

A PARTON MODEL RELATION SANS QCD MODIFICATIONS
IN LEPTON PAIR PRODUCTIONS

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ABSTRACT

It is pointed out that, in contrast to the recently discovered large QCD correction to the Drell-Yan formula, a certain quark-parton model relation between structure functions for lepton pair production is not subject to any first order QCD modification. Both parallelism and contrast to the Callan-Gross relation in deep inelastic scattering are spelled out. Implications on the lepton angular distribution for both low and high q_{\perp} are discussed. The case is made that this relation provides a unique opportunity to test the "QCD improved quark-parton model."

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Recent QCD calculations¹ reveal a large ($\sim 100\%$) order α_s correction to the Drell-Yan cross-section formula² for lepton pair production (LPP) in hadron collisions. This makes the integrated cross section a dubious testing ground for QCD corrected parton model, and raises fundamental questions about the viability of the perturbative QCD approach. It is, therefore, important to study other aspects of this approach.

There is much more to the quark-parton model and its QCD modifications in LPP than just the integrated Drell-Yan cross-section formula. The lepton angular distributions are controlled by structure functions which obey parton-model relations^{3,4} similar to those between F_1 and F_2 in Deep Inelastic Scattering (DIS). How are these relations affected by perturbative QCD corrections? The answer to this question is quite surprising: at least one of these relations--the exact counterpart of the Callan-Gross relation--is not modified at all by first order QCD corrections although individual terms in this relation may be subject to large corrections. In the rest of this note, we spell out explicitly the parallelism as well as the contrast between the DIS and LPP cases in the perturbative QCD approach and discuss the experimental implications of these results.

The LPP process is described by a tensor amplitude⁶ $W_{\mu\nu}$ corresponding to the current correlation function

$$W_{\mu\nu} = \mathcal{P} \int d^4z e^{i q \cdot z} \langle P_1 P_2 | J_\mu(z) J_\nu(0) | P_1 P_2 \rangle \quad (1)$$

similar to that of DIS. Here P_1 and P_2 are the four momenta of the colliding hadrons, $\mathcal{P} = -(P_1 + P_2)^2$, and q is the momentum of the virtual photon (hence the lepton pair). The trace of this tensor is related to the cross section (integrated over the lepton center-of-mass angles) via the formula^{2,4}

$$\frac{d\sigma}{d^4q} = \left(\frac{\alpha}{M\mathcal{P}}\right)^2 \frac{1}{(2\pi)^3} W_\mu^\mu \quad (2)$$

wherein M represents the effective mass of the lepton pair and α the fine structure constant.

The tensor $W_{\mu\nu}$ can be decomposed into four structure functions⁴ in much the same way as in DIS:

$$W_{\mu\nu} = W_1 \tilde{g}_{\mu\nu} + W_2 \tilde{P}_\mu \tilde{P}_\nu - W_3 (\tilde{P}_\mu \tilde{p}_\nu + \tilde{p}_\mu \tilde{P}_\nu) + W_4 \tilde{P}_\mu \tilde{P}_\nu \quad (3)$$

where $\tilde{g}_{\mu\nu} = g_{\mu\nu} - q_\mu q_\nu / q^2$, $P = P_1 + P_2$, $p = P_1 - P_2$, $\tilde{P}_\mu = \tilde{g}_{\mu\nu} P^\nu / \sqrt{\mathcal{P}}$, and $\tilde{p}_\mu = \tilde{g}_{\mu\nu} p^\nu / \sqrt{\mathcal{P}}$. These structure functions can be determined experimentally from Eq. (2) together with lepton angular distribution measurements. Eq. (3) closely resembles the corresponding formula defining W_1 and W_2 in DIS (where the last two terms are absent).

A basic result of the quark-parton model in DIS is the Callan-Gross relation⁵ It is usually written as:

$$W_L = W_1 + \left(\frac{\nu}{2} - 1\right) W_2 = 0 \quad (4)$$

where W_L is the helicity structure function for longitudinal virtual photon. We can recast this result in the not-so-familiar form:

$$W_\mu^\mu = 2 W_1 \quad (5)$$

In Ref. 4 we showed that the same equation follows as a general consequence of the quark-parton model in LPP. In terms of the invariant structure functions W_1 to W_4 of LPP, Eq. (5) takes the form

$$W_1 + \left(\frac{(q \cdot P)^2}{\mathcal{P}_M^2} - 1\right) W_2 - \frac{(q \cdot P)(q \cdot p)}{\mathcal{P}_M^2} W_3 + \left(1 + \frac{(q \cdot P)^2}{\mathcal{P}_M^2}\right) W_4 = 0 \quad (6)$$

whereas in terms of the helicity structure functions

$$W_L = 2 W_{\Delta\Delta} \quad (7)$$

Here $W_{\Delta\Delta}$ is the 'double-flip' structure function.⁴ These results are to be compared with Eq. (4) for DIS. Of the three equivalent versions, Eqs. (5), (6) and (7) only the first one is form-invariant when going over from LPP to DIS.

It is known that in perturbative QCD the structure functions become functions of q^2 . In addition:

i) for DIS, the Callan-Gross relation is modified by small first-order QCD terms; and

ii) for LPP, the Drell-Yan cross-section formula, which is essentially W_{μ}^{μ} (cf. Eq. (2)) is subject to very substantial first order corrections.¹ It is natural, therefore, to ask how is Eq. (5) affected by QCD effects in LPP?

If we represent the basic parton-parton amplitude by Fig. 1, then first order QCD corrections come in three forms: the "annihilation" diagrams, Fig. 2; the "Compton" diagrams, Fig. 3; and the "vertex correction" diagrams, Fig. 4. The annihilation and Compton diagrams have been studied by us⁸ and others^{9,10} in connection with high q_{\perp} events for which they should represent the dominant mechanism. It has been noticed⁸ that the parton-model structure function relation, Eq. (5), is also satisfied by these QCD diagrams. Since this result holds for all values of q_{\perp} , it means that these diagrams do not introduce any first order QCD correction to the parton relation Eq. (5) even at low q_{\perp} . We now point out that the remaining first order diagrams, Fig. 4, give rise to an amplitude with the same tensor structure as the zeroth order amplitude of Fig. 1--as any anomalous magnetic moment term from the modified vertex drop out upon taking the trace of the Dirac matrices. It follows then, even though the overall all normalization may be corrected by some factor, the decomposition into invariant structure functions is not affected and Eq. (5) remains intact.

We conclude, therefore, in LPP the parton model relation $W_{\mu}^{\mu} = 2 W_1$ is not modified by first order QCD corrections at all--in contrast to both the Callan-Gross relation (for DIS) and the Drell-Yan cross-section formula (essentially W_{μ}^{μ} , cf. Eq. (2)). This appears to be a rather remarkable result; we are not aware of any other parton model result which is not affected by QCD corrections. For this reason, we sketch in Appendix I a derivation of Eq. (5) from the diagrams Figs. 1 - 4 which is more direct than those given before.^{8,9,10} It will be interesting to find out to what extent can this result be extended to higher order diagrams.

We now make a few additional remarks on Eq. (5) and its experimental implications.

i) The relation $W_{\mu}^{\mu} = 2 W_1$ is closely related to the spin 1/2 nature of the charged parton (i.e., quark). The fact that this relation is not modified by first order QCD effects when both sides themselves may be subject to corrections of order 100% certainly represents a unique signature of the QCD-quark-parton picture.

ii) In terms of helicity structure functions, this relation takes the form $W_L = 2 W_{\Delta\Delta}$, Eq. (7). Although for LPP, the helicity structure functions depend on the choice of coordinate axes⁴ (e.g., Gottfried-Jackson, Collins-SOPER,...etc.), this relation remains frame independent--i.e., if the QCD-quark-parton model is correct the two structure functions W_L and $W_{\Delta\Delta}$ must be related by Eq. (7), for any choice of axes in the lepton pair center of mass frame. This strong result again demonstrates the significance of this relation.

We know the angular distribution of the leptons in the rest frame of

the pair is given by⁴:

$$\frac{d\sigma}{d^4q d\Omega^*} = \frac{1}{2} \frac{1}{(2\pi)^4} \left(\frac{\alpha}{M\mathcal{P}}\right)^2 [W_T(1 + \cos^2\theta^*) + W_L(1 - \cos^2\theta^*) + W_\Delta \sin 2\theta^* \cos \phi^* + W_{\Delta\Delta} \sin^2\theta^* \cos 2\phi^*] \quad (8)$$

Written in the form

$$\frac{d\sigma}{d^4q d\Omega^*} \propto 1 + \alpha \cos^2\theta^* + \beta \sin 2\theta^* \cos \phi^* + \gamma \sin^2\theta^* \cos 2\phi^* \quad (9)$$

Eq. (7) implies

$$1 - \alpha = 4\gamma \quad (10)$$

independent of the choice of axes.

iii) For $q_\perp^2 \ll M^2$, kinematic constraints require $W_{\Delta\Delta}$ to be small⁴ and of order q_\perp^2/M^2 . The dynamical relation Eq. (7) restricts W_L to stay the same as $2W_{\Delta\Delta}$, hence it must also be small. The angular distribution of the leptons should be close to $1 + \cos^2\theta^*$. (W_Δ is required by kinematics to be $O(q_\perp/M)$ and small also. Now, make the transition to the large q_\perp region. There is no reason for $W_{\Delta\Delta}$ to stay small, the angular distribution will be very different from $1 + \cos^2\theta^*$. But Eq. (7) is still in force and W_L must keep pace with the change in $W_{\Delta\Delta}$ as q_\perp increases. Note, although the rate of high q_\perp events is of order $\alpha_s(M^2)$, the predicted change in angular distribution from low to high q_\perp is of order 1 and the relation between W_L and $W_{\Delta\Delta}$ remains precise throughout. In other words, the test of QCD furnished by this relation does not concern just small correction effects; it involves quantities of order unity.

For comparison, in DIS, the Callan-Gross relation reads $W_L = 0$ in the parton limit and acquires a small α_s correction when QCD is taken into account.

Since there is no q_{\perp} variable in DIS, the transition in angular distribution discussed above simply does not exist here.

iv) So far, we have neglected all masses (m_i) and intrinsic transverse momenta ($P_{i\perp}$) of the partons inside the hadron. It is relatively easy to see that incorporating these effects in the zeroth order parton model (Fig. 1) incurs order m^2/M^2 and $p_{i\perp}^2/M^2$ corrections to the basic relation Eq. (5).⁴ In current QCD jargon, these represent "higher twist" effects. In DIS as well as elsewhere, the higher twist effects occur alongside with calculable QCD (twist two) correction terms, thereby complicating the phenomenology considerably. Because of the unique situation here in LPP where first order QCD correction is absent, deviations from Eqs. (5)-(7) can only come from the higher twist effects. Therefore, we have a cleaner source of information on the size of $\langle p_{\perp}^2 \rangle$ ---etc.

Incorporating parton intrinsic transverse momenta in first order QCD diagrams, Figs. 2-4, is much more complicated. However, here the effects are of order $\alpha_s p_{i\perp}^2/M^2$ and represent only a very small perturbation if M^2 is large.

v) One may wonder why Eq. (5) is not subject to first order QCD corrections in LPP while the same relation (i.e., Callan-Gross relation) in DIS is ---- after all, the Feynman diagrams involved in the two cases are identical except for line reversal. The technical explanation for this discrepancy lies in the fact that: in DIS, corrections to the Callen-Gross relation come from Figs. 2 and 3 as a result of integration over the momenta of the two final-state partons; by contrast, in LPP, there is only one parton in the final state--the momentum of which is fixed by conservation, hence, there is no integration involved.

vi) We mention, for completeness, that in addition to Eq. (5), there are other parton model relations⁴ which follow from the Drell-Yan mechanism for low q_{\perp} and from the QCD annihilation diagrams, Fig. 2, for high q_{\perp} .^{8,10} Those relations are interesting in their own right and, in conjunction with Eq. (5), yield definite predictions on the angular distribution of the leptons. However, these predictions do depend on the choice of helicity frames and are subject to QCD corrections, hence are not as striking as Eq. (5).

APPENDIX

In the literature, all structure function relations were derived by first calculating the full tensor amplitude. A more direct derivation of $W_{\mu}^{\mu} = 2W_1$ is given in this Appendix. Feynman gauge is used and partons are assumed to be massless.

In Fig. 1, and Fig. 2(a), the cross section tensor $W_{\mu\nu}$ are respectively proportional to

$$T_{\mu\nu}^{(1)} = \text{Tr}(\gamma_{\mu} \gamma_{P_1} \gamma_{\nu} \gamma_{P_2}) \equiv T_1^{(1)} g_{\mu\nu} + \dots \quad (\text{A1})$$

and

$$T_{\mu\nu}^{(2a)} = \text{Tr}(\gamma_{\mu} \gamma_{P_3} \gamma_{\alpha} \gamma_{P_1} \gamma^{\alpha} \gamma_{P_3} \gamma_{\nu} \gamma_{P_2}) \equiv T_1^{(2a)} g_{\mu\nu} + \dots \quad (\text{A2})$$

Using the identities

$$\gamma_{\mu} \gamma_{\alpha_1} \gamma_{\alpha_2} \dots \gamma_{\alpha_{2n+1}} \gamma^{\mu} = -2 \gamma_{\alpha_{2n+1}} \dots \gamma_{\alpha_2} \gamma_{\alpha_1} \quad (\text{A3})$$

$$\begin{aligned} \text{Tr}(\gamma_{\mu} \gamma_{\alpha_1} \gamma_{\alpha_2} \dots \gamma_{\alpha_{2n+1}} \gamma_{\nu} \gamma_{\beta_1} \gamma_{\beta_2} \dots \gamma_{\beta_{2m+1}}) &= -g_{\mu\nu} \text{Tr}(\gamma_{\alpha_1} \gamma_{\alpha_2} \dots \\ &\gamma_{\alpha_{2n+1}} \gamma_{\beta_1} \gamma_{\beta_2} \dots \gamma_{\beta_{2m+1}}) + \dots \end{aligned} \quad (\text{A4})$$

We immediately obtain the relation $T_{\mu}^{\mu} = 2T_1$ for both Fig. 1 and Fig. 2(a).

Similarly we can derive the relation for Figs. 2(b), 3(a) and 3(b).

The derivatiōn for Figs. 2(c), 2(d), 3(c) and 3(d) are more complicated in that the massless nature of the partons have to be used. We merely exhibit how this is done for Fig. 2(c). There

$$\begin{aligned} T_{\mu\nu}^{(2c)} &= \text{Tr}(\gamma_{\mu} \gamma_{P_1} \gamma_{\alpha} \gamma_{P_4} \gamma_{\nu} \gamma_{P_2} \gamma^{\alpha} \gamma_{P_3}) \\ &= -2 \text{Tr}(\gamma_{\mu} \gamma_{P_1} \gamma_{P_2} \gamma_{\nu} \gamma_{P_4} \gamma_{P_3}) = -2 g_{\mu\nu} \text{Tr}(\gamma_{P_1} \gamma_{P_2} \gamma_{P_4} \gamma_{P_3}) + \dots \end{aligned}$$

Using

$$p_3 = p_2 + p_5, \quad p_4 = p_1 - p_5, \quad p_1^2 = p_2^2 = p_5^2 = 0, \text{ we get}$$

$$\begin{aligned} T_1^{(2c)} &= -8 (p_1 \cdot p_2 p_3 \cdot p_4 - p_1 \cdot p_4 p_2 \cdot p_3 + p_1 \cdot p_3 p_2 \cdot p_4) \\ &= -16 p_1 \cdot p_2 \{p_1 \cdot p_2 - p_2 \cdot p_5 + p_1 \cdot p_5\} = -16 p_1 \cdot p_2 p_3 \cdot p_4 = \frac{1}{2} T_\mu^{(2c)\mu} \end{aligned}$$

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FIGURE CAPTIONS

1. Lowest order diagram for LPP cross section.
2. Order α_s "annihilation diagrams" for LPP cross section.
3. Order α_s "Compton diagrams" for LPP cross section.
4. Order α_s "vertex correction diagrams" for LPP cross section.

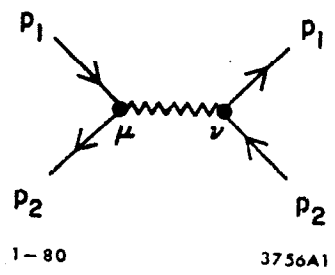


Fig. 1

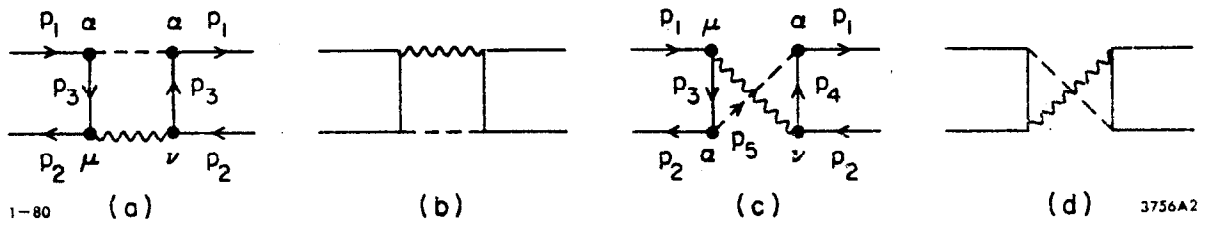


Fig. 2

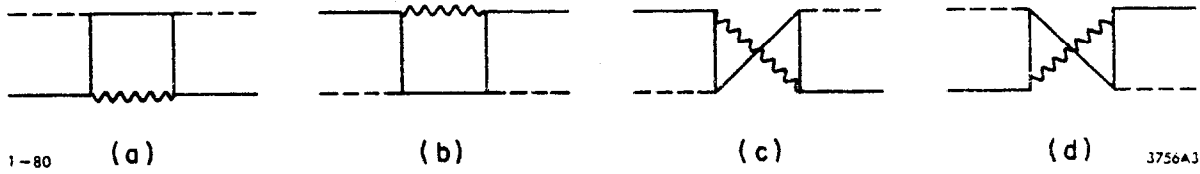
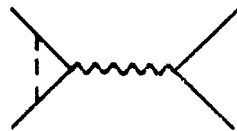
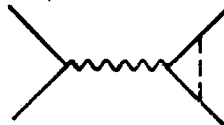


Fig. 3



(a)

1-80



(b)

3756A4

Fig. 4