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HEAVY LEPTONS*

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ABSTRACT

Experimental and theoretical aspects of heavy leptons are reviewed.

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I. Introduction

After muon was discovered in 1937 by Neddermeyer and Anderson[1] and Street and Stevenson[2] one of the greatest puzzles in physics was why muon should exist. One can understand why the electron should exist because it is responsible for all the chemical reactions and thus is also responsible for the existence of life. Pions are responsible for binding the nucleus together. Nobody could think of any good reason why μ should exist. The only use of muons I can think of is to make the π lifetime shorter thus making cosmic rays less lethal. Since muons exist in nature for no apparent reason, one expects that other heavy lepton might also exist in nature. All early theoretical works[3,4] up to 1971 seem to treat a heavy lepton as sequential, namely, it has its own leptonic number (L-ness) different from that of e (e-ness) or μ (μ-ness) and its own massless neutrino ν_L different of ν_e or ν_μ; furthermore, the charged current $(L^\pm \nu_L)$ is coupled to the same $W^\pm$ boson which is coupled to all other known charged weak currents; and the coupling is V-A with magnitude equal to that between W and $(e^\pm \nu_e)$. With this assumption the decay widths into various channels were calculated[3,4] and the energy-angle distributions of decay products of L^+ and L^- in the e^+e^- collisions were investigated[4] in great detail theoretically in 1971 which proved to be of great use in the design and analysis of the experiments later. The models of heavy leptons which were nonsequential were later proposed by Georgi and Glashow[5] (1972) and by Bjorken and Llellewyn Smith[6] (1973). Even though later experiments showed that the τ lepton was a sequential type, these latter investigations served to focus attention on various different possibilities and the ways to distinguish among different models.

Before the discovery of the anomalous μe events in the e^+e^- collision in 1975 several groups attempted to search for a heavy lepton. In 1963 D. H. Coward et al.[7] used the 1 GeV Stanford Electron Accelerator to search for pair produced charged particles[8] whose mass is less than the muon mass. They ruled out the existence of any but very short-lived particles in the range from 1 to 175 Me. In 1968 A. Barna et al.[9] used the 18 GeV electron beam from SLAC to look for pair produced charged particles. They did not find τ because the cross section is too small[8] and the lifetime is too short.[4] In 1970 V. Alles-Borelli[9] searched for the anomalous μe events using the e^+e^- colliding beam ring (800 to 1000 MeV each beam) at Frascati. They did not find any and concluded that heavy lepton with mass less than 780 MeV does not exist. In 1972 a beam-dump experiment was carried out[11] at SLAC by D. Dorfan, D. Fryberger, J. M. Gaillard, D. Kreinick, A. Mann, A. Rothenberg, M. Schwartz, and T. Zipf. In principle τ pair could have been produced by photons and these τ pair would decay quickly producing ν_τ and $\bar{\nu}_\tau$, which would travel through 55 meters of dirt and reach the neutrino detector. They did not find any because not enough τ's could have been produced by 20 GeV electrons for this particular experiment.

In 1975 M. Perl et al.[12] made the original observation of anomalous $e\mu$ events of the form

$$e^+ + e^- \to e^\pm + \mu^\mp + \geq 2 \text{ undetected particles.} \qquad (1.1)$$

In order to preserve the conservation of the e-ness and the μ-ness quantum numbers ν_e and ν_μ must be contained in "≥ 2 undetected particles" in (1.1). Thus these events must be due to the weak decay of a pair of new particles such as a pair of heavy leptons $L^\pm$

$$e^+ + e^- \to L^+ + L^- \qquad \begin{cases} L^+ \to e^+ + \nu_e + \bar{\nu}_L \\ L^- \to \mu^- + \nu_\mu + \nu_L \end{cases} \tag{1.2}$$

or a pair of vector bosons

$$e^+ + e^- \to W^+ + W^- \qquad \begin{cases} W^+ \to e^+ + \nu_e \\ W^- \to \mu^- + \bar{\nu}_\mu \end{cases} \tag{1.3}$$

The possibility of the parent particles being a pair of spin 0 particles was ruled out because the ratio of e + hadrons to μ + hadrons events is very close to unity and definitely not $(M_e/M_\mu)^2$ which would be the case if the parents were spin 0 particles.

In their 1976 paper Perl et al. ruled out the two body decay mode, Eq. (1.3), using the energy distribution of e and μ in (1.1). The energy distribution of e from the two body decay $W \to e + \nu_e$ is a δ function in the rest frame of W but has a flat top in the moving frame of W, whereas in the three body decay $L \to \nu_L + e + \nu_e$ it has a round top whose exact shape depends upon the dynamics (such as V+A or V-A; see Sec. II) and the ends points are determined by the mass of ν_L

$$E_e^{\substack{max \\ min}} = \frac{M_L^2 - M_{\nu_L}^2}{2\left[E_L \mp \left(E_L^2 - M_L^2\right)^{1/2}\right]} \tag{1.4}$$

which can be used to determine the mass of M_{ν_L}. For completeness the energy distribution of e from the semileptonic decay of charmed particle $D^{\pm} \to$ hadron + e + ν_e has also a round top and the end points are also given by Eq. (1.4) except now M_{ν_L} is replaced by the hadronic mass in the final state and M_L is replaced by M_D. By 1977 Perl et al. were able to show that the τ particle is consistent with being a spin $\frac{1}{2}$ lepton with its own leptonic number and its own associated neutrino ν_τ whose mass is less than 0.6 GeV; V-A coupling is favored over V+A for the (τ, ν_τ) current; the leptonic branching ratios are 0.186 ± 0.018 ± 0.028 from the eμ events and 0.175 ± 0.027 ± 0.030 from the μX events where the first error is statistical and the second is systematic; and the mass is 1.90 ± 0.10 GeV. Also by 1977 PLUTO[15] and DASP[16] Groups at DESY also saw the evidence of τ. The semileptonic decay modes such as $\tau \to \nu_\tau + \rho$, $\tau \to \nu_\tau + A_1$ and $\tau \to \nu_\tau$ + hadron inclusive were also observed and found to agree with theoretical calculations. At 1977 Hamburg conference DASP Group[16] reported that the very crucial mode $\tau \to \nu_\tau + \pi$ was missing, the branching ratio being $B_\pi = .02 \pm .025$ instead of the theoretical value of $B_\pi = .10$ (see Table 1). The problem of B_π was clarified a year later by SLAC-LBL Group, DELCO Group, DASP Group and PLUTO Group at the 1978 Tokyo conference.[17]

During 1978, three years after the discovery of the anomalous μe events, the τ finally became accepted even by the most hardheaded doubters. The work of the DELCO Group[18] resolved many of the remaining doubts about the existence of τ and showed that it is indeed a spin $\frac{1}{2}$ lepton, having its own lepton number "τ-ness" and associated neutrino ν_τ and furthermore the $\tau\nu_\tau W$ vertex is V-A where the charged weak vector

Table 1. Branching Ratios (in %) of Heavy Lepton (Sequential)

Decay Mode \ M_L (GeV)	1.8	8.0	10.0	12.0	14.0
$\nu_L + \nu_e + e^-$	17.62	11.20	10.85	10.72	10.57
$\nu_L + \nu_\mu + \mu^-$	17.15	11.20	10.85	10.72	10.57
$\nu_L + \nu_\tau + \tau^-$	0	7.40	8.29	8.87	9.20
$\nu_L + \pi^-$	10.52	0.34	0.21	0.15	0.10
$\nu_L + K^-$	0.66	0.03	0.02	0.01	0.01
$\nu_L + \rho^-$	21.45	0.75	0.47	0.32	0.23
$\nu_L + K^{*-}$	1.46	0.06	0.04	0.02	0.02
$\nu_L + A_1^-$	8.71	0.39	0.24	0.17	0.12
$\nu_L + Q^-$	0.38	0.03	0.02	0.01	0.01
$\nu_L + \bar{u}d > 1.1$ GeV	20.55	34.10	32.91	32.66	32.41
$\nu_L' + \bar{u}s > 1.1$ GeV	1.49	2.50	2.41	2.41	2.36
$\nu_L + \bar{c}s > 2.$ GeV	0	30.80	31.01	31.28	31.38
$\nu_L + \bar{c}d > 2$ GeV	0	2.24	2.28	2.29	2.28
Total Rate in 10^{10} sec^{-1}	368	1.00×10^6	3.16×10^6	7.96×10^6	1.74×10^7

boson $W^{\pm}$ is the same one which couples to all other known charged weak currents.

We have described the brief history of τ from the initial discovery of μe anomally in e^+e^- collision to its final confirmation. As we have seen above, it took slightly over three years of concentrated effort by four large experimental groups (LBL-SLAC, DELCO, DASP and PLUTO) to establish the existence and investigate the properties of τ after its initial discovery. The reasons for taking such a long time are 1) the τ lifetime is too short (theoretical estimate 2.6×10^{-13} sec) to be seen directly; 2) its decay products always contain at least one neutrino ν_τ, thus making it impossible to be identified through a bump in the invariant mass of the decay product; and 3) the production and the subsequent decay of a pair of τ have several unique characteristics which were thoroughly investigated theoretically[3-6] long before 1975 and all these unique characteristics must be (they were!) present in order to establish the τ's identity.

II. Properties of τ

In this section we summarize the theoretical arguments and experimental evidence concerning the properties of τ. Part of the contents of this section can be found in my unpublished note[19] SLAC-PUB-2105 (April 1978).

A. Why is τ a spin $\frac{1}{2}$ lepton?

In Ref. 19 a chain of argument is given to show that τ is not only consistent with being a spin $\frac{1}{2}$ lepton but one can deduce from the available experimental evidence that it cannot be anything else. The argument goes as follows:

i) τ cannot be a baryon. If τ were a baryon its decay products would contain one nucleon. The missing neutral in the decay $\tau \rightarrow e_\nu$ + neutral has a mass upper limit[18] of .25 GeV. Hence "neutral" cannot contain a nucleon and thus τ cannot be a baryon. (We assume baryon number conservation.)

ii) τ cannot be a boson. A boson and its antiparticle have the same parity. Since the virtual photon has quantum number $J^P = 1^-$, the orbital angular momentum of the boson-antiboson pair cannot be in the s state. Experimental results[18] clearly show the s-wave threshold behavior of τ events. Two bosons can be produced in the s state only if they have opposite parity; for example, 0^-+1^+, 0^++1^-, 1^-+1^+, etc. Only one of the two particles produced can be stable against strong and electromagnetic interactions. (For example in the production of $D+D^*$ only D is stable.) This means that if τ events were due to the production of two bosons with opposite parity, their decay would <u>always</u> be accompanied by γ's and π^0's. Experimentally this seems to have been ruled out.[12]

Now if a particle is neither a baryon nor a boson it can only be a non-strongly interacting particle with a half integer spin $\frac{1}{2}, \frac{3}{2}, \frac{5}{2} \ldots$ because no hadron with half integer spin and no baryonic number exists in nature. Thus in order to show that τ is a spin $\frac{1}{2}$ lepton we need to show only that it is not a non-hadron with spin $\geq \frac{3}{2}$.

iii) τ cannot be a point-like particle with spin equal to or greater than $\frac{3}{2}$. The s-dependence of the cross section for high-spin particle production is at least two powers of s more divergent than that for spin $\frac{1}{2}$ particles when the energy is far above the threshold. The energy dependence of the τ production cross section excludes such a steep energy dependence. (See Fig. 1 and Fig. 2.)

In Fig. 1, the experimental result[18] of the DELCO Group for $e^+ + e^- \rightarrow e +$ one charged prong ($\neq e$) + no detected photons is plotted. The quantity plotted is the ratio R which is the cross section of interest divided by the muon-pair production cross section. The solid line is $0.11 \times R_\tau(\text{spin } \frac{1}{2})$, where $R_\tau(\text{spin } \frac{1}{2})$ is the ratio of the spin $\frac{1}{2}$ heavy lepton cross section to the muon cross section:

$$R_\tau(\text{spin } \tfrac{1}{2}) = \sigma_\tau/\sigma_\mu = \beta \, \frac{3-\beta^2}{2} \tag{2.1}$$

The factor 0.13 comes from a theoretical estimate of the branching ratios (see Table 1):

$$\begin{aligned} &2 \times \left(B_\mu + B_\pi + B_K + \tfrac{1}{3} B_\rho\right) \times B_e \\ &\quad = 2 \times (0.17 + 0.11 + 0.007 + 0.21/3) \times 0.176 = 0.13 \end{aligned} \tag{2.2}$$

The factor $\frac{1}{3}$ in front of B_ρ comes from the fact that the probability of missing the π^o in the decay $\rho^- \rightarrow \pi^- + \pi^o$ is $\frac{1}{3}$ for the detector used. We observe that the theoretical curve has the right shape and magnitude. The correct shape implies that τ is a spin $\frac{1}{2}$ particle with unit form factor and no anomalous magnetic moment, whereas the correct magnitude implies that the assumptions made in the calculation of branching ratios are right. The decay branching ratios given in Table 1 are the updated version of a similar table published in my 1971 paper (see Section III).

In Fig. 2 four curves are plotted, each representing the production cross section (divided by the muon cross section) of a point-like particle of a particular spin. The curve labeled $s = \frac{1}{2}$ represents R for the production of spin $\frac{1}{2}$ particles with no anomalous magnetic moment (Eq. (2.1)). The curve labeled s=1, κ=0 represents R for a spin 1 particle with no anomalous quadrupole moment and gyromagnetic ratio equal to one.[20] This corresponds to $\kappa=0$, $\lambda=0$ in the notation of Ref. 20. $R_\tau(\text{spin } 1, \kappa=0) = \beta^3(0.75 + \gamma^2)$. This choice of κ and λ gives the least divergent asymptotic behavior. Any other choice of κ and λ values will yield an asymptotic s dependence equal to that of the next case. The curve labeled s=1, κ=1 represents R for a spin 1 particle with no anomalous quadrupole moment and a gyromagnetic ratio equal to two. Yang-Mills particles have this property. This corresponds to $\kappa=1$ and $\lambda=0$ in the notation of Ref. 20. $R_\tau(\text{spin } 1, \kappa=1) = \beta^3(0.75 + 5\gamma^2 + \gamma^4)$. This has a p wave threshold behavior and asymptotic behavior $\propto s^2$. The curve labeled $s=\frac{3}{2}$, A=1, B=C=D=0 represents R for a spin $\frac{3}{2}$ particle with the least divergent asymptotic behavior. A spin $\frac{3}{2}$ particle can have four multipoles,[21] thus we need four arbitrary numbers to describe its electromagnetic interactions. Let us write the vertex function as

$$\langle P_1|J_\mu(o)|P_2\rangle = \bar{U}_\alpha(P_1)\Big\{g_{\alpha\beta}\Big[A\gamma_\mu + B(P_1 - P_2)_\mu\Big] + P_{1\beta}P_{2\alpha}\Big[C\gamma_\mu + D(P_1 - P_2)_\mu\Big]\Big\}v_\beta(P_2)$$

where A, B, C, D are four constants related to the four multipoles. U_α and v_β are the vector-spinors of Rarita and Schwinger[22] representing spin $\frac{3}{2}$ particle and antiparticle, respectively. The general expression for the cross section is rather long. However the results of the calculation show that the choice B=C=D=0 yields the least divergent result when s is large. Letting A=1, which corresponds to unit charge, we obtain

$$R_\tau(\text{spin } \tfrac{3}{2}) = (\beta/9)(15\gamma^{-2} + 30\beta^2 + 40\gamma^2\beta^4 + 16\gamma^4\beta^6) \quad ,$$

where β and γ are velocity and E/M of τ. Since a spin $\frac{3}{2}$ particle is a fermion it has an s wave threshold behavior. The asymptotic behavior is the same as in the spin 1 case. This is to be expected because a spin $\frac{3}{2}$ particle can be regarded as a direct product of spin $\frac{1}{2}$ and spin 1 states, hence its asymptotic behavior must be at least as divergent as that of spin 1 state. This argument can be generalized to higher spins. From Fig. 1 and Fig. 2 we conclude that τ events cannot be due to the decay of high spin ($s \geq \frac{3}{2}$) point-like fermions. Thus we conclude by elimination that τ must be a spin $\frac{1}{2}$ particle with no baryonic number and hence it must be a heavy lepton.

Before 1978, the most serious doubt about the τ events was that they might be from the semileptonic decay of some charmed state. This could be ruled out from argument (i) and (ii) given above but there are more direct evidence against such possibility. Both DASP and DELCO Groups have shown that τ events also occur below the threshold of D; the total hadronic cross section has many bumps and valleys, but the τ events do not fluctuate with σ_{total}; there is the observation made by G. Feldman[13] who investigated k^o's accompanying μe events and found that the μe events could not all have been due to the decay of D particles.

B. <u>$\tau^\pm$ is a sequential lepton having its own leptonic number different from that of $e^\pm$, $e^\mp$, $\mu^\pm$ or $\mu^\mp$</u>

1. τ^- cannot have the same leptonic number as that of e^-, otherwise the electromagnetic decay $\tau^- \to e^- + \gamma$ will be order $\alpha^{-1}(M_W/M_\tau)^4 \sim 10^8$ greater than the weak decay modes $\tau^- \to \nu_\tau + \mu^- + \nu_\mu$ and $\tau^- \to \nu_\tau$ + hadrons,

and the result is the μe events or μ + hadron events in the decay of a pair of τ's could not have been seen. Similar argument can be used to rule out τ^- having the same leptonic number as that of μ^-.

2. τ^- cannot have the same leptonic number as that of e^+. If τ^- and e^+ share the same leptonic number, then ν_τ would be identical to $\bar{\nu}_e$ and the rate of $\tau^- \to \nu_\tau + e^- + \bar{\nu}_e$ would be twice[6] that of $\tau^- \to \nu_\tau + \mu^- + \bar{\nu}_\mu$, hence $e^+ + e^- \to e$ + hadron would have twice the rate of $e^+ + e^- \to \mu$ + hadron. Experimentally these two reactions have roughly the same rate, which rules out also τ^- to have the same leptonic number as that of μ^+ as well as that of e^+. The factor of two mentioned above is true only if the detector has 100% detection efficiency. Since energy angle distributions in two cases are different, some correction must be made when the detection efficiency is less than 100%. This subject is treated in detail in Ref. 23.

C. V-A vs V+A

The best evidence for the current (τ^-, ν_τ) being V-A is given by the DELCO Group[18] as shown in Fig. 3.

Ignoring the mass of neutrinos and the radiative corrections, the energy distribution of an electron from the decay of a heavy lepton can be conveniently written in terms of Michel parameter ρ ($x = p/p_{max}$ in the rest system of τ)

$$\frac{d\Gamma}{\Gamma dx} = 4x^2 \left\{ 3(1-x) + 2\rho \left(\frac{4x}{3} - 1 \right) \right\} \qquad (2.3)$$

where ρ can be written in terms[23] of g_L and g_R representing the coupling constants to V-A and V+A currents respectively:

$$\rho = \frac{3}{4} \frac{g_L^2}{g_L^2 + g_R^2} \quad . \tag{2.4}$$

If the current is V-A, we have $g_L = 1$ and $g_R = 0$, hence $\rho = \frac{3}{4}$; if the current is V+A, we have $g_L = 0$ and $g_R = 1$, hence $\rho = 0$; if the current is either pure V or pure A we have $g_L = g_R$, hence $\rho = \frac{3}{8}$. We notice in Eq. (2.3) that the larger ρ is the harder the spectrum. (See Fig. 4.) When the radiative corrections to the spectrum are applied, the high energy component is depleted and the low energy component is increased thus effectively reducing ρ.

D. Mass of ν_τ

M_{ν_τ} can in principle be measured by using any of the decay channels whose final state can be calculated and measured reliably, such as $\tau \to \nu_\tau + \mu + \nu_\mu$, $\nu_\tau + e + \nu_e$, $\nu_\tau + \rho$, $\nu_\tau + \pi$, $\cdots$. The effect of finite mass of ν_τ is also to deplete the high energy component of particle spectrum and increase the low energy component. The effect is similar to the radiative corrections. The DELCO Group[18] used the channel $\tau \to \nu_\tau + e + \nu_e$ and obtained an upper limit $M_{\tau_\nu} < 250$ MeV. The formula for the spectrum of $\tau \to \nu_\tau + e + \nu_e$ with finite M_{ν_τ} and arbitrary combination of V and A can be found in Ref. 23.

E. How many kind of $W^\pm$'s, spin of ν_τ and the existence of charged Higg's boson

In our calculation[4] of decay rates to different channels we have assumed, beside V-A, $M_{\tau_\nu} = 0$ and sequential lepton, the following:

1. All charged currents in the weak interactions are mediated by one kind of $W^{\pm}$, also there is no charged Higg's particle mediating the charged currents.

2. Spin of ν_τ is 1/2 not $\geq 3/2$.

Suppose there were two kinds of $W^{\pm}$, say $W_A^{\pm}$ and $W_B^{\pm}$, (τ, ν_τ) couples only to $W_B^{\pm}$, but (μ, ν_μ) and (e, ν_e) couple to both $W_A^{\pm}$ and $W_B^{\pm}$. Under this circumstance we would not expect the branching ratios listed in Table 1 to agree with experiments. Now suppose there is a charged Higg's scalar particle which is characterized by its coupling constant being proportional to the mass of the particles it is coupled to. In this case the width $\tau \to \nu_\tau + \mu + \nu_\mu$ would be larger than the width of $\tau \to \nu_\tau + e + \nu_e$ and the rate for $\tau \to \nu_\tau + \pi$ would not be simply related to the rate of $\pi \to \mu + \nu_\mu$ as we have assumed in constructing Table 1. Now if spin of ν_τ were $\frac{3}{2}$ and $M_{\nu_\tau} = 0$ then $\tau \to \pi + \nu_\tau$ would be forbidden. If $M_{\nu_\tau} \neq 0$, then no particular prediction can be made because there will be too many adjustable parameters. The available data on the τ decay seem to agree with the standard model calculation shown in Table 1, hence these assumptions are likely to be correct.

F. What else can we learn from τ?

As long as one accepts the Standard Model, the decay properties of four channels, $\tau \to \nu_\tau + e + \nu_e$, $\tau \to \nu_\tau + \mu + \nu_\mu$, $\tau \to \nu_\tau + \pi$ and $\tau \to \nu_\tau + k$ are uniquely determined. Hence we can use these four channels to test the Standard Model.

With one additional hypothesis, CVC, one can relate[4] the decay $\tau \to \nu_\tau + 2n$ pions to the cross section $e^+ + e^- \to 2n$ pions:

$$\Gamma(\tau \to \nu_\tau + 2\text{ n pion}) = \frac{G^2}{(2\pi)^2\,(2M_\tau)^3}\,\frac{\cos^2\theta_2}{4\pi^2\alpha^2}\int_0^{M_\tau^2}(M_\tau^2-q^2)^2\,(M_\tau^2+2q^2)$$

$$\times\, q^2\,\sigma_{e^+e^- \to 2n\text{ pions}}(q^2)\,dq^2 \qquad (2.5)$$

This formula relates, for example, $\tau^+ \to \nu_\tau + \rho^+$ to $e^+e^- \to \rho^o$ and $\tau^+ \to \nu_\tau + \rho'^+$ to $e^+e^- \to \rho'^o$. The relation is violated by higher order electromagnetic interactions, for example, the regular radiative corrections or special effects, such as the ω-ρ mixing in the e^+e^- interaction. Aside from these small corrections the above formula can be used to test the validity of CVC. Since CVC is quite an established theory we can also use Eq. (2.5) to make additional tests of the validity of the Standard Model. It is amusing to note that in the case of ρ, the experiment[24] on $\tau^\pm \to \nu_\tau + \rho^\pm$ is much cleaner than the experiment[25] on $e^+ + e^- \to \rho^o$ because the latter is marred by the ρ-ω interference mentioned above.

Using the narrow width approximation for ρ in Eq. (2.5) we can write $\Gamma(\tau \to \nu_\tau + \rho)$ in terms of $\Gamma(\rho \to e^+e^-)$:

$$\Gamma(\tau \to \nu_\tau + \rho) = \Gamma(\rho \to e^+e^-)\,\frac{3}{2}\,\frac{G^2\cos^2\theta_c\,M\rho}{(4\pi\alpha)^2}\,M_\tau^3\,(1-M_\rho^2/M_\tau^2)^2$$

$$\times\,(1+2M_\rho^2/M_\tau^2). \qquad (2.6)$$

According to Benaksas et al.[25] $\Gamma(\rho \to e^+ e^-) = (5.8 \pm 0.5)$ KeV but the world average as given by the Particle Data Group[26] gives $\Gamma(\rho \to e^+e^-) = (6.7 \pm 0.8)$ KeV. In my earlier works[4,19] I used the latter value which is 16% higher than the former. Experimental result[24] of τ decay show that using the former agrees better with experiment, so in constructing Table 1 the former value was used. Gilman and Miller[27] used the experimental value for $\sigma(e^+ + e^- \to \pi^+ + \pi^-)$ in Eq. (2.5) without making the narrow width approximation. He obtained $\Gamma(\tau \to \nu_\tau + \rho) = 1.2\Gamma(\tau \to \nu_\tau + e + \nu_e)$ at $M_\tau = 1.8$ GeV whereas using the narrow width approximation we obtain 1.21 for the ratio of the two width, hence the narrow width approximation is adequate.

There are four different kinds of currents which can contribute to the hadronic final states of τ decay:

$\Delta S = 0$, Vector current , $G = +$

$\Delta S = 0$, Axial vector current , $G = -$

$\Delta S = 1$, Vector current

$\Delta S = 1$, Axial vector current

Only the first one is conserved and thus can be related to $\sigma(e^+ + e^- \to$ isovector states) as shown above. The second current has $G = -$ hence it has odd number of pions in its final states. The last two are a factor $\tan^2\theta_C$ smaller in magnitude than the first two. These four currents are related by Weinberg's sum rules[28] and Das-Mathur-Okubo sum rules[29] as explained in some detail in my 1971 paper.[4] All these sum rules say are quite simple: the spectral functions of all four currents become equal at high energies, and at low energies they are unequal because of the existence of pseudo scalar particles such as π's and K's. In the parton model these spectral functions are calculated by assuming that the W^- is coupled to free $\bar{u}d$ quarks for the first two currents and it is coupled to the free $\bar{u}s$ quarks in the last two currents. The QCD radiative corrections cause some logarithmic increase[30] at lower q^2 value compared with the parton model. The continuum contribution in Table 1 are calculated according to Q.C.D. Thus heavy lepton research can also teach us something about Q.C.D.

III. Heavier Leptons?

The most intriguing question in high energy physics today is whether the number of leptonic species and quark flavors is finite. There is some argument[31] against having too many species of very light neutrinos based on the Helium abundance in the universe. We also notice that if there are too many kinds of light neutrinos the width of Z^o would eventually exceed its mass, making it very unlikely that the phenomenology of weak neutral current, which ignores the Z_o width, to have any validity at all. On the other hand suppose nature has, for example, "four generations" of leptons and quarks, then one has to ask why the number "four" is picked over all other possible numbers? The easiest way out is that there are infinitely many generations of leptons and quarks and the neutrino associated with each species of lepton is progressively massive. If there are infinitely many species of leptons, the mass of each must obey certain regularities. After all the nature cannot be completely random. A random world can never be able to produce a highly intricate system such as our biological world for example. We have now three generations of leptons: (e, ν_e), (μ, ν_μ) and (τ, ν_τ). We still cannot see any rule governing the masses of these particles. If we find a few more generations we may be able to figure out the rules. Preliminary results[32] from PETRA seem to indicate that there is no new lepton with mass below $\sim$10 GeV. From Table I we see that the single prong hadronic events are no longer the dominant decay modes when the lepton mass is larger than 10 GeV. The pure leptonic decay modes, $L \to \nu_L + e + \nu_e$, $\nu_L + \mu + \nu_\mu$, $\nu_L + \tau + \nu_\tau$, . . . , remain prominent. The branching fraction into each hadronic generation is about three times the branching fraction into each leptonic generation due to the color factor. Small deviation from this rule is due to the phase space factor and higher order QCD corrections.

APPENDIX A. CALCULATION OF BRANCHING FRACTIONS

In Table I, we give the branching ratios and the total decay rates of L for various values of M_L. The formulas[4] used to calculate them are given below. (All masses are in units of GeV.)

$$\Gamma(L \to \nu_L + \nu_e + e) = \frac{G^2 M_L^5}{3\times 2^6 \pi^3} \quad \text{where} \quad G = 1.02 \times 10^{-5}/M_p^2$$

$$= 3.43 \times 10^{10}\, M_L^5 \text{ sec}^{-1} \qquad \text{(A.1)}$$

$$\Gamma(L \to \nu_L + \nu_\mu + \mu) = \Gamma(L \to \nu_L + \nu_e + e)\ (1 - 8y + 8y^3 - y^4 - 12y^2 \ln y)$$

$$\text{where } y = M_\mu^2/M_L^2 \ . \qquad \text{(A.2)}$$

$$\Gamma(L \to \nu_L + \nu_\tau + \tau) = \text{the same as above with } y = M_\tau^2/M_L^2 \qquad \text{(A.3)}$$

$$\Gamma(L \to \nu_L + \pi) = \Gamma(\pi \to \mu + \nu)\ \frac{M_L^3\,(1 - M_\pi^2/M_L^2)^2}{2M_\pi M_\mu^2\,(1 - (M_\mu/M_\pi)^2)^2}$$

$$= 6.71 \times 10^{10}\, M_L^3 (1 - M_\pi^2/M_L^2)^2 \text{ sec}^{-1} \qquad \text{(A.4)}$$

$$\Gamma(L \to \nu_L + K) = \Gamma(L \to \nu_L + \pi)\ \tan^2 \theta_c (1 - M_k^2/M_L^2)^2/(1 - M_\pi^2/M_L^2$$

$$= 0.49 \times 10^{10}\, M_L^3 (1 - M_k^2/M_L^2)^2 \text{ sec}^{-1} \qquad \text{(A.5)}$$

$$\Gamma(L \to \nu_L + \rho) = \Gamma(\rho \to ee)\ \frac{3}{2}\, \frac{G^2 \cos^2 \theta_c\, M_\rho}{(4\pi\alpha)^2}\, M_L^3\ (1 - M_\rho^2/M_L^2)^2 (1 + 2M_\rho^2/M_L^2)$$

$$= 14.8 \times 10^{10}\, M_L^3 (1 - M_\rho^2/M_L^2)^2\ (1 + 2M_\rho^2/M_L^2) \text{ sec}^{-1} \qquad \text{(A.6)}$$

$$\Gamma(L \to \nu_L + A_1) = 7.67 \times 10^{10}\ M_L^3 (1 - M_{A_1}^2/M_L^2)^2 (1 + 2\ M_{A_1}^2/M_L^2)\ \text{sec}^{-1} \tag{A.7}$$

where 7.67 comes from $14.8 \times (M_\rho^2/M_{A_1}^2)$

$$\Gamma(L \to \nu_L + k^*) = 1.08 \times 10^{10}\ M_L^3 (1 - M_{k*}^2/M_L^2)^2\ (1 + 2\ M_{k*}^2/M_L^2)\ \text{sec}^{-1} \tag{A.8}$$

where 1.08 comes from $14.8 \times \tan^2 \theta_c$

$$\Gamma(L \to \nu_L + Q) = 0.51 \times 10^{10}\ M_L^3 (1 - M_Q^2/M_L^2)^2\ (1 + 2\ M_Q^2/M_L^2)\ \text{sec}^{-1} \tag{A.9}$$

where 0.51 comes from $14.8 \times \tan^2 \theta_c\ (M_{k*}^2/M_Q^2)$

Various contributions from $L \to \nu_L +$ hadron continuum can be calculated[4] from the parton model with a logarithmic correction due to asymptotically free gauge theory.[30]

$$\begin{aligned}\Gamma(L \to \nu_L + u\bar{d} > 1.1\ \text{Gev}) &= \Gamma(L \to \nu_L + \nu_e + e) \times 3 \times \cos^2 \theta_c\, F(\Lambda, N, M_L) \\ &= 9.59 \times 10^{10}\ M_L^5\ F(\Lambda, N, M_L)\ \text{sec}^{-1}\ .\end{aligned} \tag{A.10}$$

$$\text{where } F(\Lambda, N, M_L) = 2 \int_{\Lambda^2/M_L^2}^{1} (1-x)^2 (1+2x) \left[1 + \frac{12}{(33-2N)\ln(4x\, M_L^2)}\right] dx \tag{A.11}$$

Λ is the threshold of the continuum and we have chosen $\Lambda = 1.1$ Gev for hadronic continuum formed by u and $\bar{d}$ quarks. N is the number of flavours which can participate in the interaction for $s = M_L^2 x$. We have used N = 3 if S $\leq$ 4 Gev^2, and N = 4 if s > 4 Gev^2.

$$\Gamma(L \to \nu_L + u\bar{s} > 1.1\ \text{Gev}) = \Gamma(L \to \nu_L + u\bar{d} > 1.1\ \text{Gev})\ \tan^2 \theta_c \tag{A.12}$$

$$\Gamma(L \to \nu_L + c\bar{s} > 2\ \text{Gev}) = \Gamma(L \to \nu_L + u\bar{d} > 2\ \text{Gev}) \tag{A.13}$$

$$\Gamma(L \to \nu_L + c\bar{d} > 2\text{ Gev}) = \Gamma(L \to \nu_L + u\bar{d} > 2\text{ Gev}) \tan^2 \theta_c \qquad \text{(A.14)}$$

Equation (A.1) through equation (A.9) were derived in reference 4. Slightly different numerical values from those obtained previously are due mostly to the improved accuracy of an electronic calculator over a slide rule. We have also used $\Gamma(\rho \to e^+e^-) = 5.8$ Kev instead of 6.7 Kev in Eq. (A.6). The contributions due to hadron continuum have to be increased drastically from the previous estimates. The reason is when that paper was written in 1971, I was not sure how much faith to put in the total cross section data of the e^+e^- reaction from Frascati and thus the value of $R = \sigma(e^+ + e^- \to \text{hadron})/\sigma(e^+ + e^- \to \mu^+\mu^-)$ used was 2/3 which was derived from the quark parton model without color.

The parton model with three colors and the logarithmic correction due to asymptotically free gauge theory gives

$$R\text{ (asymptotically free gauge theory)} = \frac{2}{3} \times 3 \times \left(1 + \frac{12}{27 \ln 4s}\right)$$

$$= 2.43 \text{ for } s = 2\text{ Gev}^2$$

We have used this theory to calculate all of our continuum contributions shown in Eq. (A.10) through Eq. (A.14). The factor F introduced in Eqs. (A.10) and (A.11) is the correction factor to the parton model due to the finite threshold ($\Lambda \neq 0$) and higher order QCD corrections. To see this we note that if the logarithmic correction term in Eq. (A.11) is ignored the integration can be carried out easily and one obtains

$$2\int_{\Lambda^2/M_L^2}^{1} (1-x)^2(1+2x)\,dx = (1-2y+2y^3-y^4) \underset{y\to 0}{\to} 1\,,$$

where $y = \Lambda^2/M_L^2$.

This factor was derived in Eq. (3.40) of ref. 4.

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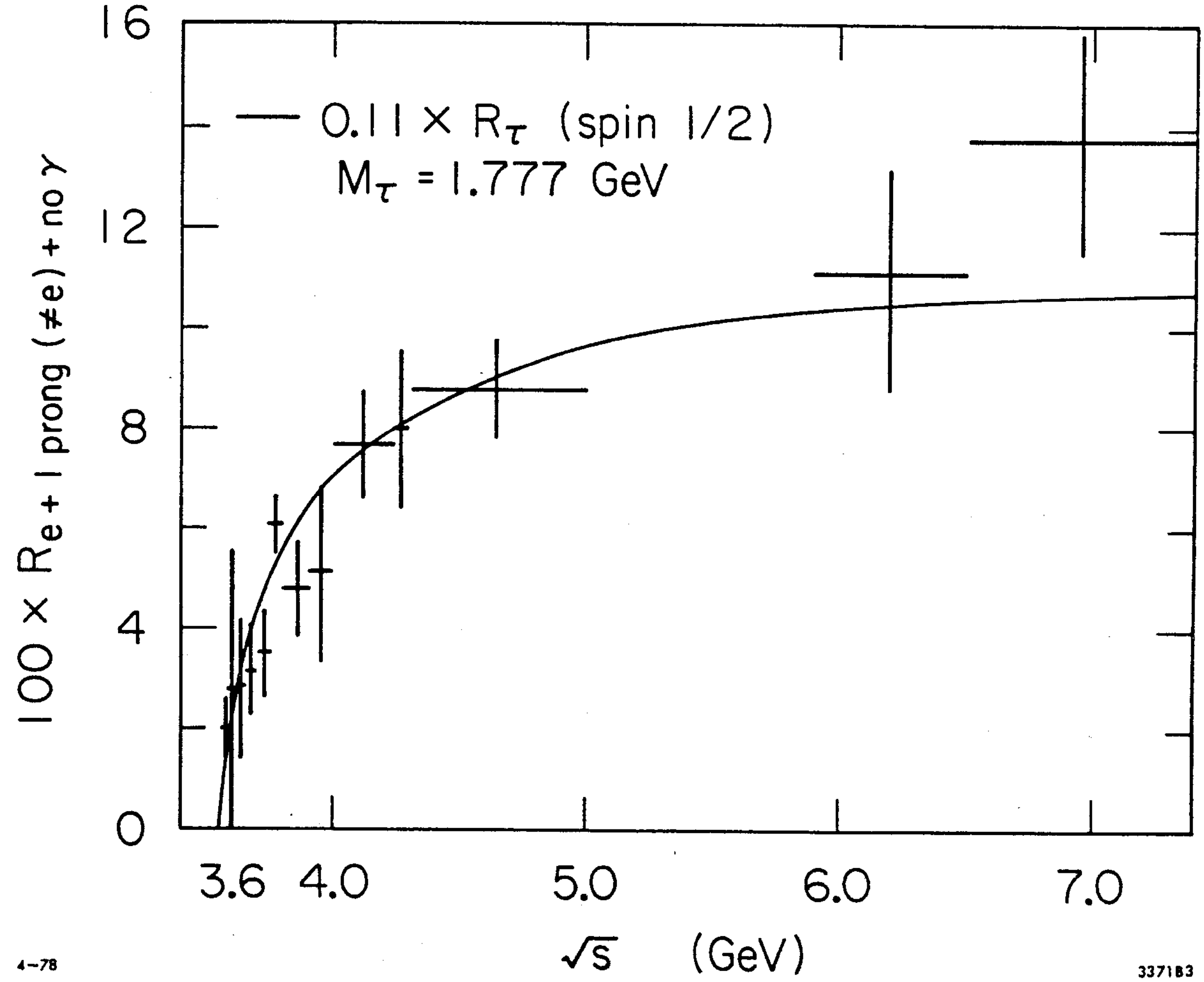
FIGURE CAPTIONS

1. Experimental values for the cross section/σ_μ for $e^+ + e^- \to e + 1$ charged prong ($\neq$e) + no detected photon from DELCO versus theoretical estimate (solid line) assuming τ to be a spin $\frac{1}{2}$ particle. Notice the threshold s wave behavior and the energy dependence of the cross section at high energies.

2. Energy dependence of $R_\tau = \sigma_\tau/\sigma_\mu$ assuming τ to be spin $\frac{1}{2}$; spin 1, κ=0; spin 1, κ=1 and spin $\frac{3}{2}$, A=1, B=C=D=0. Notice that the scale of the ordinate is linear from 0 to 1, but it is logarithmic above 1.0.

3. The normalized electron energy spectrum obtained by DELCO in the energy range, $3.57 < E_{cm} < 7.5$ GeV (excluding ψ''). The radiatively-corrected fits for V-A (solid) and V+A (dashed) show χ^2/dof of 15.9/17 and 53.7/17, respectively.

4. The (non-radiatively-corrected) normalized electron energy spectrum in the τ rest-frame for the decay $\tau^- \to \nu_\tau e^- \bar{\nu}_e$ under several space-time assumptions for the $\tau - \nu_\tau$ coupling.



0.11 × R_τ (spin 1/2)
M_τ = 1.777 GeV
100 × R_e + 1 prong (≠e) + no γ
16
12
8
4
0
3.6
4.0
5.0
6.0
7.0
√s (GeV)
4-78
337183

Fig. 1

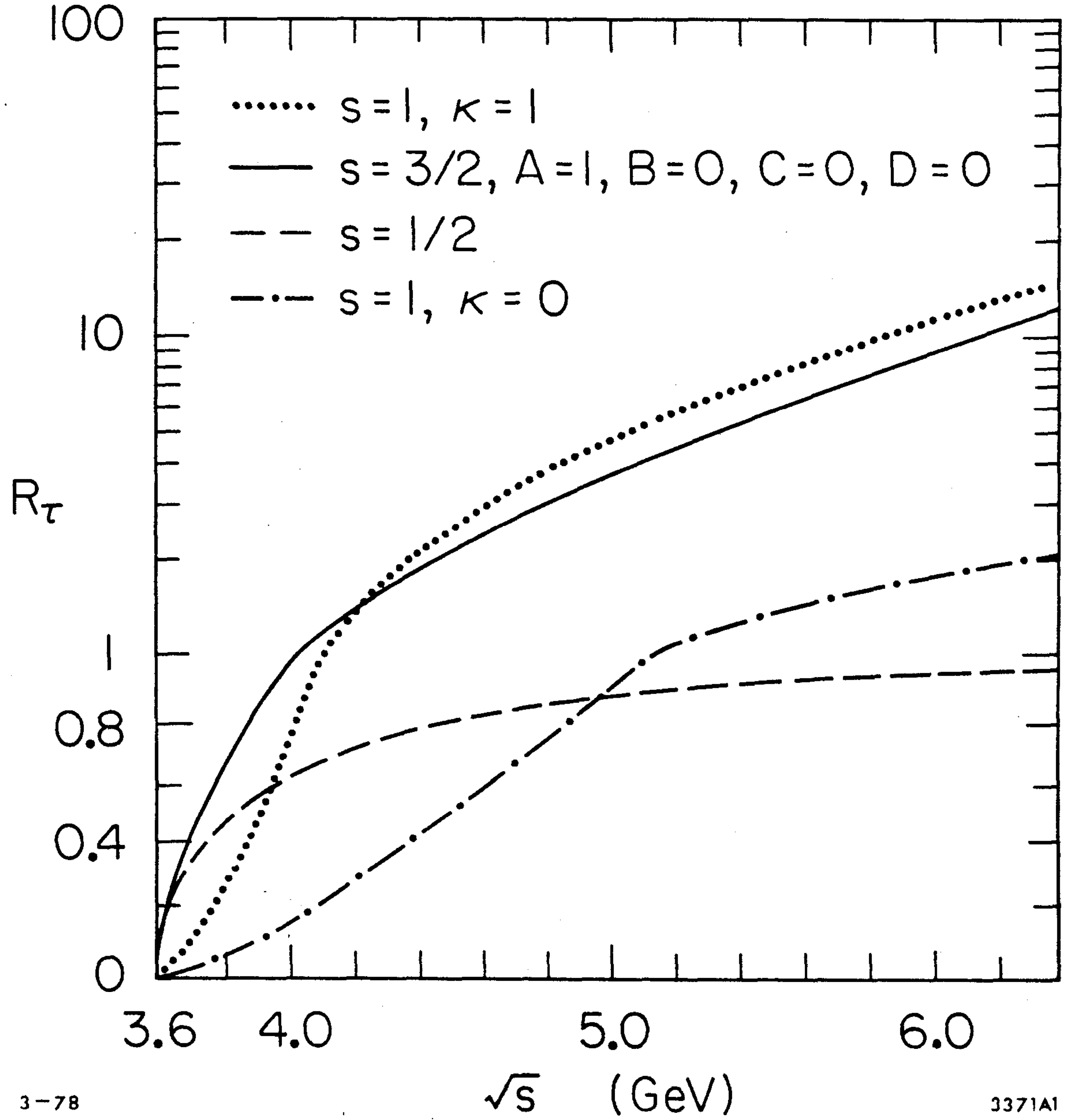

100
10
1
0.8
0.4
0
R$_\tau$
s = 1, κ = 1
s = 3/2, A = 1, B = 0, C = 0, D = 0
s = 1/2
s = 1, κ = 0
3.6
4.0
5.0
6.0
√s (GeV)
3-78
3371A1

Fig. 2

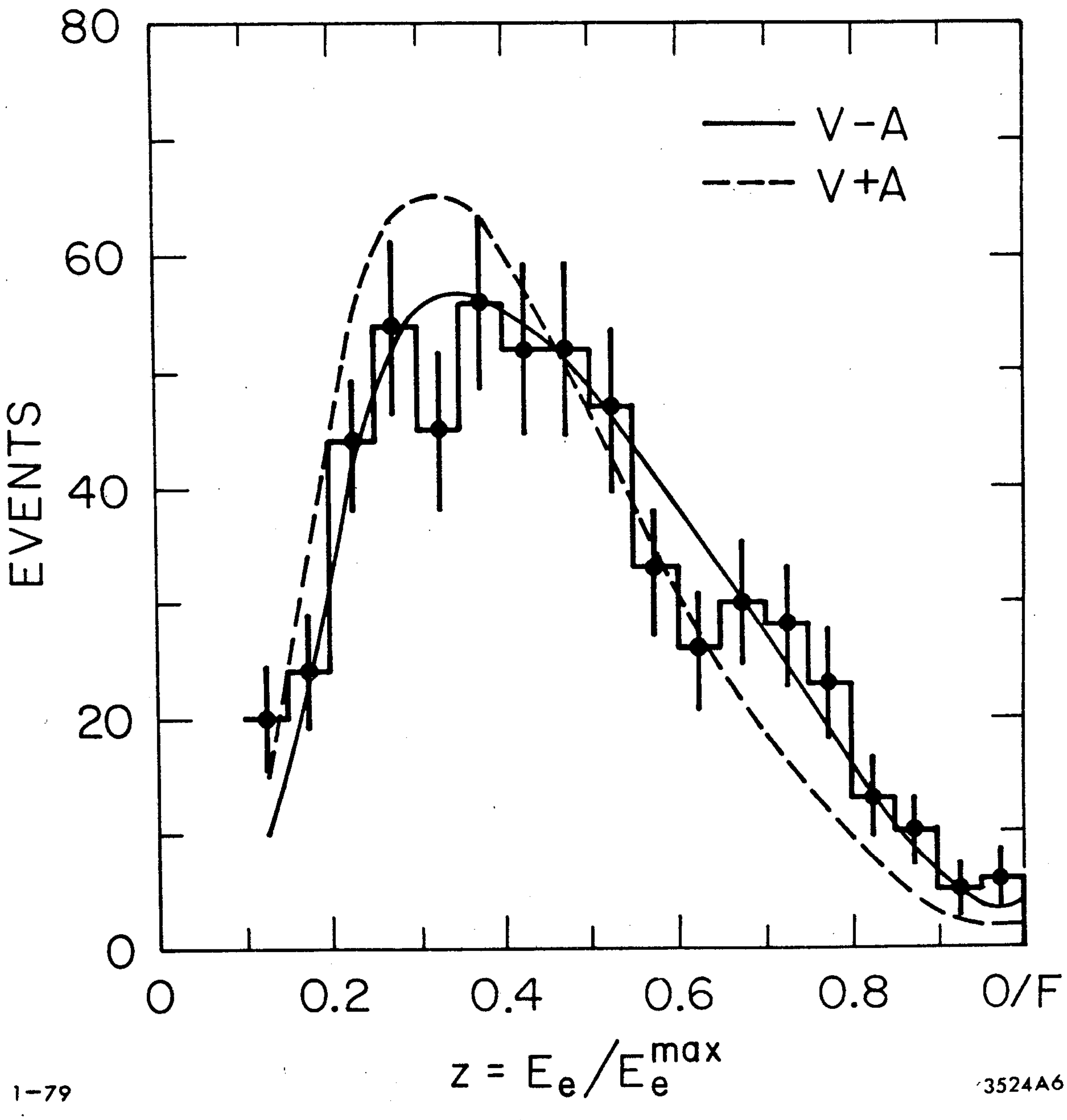


Fig. 3

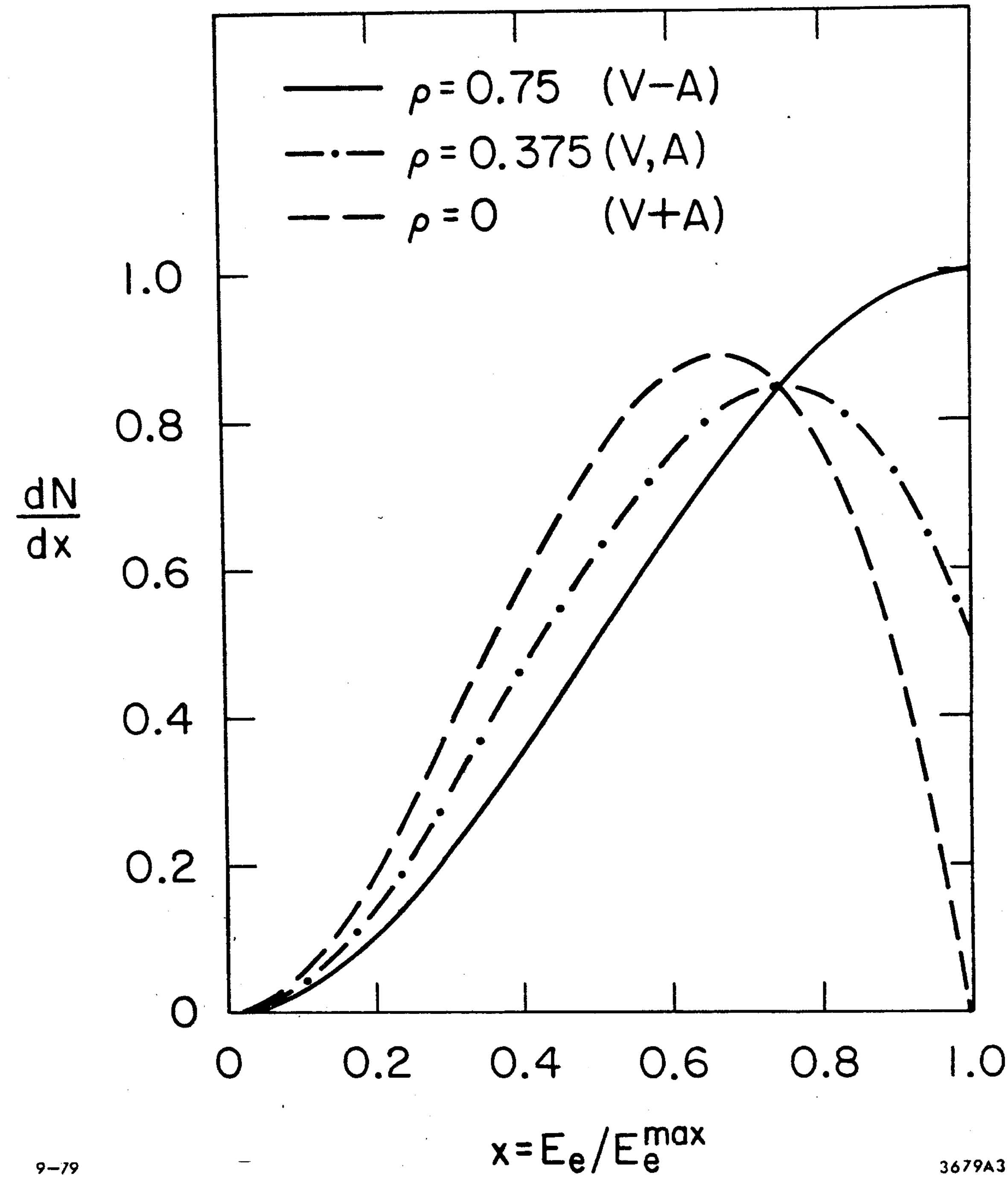

ρ = 0.75 (V−A)
ρ = 0.375 (V, A)
ρ = 0 (V+A)
dN/dx
1.0
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1.0
x = E_e/E_e^max
9–79
3679A3

Fig. 4