^2}\right)^{-n\gamma_F/2\beta_1} \int \prod_{i=1}^{n} \left\{d^2\vec{k}_\perp^{(i)}\,\theta(Q^2-\vec{k}_\perp^{2(i)})\right\} \delta^{(2)}\left(\sum \vec{k}_\perp^{(j)}\right) \Psi(x_i,\vec{k}_\perp^{(i)}) \tag{4}$$

where $\Psi(x_i,\vec{k}_\perp^{(i)})$ is the positive energy projection of the Bethe-Salpeter wave function on the null plane,

$$\Psi_M(x_i,\vec{k}_\perp^{(i)}) \sim \text{F.T.}\left[\langle 0 \mid T\psi(z_1)\bar{\psi}(z_2) \mid M\rangle\right]_{z_1^+ = z_2^+} \tag{5}$$

$$\Psi_B(x_i,\vec{k}_\perp^{(i)}) \sim \text{F.T.}\left[\langle 0 \mid T\psi(z_1)\psi(z_2)\psi(z_3) \mid B\rangle\right]_{z_1^+ = z_2^+ = z_3^+} \tag{6}$$

(F.T. stands for Fourier transform (see eq. (14)) and ψ are the quark fields). The factor $(\log Q^2/\Lambda^2)^{-n\gamma_F/2\beta_1}$ in eq. (4) is due to the vertex and propagator corrections to T_H (see fig. 1); both T_H and $\phi(x_i,Q^2)$ have zero anomalous dimensions because this factor is included here rather than in eq. (3). Note that in general γ_F, the anomalous dimension of the quark field, is gauge dependent. In the analysis presented here we shall work in the light-cone gauge [6] $A^+ = \eta\cdot A = 0$, although the final results are gauge-invariant.

For mesons the behavior of $\phi(x_i, Q^2)$ at fixed x_i as $Q^2 \to \infty$ is dominated by the behavior of the operator $T\psi(z)\bar{\psi}(0)$ for $z^2 = -z_\perp^2 = O(1/Q^2) \to 0$. This light-cone region can be studied using the usual operator product expansion. Despite the fact that $\psi(z)\bar{\psi}(0)$ is not itself gauge-invariant, it is interesting that only gauge-invariant operators actually contribute to the matrix element in (5) evaluated in light-cone gauge. To see this, ignore external derivatives for the moment and note that $T\psi(z)\bar{\psi}(0)$ has an operator product expansion of the form

$$T\psi(z)\bar{\psi}(0) \sim \sum_n \tilde{c}_n(z^2 - i\varepsilon)\,\Gamma_{(i)}\; z^{\mu_1}\ldots z^{\mu_n}\left(\bar{\psi}\Gamma_{(i)}\overleftrightarrow{D}_{\mu_1}\ldots\overleftrightarrow{D}_{\mu_n}\psi\right)$$

$$+ \sum_{\substack{n,m \\ (m\neq 0)}} \tilde{c}_{nm}(z^2 - i\varepsilon)\,\Gamma_{(i)}\; z^{\mu_1}\ldots z^{\mu_n}\, z^{\nu_1}\ldots z^{\nu_m}$$

$$\times\left(\bar{\psi}\Gamma_{(i)}\overleftrightarrow{D}_{\mu_1}\ldots\overleftrightarrow{D}_{\mu_n}\, A_{\nu_1}\ldots A_{\nu_m}\psi\right) + \ \ldots \tag{7}$$

where the $\Gamma_{(i)}$ are the 16 Dirac matrices. In general $\tilde{c}_{nm} \neq 0$ since $\psi(z)\bar{\psi}(0)$ is not gauge-invariant. However, for the matrix element $\langle 0 \,|\, T\psi(z)\bar{\psi}(0) \,|\, \pi\rangle$ each operator A_ν always leads to a factor of η_ν ($p_{\pi\nu}$ and η_ν are the only available 4-vectors, and $A^+ = 0$ rules out $p_{\pi\nu}$ since $p_\pi^+ \neq 0$). Since $\eta \cdot z = z^+ = 0$, no operator which explicitly contains A_ν can contribute to the meson wave function (5). This leaves only the standard operators $\bar{\psi}\,\Gamma\overleftrightarrow{D}_{\mu_1}\ldots\overleftrightarrow{D}_{\mu_n}$ (and their external derivatives, as discussed below) in the operator product expansion. It is also for this reason that only the $q\bar{q}$ wave function is required in eq. (2) for the leading power behavior.

In general the functions $f(x_i,y_i)$ in T_H are singular at the endpoints of the x_i and y_i integrations. If the wave functions $\phi(x_i,Q^2)$ were constant, then the integration in eq. (2) would diverge logarithmically for the meson form factor. However, the hermiticity of the kinetic energy operator $\sum_i [(k_\perp^2+m^2)/x]_i$ for a composite state guarantees that $\phi(x_i,\lambda^2) \sim (1-x_i)^\varepsilon$ with $\varepsilon > 0$ as $x_i \to 1$ for any λ^2. This ensures that the meson form factor in QCD is not dominated by the endpoint (large distance) region of the x_i integration, and that the short distance domain of the operator $\psi\bar{\psi}$ controls its asymptotic behavior [7]. Further, the compositeness condition ensures the existence of the evolution equations derived in ref. 1 and the convergence of the polynomial expansions for $\phi_M(x_i,Q^2)$ as in eq. (20).

It is important to observe that the large Q^2 behavior of the non-singlet structure function moments is controlled by the same singularities which appear in eq. (7) for helicity zero mesons. Indeed, aside from flavor factors, the same operators $\mathcal{O}_{(n)}$ dominate the operate expansions of $\psi(z/2)\bar{\psi}(-z/2)$ and $J_\mu(z/2)J_\nu(-z/2)$:

$$\psi(z/2)\bar{\psi}(-z/2) \sim \sum_n \tilde{c}_n(z^2 - i\varepsilon z_0) \sum_{m\geq n} \Gamma_\alpha^{(i)} z_{\alpha_1}\cdots z_{\alpha_m} \mathcal{O}_{(n)(i)}^{\alpha_1\cdots\alpha_m\alpha} \tag{8}$$

and

$$J_\mu(z/2)J_\nu(-z/2) \sim \sum_n \frac{c_n(z^2 - i\varepsilon z_0)}{(z^2 - i\varepsilon z_0)^2} \sum_{m\geq n} \Gamma_{\mu\nu\alpha\beta}^{(i)} z_{\alpha_1}\cdots z_{\alpha_m} z^\beta \mathcal{O}_{(n)(i)}^{\alpha_1\cdots\alpha_m\alpha} \tag{9}$$

where

$$\Gamma_\alpha^{(i)} = \begin{cases} \gamma_\alpha & i=1 \\ \gamma_\alpha\gamma_5 & i=2 \end{cases} \quad , \tag{10}$$

$$\Gamma^{(i)}_{\mu\nu\alpha\beta} = \begin{cases} g_{\nu\alpha}g_{\mu\beta} + g_{\nu\beta}g_{\mu\alpha} - g_{\mu\nu}g_{\alpha\beta} & (F_1,F_2) \\ \epsilon_{\mu\nu\alpha\beta} & (F_3) \end{cases} \tag{11}$$

and [8]

$$\mathcal{O}^{\alpha_1\ldots\alpha_m\alpha}_{(n)(i)} = \sum_{k=0}^{n} d_{mnk}\, \partial^{\alpha_{k+1}}\ldots\partial^{\alpha_m}\, \bar{\psi}(0)\, \Gamma^{\alpha}_{(i)}\, \overleftrightarrow{D}_{\alpha_1}\ldots\overleftrightarrow{D}_{\alpha_k}\, \psi(0) \ . \tag{12}$$

Only $\Gamma^{(2)}_{\alpha}$ is relevant for the pseudoscalar meson wave function, and only $\Gamma^{(1)}_{\alpha}$ contributes to the helicity-zero vector mesons wave function. The anomalous dimensions of $\mathcal{O}_{(n)(1)}$ and $\mathcal{O}_{(n)(2)}$ are the same. The overall factor of $(1/z^2)^2$ in eq. (9) is due to the canonical dimension of $J_\mu J_\nu$; as defined here, both c_n and $\tilde{c}_n$ have zero canonical dimensions.

The distinguishing characteristic between the moment and wave function analyses is just the difference between forward and non-forward matrix elements, respectively. For the moments, the forward matrix element $\langle p \mid J_\mu J_\nu \mid p\rangle$ has contributions only from terms having no external derivatives (i.e., one term for each n with $m = n = k$). In contrast the wave function receives contributions from all terms in eq. (12). Fourier transforming eq. (8) we obtain the distribution amplitude [eq. (4)]

$$\phi(x_i,Q^2) = \sum_{n=0}^{\infty} a_n\, \phi_n(x_i) \left(\ln \frac{Q^2}{\Lambda^2}\right)^{-\gamma_n/2\beta_1} \tag{13}$$

Since $\phi(x_i,Q^2)$ was defined to have no overall anomalous dimension, the γ_n appearing here are just the anomalous dimensions of the operator $\mathcal{O}_{(n)}$, i.e., precisely the anamolous dimensions controlling the moments of the non-singlet structure function, where $\langle p \mid \mathcal{O}^{\alpha_1\ldots\alpha_m\alpha}_{(n)(1)} \mid p\rangle \propto$
$\langle p \mid \bar{\psi}\gamma^{\alpha}\overleftrightarrow{D}^{\alpha_1}\ldots\overleftrightarrow{D}^{\alpha_n}\psi \mid p\rangle$.

The functional dependence on x_i in eq. (13) reflects the fact that the q and $\bar{q}$ do not have the same light-cone momentum fractions in the non-forward matrix element. Explicitly ($x \equiv x_1 - x_2$)

$$\Psi_M(x_i,\vec{k}_\perp) \equiv \int \frac{d^2 z_\perp}{16\pi^3} e^{-i\vec{k}_\perp \cdot \vec{z}_\perp} \int dz^- \; e^{(i/2)xz^-p^+}$$

$$\times \; \langle 0 \mid T \; \bar{\psi}(-z/2) \, p^+ \Gamma^- \psi(z/2) \mid p\rangle \Big|_{z^+=0}$$

$$= \sum_n \bar{c}_n(\vec{k}_\perp^2) \sum_{m \geq n} a_{mn} \int dz^- \; e^{(i/2)xz^-p^+} (p^+z^-)^m \qquad (14)$$

where

$$a_{mn} = \sum_{k=0}^{n} d_{mnk} \, b_k$$

and b_k is the normalization factor in

$$\langle 0 \mid \bar{\psi}(0) \, \Gamma_\alpha^{(i)} \overleftrightarrow{D}_{\alpha_1} \cdots \overleftrightarrow{D}_{\alpha_k} \psi(0) \mid p\rangle = b_k \, p_{\alpha_1} \cdots p_{\alpha_k} \, p_\alpha \quad . \qquad (15)$$

The large $k_\perp$ behavior of the (gauge-dependent) Bethe-Salpeter equation is then given by eq. (14) where

$$\bar{c}_n(\vec{k}_\perp^2) \propto \frac{\alpha_s(\vec{k}_\perp^2)}{\vec{k}_\perp^2} \left(\ell n \, \frac{\vec{k}_\perp^2}{\Lambda^2} \right)^{-(\gamma_n - 2\gamma_F)/2\beta_1} \qquad (16)$$

To one-loop order the conformal invariance of the theory at short distance is broken only in the singular functions $\bar{c}_n$ and not in the coefficients a_{mn} of eq. (14). Thus in leading order these coefficients can be determined using conformal invariance and are independent of the details of the theory. In particular, we can use the following result derived in refs. 9 and 10 for scalar field theory:

$$\sum_{m=n}^{\infty} a_{mn}(p^+z^-)^m \propto (p^+z^-)^n e^{-(i/2)p^+z^-}\int_0^1 du\, [u(1-u)]^{n+1} e^{iup^+z^-} \tag{17}$$

Combining eqs. (14) and (17), we get

$$\Psi(x,\vec{k}_\perp^2) \sim \sum_n d_n \bar{c}_n(\vec{k}_\perp^2) \frac{\partial^n}{\partial x^n}(1-x^2)^{n+1} \quad . \tag{18}$$

Here

$$\frac{\partial^n}{\partial x^n}(1-x^2)^{n+1} \propto (1-x^2)\, C_n^{3/2}(n) \tag{19}$$

where the $C_n^{3/2}(x)$ are the Gegenbauer polynomials. The distribution amplitude of eqs. (4) and (13) is thus

$$\phi_M(x_i,Q^2) = x_1 x_2 \sum_n a_n\, C_n^{3/2}(x_1 - x_2)\left(\ell n\, \frac{Q^2}{\Lambda^2}\right)^{-\gamma_n/2\beta_1} \tag{20}$$

in agreement with the evolution equation derivation given in ref. [1].

The coefficients a_n in eq. (20) are the matrix elements of the local operators appearing in eq. (15). In particular the coefficient a_0 of the leading term ($n = 0$) for pions is proportional to $\langle 0 \mid \bar{\psi}(0)\gamma_\mu\gamma_5(\tau_+/2)\psi(0) \mid \pi\rangle = f_\pi p_\mu$, where f_π is determined by the decay rate for $\pi \to \mu\nu$. Thus the leading term is completely normalized [5]. Similarly, the decay $\rho^0 \to \ell\bar{\ell}$ can be used to normalize the asymptotic distribution amplitude and form factor for helicity-zero ρ-mesons [11]. For the transverse ρ, the local operators are built on the spin-flip operator $\bar{\psi}\sigma_{\alpha\beta}(\tau_+/2)\psi$ in analogy to eq. (12). Since the anomalous dimension of $\bar{\psi}\sigma_{\alpha\beta}\psi$ is $2\hat{\gamma}_F$ where $\hat{\gamma}_F = C_F/16\pi^2$, the factor $(\ell n\, Q^2/\Lambda^2)^{-\hat{\gamma}_F/\beta_1}$ will appear in the asymptotic form factor. More significantly, the corresponding hard scattering amplitude T_H vanishes with an extra power of m/Q because of the necessity for helicity-flip.

The asymptotic behavior of the baryon form factors can similarly be calculated in terms of the anomalous dimensions of towers of operators based on three quark operators [12]. Again, asymptotically it is the operator with the least number of derivatives which has the lowest anomalous dimension. In this case it is $\hat{\gamma}_F/\beta_1$ for the helicity 1/2 and $3\hat{\gamma}_F/\beta_i$ for the helicity 3/2 baryons. Notice, however, that the integrations over the light-cone momentum fractions in eq. (2) would diverge linearly if the wave function were replaced by a constant. Since compositeness only insures that $\phi(x_i) \sim (1-x_i)^\varepsilon$ as $x_i \to 1$ for $\varepsilon > 0$, endpoint singularities are possible, and the proof of the short distance dominance of the nucleon form factor is more subtle. However, as shown in ref. [1], each leading twist contribution to the operator product expansion for $\psi\psi\psi$ leads to a contribution to $\phi_B(x_i,Q^2)$ which is of the form $x_1x_2x_3$ times a polynomial. The sum of such terms is convergent and yields a wave function $\phi_B(x_i,Q^2)$ which vanishes as $(1-x_i)^{2-\delta(Q^2)}$ where $\delta(Q^2)$ vanishes monotonically as $Q^2 \to \infty$ [13]. Thus the region of finite x_i yields a contribution to the form factor which is dominated by the short distance domain. There remains the potentially dangerous region where some of the x_i are infinitesimally small, e.g., $x_2,x_3 \sim O(m/Q)$. A detailed analysis shows that this kinematic region is suppressed by at least two powers of $\alpha_s(Q^2)$. Such contributions correspond to quasi-on-shell quark scattering with $k^2 \sim O(mQ)$ and are further suppressed by a Sudakov-type form factor at the photon-quark vertex [14].

Thus the baryon form factor in QCD, like the meson form factor, is not dominated by the endpoint region in the x_i integration, and the short distance structure of the operator products controls the asymptotic behavior.

In this letter we have shown that the results obtained previously [1] for the form factors of hadrons, can be quite naturally understood in terms of the operator product expansion. In particular, we see that the exponents which appear in eq. (1), which originally were obtained by solving the bound state equations explicitly, are just the anomalous dimensions of familiar operators.

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[6] Note that in $A^+ = 0$ gauge γ_F is formally divergent since it contains terms proportional to $\int dk_+/k_+$. However, γ_F does not appear in the final gauge-invariant form factor -- the explicit dependence on γ_F is cancelled by Ψ. In ref. [1] all effects due to the divergent part of γ_F are absorbed into the definition of a finite potential for the evolution equation.

[7] This is not the case in QCD in two dimensions, where $x_i \to 0$ dominate because of the quadratic divergence in T_H. See M. B. Einhorn, Phys. Rev. D14 (1976) 3451. Thus the form factor behaves like $(1/Q^2)^{\delta}$, $0 < \bar{\delta} < 1$. Note that dimensional counting as defined in ref. [2] predicts a (g^2/Q^2) behavior. T. Appelquist and E. Poggio, Phys. Rev. D10 (1970) 3280, have analyzed the case of ϕ^3 in six dimensions. In fact, we follow their outline in our discussion. Here the dimensional counting laws suggest a $1/Q^4$ behavior. However, T_H again has a quadratic singularity as $x_i \to 0$. Thus here the "meson" wave function would have to vanish as $x_i^{(1+\varepsilon)}$ to get the dimensional counting rule.

[8] That $m \geq n$ follows from the fact that all operators appearing in the n-th combination with well-defined dimensions have at least n factors of z's. That $n \geq k$ follows from the fact that a spin n operator mixes with operators of lower spin only. As usual we can symmetrize and remove trace terms to select operators of definite spin; the trace terms correspond to higher twist operators, which for the wave function matrix element are suppressed at short distance by powers of $z^2 = -z_\perp^2 = O(1/Q^2)$ and thus give power-law suppressed contributions to the form factor.

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[13] This is a consequence of the fact that the anomalous dimensions increase as log n. For example, for mesons, $\phi_M(x_i,Q^2) \sim (x_1x_2)^{1-\delta(Q^2)}$ as $x_i \to 0$ where

$$\delta(Q^2) = \delta(Q_o^2) - \frac{4C_F}{\beta} \ell n \frac{\ell n\, Q^2/\Lambda^2}{\ell n\, Q_o^2/\Lambda^2}$$

for all $Q^2 \le Q_c^2$ where $\delta(Q_c^2) = 0$. For $Q^2 > Q_c^2$, $\phi_M(x_i,Q^2) \sim x_1x_2$ as $x_i \to 0$. The corresponding evolution of the baryon distribution amplitude ϕ_B removes the potential logarithmic singularity from the region $1 \gg (1-x_1) \gg m/Q$ noted by Duncan and Mueller [3].

[14] We wish to thank A. H. Mueller for a discussion on this point. See also A. Duncan and A. H. Mueller, Colombia preprint CU-TP-172 (1979). This suppressed contribution to the form factor corresponds to the calculation of R. Coquereaux and E. deRafael, Phys. Lett. 74B (1978) 105. An analogous factor suppresses the Landshoff pinch contribution to large momentum transfer hadron-hadron scattering [1]. These points will be discussed in more detail in a future publication.

FIGURE CAPTION

Fig. 1(a). Meson form factor.

1(b). Baryon form factor (+ ... stands for all other connected Born graphs).

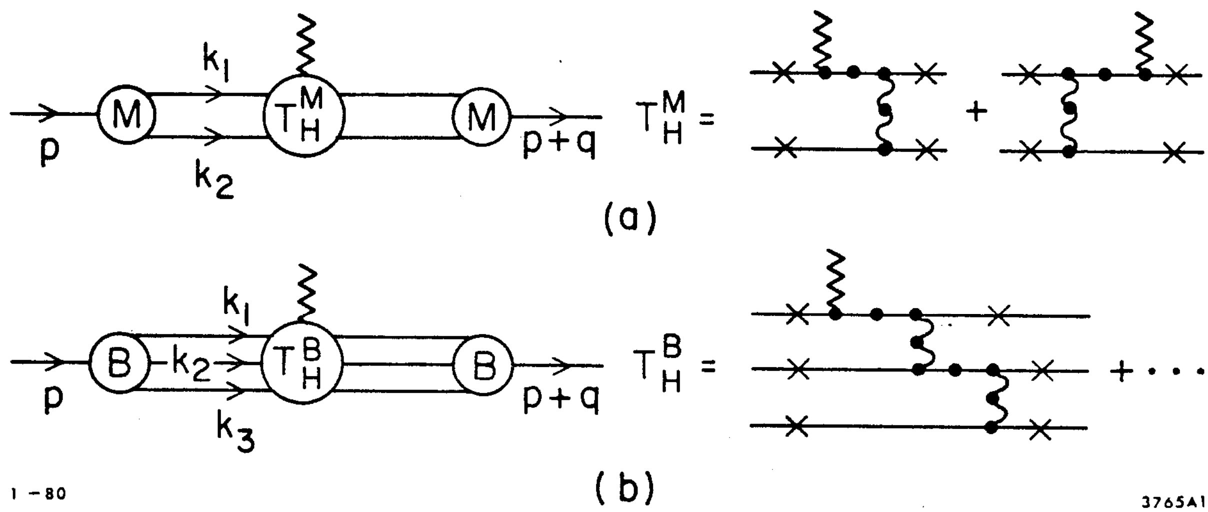

Fig. 1