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WHAT IF THE TOP QUARK HAS MASS M > 19 GeV? *

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Recent results from PETRA show no indication for the production of the top quark up to the energy $\sqrt{s}$ = 31.6 GeV. In the next few months, the modified PETRA will reach $\sqrt{s} \sim 38$ GeV approximately equal to the maximum energy of PEP in its present configuration. Further increases of c. m. energy of either machine requires a substantial increase of RF power and may be prohibitive in both construction costs and power consumption. If threshold for production of the top quark is not reached by 38 GeV, it may be worthwhile to investigate the alternative options for reaching higher energies before the next generation of accelerators becomes operational. In the following, I will summarize the calculations for the system discussed originally by Csonka and Rees,[1] in which the electron beam from the linear accelerator collides with a beam of positrons in the PEP storage ring. Recent work on single pass collider at SLAC has clarified some of the properties of the SLAC e^- beam operating with a single bunch, so that luminosity estimates can be made with increasing confidence. The system can maintain at the same time the normal PEP colliding-beam operation of the storage ring together with high energy collisions in one special area. Hadronic event rates of about one per hour due to the annihilation process appear feasible.

ENERGY RANGE

If positrons are stored in the PEP ring at energy E_1 and the electron beam from the linear accelerator has energy E_2, then the c.m. energy of e^+e^- collisions is given by:

$$\sqrt{s} \simeq 2\sqrt{E_1 E_2} \quad .$$

The dependence of $\sqrt{s}$ on the energy of the linac is shown in Fig. 1.

For a 15 GeV stored positron beam and SLED electrons of 34 GeV, $\sqrt{s}$ = 45 GeV; while for the electrons energy of 45 GeV one can reach $\sqrt{s}$ = 52.5 GeV.

LUMINOSITY

General formalism

Calculations concerning the collisions of the linac beam with stored positrons were made by Csonka and Rees[1] for the case in which both beams are cylindrical at the interaction point. The luminosity of such interactions depends on the number of electrons and positrons in the corresponding bunches. For a high intensity linac beam the luminosity is limited by beam-beam interaction effects which result in a blow-up of the transverse dimension of the stored bunches. The maximum luminosity for $N_1 = N_c$ is given by

$$L_{MAX} = \frac{2\gamma f N_s}{4\pi r_e \beta \left[f \tau \ln (4/3)\right]^{1/2}} \tag{1}$$

and

$$N_c = \frac{2\gamma \sigma_r^2}{r_e \beta \left[f \tau \ln (4/3)\right]^{1/2}} \tag{2}$$

where

r_e = classical radius of electron $\left[2.8 \; 10^{-13} \text{ cm}\right]$

f = linac repetition rate $\left[180 \text{ sec}^{-1}\right]$

N_s = number of stored positrons

τ = transverse normal mode radiation damping time

β = local reduced betatron wavelength

$\gamma = \frac{E}{m}$ for stored positrons

N_1 = number of electrons in the linac pulse

σ_r = standard deviation of the beam distribution

The luminosity is given by the expression

$$L = L_{MAX} \frac{N_c}{2N_1} \left\{ \left[1 + \left(\frac{2N_1}{N_c} \right)^2 \right]^{\frac{1}{2}} - 1 \right\} . \tag{3}$$

For higher intensity of the linac bunch $N_1 > N_c$ the beam of positrons is blown up in transverse dimensions due to beam-beam interactions and may hit the storage ring vacuum pipe.

For the linac beam intensity much smaller than the critical linac bunch population parameter ($N_1 << N_c$) one can neglect the beam disruption effects. The formula for the luminosity is then simply

$$L = \frac{fN_1N_s}{I} \tag{4}$$

where I is the overlap integral of the two-dimensional density distribution of the bunches. The derivation and general form of such an integral is given in the appendix. For circular beams of equal sizes described by the Gaussian density distributions centered at the same x,y point

$$I = \frac{1}{4\pi\sigma_r^2} . \tag{5}$$

Example

Let us assume as an example a system in which only one bunch of positrons and one or two bunches of electrons circulate in the PEP ring and provide e^+e^- collisions in two or four intersection regions. In another intersection region, the stored bunch of positrons may collide with an intense pulse of electrons from the linear accelerator. For the standard PEP configuration and 15 GeV stored beams, the relevant parameters are:[2]

$$N_s = 2.49 \cdot 10^{12}$$

$$\gamma = 3 \cdot 10^4$$

$$\tau = 0.00816 \text{ sec}$$

$$\beta_y = 11 \text{ cm}$$

$$N_1 = 5 \cdot 10^{10}$$

If he can arrange the horizontal dimensions of the beam in one of the intersections to be equal to its vertical dimensions ($\beta_x = \beta_y$) without changing other parameters, then the maximum luminosity of the system is $L_{MAX} = 1.07 \cdot 10^{30}$. The dependence of the critical parameter N_c on the transverse dimension of the round beam is shown in Fig. 2. For $\sigma_r = 0.06$ cm the value of N_c is $N_c = 1.08 \cdot 10^{14}$, while for $\sigma_r = 0.003$ cm $N_c = 2.7 \cdot 10^{11}$. The corresponding luminosities for N_1 electrons in the linac pulse are $L = 4.0 \cdot 10^{26}$ cm^{-2} sec^{-1} and $L = 1.9 \cdot 10^{29}$ cm^{-2} sec^{-1} respectively (see Fig. 3). Since the values of the parameter N_c are much higher than the assumed intensity of the linac beam, the results of the calculations using formula (4) are the same as above.

The resulting luminosity values are also expected[3] not to be significantly different for the case in which the PEP beam has much larger horizontal than vertical dimensions ($\beta_x >> \beta_y$) while the linac beam is round and of the size of the stored beam vertical dimension.

A Few Specific Calculations

From the previous example it is clear that in order to obtain higher luminosity one has to reduce σ_r. Such reduction may be done by various methods but for obvious reasons minimum modifications to the magnets and quadrupoles configuration are preferred. One of such minimal modifications is to reduce the β function at the intersection point while keeping the emittance of the stored beam constant. The standard deviation of the transverse beam distribution is related to the emittance through the relations:

$$\sigma_{x\beta}^2 = \frac{1}{1+k^2} \beta_x E_x ,$$

$$\sigma_{y\beta}^2 = \frac{k^2}{1+k^2} \beta_y E_y ,$$

where k is the x - y coupling coefficient $k \geq 0.1$. The following Table I shows the values of the luminosity obtained when the β of the interaction region is reduced to 1 cm and with both electron and positron bunches circulating in the machine. All the luminosity values may be increased approximately by a factor of two if only the beam of positrons is stored in the PEP ring (since the increased available RF power allows then for doubling the number of stored particles in a single bunch).

Table I		PEP Configuration		
		I	II	III
Energy, GeV	E	15	15	18
Betatron tunes				
Horizontal	ν_x	21.25	21.25	30.76
Vertical	ν_y	18.75	18.75	18.75
Emittance, cm-mrad				
Horizontal	E_x	.0126	.0126	.00479
Vertical	E_y	.0080	.00365	.000063
Number of stored particles	N_s	$2.49 \cdot 10^{12}$	$1.53 \cdot 10^{12}$	$5.1 \cdot 10^{11}$
Beta, cm				
Horizontal	β_x	280	380	340
Vertical	β_y	11	11	11
Beam size, cm				
Horizontal	$\sigma_{x\beta}$	.0595	.0692	.0404
Vertical	$\sigma_{y\beta}$	.003	.002	.00083
Assuming $\beta_x = \beta_y = 1$ cm				
Beam size, cm				
Horizontal	$\sigma_{x\beta}$	.00355	.00354	.00219
Vertical	σ_y	.0009	.0006	.00025
Critical parameter N_c for vertical beam size		$2.6 \cdot 10^{11}$	$1.2 \cdot 10^{11}$	$3.2 \cdot 10^{10}$
Number of electrons in a linac bunch		$5 \cdot 10^{10}$	$5 \cdot 10^{10}$	$5 \cdot 10^{10}$
Luminosity* L, cm^{-2} sec^{-1}		$2.2 \cdot 10^{30}$	$3 \cdot 10^{30}$	$1.0 \cdot 10^{30}$†

*Formula (4) †Formulae (1) - (3)

Further and more complicated modifications of the system are discussed by Rees and Wiedmann[3] who had shown that by reducing the vertical emittance of the PEP beam to $E = 1.6 \cdot 10^{-9}$ meter-radians and using the horizontal-vertical coupling $k = (E_y / E_x)^{1/2} = .17$ it is possible to reach the luminosity $L = 9 \cdot 10^{30}\ cm^{-2}\ sec^{-1}$.

Depending on the shape of the beta function of the stored beam a small increase in the luminosity may be also obtained by allowing two consecutive bunches of the linac beam to collide with the same bunch of positrons at PEP. The collision points would be separated by about 10 cm in space which is easily distinguishable by any track reconstruction program.

CROSS SECTIONS AND RATES

The cross-section estimates for various processes in e^+e^- collisions at $\sqrt{s}$ = 40 and 50 GeV are summarized in the following table.

Table 2

Process	$\sqrt{s}$ = 40 GeV	$\sqrt{s}$ = 50 GeV
$\sigma_{\mu\mu}(e^+e^- \to \gamma \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s} \approx \frac{87}{s[GeV^2]}$ nb *	.0544 nb	.0348 nb
$\sigma_{AN}(e^+e^- \to \gamma \to \text{hadrons}) \approx R \cdot \sigma_{\mu\mu}$	~.32 nb	~.25 nb
$\sigma_{WEAK} \sim \frac{G^2 s}{\pi} = 1.3 \cdot 10^{-38}\ s[GeV^2]\ cm^2$ *	.0208 nb	.0325 nb
$\sigma_{2\gamma}(e^+e^- \to e^+e^- X) = \frac{\sigma_{\gamma\gamma \to x}}{2s} N_1 N_2 f\left(\frac{s}{so}\right)$ †	~200-400 nb	~250-500 nb
*Ref. 4 †Ref. 5		

For the annihilation process the luminosity of 10^{30} cm^{-2} sec^{-1} would result in about one hadronic event per hour. At the same time there will be about 1000 hadronic events per hour due to the two-photon processes. Due to the energy spectrum of the photons, these events will not constitute any serious background to the annihilation events and may be clearly separated out by tagging of the final state electrons and by the low visible energy of the hadronic secondaries.

SUMMARY

The calculations of the luminosity in the previous sections show that it is possible to attain the luminosity of a few times 10^{30} cm^{-2} sec^{-1} for the collisions of the SLED beam of electrons with stored positrons in PEP. The efficiencies of the data taking would probably scale down the luminosity by the usual factor of $\sim 1/3$ to the effective value of about $L = 1 \cdot 10^{30}$ cm^{-2} sec^{-1}. A few hundred hours of running at the top energy would be, therefore, sufficient to establish whether the threshold for the top quark had been passed.

A possible experimental design may allow for the continuation of two or four other experiments at PEP in other intersections. It would require a construction of a beam transport line to bring the electrons from the linac to one of the PEP intersection regions, some modifications of the storage ring in the interaction region in order to reduce the β at the intersection point, and a beam dump for the linac beam after the collision. On the other hand, this design could use any of the existing large solid angle detectors without any major modification.

REFERENCES

1. P. L. Csonka and J. R. Rees, Nucl. Instr. and Methods 96, 149 (1971).
2. PEP Design Handvook, H. Wiedman, editor, 1977.
3. J. R. Rees and H. Wiedmann, PEP-Note 324 (1979).
4. J. Ellis and M. K. Gaillard, "Physics with Very High Energy e^+e^- Colliding Beams," CERN 76-18.
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APPENDIX

Overlap Integral of the Two-Dimensional Density Distribution of Two Beams

Assume beams with the density distribution of the form of two-dimensional Gaussian function described by their standard deviations σ of the form

$$\begin{vmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{vmatrix} \quad .$$

$$I(x,y) = N\, e^{-\frac{(x-x^1)^2 \sigma_y^2 - 2(x-x^1)(y-y^1)\sigma_{xy} + (y-y^1)^2 \sigma_x^2}{2D}}$$

where x^1, y^1 define the center of the distribution,

$$D = \sigma_x^2 \sigma_y^2 - \sigma_{xy}^2$$

and the normalization factor N is provided by the integral:

$$\int_{-\infty}^{+\infty} dy \int_{-\infty}^{+\infty} dx \cdot I(x,y) = 1 \quad .$$

After integration the normalization factor is:

$$N = \frac{1}{2\pi\sqrt{D}} = \frac{1}{2\pi\sqrt{\sigma_x^2 \sigma_y^2 - \sigma_{xy}^2}} \quad .$$

The constants x^1 and y^1 represent the offset of the center position of the Gaussian distribution.

<u>The overlap integral</u>

$$I = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} dx\, dy\; I_1(x,y)\; I_2(x,y)$$

$$= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} dx\, dy \exp\left\{ -\left[\frac{(x-x_1)^2 \sigma_{1y}^2 - 2(x-x_1)(y-y_1)\sigma_{1xy} + (y-y_1)^2 \sigma_{1x}^2}{2D_1} + \frac{(x-x_2)^2 \sigma_{2y}^2 - 2(x-x_2)(y-y_2)\sigma_{2xy} + (y-y_2)^2 \sigma_{2x}^2}{2D_1} \right] \right\}$$

where

$$N_1 = \frac{1}{2\pi\sqrt{D_1}} \quad , \qquad N_2 = \frac{1}{2\pi\sqrt{D_2}}$$

$$D_1 = \sigma_{1x}^2\, \sigma_{1y}^2 - \sigma_{xy}^2 \quad , \qquad D_2 = \sigma_{2x}^2\, \sigma_{2y}^2 - \sigma_{2xy}^2$$

The solution is given by:

$$\boxed{I = \frac{1}{2\pi\sqrt{D_1 + D_2 + \sigma_{2y}^2\, \sigma_{1x}^2 + \sigma_{2x}^2\, \sigma_{1y}^2 - 2\sigma_{1xy}\, \sigma_{2xy}}}\; e^{-\frac{R}{2D_1D_2}}}$$

where

$$R = f - \frac{b^2}{a} - \frac{\left(e + \frac{bc}{a}\right)^2}{d - \frac{c^2}{a}}$$

$$a = D_2\sigma_{1y}^2 + D_1\sigma_{2y}^2$$

$$b = x_1D_2\sigma_{1y}^2 + x_2D_1\sigma_{2y}^2 - y_1D_2\sigma_{1xy} - y_2D_1\sigma_{2xy}$$

$$c = D_2\sigma_{1xy} + D_1\sigma_{2xy}$$

$$d = D_2\sigma_{1x}^2 + D_1\sigma_{2x}^2$$

$$e = y_1D_2\sigma_{2x}^2 + y_2D_1\sigma_{2x}^2 - x_1D_2\sigma_{1xy} - x_2D_1\sigma_{2xy}$$

$$f = x_1^2D_2\sigma_{1y}^2 - 2x_1y_1D_2\sigma_{1xy} + y_1^2D_2\sigma_{1x}^2 + x_2^2D_1\sigma_{2y}^2 - 2x_2y_2D_1\sigma_{2xy} + y_2^2D_1\sigma_{2x}^2$$

When the centers of the beams coincide, i.e., $x_1 = x_2$ and $y_1 = y_2$, then the value of parameter R is equal to zero.

Special cases

(When centers of the beams coincide, i.e., $x_1 = x_2$ and $u_1 = y_2$.)

1. Beams circular and equal sizes

$$\sigma_{1x}^2 = \sigma_{1y}^2 = \sigma_{2x}^2 = \sigma_{2y}^2 = \sigma^2 , \quad \sigma_{1xy} = \sigma_{2xy} = 0$$

$$\boxed{I = \frac{1}{4\pi\sigma^2}}$$

2. Beams equal sizes, ellipsoidal, uncorrelated

$$\sigma_{1x}^2 = \sigma_{2x}^2 = \sigma_x^2$$

$$\sigma_{1y}^2 = \sigma_{2y}^2 = \sigma_y^2$$

$$\sigma_{1xy} = \sigma_{2xy} = 0$$

$$\boxed{I = \frac{1}{4\pi\sigma_x\sigma_y}}$$

3. One beam ellipsoidal, one circular, uncorrelated

$$\boxed{I = \frac{1}{2\pi\sqrt{\sigma_x^2\,\sigma_y^2 + \sigma^2(\sigma^2 + \sigma_x^2 + \sigma_y^2)}}}$$

In every case, the relative displacement of the centers of the beam introduces the decrease of the luminosity, controlled by the exponent of the general solution (see Fig. 4).

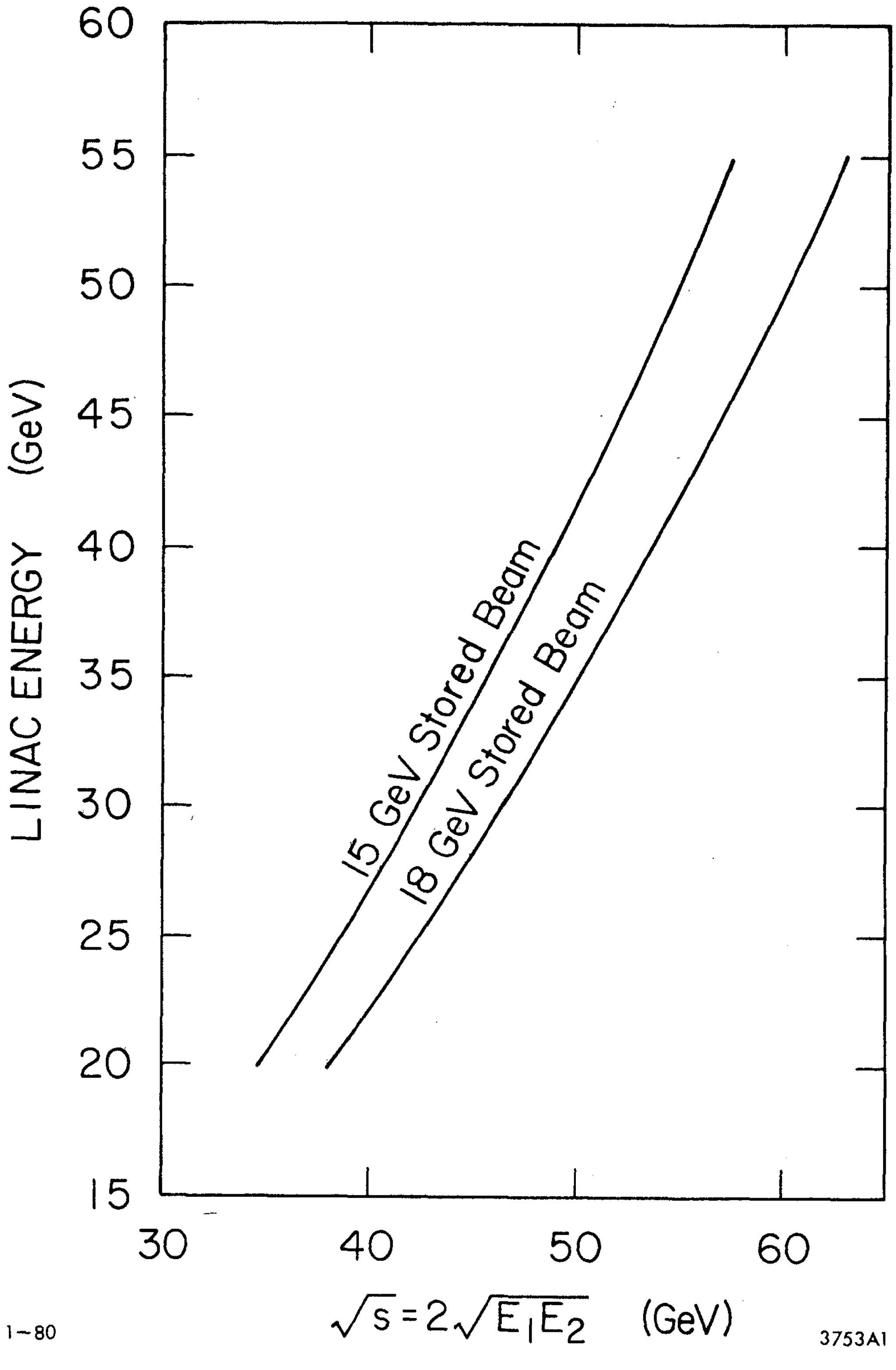
FIGURE CAPTIONS

1. The $\sqrt{s}$ dependence on the energy of the linac beam for the 15 and 18 GeV beam stored in the PEP ring.

2. Dependence of the critical parameter N_c on the radial dimensions of the beams for the case discussed in Ref. 1.

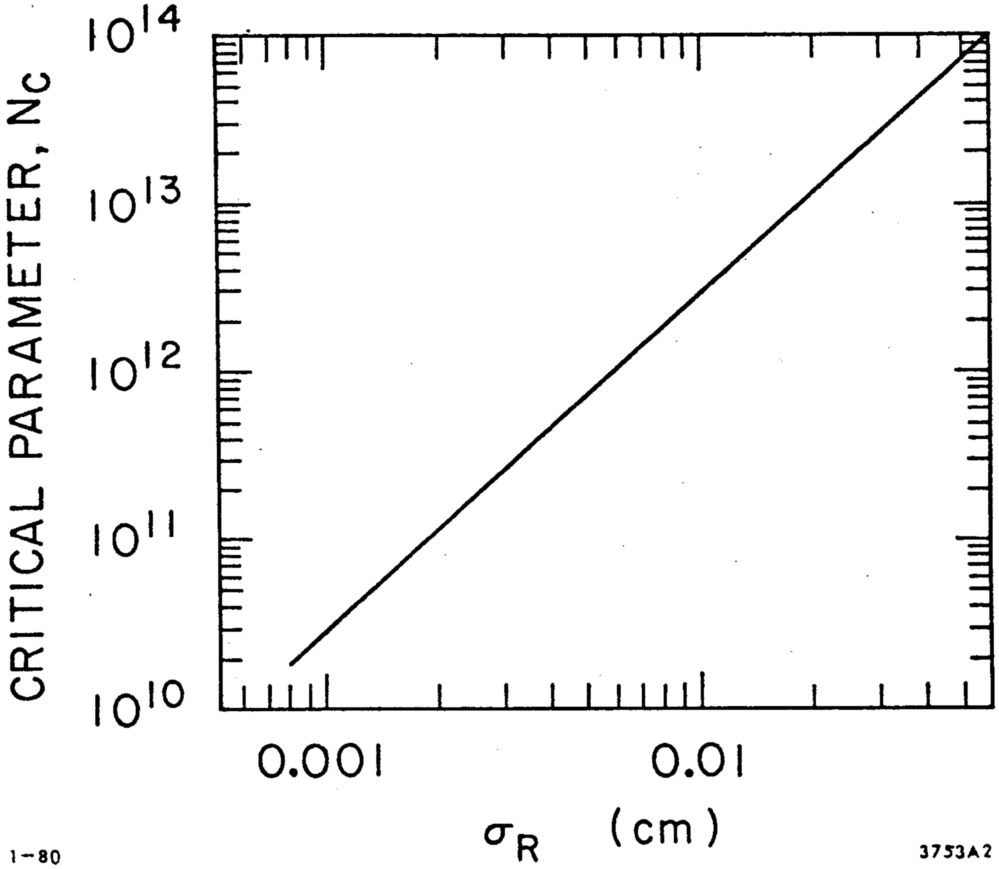
3. Luminosity dependence on the radial dimensions of the beams for the three values of the number of electrons in the linac pulse.

4. Fractional drop in the luminosity due to misplacement of the centers of the colliding beams.



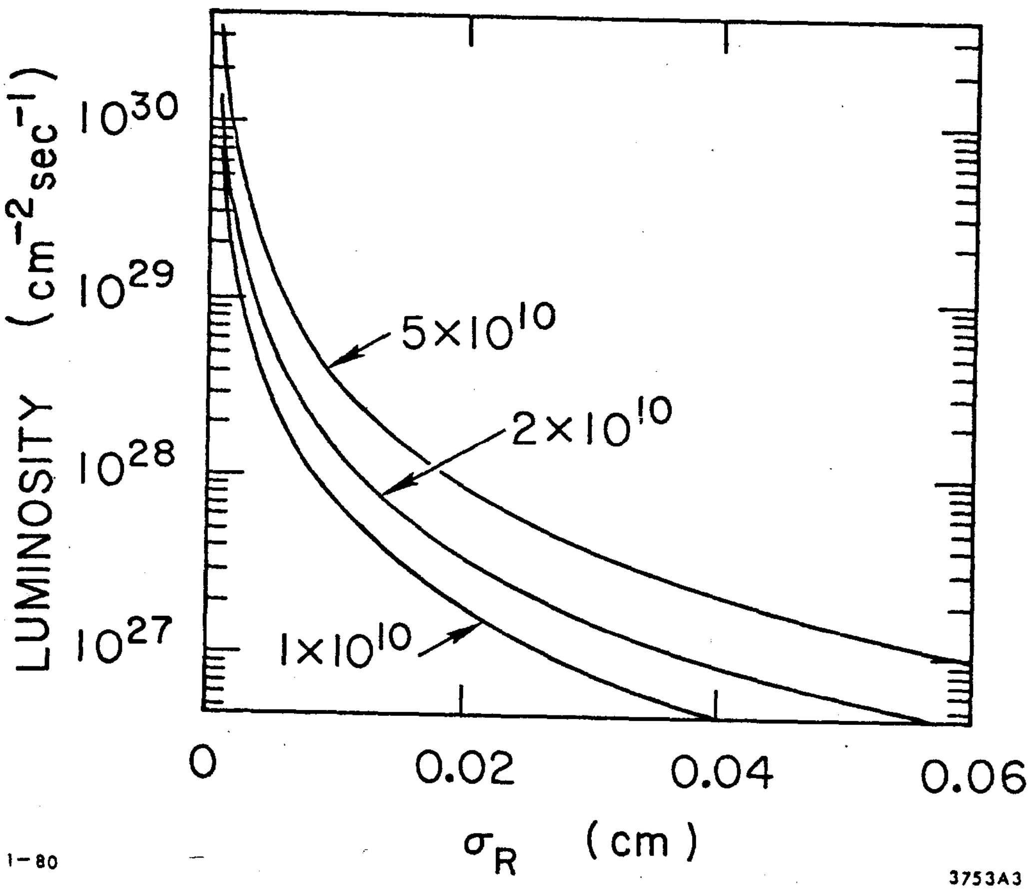
LINAC ENERGY (GeV)
60
55
50
45
40
35
30
25
20
15
30
40
50
60
$\sqrt{s} = 2\sqrt{E_1 E_2}$ (GeV)
15 GeV Stored Beam
18 GeV Stored Beam
1-80
3753A1

Fig. 1



CRITICAL PARAMETER, N_C
10^14
10^13
10^12
10^11
10^10
0.001
0.01
σ_R (cm)
1-80
3753A2

Fig. 2



LUMINOSITY (cm⁻² sec⁻¹)
10^30
10^29
10^28
10^27
5×10^10
2×10^10
1×10^10
0
0.02
0.04
0.06
σ_R (cm)
1-80
3753A3

Fig 3

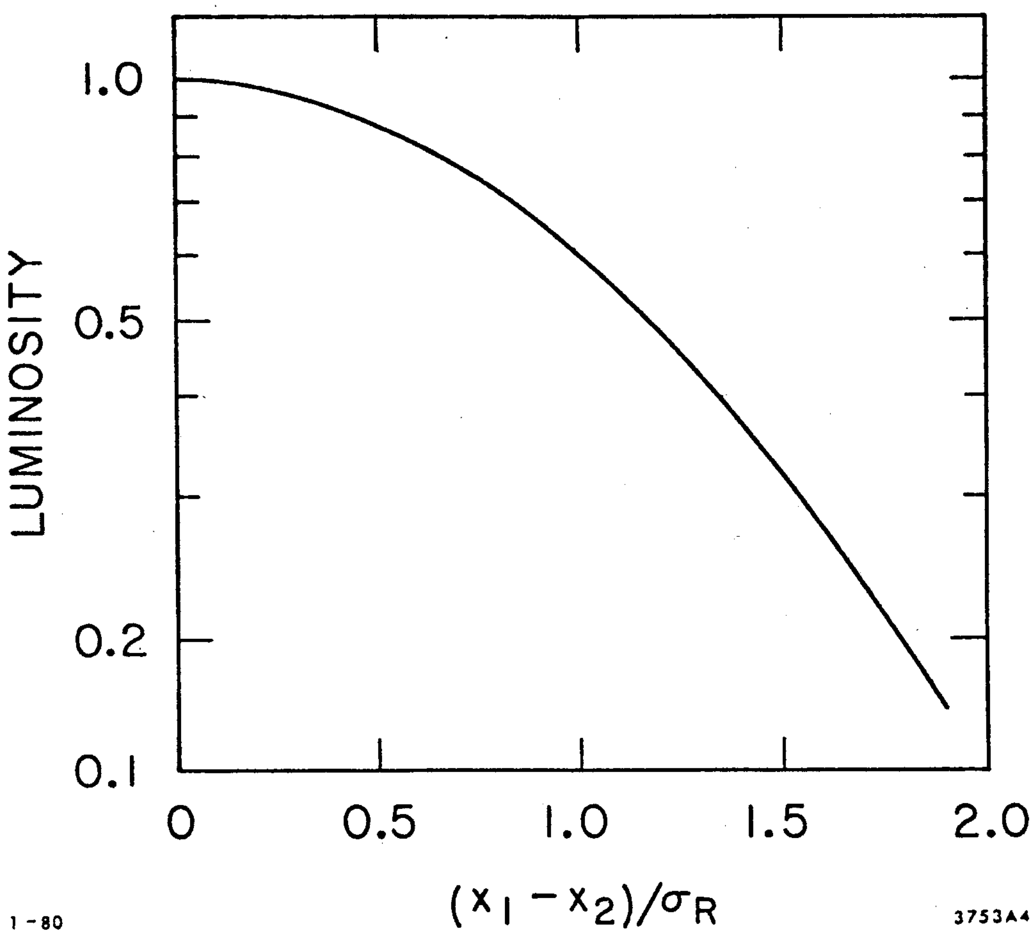


Fig. 4