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A POSSIBLE ELECTRO-WEAK MODEL IN SU(3) × U(1)*

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Abstract

A model of electro-weak interaction involving leptons and quarks in SU(3) × U(1) gauge theory is constructed by means of the helicity mixed representation. This model is left-right symmetrical before spontaneous symmetry breaking and anomaly free. As a limiting case, it gives the same results as those of Weinberg-Salam model in the low energy range. The Weinberg angle is bound by $\sin^2\theta_W \le \frac{1}{4}$. A new conserved quantum number S_W called the weak strangeness is introduced in this model. The Kobayashi-Maskawa expression of Cabibbo mixing for quarks may be obtained in the model generalized to include three generations of fermions.

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1. Introduction and Motivation

Recent neutrino induced elastic and deep inelastic neutral current data are in agreement with expectations based on the simple $SU(2) \times U(1)$ gauge model of Weinberg-Salam [1,2]. The Weinberg angle θ_W is shown to be slightly less than 30°.

The motivation of the present paper is to investigate the following questions:

1. Whether $\sin^2\theta_W \lesssim \frac{1}{4}$ has special physical meaning?
2. Are there possibilities other than $SU(2) \times U(1)$?

The requirements for the new model to be constructed are:

1. It gives the same observable results as the Weinberg-Salam model in the low energy range.
2. There might be some predictions different from the usual $SU(2) \times U(1)$ model, which can be tested in the near future.
3. The model is right-left symmetrical before spontaneous symmetry breaking.
4. The model is anomaly free.

It will be shown in the present paper that such possibility indeed exists for gauge group $SU(3) \times U(1)$ in which the value of $\sin^2\theta_W$ cannot exceed $\frac{1}{4}$. In this model a global $U(1)$ symmetry is also postulated which turns into a conservation law called weak strangeness conservation after spontaneous symmetry breaking. Conservation of weak strangeness ensures that the additional gauge bosons have no direct coupling with the ordinary fermions and makes our model different from that of B. W. Lee and S. Weinberg [3].

This model contains right-handed neutrinos, heavy fermions and bosons yet unobserved. Their number is rather large and we are aware of the possibility that they might cause difficulties in problems not discussed in the present paper. We shall discuss them in later publications.

The paper is outlined as follows. In Sections 2 and 3 we discuss the transformation properties. In Section 4 the model is presented with a brief discussion of the left-right symmetry and the cancellation of the anomalies. In Section 5 the mass spectrum is obtained from spontaneous symmetry breaking. In Section 6 the weak currents are obtained which tends to that of Weinberg-Salam model as $\sin^2\theta_W \to \frac{1}{4}$. Section 7 is devoted to the conservation of weak strangeness and its experimental implications. In Section 8 generalization to more generation of fermions is briefly discussed. The section contains the summary and some conclusions.

2. Transformation Properties of Fermions and Gauge Fields

In order to preserve the left-right symmetry before spontaneous symmetry breaking, we introduce an unified description of transformation properties for two chiral components of fermions. In the chiral representation if three left-handed fermions form a triplet representation $\underline{3}$ of SU(3) group, then the corresponding right-handed fermions form $\underline{3}^*$ which is the conjugate representation of $\underline{3}$. We will embed γ_5 into some of generators to describe the transformation properties of both $\underline{3}$ and $\underline{3}^*$ in an unified formula simultaneously. Since the mass terms of fermions connect the left-handed components to the right-handed components, the unified description is rather useful for many physical discussions and makes them simple and brief. We formulate generally this kind of description in the following. The connection between this description and usual notation is given in Appendix B.

The generators $\hat{I}_i$, $i=1,\dots,8$ of the SU(3) group can be composed into two sets $\hat{I}_a$ and $\hat{I}_\alpha$, where the choice of a and α is one of the seven possibilities listed in Table I. We now define operators $\hat{I}_i^{(\varepsilon)}$ by

$$\hat{I}_a^{(\varepsilon)} = \varepsilon\hat{I}_a \quad , \quad \hat{I}_\alpha^{(\varepsilon)} = \hat{I}_\alpha \tag{2.1}$$

where ε commutes with $\hat{I}_i$ and satisfies the relation $\varepsilon^2=1$. One easily verifies that the Lie algebra for $\hat{I}_i^{(\varepsilon)}$ are the same as that for $\hat{I}_i$. Four possible choices for ε: $\varepsilon=+1$, -1, γ_5, $-\gamma_5$ will be used below. They are denoted simply by +, −, 5, −5 respectively.

The first choice for a and α listed in Table I will be used in the following. The second, third and fourth choices will give essentially similar results. The generator of the U(1) gauge field will be denoted by $\hat{Y}$.

Besides the conservation of various fermion numbers there is another global U(1) symmetry whose generator will be denoted by $\hat{S}$. This global U(1) will combine with an Abelian subgroup in SU(3) × U(1) to give a new quantum number S_w after spontaneous symmetry breaking. This new quantum number S_w is called weak strangeness and will be discussed in detail below.

For simplicity, suppose there is only one generation of fermions (leptons and quarks), the model involving several generations will be discussed later. The leptons form two $Y=0$ triplets of Dirac spinor in SU(3) and are denoted by ψ_ℓ and ψ_ℓ^h respectively. The representations for quarks are two triplets ψ_q and ψ_q^h with $Y=\frac{2}{3}$ together with a couple of $Y = \frac{2}{3}$ singlets u and u^h. Transformation properties for fermions are respectively

$$\begin{aligned}
\psi \rightarrow \psi' &= U^{(5)}(\xi_j(x))\, e^{i\hat{Y}\theta(x)}\, e^{i\hat{S}\gamma_5\eta}\,\psi \quad ,\\
\psi^h \rightarrow \psi^{h'} &= U^{(-5)}(\xi_j(x))\, e^{i\hat{Y}\theta(x)}\, e^{i\hat{S}(-\gamma_5)\eta}\,\psi^h \quad ,\\
u \rightarrow u' &= e^{i\hat{Y}\theta(x)}\, e^{i\hat{S}\gamma_5\eta}\, u \quad ,\\
u^h \rightarrow u^{h'} &= e^{i\hat{Y}\theta(x)}\, e^{i\hat{S}(-\gamma_5)\eta}\, u^h \quad ,
\end{aligned} \tag{2.2}$$

where

$$U^{(\varepsilon)}(\xi_j(x)) = \exp\left\{ i\hat{I}_j^{(\varepsilon)}\, \xi_j(x) \right\} \quad . \tag{2.3}$$

In (2.2) and (2.3) $\xi_j(x)$, $j = 1,\ldots,8$, $\theta(x)$ and η are group parameters for the local $SU(3) \times U(1)$ and the global $U(1)$ respectively. The generator $\hat{S}$ will take value $\frac{1}{6}$ for fermion triplets and $-\frac{1}{2}$ for fermion singlet.

One note that ψ^h and u^h transform just as ψ and u with $-\gamma_5$ instead of γ_5, so we may call ψ^h the helicity conjugation of ψ and vice versa.

There are nine gauge fields $A_\mu^j(x)$, $j = 1,\ldots,8$ of SU(3) and $B_\mu(x)$ of U(1). Define

$$\hat{A}_\mu^{(\varepsilon)} = igA_\mu^j(x)\, \hat{I}_j^{(\varepsilon)} \tag{2.4}$$

$\hat{A}_\mu^{(\varepsilon)}$ transform under SU(3) in the following way

$$\hat{A}_\mu^{(\varepsilon)} \rightarrow \hat{A}_\mu^{(\varepsilon)'} = U^{(\varepsilon)}(\xi_j(x))\left(\partial_\mu + \hat{A}_\mu^{(\varepsilon)}(x)\right)U^{(\varepsilon)+}(\xi_j(x)) \tag{2.5}$$

The covariant derivatives for the fermions are easily constructed. They are

$$D_\mu \psi = \left(\partial_\mu + \hat{A}_\mu^{(5)} + \frac{i}{2} g' \hat{Y} B_\mu\right)\psi \quad ,$$

$$D_\mu \psi^h = \left(\partial_\mu + \hat{A}_\mu^{(-5)} + \frac{i}{2} g' \hat{Y} B_\mu\right)\psi^h \quad ,$$

$$D_\mu u = \left(\partial_\mu + \frac{i}{2} g' \hat{Y} B_\mu\right) u \quad ,$$

$$D_\mu u^h = \left(\partial_\mu + \frac{i}{2} g' \hat{Y} B_\mu\right) u^h \quad . \tag{2.6}$$

The invariant Lagrangian involving fermions and gauge fields can be written as

$$\begin{aligned}\mathscr{L}_{FG} = & -\frac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \\ & + \bar{\psi}_\ell \gamma^\mu \left(\partial_\mu + \hat{A}_\mu^{(5)}\right)\psi_\ell + \bar{\psi}^h_\ell \gamma^\mu \left(\partial_\mu + \hat{A}_\mu^{(-5)}\right)\psi^h_\ell \\ & + \bar{\psi}_q \gamma^\mu \left(\partial_\mu + \hat{A}_\mu^{(5)} + i\frac{g'}{2}\hat{Y}B_\mu\right)\psi_q + \bar{\psi}^h_q \gamma^\mu \left(\partial_\mu + \hat{A}_\mu^{(-5)} + i\frac{g'}{2}\hat{Y}B_\mu\right)\psi^h_q \\ & + \bar{u}\gamma^\mu\left(\partial_\mu + i\frac{g'}{2}\hat{Y}B_\mu\right)u + \bar{u}^h\gamma^\mu\left(\partial_\mu + i\frac{g'}{2}\hat{Y}B_\mu\right)u^h \quad .\end{aligned} \tag{2.7}$$

3. Transformation Properties of Higgs Field

We choose Higgs multiplet Φ to be a 3×3 matrix. It transforms as

$$\Phi \rightarrow \Phi' = e^{i\hat{Y}\theta(x)} \, e^{i\hat{S}\eta} \, U^{(-)} \, \Phi \, U^{(+)+}$$

$$\Phi^+ \rightarrow \Phi^{+\prime} = e^{-i\hat{Y}\theta(x)} \, e^{-i\hat{S}\eta} \, U^{(+)} \, \Phi^+ \, U^{(-)+} \tag{3.1}$$

Since

$$U^{(\varepsilon)*} = U^{(-\varepsilon)} \quad , \quad \tilde{U}^{(\varepsilon)} = U^{(-\varepsilon)+} \tag{3.2}$$

for our choice of a and α, we obtain from (3.1)

$$\Phi^* \rightarrow \Phi^{*\prime} = e^{-i\hat{Y}\theta(x)} \, e^{-i\hat{S}\eta} \, U^{(+)} \, \Phi^* \, U^{(-)+}$$

$$\tilde{\Phi} \rightarrow \tilde{\Phi}' = e^{i\hat{Y}\theta(x)} \, e^{i\hat{S}\eta} \, U^{(-)} \, \tilde{\Phi} \, U^{(+)+} \tag{3.3}$$

i.e., $\tilde{\Phi}$ transforms in the same way as Φ. Thus Φ decomposes into two irreducible representation by the following additional conditions

$$\Phi^{(6)} = \tilde{\Phi}^{(6)} \quad , \quad \underline{6}$$
$$\Phi^{(3)} = -\tilde{\Phi}^{(3)} \quad , \quad \underline{3} \tag{3.4}$$

$\Phi^{(6)}$ and $\Phi^{(3)}$ can be expressed as

$$\Phi^{(6)} = \frac{1}{\sqrt{6}}\,\phi^{o} + \lambda_a \phi^{a}$$
$$\Phi^{(3)} = \lambda_\alpha \phi^{\alpha} \tag{3.5}$$

where ϕ^{o}, ϕ^{a} and ϕ^{α} are complex in general.

The triplet $\Phi^{(3)}$ can also be expressed in the column form

$$\Phi^{(3)} = \begin{pmatrix} \phi^7 \\ -\phi^5 \\ \phi^2 \end{pmatrix}$$

The transformation rule (3.1) can be expressed in this form as

$$\Phi^{(3)} \to \Phi^{(3)\prime} = e^{i\hat{Y}\theta(x)}\, e^{i\hat{S}\eta}\, U^{(+)}\, \Phi^{(3)}$$
$$\Phi^{(3)+} \to \Phi^{(3)} = e^{-i\hat{Y}\theta(x)}\, e^{-i\hat{S}\eta}\, \Phi^{(3)+}\, U^{(+)+} \tag{3.6}$$

and (3.2) as

$$\Phi^{(3)^*} \to \Phi^{(3)^{*\prime}} = e^{-i\hat{Y}\theta(x)}\, e^{-i\hat{S}\eta}\, U^{(-)}\, \Phi^{(3)^*}$$
$$\tilde{\Phi}^{(3)} \to \tilde{\Phi}^{(3)\prime} = e^{i\hat{Y}\theta(x)}\, e^{i\hat{S}\eta}\, \tilde{\Phi}^{(3)}\, U^{(-)+} \tag{3.7}$$

The covariant derivatives of Higgs fields are

$$D_\mu \Phi = \partial_\mu \Phi + \hat{A}_\mu^{(-)} \Phi - \Phi \hat{A}_\mu^{(+)} + i\frac{g'}{2}\hat{Y} B_\mu \Phi \tag{3.8}$$

for matrix form of both $\Phi^{(6)}$ and $\Phi^{(3)}$ and

$$D_\mu \Phi = \left(\partial_\mu + \hat{A}_\mu^{(+)} + i\frac{g'}{2}\hat{Y}B_\mu\right)\Phi \qquad (3.9)$$

for column form of $\Phi^{(3)}$ respectively.

In order to see how to generate the mass spectrum for quarks let us consider the Lagrnagian interaction involving quarks and Higgs field

$$\begin{aligned} & f\,\bar{\psi}\,\Phi\,P_+\,\psi \quad + f^*\,\bar{\psi}\,\Phi^+\,P_-\,\psi \\ + \; & f_1\,\bar{u}\,\Phi^+\,P_+\,\psi \quad + f_1^*\,\bar{\psi}\,\Phi\,P_-\,u \\ + \; & f_2\,\bar{u}\,\tilde{\Phi}\,P_-\,\psi \quad + f_2^*\,\bar{\psi}\,\Phi^*\,P_+\,u \\ + \; & f^h\,\bar{\psi}^h\,\Phi\,P_-\,\psi^h \quad + f^{h*}\,\bar{\psi}^h\,\Phi^+\,P_+\,\psi^h \\ + \; & f_1^h\,\bar{u}^h\,\Phi^+\,P_-\,\psi^h + f_1^{h*}\,\bar{\psi}^h\,\Phi\,P_+\,u^h \\ + \; & f_2^h\,\bar{u}^h\,\tilde{\Phi}\,P_+\,\psi^h \quad + f_2^{h*}\,\bar{\psi}^h\,\Phi^*\,P_-\,u^h \end{aligned} \qquad (3.10)$$

In (3.10) $P_\pm = \frac{1}{2}(1 \pm \gamma_5)$, which are necessary for the quark Higgs coupling to be invariant.

We note that any one multiplet such as ψ or u contains same portions of left-handed and right-handed components. The left-right symmetry requires that f_1 and f_2 be of the same magnitude. Suitable choice of the phase of Φ makes both f_1 and f_2 real and we may define a parity transformation P to realize the left-right symmetry which distinguishes two kinds of $\Phi^{(3)}$ as

$$\begin{aligned} & \psi \to \gamma_4 \psi \quad , \quad u \to \gamma_4 u \quad , \\ P: \quad & \Phi_A^{(3)} \to -\Phi_A^{(3)*} \\ & \Phi_B^{(3)} \to \Phi_B^{(3)*} \end{aligned} \qquad (3.11)$$

(3.11) holds for $\Phi^{(3)}$ in the column form.

The P-invariance require that

$$f_1 = -f_2 \qquad \text{for } \Phi_A^{(3)} \qquad ;$$

$$f_1 = f_2 \qquad \text{for } \Phi_B^{(3)} \qquad . \tag{3.12}$$

Of course, for leptons we get $f_1 = f_2 = 0$ since there is no singlet for leptons. These properties are sufficient for the model to be left-right symmetrical.

4. A Possible Model in SU(3) × U(1)

Now we construct a left-right symmetrical model involving one generation of fermions. Fields belonging to this model are

leptons : ψ_ℓ, ψ_ℓ^h

quarks : ψ_q, u, ψ_q^h, u^h

gauge fields: A_μ^j $(j = 1, \ldots, 8)$, B_μ

Higgs fields: $\underline{3}$: Φ_A, Φ_B, Φ_Y ; $\underline{6}$: Φ

where Φ_A, Φ_B and Φ have quantum numbers $Y = 0$ and $S = -\frac{1}{3}$, while $Y = -1$, $S = \frac{5}{3}$ for Φ_Y, Φ_A and Φ_B have opposite behaviors under P transformation given above. Φ_Y is free under P transformation because it does not couple directly with fermions.

The invariant Lagrangian has the form

$$\mathscr{L} = \mathscr{L}_{FG} + \mathscr{L}_H + \mathscr{L}_{FH} - V \tag{4.1}$$

where $\mathscr{L}_{FG}$ is the Lagrangian given in (2.7) and

$$\mathscr{L}_H = \mathrm{tr}(D_\mu\Phi_A)^+(D^\mu\Phi_A) + \mathrm{tr}(D_\mu\Phi_B)^+(D^\mu\Phi_B)$$
$$+ \mathrm{tr}(D_\mu\Phi_Y)^+(D^\mu\Phi_Y) + \mathrm{tr}(D_\mu\Phi)^+(D^\mu\Phi) \qquad (4.2)$$

$$\mathscr{L}_{FH} = f_{A\ell}\left(\bar{\psi}_\ell\Phi_A P_+\psi_\ell + \bar{\psi}_\ell\Phi_A^+ P_-\psi_\ell\right)$$
$$+ f_\ell\left(\bar{\psi}_\ell\Phi P_+\psi_\ell + \bar{\psi}_\ell\Phi^+ P_-\psi_\ell\right)$$
$$+ f_{Aq}\left(\bar{\psi}_q\Phi_A P_+\psi_q + \bar{\psi}_q\Phi_A^+ P_-\psi_q\right)$$
$$+ f_q\left(\bar{\psi}_q\Phi P_+\psi_q + \bar{\psi}_q\Phi^+ P_-\psi_q\right)$$
$$+ f_{Au}\left(\bar{u}\Phi_A^+ P_+\psi_q + \bar{\psi}_q\Phi_A P_- u - \bar{u}\tilde{\Phi}_A P_-\psi_q - \bar{\psi}\Phi_A^* P_+ u\right)$$
$$+ f_{Bu}\left(\bar{u}\Phi_B^+ P_+\psi_q + \bar{\psi}_q\Phi_B P_- u + \bar{u}\tilde{\Phi}_B P_-\psi_q + \bar{\psi}\Phi_B^* P_+ u\right)$$
$$+ \left(\psi \to \psi^h,\ u \to u^h,\ P_\pm \to P_\mp,\ f \to f^h \text{ terms}\right) \qquad (4.3)$$

where f's are real by a suitable choice of phase factors. Renormalizability restricts the self interaction potential V of Higgs fields to be a polynomial of Higgs fields with powers ≤ 4. Owing to the global U(1) symmetry only terms with even power are allowed. The explicit form of V used in the present model is given in Appendix A. This Lagrnagian has left-right symmetrical form as it is invariant under the following parity transformation

$$\psi \to \gamma_4\psi, \quad u \to \gamma_4 u, \quad \Phi_A \to -\Phi_A^*, \quad \Phi_B \to \Phi_B^*$$

Now we discuss the problem of triangular anomaly in this model. The typical triangular anomaly is shown in fig. 1. Two of the vertices are vector vertices and the third is axial-vector one. In our model, both vector and axial-vector vertexes are discribed uniformly by generators $\hat{I}_i^{(\pm 5)}$. The Feynman amplitude of this diagram is

$$S_{\lambda\mu\nu}(p,q) = (2\pi)^{-4}\int d^4k \ \mathrm{tr}\ \mathrm{Sp}\Big[\gamma^{\lambda} I_k^{(5)}(\gamma,q+k)\gamma^{\nu}\hat{I}_j^{(5)}(\gamma,k)$$
$$\gamma^{\mu}\hat{I}_i^{(5)}(\gamma,k-p)\Big]\Big[(q+k)^2 k^2 (k-p)^2\Big]^{-1}$$
$$+ (2\pi)^{-4}\int d^4k \ \mathrm{tr}\ \mathrm{Sp}\Big[\gamma^{\lambda}\hat{I}_k^{(-5)}(\gamma,q+k)\gamma^{\nu}\hat{I}_j^{(-5)}(\gamma,k)$$
$$\gamma^{\mu}\hat{I}_i^{(-5)}(\gamma,k-p)\Big]\Big[(q+k)^2 k^2 (k-p)^2\Big]^{-1} \qquad (4.4)$$

where $(\gamma,k) = \gamma^{\mu}k_{\mu}$, Sp means trace for Dirac matrices and tr means trace for matrices in $SU(3)\times U(1)$ space. In (4.4), $\hat{I}_i^{(\pm 5)}$, $i=1,\ldots,8$ are given above and $\hat{I}_o^{(\pm 5)}$ is define as

$$\hat{I}_o^{(5)} = \hat{I}_o^{(-5)} = \frac{1}{2}\hat{Y} \qquad (4.5)$$

Two terms in (4.4) are contributions of ψ_{ℓ}, ψ_q, u and ψ_{ℓ}^h, ψ_q^h, u^h respectively.

Since all $\hat{I}_i^{(\pm 5)}$ commute with $\gamma\gamma$, (4.4) can be simplified as

$$S_{\lambda\mu\nu}(p,q) = (2\pi)^{-4}\int d^4k \ \mathrm{Sp}\Big\{\mathrm{tr}\Big[\hat{I}_k^{(5)}\hat{I}_j^{(5)}\hat{I}_i^{(5)} + \hat{I}_k^{(-5)}\hat{I}_j^{(-5)}\hat{I}_i^{(-5)}\Big]$$
$$(\gamma,q+k)\gamma^{\nu}(\gamma,k)\gamma^{\mu}(\gamma,k-p)\gamma^{\lambda}\Big\}\Big[(q+k)^2 k^2 (k-p)^2\Big]^{-1} \qquad (4.6)$$

The factor

$$T = \mathrm{tr}\Big[\hat{I}_k^{(5)}\hat{I}_j^{(5)}\hat{I}_i^{(5)} + \hat{I}_k^{(-5)}\hat{I}_j^{(-5)}\hat{I}_i^{(-5)}\Big] \qquad (4.7)$$

is important in the discussion of anomaly. The existence of anomaly requires $T \sim \gamma_5$, i.e.,

$$\mathrm{Sp}(\gamma_5 T) \neq 0 \qquad (4.8)$$

hence there should be odd number of the "a" type vertices. Furthermore as $\hat{I}_a^{(-5)} = -\hat{I}_a^{(5)}$, $\hat{I}_{\alpha}^{(-5)} = \hat{I}_{\alpha}^{(5)}$, we get

$$T = tr\left[\hat{I}_k^{(5)}\hat{I}_j^{(5)}\hat{I}_i^{(5)} - \hat{I}_k^{(5)}\hat{I}_j^{(5)}\hat{I}_i^{(5)}\right] = 0$$

So the model is anomaly free exactly.

5. Spontaneous Symmetry Breaking and the Mass Spectrum

The self interaction potential of Higgs fields is chosen to be (see Appendix A)

$$\begin{aligned} V = & -a_A\Phi_A^+\Phi_A + b_A(\Phi_A^+\Phi_A)^2 - a_B\Phi_B^+\Phi_B + b_B(\Phi_B^+\Phi_B)^2 \\ & - a_Y\Phi_Y^+\Phi_Y + b_Y(\Phi_Y^+\Phi_Y)^2 - atr\Phi^+\Phi + b(tr\Phi^+\Phi)^2 \\ & + c\Phi_A^+\Phi_Y\Phi_Y^+\Phi_A - d(\Phi_A^+\Phi_B + \Phi_B^+\Phi_A)^2 \\ & - etr\left[(\Phi\Phi_A^+ + \Phi_A\Phi^+)\Phi_Y\Phi_Y^+\right] \end{aligned} \tag{5.1}$$

where all coefficients are positive. The vacuum expectation value are

$$\langle\Phi_A\rangle_o = \begin{pmatrix} \nu_A \\ 0 \\ 0 \end{pmatrix}, \qquad \langle\Phi_B\rangle_o = \begin{pmatrix} \nu_B \\ 0 \\ 0 \end{pmatrix},$$
$$\langle\Phi_Y\rangle_o = \begin{pmatrix} 0 \\ 0 \\ \nu_Y \end{pmatrix}, \qquad \langle\Phi\rangle_o = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{\nu}{\sqrt{2}} \\ 0 & \frac{\nu}{\sqrt{2}} & 0 \end{pmatrix} \tag{5.2}$$

respectively.

After spontaneous breaking of symmetry, eight components of gauge fields become massive while the nineth one corresponding to photon remains massless.

Let us perform infinitesimal gauge transformation for the Higgs fields near their vacuum expectation values, which can be expressed as

$$\Phi'_{A,B} = \Phi_{A,B} + \frac{i}{2}\nu_{A,B}\begin{pmatrix} \xi^3 + \frac{\xi^8}{\sqrt{3}} \\ \xi^1 + i\xi^2 \\ \xi^4 + i\xi^5 \end{pmatrix} , \quad \Phi'_Y = \Phi_Y + \frac{i}{2}\nu_Y \begin{pmatrix} \xi^4 - i\xi^5 \\ \xi^6 - i\xi^7 \\ -\frac{2}{\sqrt{3}}\xi^8 - 2\theta \end{pmatrix}$$

$$\Phi' = \Phi + \frac{-i}{2\sqrt{2}}\nu \begin{pmatrix} 0 & \xi^4 + i\xi^5 & \xi^1 + i\xi^2 \\ \xi^4 + i\xi^5 & 2(\xi^6 + i\xi^7) & -\left(\xi^3 + \frac{\xi^8}{\sqrt{3}}\right) \\ \xi^1 + i\xi^2 & -\left(\xi^3 + \frac{\xi^8}{\sqrt{3}}\right) & 2(\xi^6 - i\xi^7) \end{pmatrix} \tag{5.3}$$

It is possible to eliminate eight components in Φ_A and Φ_Y by a suitable choice of ξ^j and θ after spontaneous symmetry breaking and get the remaining components as

$$\Phi_A = \begin{pmatrix} \nu_A + \phi^o \\ 0 \\ 0 \end{pmatrix} , \quad \Phi_B = \begin{pmatrix} \nu_B + \phi_1^o + i\phi_1'^o \\ \phi_2^- \\ \phi_3^+ \end{pmatrix} , \quad \Phi_Y = \begin{pmatrix} \chi^- \\ 0 \\ \nu_Y + \chi^o \end{pmatrix} ,$$

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\,\phi_{11}^o & \phi_{12}^+ & \phi_{13}^- \\ \phi_{12}^+ & \sqrt{2}\,\phi_{22}^{++} & \nu + \phi_{23}^o + i\phi_{23}'^o \\ \phi_{13}^- & \nu + \phi_{23}^o + i\phi_{23}'^o & \sqrt{2}\,\phi_{33}^{--} \end{pmatrix} \tag{5.4}$$

where ϕ^o, χ^o, ϕ_1^o, $\phi_1'^o$, ϕ_{23}^o and $\phi_{23}'^o$ are real and the others complex. All of them will be heavy Higgs bosons, since the Higgs potential has no additional higher global symmetry, no pseudo-goldstone appears after spontaneous breaking of symmetry. In other words, for given $\langle\Phi_A\rangle_o$ and $\langle\Phi_Y\rangle_o$, all degeneracy on the minimum of V are removed.

Φ_Y does not couple directly with fermions owing to the conservation of Y. $\phi_3^{\pm}$, ϕ_{11}^{o}, ϕ_{11}^{o*}, ϕ_{12}^{+}, ϕ_{12}^{-}, ϕ_{22}^{++}, ϕ_{22}^{--}, ϕ_{33}^{--} and ϕ_{33}^{++} do not couple with ordinary fermions owing to the conservation of weak strangeness S_w discussed below.

One local U(1) and on global U(1) symmetry remain unbroken. Their generators are the charge:

$$\hat{Q} = \hat{I}_3 - \sqrt{3}\,\hat{I}_8 + \hat{Y} \tag{5.5}$$

and the weak strangeness:

$$\hat{S}_w = \frac{2}{\sqrt{3}}\,\hat{I}_8 + \hat{Y} + \hat{S} \tag{5.6}$$

respectively, which are conserved quantum numbers. After sponteneous symmetry breaking, all physical particles are eigenstates of charge $\hat{Q}$ and weak strangeness $\hat{S}_w$ as shown below.

For fermions we must replace (5.5) and (5.6) by

$$\hat{Q} = \hat{I}_3^{(\pm 5)} - \sqrt{3}\,\hat{I}_8^{(\pm 5)} + \hat{Y} \tag{5.7}$$

$$\hat{S}_w = \frac{2}{\sqrt{3}}\,\hat{I}_8^{(\pm 5)} + \hat{Y} + \hat{S}^{(\pm 5)} \tag{5.8}$$

on account of the transformation properties of ψ and ψ^h introduced above. For example, ψ_ℓ, ψ_q and u can be expressed as

$$\psi_\ell = \begin{pmatrix} L^{o}_{1/2} + R^{o}_{-1/2} \\ L^{-}_{1/2} + R^{+}_{-1/2} \\ L^{+}_{-1/2} + R^{-}_{1/2} \end{pmatrix}, \quad \psi_q = \begin{pmatrix} L^{2/3}_{7/6} + R^{2/3}_{1/6} \\ L^{-1/3}_{7/6} + R^{5/3}_{1/6} \\ L^{5/3}_{1/6} + R^{-1/3}_{7/6} \end{pmatrix}, \quad u = \left(L^{2/3}_{1/6} + R^{2/3}_{7/6}\right) \tag{5.9}$$

where $L^Q_{S_w}$ and $R^Q_{S_w}$ denote the left-handed and the right-handed fermions with charge Q and weak strangeness S_w respectively.

After spontaneous symmetry breaking the mass terms of the fermions are derived from $\mathscr{L}_{FH}$. For ψ_ℓ, ψ_q and u, they have the form given in (4.3) and the masses are determined by the following expression

$$
\begin{aligned}
&\frac{1}{\sqrt{2}}\left(f_\ell v + f_{A\ell} v_A\right)\left(\bar{R}^-_{1/2} L^-_{1/2} + \bar{L}^-_{1/2} R^-_{1/2}\right) \\
+ &\frac{1}{\sqrt{2}}\left(f_\ell v - f_{A\ell} v_A\right)\left(\bar{R}^+_{-1/2} L^+_{-1/2} + \bar{L}^+_{-1/2} R^+_{-1/2}\right) \\
+ &\frac{1}{\sqrt{2}}\left(f_q v + f_{Aq} v_A\right)\left(\bar{R}^{-1/3}_{7/6} L^{-1/3}_{7/6} + \bar{L}^{-1/3}_{7/6} R^{-1/3}_{7/6}\right) \\
+ &\frac{1}{\sqrt{2}}\left(f_q v - f_{Aq} v_A\right)\left(\bar{R}^{5/3}_{1/6} L^{5/3}_{1/6} + \bar{L}^{5/3}_{1/6} R^{5/3}_{1/6}\right) \\
+ &\left(f_{Bu} v_B + f_{Au} v_A\right)\left(\bar{R}^{2/3}_{7/6} L^{2/3}_{7/6} + \bar{L}^{2/3}_{7/6} R^{2/3}_{7/6}\right) \\
+ &\left(f_{Bu} v_B - f_{Au} v_A\right)\left(\bar{R}^{2/3}_{1/6} L^{2/3}_{1/6} + \bar{L}^{2/3}_{1/6} R^{2/3}_{1/6}\right)
\end{aligned}
\tag{5.10}
$$

The charged fermions with $S_w = 1/2$ and $7/6$ are made light while those with $S_w = -1/2$ and $1/6$ very heavy by means of suitable combination of "f"s and "v"s. In addition, we assume that the coupling constants for ψ^h_ℓ, ψ^h_q and u^h are much larger than those for ψ_ℓ, ψ_q and u to make the masses of charged components of ψ^h_ℓ, ψ^h_q and u^h very heavy. Thus the lgiht fermions remaining after spontaneous symmetry breaking are

$$
\begin{aligned}
&\psi_\ell \to \begin{pmatrix} \nu_L + \nu_R \\ e_L \\ e_R \end{pmatrix}, \quad \psi_q \to \begin{pmatrix} u_L \\ d_L \\ d_R \end{pmatrix}, \quad u \to (u_R) \\
&\psi^h_\ell \to \begin{pmatrix} \nu^h_R + \nu^h_L \\ - \\ - \end{pmatrix}, \quad \psi^h_q \to \begin{pmatrix} - \\ - \\ - \end{pmatrix}, \quad u^h \to (-) \quad .
\end{aligned}
\tag{5.11}
$$

We note that there are four massless neutrinos appearing in the present model. However, as will be discussed below, there is no contradiction in comparison with experiments.

The charges and the weak strangeness of all bosons are given in Table II.

The mass terms of the vector gauge bosons are easily derived from (4.2). They are

$$\begin{aligned} &\frac{1}{2} g^2\left(|\nu_A|^2 + |\nu_B|^2\right)\left(W^+W^- + V^+V^- + \frac{2}{3} Z^{o2}\right) \\ &+ \frac{1}{2} g^2|\nu|^2\left(W^+W^- + V^+V^- + 4U^{++}U^{--} + \frac{2}{3} Z^{o2}\right) \\ &+ \frac{1}{2} g^2|\nu_Y|^2\left(V^+V^- + U^{++}U^{--}\right) \\ &+ \frac{1}{4} g^2|\nu_Y|^2\left(\frac{1}{\sqrt{3}} Z^o + \frac{1}{\sin\varphi} Z'^o\right)^2 \end{aligned} \tag{5.12}$$

where

$$\begin{aligned} W^\pm &= \frac{1}{\sqrt{2}} (A' \mp iA^2) , \quad V^\pm = \frac{1}{\sqrt{2}} (A^4 \pm iA^5) , \\ U^{\pm\pm} &= \frac{1}{\sqrt{2}} (A^6 \pm iA^7) , \quad Z^o = \frac{1}{2} (\sqrt{3} A^3 + A^8) , \\ Z'^o &= -\frac{1}{2} (A^3 - \sqrt{3} A^8) \sin\varphi + B\cos\varphi \end{aligned} \tag{5.13}$$

and

$$\sin\varphi = \frac{g}{\sqrt{g^2 + g'^2}} , \quad \cos\varphi = \frac{g'}{\sqrt{g^2 + g'^2}} \tag{5.14}$$

From (5.14) and (5.15) we obtain:

(1) The photon field

$$A = \frac{1}{2}(A^2 - \sqrt{3} A^8) \cos\varphi + B\sin\varphi \tag{5.15}$$

is massless as required.

(2) The masses of the charged vector bosons are

$$m_W^2 = \frac{1}{2} g^2\left(|\nu_A|^2 + |\nu_B|^2 + |\nu|^2\right)$$

$$m_U^2 = \frac{1}{2} g^2\left(|\nu_Y|^2 + 4|\nu|^2\right)$$

$$m_V^2 = \frac{1}{2} g^2\left(|\nu_A|^2 + |\nu_B|^2 + |\nu|^2 + |\nu_Y|^2\right) \qquad (5.16)$$

From (5.16) the inequality

$$m_W^2 < m_V^2 < m_W^2 + m_U^2 \quad , \quad m_U^2 < m_V^2 + 3m_W^2 \qquad (5.17)$$

holds. Let us introduce a new parameter

$$v \equiv \frac{|\nu_Y|^2}{|\nu_A|^2 + |\nu_B|^2 + |\nu|^2} = \frac{m_V^2}{m_W^2} - 1 \qquad (5.18)$$

which will be useful in the following.

The Z and Z' bosons are not eigenstates of the mass matrix. Let the true neutral vector bosons be Z_1 and Z_2, they are related to Z and Z' by a rotation

$$Z = \cos\alpha Z_1 + \sin\alpha Z_2$$

$$Z' = -\sin\alpha Z_1 + \cos\alpha Z_2 \qquad (5.19)$$

Diagonizing the mass matrix we have

$$m_{Z_1}^2 = m_Z^2\left[1 + \frac{v}{4}\left(1 - \frac{\sqrt{3}\,\mathrm{tg}\alpha}{\sin\varphi}\right)^2\right]$$

$$m_{Z_2}^2 = m_Z^2\left[\mathrm{tg}^2\alpha + \frac{v}{4}\left(\mathrm{tg}\alpha + \frac{\sqrt{3}}{\sin\varphi}\right)^2\right] \qquad (5.20)$$

where

$$m_Z^2 = \frac{4}{3} m_W^2 \cos^2\alpha \qquad (5.21)$$

and

$$\mathrm{tg}2\alpha = \frac{2\sqrt{3}\, v \sin\varphi}{3v - (4+v)\sin^2\varphi} \qquad (5.22)$$

We note that $\mathrm{tg}\alpha = \frac{1}{\sqrt{3}} \sin\varphi$, $m_{Z_1}^2 = m_W^2/\cos^2\theta_w$ and $m_U^2, m_V^2, m_{Z_2}^2 \to \infty$ in the limiting case $v \to \infty$ for arbitrary value of $\sin\varphi$. The neutral current $J_{N,\mu}$ between fermions can be written as $J_{N,\mu} = J_\mu^3 - \sin^2\theta_w J_\mu^{e.m.}$. Therefore, all observable results in this limiting case are exactly the same as those in Weinberg-Salam model. Another interesting limiting case is

$$\sin\varphi \ll 1 \quad , \quad \frac{\sin^2\varphi}{v} \ll 1 \qquad (5.23)$$

In this limiting case eq. (5.22) becomes

$$\mathrm{tg}\alpha = \frac{1}{\sqrt{3}} \sin\varphi \left(1 + \frac{4}{3v} \sin^2\varphi + O(\sin^4\varphi)\right) \qquad (5.24)$$

Substituting eq. (5.24) into (5.20) and (5.21), we obtain

$$m_{Z_1}^2 = m_Z^2 \left(1 + \frac{4}{9v} \sin^4\varphi + O(\sin^6\varphi)\right)$$

$$m_{Z_2}^2 = m_Z^2 \frac{3v}{4\sin^2\varphi} \left(1 + \frac{2}{3} \sin^2\varphi + O(\sin^4\varphi)\right)$$

$$\gg m_{Z_1}^2 \quad \text{for samll} \quad \frac{\sin^2\varphi}{v} \qquad (5.25)$$

and

$$m_Z^2 = \frac{4}{3} m_W^2 \left(1 - \frac{1}{3} \sin^2\varphi + O(\sin^4\varphi)\right) \qquad (5.26)$$

Comparing with the mass formula in Weinberg-Salam model

$$m_Z^2 = m_W^2 / \cos^2\theta_w \qquad (5.27)$$

We obtain to order $\sin^2\varphi$ that

$$\sin^2\theta_W = \frac{1}{4}\cos^2\varphi \le \frac{1}{4} \tag{5.28}$$

This is an encouraging result since recent experiments require that $\sin^2\theta_W \le \frac{1}{4}$. Of course, this result is model dependent, one obtains different values of $\sin^2\theta_W$ when different representations for fermions and Higgs are chosen. But in this model, 1/4 is the limit value for $\sin^2\theta_W$ and thus has special meaning.

6. Interactions between the Fermions and the Gauge Bosons

The gauge interaction Lagrangian for fermions can be written as

$$\begin{aligned}\mathscr{L} &= \bar{\psi}_\ell\gamma^\mu\left(\partial_\mu + \hat{A}_\mu^{(5)}\right)\psi_\ell + \bar{\psi}_\ell^h\gamma^\mu\left(\partial_\mu + \hat{A}_\mu^{(-5)}\right)\psi_\ell^h \\ &+ \bar{\psi}_q\gamma^\mu\left(\partial_\mu + \hat{A}_\mu^{(5)} + \frac{i}{2}g'\hat{Y}B_\mu\right)\psi_q + \bar{\psi}_q^h\gamma^\mu\left(\partial_\mu + \hat{A}_\mu^{(-5)} + \frac{i}{2}g'\hat{Y}B_\mu\right)\psi_q^h \\ &+ \bar{u}\gamma^\mu\left(\partial_\mu + \frac{i}{2}g'\hat{Y}B_\mu\right)u + \bar{u}^h\gamma^\mu\left(\partial_\mu + \frac{i}{2}g'\hat{Y}B_\mu\right)u^h\end{aligned} \tag{6.1}$$

We shall discuss the interactions between the light fermions and the gauge vector bosons first. For this purpose, the terms involving heavy fermions are neglected and the following substitutions are made in (6.1)

$$\psi_\ell \to \begin{pmatrix} \nu_L + \nu_R \\ e_L \\ e_R \end{pmatrix} = \psi'_\ell + \begin{pmatrix} \nu_R \\ 0 \\ 0 \end{pmatrix}, \quad \psi_q \to \begin{pmatrix} u_L \\ d_L \\ d_R \end{pmatrix} = \psi'_q, \quad u \to u_R,$$

$$\psi_\ell \to \begin{pmatrix} \nu_R^h + \nu_L^h \\ 0 \\ 0 \end{pmatrix}, \quad \psi_q^h \to \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \quad u^h \to 0 \tag{6.2}$$

As $\bar{L}\gamma^{\mu}R = \bar{R}\gamma^{\mu}L = 0$, one immediately see that $A_{\mu}[4,5,6,7]$ do not couple directly to either $\bar{\psi}_{\ell}'\gamma^{\mu}\psi_{\ell}'$ or $\bar{\psi}_{q}'\gamma^{\mu}\psi_{q}'$. It is also easily verified that

$$\begin{aligned}\bar{\psi}'\gamma^{\mu}A_{\mu}^{(5)}\psi' &= ig\bar{\psi}'\,\frac{\gamma^{\mu}}{2}\left(\gamma_5\hat{\lambda}_1A_{\mu}^1 + \hat{\lambda}_2A_{\mu}^2 + \gamma_5\hat{\lambda}_3A_{\mu}^3 + \gamma_5\hat{\lambda}_8A_{\mu}^8\right)\psi' \\ &= ig\bar{\psi}'\,\frac{\gamma^{\mu}}{2}\left(\hat{\lambda}_1A_{\mu}^1 + \hat{\lambda}_2A_{\mu}^2 + \hat{\lambda}_3A_{\mu}^3 + \hat{\lambda}_8'A_{\mu}^8\right)\psi' \qquad (6.3)\end{aligned}$$

where ψ' denotes either ψ_{ℓ}' or ψ_{q}', $\hat{\lambda}_i$, $i = 1,\ldots,8$ are the usual Gell-Mann matrices for SU(3) and

$$\hat{\lambda}_8' = \frac{1}{\sqrt{3}}\begin{pmatrix}1 & & \\ & 1 & \\ & & 2\end{pmatrix} \qquad (6.4)$$

is just the generator used by Ne'eman, Fairlie and others [4] in their graded SU(2|1) gauge group formulation. It occurs here simply as a result of the differences in the action of γ_5 on left-handed and right-handed fermions, i.e.,

$$\gamma_5 L = L \quad , \quad \gamma_5 R = -R$$

In terms of the physical gauge fields found in the previous section, the interactions with $W^{\pm}$ and the photon have the usual form with

$$e = \frac{1}{2}\, g\, \cos\varphi \qquad (6.5)$$

which indicates that $\sin^2\theta_W = \frac{1}{4}\cos^2\varphi$ in the charged current sector. This is in agreement with that obtained from the mass of Z_1 boson in the limiting case of small $\sin^2\varphi$.

The additional charged interaction involving ν_R is

$$-\frac{ig}{\sqrt{2}}\left(\bar{\nu}_R\gamma^{\mu}V_{\mu}^{+}e_R + \bar{e}_R\gamma^{\mu}V_{\mu}^{-}\nu_R\right) \qquad (6.6)$$

Since m_V is probably much heavier than m_W on account of experimental limitation, one may expect the contribution of V boson is much less than that of W boson in the low energy processes. But in the high energy range, the influence of such term might be observed experimentally.

There is another possibility for choice of leptons which gives the usual form of charged current for ordinary fermions. One may propose the existence of singlets s and s^h for leptons. The charge of s is neutral and the weak strangenesses are 1/2 for right-handed component and -1/2 for left-handed component respectively. ν_R will couple with s_L after spontaneous symmetry breaking and get heavy mass just similar to that for corresponding quarks. However, the coupling between ν_L and s_R can be chosen to be vanished by suitable choice of coupling constants among Higgs fields and leptons. In this case, the remaining light neutral leptons are ν_L, s_R, ν_R^h and s_L^h. Since s_R belongs to the singlet, it will decouple with any component of SU(3) gauge field. The charged current involving V corresponding to (6.6) should vanish in the low energy range and the lightest weak strange boson should be stable. One may expect to find in high energy experiment a heavy weak strange boson, perhaps, the double charged U boson.

The interaction Lagrangian of ψ' or ψ_q' with the neutral vector bosons has the form

$$ig\bar{\psi}'\gamma^\mu \left\{ Z_{1\mu}\left[\frac{1}{2\sqrt{3}}\,(2\hat{\lambda}_3 - \hat{Q} + \hat{Y})\cos\alpha + \frac{1}{2}\left(\sin\varphi\hat{Q} - \frac{1}{\sin\varphi}\hat{Y}\right)\sin\alpha\right] \right.$$

$$\left. + \ Z_{2\mu}\left[\frac{1}{2\sqrt{3}}\,(2\hat{\lambda}_3 - \hat{Q} + \hat{Y})\sin\alpha - \frac{1}{2}\left(\sin\varphi\hat{Q} - \frac{1}{\sin\varphi}\hat{Y}\right)\cos\alpha\right] \right\}\psi' \qquad (6.7)$$

In the limiting case of small $\sin\varphi$, this interaction can be greatly simplified. The effective Lagrangian for the interchange of a neutral vector bosons between fermions can be written as

$$\mathscr{L}_{\rm eff} = 4\frac{G_F}{\sqrt{2}}\left[J_{N_1\mu}J^{\mu}_{N_1} + J_{N_2\mu}J^{\mu}_{N_2}\frac{4}{9v}\right] \tag{6.8}$$

where

$$J_{N_1\mu} = J^3_\mu - \sin^2\theta_W J^{e.m.}_\mu - \frac{4}{3v}\left(1-4\sin^2\theta_W\right)J^Y_\mu \tag{6.9}$$

and

$$J_{N_2\mu} = J^Y_\mu\cos^2\theta_W + \left(1-4\sin^2\theta_W\right)\left(J^3_\mu - J^{e.m.}_\mu\right) \tag{6.10}$$

with

$$\sin^2\theta_W = \frac{1}{4}\cos^2\varphi$$

are two neutral currents coupled to the Z_1 and Z_2 bosons. In the case of $\sin^2\theta_W = 1/4$, the first neutral current has exactly the same form as in the simple gauge theory; the second one is a pure vector current for quarks which is difficult to observe owing to the interference with strong interaction. Recent experiments tell us that $\sin^2\theta_W$ is slightly less than 1/4, the present model provides a small correction (depending on the value v) in the neutral currents of fermions. This small effect could be measured by more accurate experiments.

The additional neutral interaction involving ν_R, ν^h_R and ν^h_L can be written explicitly as

$$\frac{i}{\sqrt{3}}\,g\left(-\bar{\nu}_R\gamma^\mu\nu_R - \bar{\nu}^h_L\gamma^\mu\nu^h_L + \bar{\nu}^h_R\gamma^\mu\nu^h_R\right)\left(Z_{1\mu}\cos\alpha + Z_{2\mu}\sin\alpha\right) \tag{6.11}$$

which is difficult to observe directly in the low energy range. When free Z_1 is observed in high energy processes, the branching ratio $Br = \Gamma(Z_1\to\nu\bar{\nu})/\Gamma(Z_1\to \text{all})$ may be used to check this model since Br should be four times of that predicted in usual model.

7. Weak Strangeness and Weak Strange Particles

The weak strangeness S_w is a conserved quantum number in this model. Owing to the conservation of lepton number and quark number, one may re-define the weak strangeness for fermions as

$$S_w = 0 \quad , \qquad \text{for} \qquad \psi'_\ell,\ \psi'_q,\ u_R,\ \nu^h_R,$$

$$S_w = -1 \quad , \qquad \text{for} \qquad \nu_R,\ \nu^h_L\ .$$

Since ν^h_R and ν^h_L only couple with neutral vector boson in the above approximation, they will be unimportant in the following discussion on weak strangeness. It is sufficient to consider only one weak strange lepton ν_R. The weak strange bosons are $V^\pm$ and $U^{\pm\pm}$ together with many kinds of weak strange Higgs as shown above. These bosons are heavy and probably heavier than $W^\pm$ and Z_1. The conservation of weak strangeness requires that:

(1) Weak strange particles can be produced only in pair.

(2) They do not couple directly with ordinary quarks and some of the single charged weak strange bosons can couple with electron and ν_R.

(3) They are weak decaying and ended in final state with the lightest weak strange particle.

It is interesting to discuss the decaying behavior of U since it might be the lightest weak strange bosons. If $m_U < m_W + m_V$, U might decay through the virtual V boson

$$U^{++} \longrightarrow W^+ + V^+$$

$$V^+ \longrightarrow e^+ + \nu_R \quad (\mu^+ + \nu_{\mu R}, \ldots)$$

$$W^+ \longrightarrow e^+ + \nu_L \quad (\mu^+ + \nu_{\mu L}, \ldots, u + \bar{d}, \ldots)$$

It is an electro-weak process of order two with the propagator of V boson.

If $m_U < m_W$, U^{++} will decay through the electro-weak process of order three with both W and V virtual. So it may appear as a long lived particle with double charge decaying possibly into two positively charged lepton.

If $m_U > m_V$, one may expect to discover V boson earlier than U boson. In this case, V boson will decay into electron and ν_R through the V+A current with a short life time

$$V^+ \rightarrow e^+ + \nu_R \ (\mu^+ + \nu_{\mu R}, \ldots)$$

Decay of V particle into ordinary hadrons is forbidden in the present model.

8. Model Involving Several Generations of Fermions

Three generations of leptons (ν_e, e), (ν_μ, μ) and (ν_τ, τ) have been discovered experimentally and it is probably the same for quarks. There might be symmetrical properties between various generations. An important problem is the Cabibbo mixing among various generations of quarks. This problem has been discussed by many authors [5].

It is easy to generalize present model to include several generations, for example, three generations of fermions by the method of Kobayashi and Maskawa [5]. Denote the ith generation of fermions by

$$\psi_{\ell i},\ \psi_{qi},\ u_i,\ \psi^h_{\ell i},\ \psi^h_{qi},\ u^h_i$$

Because of the degeneracy of various generations, we must consider all possible coupling term among these generations in $\mathscr{L}_{FH}$. For example, the term

$$f_{A\ell}\left(\bar{\psi}_\ell \Phi_A P_+ \psi_\ell + \bar{\psi}_\ell \Phi_A^+ P_- \psi_\ell\right)$$

must be replaced by

$$f_{A\ell ij}\left(\bar{\psi}_{\ell i} \Phi_A P_+ \psi_{\ell j} + \bar{\psi}_{\ell j} \Phi_A^+ P_- \psi_{\ell i}\right)$$

etc.

Let us denote the $Q = -1$ lepton, $Q = 2/3$ quark and $Q = -1/3$ quark of ith generation by e_i, u_i and d_i respectively. It is easy to derive the mass matrices among various generations from above generalization. They can be expressed simply as

$$m_{eij}\bar{e}_i e_j + m_{uij}\bar{u}_i u_j + m_{dij}\bar{d}_i d_j \tag{8.1}$$

where all mass matrices are Hermitian.

For leptons, we can introduce an unitary transformation to make the mass matrix diagonal and obtain

$$m_{eij}\bar{e}_i e_j \rightarrow m_e \bar{e}e + m_\mu \bar{\mu}\mu + m_\tau \bar{\tau}\tau \tag{8.2}$$

Since all neutrinos are massless, they will remain massless under the same unitary transformation. Of course, the kinetic energy term of leptons

$$\psi_{\ell i}\gamma^\mu\left(\partial_\mu + \hat{A}_\mu^{(5)}\right)\psi_{\ell i}$$

is invariant under such transformation. From this rearrangement of leptons, we conclude that for leptons:

(1) There exist three kind of charged leptons with various masses and one may identify them as e, μ and τ.

(2) There are three neutrinos corresponding to three charged leptons respectively. All neutrinos are massless.

(3) Three kinds of leptonic numbers can be introduced to distinguish three generations of leptons. They are conserved individually. There is no Cabibbo mixing appearing among leptons.

For quarks, the unitary transformations to diagonalize the mass matrices of u_i and d_i are different in general. So if we introduce an unitary transformation to make u_i diagonal

$$m_{uij}\,\bar{u}_i\,u_j \rightarrow m_u\,\bar{u}\,u + m_c\,\bar{c}\,c + m_t\,\bar{t}\,t \tag{8.3}$$

d_i will transform into d_i' (d',s',b') but remain undiagonal in general. The connection between d_i' (d',s',b') and diagonal d_i (d,s,b) is described by an unitary transformation, i.e.,

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = U \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{8.4}$$

There are nine parameters appearing in the 3×3 unitary matrix U. Owing to the arbitrariness of relative phases, five of them can be removed by suitable choice of relative phases among these six states. The remaining matrix with four parameters is the well-known Kobayashi-Maskawa general expression of Cabibbo mixing. Therefore, for quark:

(1) Both u_i and d_i get masses by Higgs mechanism in contrast to neutrinos which are massless.

(2) In general, u_i and d_i will not diagonalized simultaneously, it leads to the Cabibbo mixing.

(3) The Cabibbo mixing can be described by a 3×3 unitary matrix with four parameters.

Of course, all above discussions is easy to extend into the case with more generations of fermions.

9. Summary and Several Remarks

In the previous sections we proposed an electro-weak model in $SU(3) \times U(1)$. The main results of this model can be summarized as:

(1) The nature is left-right symmetrical before spontaneous symmetry breaking. The left-right asymmetry is caused by the spontaneous symmetry breaking.

(2) This model is anomaly free exactly.

(3) As a limiting case, it gives the same results as those of the Weinberg-Salam model and is in agreement with present experiments. The Weinberg angle θ_W is bound by the relation $\sin^2\theta_W \leq \frac{1}{4}$ and the neutral current for ordinary fermions reduces to that of Weinberg-Salam model exactly in the limit of $\sin^2\theta_W = \frac{1}{4}$.

(4) There are some small deviation from the Weinberg-Salam model about the predictions in neutral currents, where $\sin^2\theta_W$ is slightly less than 1/4 which can be verified by more accurate experiments in the near future.

(5) A new conserved quantum number S_W, called the weak strangeness, is introduced in the present model. Furthermore, there exist many weak strange particles with non vanishing weak strangeness. Three of them, the right-handed neutrino ν_R, the vector bosons V and U are more interesting, because they might be lighter than the others. The weak

strange particles can be producted only in pairs and decay through the electro-weak interaction with ν_R in the final states. These predictions are the common results of a class of models analogous to the present one.

(6) The number of fermions is four times the number of the known fermions. After spontaneous symmetry breaking, about three quarters of them get heavy masses and cannot be observed in the present energy accelerator experiments. Since all four kinds of neutrinos are massless, the influence of them ought to be considered. ν_R^h and ν_L^h couple only with neutral vector boson and give no influence in charged current. They are very difficult to be discovered experimentally. The right-handed neutrino ν_R can couple with V and e_R via the V+A current. The coupling constant of this vertex is the same as that for W boson. However, since both the vertexes of W and V will give contribution to the decay of muon

$$\mu^- \rightarrow \nu_{\mu L} + W^- \;\; \llcorner\!\rightarrow e^- + \tilde{\nu}_{eL} \qquad\qquad \mu^- \rightarrow \nu_{\mu R} + V^- \;\; \llcorner\!\rightarrow e^- + \tilde{\nu}_{eR}$$

one may estimate the mass ratio m_V/m_W by the decay parameters for muon $|g_A/g_V| = 0.85 {+0.33 \atop -0.11}$ and $\phi = 180^\circ \pm 15^\circ$ obtained experimentally. It seems that if m_V/m_W is of the order 3 or more, there is no contradiction with the present experiments. More accurate measurement of decay parameters for muon is important for estimation of the mass of V boson in the present model. Of course, the existence of four kinds of neutrinos might be observed in higher energy experiments.

In addition, for the another choice of leptons discussed in Section 6 the V+A charged current involving V should vanish in the low energy range and the lightest weak strange boson should be stable.

(7) This model can be extended to include several generations of fermions. The mixing among various generations of fermions appears naturally after spontaneous symmetry breaking and leads to the well-known Kobayashi-Maskawa description of the Cabibbo mixing.

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APPENDIX A

The Form of Higgs Potential

The second order term in potential V for a Higgs field Φ is unique and can be expressed as

$$\mathrm{tr}(\Phi\Phi^+)$$

The number of the fourth order term is restricted by the following theorem.

<u>Theorem</u>: If there are m irreducible representations with various transformation properties decomposed from the direct production $\Phi \times \Phi^+$ for a given irreducible representation Φ of an unitary group G, the number of independent invariant terms constructed by four Φ and Φ^+ should be less than m.

<u>Proof</u>: Let $\Phi = (\phi_1,...,\phi_n)$ be a n-dimensional representation of unitary group G. There is only one invariant of second order and we denote it by

$$I_o = \sum_{i=1}^{n} \phi_i \phi_i^*$$

Assume the direct product $\Phi \times \Phi^+$ is decomposed into m irreducible representations $D_1,...,D_m$ with dimensions $N_1,...,N_m$ respectively. We use I_{N_j} to denote the fourth order invariants tr $D_j D_j^+$ and define the normalization of I_{N_j} to be N_j, i.e., I_{N_j} is the sum of the squares of magnitudes of all component- appearing in D_j. Then we have

$$\sum_{j=1}^{m} N_j = n^2$$

$$I_1 = \frac{1}{n} I_o^2 , \quad (N_1 = 1)$$

and

$$\sum_{j=1}^{m} I_{N_j} = \sum_{i=1}^{n}\sum_{k=1}^{n} (\phi_i\phi_k^*)(\phi_i\phi_k^*)^* = I_o^2 = nI_1$$

thus one can use this relation to eliminate one of the invariants and only m-1 of I_{N_j}, $j = 1,\ldots,m$ or less are linear independent.

According to this theorem, $\Phi^{(3)}$ has an unique fourth order invariant and $\Phi^{(6)}$ has two.

In the present model four Higgs multiplets are introduced. Their properties are listed in following Table:

	$\Phi_A^{(3)}$	$\Phi_B^{(3)}$	$\Phi_Y^{(3)}$	$\Phi^{(6)}$
Dimension	3	3	3	6
Y	0	0	-1	0
S	-1/3	-1/3	5/3	-1/3
P	A	B		

First, we discuss the minimum of self interaction potential for a three dimensional Higgs field $\Phi^{(3)}$ (denoted by Φ)

$$V = -a\Phi^+\Phi + b(\Phi^+\Phi)^2 \qquad (A.1)$$

The minimum takes place at

$$\Phi^+\Phi = \frac{a}{2b} \qquad (A.2)$$

so it determines only the magnitude of the vector and there is a degeneracy along the relative direction and the phase of the components.

If there are two three dimensional Higgs fields Φ and Φ', both Φ and Φ' have self interacting potentials of form (A.1) and get minima at

$\Phi^+\Phi = a/2b$ and $\Phi'^+\Phi' = a'/2b'$ respectively. Now we introduce some additional term to generate the correlation between Φ and Φ' and reduce the degeneracy. If the quantum number Y of Φ is different to that of Φ', the coupling terms allowed by Y conservation are

$$c\Phi^+\Phi\Phi'^+\Phi' + c'\Phi^+\Phi'\Phi'^+\Phi \tag{A.3}$$

It is interesting for us to discuss the second term since it can reduce the degeneracy of the minimum. This term can be expressed as

$$c' X^2 X'^2 |S|^2 \tag{A.4}$$

where

$$X = \sqrt{\Phi^+\Phi} \quad , \qquad X' = \sqrt{\Phi'^+\Phi'} \quad , \qquad \Phi^+\Phi' = XX'S \quad \text{and} \quad |S|^2 \le 1$$

Thus the minimum takes place at

$$|S|^2 = 0 \quad , \quad \text{when} \quad c' > 0 \quad ;$$

$$|S|^2 = 1 \quad , \quad \text{when} \quad c' < 0 \quad .$$

In the present model we select $c' > 0$ for $\Phi = \Phi_A$ and $\Phi' = \Phi_Y$ in order to make the minimum to take place at different components for Φ_A and Φ_Y, i.e., the not vanishing vacuum expectation values take place at the first component of Φ_A and the third component of Φ_Y.

Now we turn to the correlation between Φ_A and Φ_B. They have the same quantum number $Y = 0$. We hope that the not vanishing vacuum expectation values of this two triplets take place at same component, i.e., the real part of the first component. A coupling term of power two may be introduced for this purpose. It is

$$d\left(\Phi_A^+\Phi_B + \Phi_B^+\Phi_A\right) \tag{A.5}$$

where d is real. With the notation used above it can be expressed as

$$2d\, X_A X_B \,\mathrm{Re}\, S \tag{A.6}$$

The minimum takes place at

$$\mathrm{Re}S = -1 \quad , \quad \text{when} \quad d > 0 \quad ;$$

$$\mathrm{Re}S = +1 \quad , \quad \text{when} \quad d < 0 \quad .$$

Since $S = e^{i\alpha}\cos\theta$ ($|\alpha| \le \pi/2$), $\mathrm{Re}S = \mp 1$ means that the not vanishing vacuum expectation values of Φ_A and Φ_B take place at the same component with relative phase π or 0 respectively. Thus, if the vacuum expectation value of Φ_A is real, that of Φ_B should be real too.

The self interaction potential of $\Phi^{(6)}$ can be written as

$$V = -a\,\mathrm{tr}\,\Phi^+\Phi + b(\mathrm{tr}\,\Phi^+\Phi)^2 + b'\,\mathrm{tr}\,\Phi^+\Phi\Phi^+\Phi \tag{A.7}$$

In the case of $b' = 0$, the minimum takes place at $\mathrm{tr}\,\Phi^+\Phi = a/2b$ and is degenerate for all six components of Φ.

A coupling term

$$e\,\mathrm{tr}\left[(\Phi\Phi_A^+ + \Phi_A\Phi^+)\,\Phi_Y\,\Phi_Y^+\right] \tag{A.8}$$

can be adopted to remove the degeneracy. If the not vanishing vacuum expectation values of Φ_A and Φ_Y take place at the real parts of the first component of Φ_A and the third component of Φ_Y respectively, (A.8) can be written simply as

$$-\frac{1}{2}\,e\phi_1\chi_3^2\,\mathrm{Re}\,\phi_{23} \tag{A.9}$$

The minimum takes place at $\phi_{11} = \phi_{22} = \phi_{33} = \phi_{12} = \phi_{13} = \mathrm{Im}\phi_{23} = 0$, the sign of ϕ_{23} depends on e and ϕ_1.

From the above discussion, we may construct a self interaction potential of Higgs fields to generate the expected spontaneous symmetry breaking. It has the form

$$\begin{aligned} V = & -a_A \Phi_A^+ \Phi_A + b_A \left(\Phi_A^+ \Phi_A\right)^2 - a_B \Phi_B^+ \Phi_B + b_B \left(\Phi_B^+ \Phi_B\right)^2 \\ & - a_Y \Phi_Y^+ \Phi_Y + b_Y \left(\Phi_Y^+ \Phi_Y\right)^2 - a\,\mathrm{tr}\,\Phi^+ \Phi + b\left(\mathrm{tr}\,\Phi^+ \Phi\right)^2 \\ & + c\Phi_A^+ \Phi_Y \Phi_Y^+ \Phi_A - d\left(\Phi_A^+ \Phi_B + \Phi_B^+ \Phi_A\right)^2 \\ & - e\,\mathrm{tr}\left[\left(\Phi \Phi_A^+ + \Phi_A \Phi^+\right) \Phi_Y \Phi_Y^+\right] \end{aligned} \tag{A.10}$$

where all coefficients are positive. The vacuum expectation values of Higgs fields are

$$\langle \Phi_A \rangle_o = \begin{pmatrix} \nu_A \\ 0 \\ 0 \end{pmatrix} , \qquad \langle \Phi_B \rangle_o = \begin{pmatrix} \nu_B \\ 0 \\ 0 \end{pmatrix} ,$$

$$\langle \Phi_Y \rangle_o = \begin{pmatrix} 0 \\ 0 \\ \nu_Y \end{pmatrix} , \qquad \langle \Phi \rangle_o = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \nu/\sqrt{2} \\ 0 & \nu/\sqrt{2} & 0 \end{pmatrix}$$

respectively. The values of ν_A, ν_B, ν_Y and ν depend on the coefficients in (A.10).

APPENDIX B

The Connection Between the Helicity Mixed Representation and the Usual Notation

In models of electro-weak interaction we deal with left-handed and right-handed fermions separately. One simple way to realize this requirement is to adopt the chiral group. In order to preserve the left-right symmetry before spontaneous symmetry breaking one may adopt the direct product of two chiral groups. Another attractive possibility considered in our model is to require the group to be left-right symmetrical, i.e., the left-handed and right-handed components are conjugate to each other in their representations. For example, if the left-handed components form a $\underline{3}$ representation of SU(3) group, we use a $\underline{3}^*$ representation (conjugate representation of $\underline{3}$) to describe the corresponding right-handed components.

For simplicity we discuss the SU(3) description for the first generation of leptons in our model. For triplets

$$\begin{array}{cccc} \psi_L & \psi_R & \psi_R^h & \psi_L^h \\ \underline{3} & \underline{3}^* & \underline{3} & \underline{3}^* \end{array} \tag{B.1}$$

are introduced, where ψ^h is the helicity conjugate of ψ as shown in Section 2. We note that ψ_R^h has the same transformation property as ψ_L under SU(3). Using the usual notations their transformation properties under SU(3) can be expressed as

$$\begin{aligned} \psi_L \to \psi_L' &= U\psi_L \quad , & \psi_R \to \psi_R' &= U^*\psi_R \quad , \\ \psi_R^h \to \psi_R^{h'} &= U\psi_R^h \quad , & \psi_L^h \to \psi_L^{h'} &= U^*\psi_L \quad , \end{aligned} \tag{B.2}$$

where $U = U^{(+)}(\xi_j(x))$ as shown in (2.3) which is the usual transformation matrix of SU(3) group. Using (B.2) one may discuss our model in the usual notation. If we use the notation given in Section 2, from (3.2) the transformation (B.2) can be rewritten as

$$\psi_L \to \psi_L' = U^{(+)}\psi_L \quad , \qquad \psi_R \to \psi_R' = U^{(-)}\psi_R \quad ,$$
$$\psi_R^h \to \psi_R^{h'} = U^{(+)}\psi_R^h \quad , \qquad \psi_L^h \to \psi_L^{h'} = U^{(-)}\psi_L^h \quad , \tag{B.3}$$

or simply

$$\psi \to \psi' = U^{(5)}\psi \quad , \quad \left(\psi = \psi_L + \psi_R\right) \quad ,$$
$$\psi^h \to \psi^{h'} = U^{(-5)}\psi^h \quad , \quad \left(\psi^h = \psi_L^h + \psi_R^h\right) \quad . \tag{B.4}$$

This is just the SU(3) part of (2.2) for leptons. This means that our description is consistent with the usual one.

Since both ψ_L and ψ_R are $\underline{3}$ representations of SU(3), they can couple to scalar multiplets of either $\underline{3}$ or $\underline{6}^*$ only. Both $\underline{3}^*$ scalars and $\underline{6}$ scalars can be expressed as a field with two SU(3) indexes ϕ_{ij} with $\phi_{ij} = -\phi_{ji}$ for $\underline{3}^*$ and $\phi_{ij} = \phi_{ji}$ for $\underline{6}$. In the usual notation ϕ_{ij} transforms as

$$\phi_{ij} \to \phi_{ij}' = U_{ii'}U_{jj'}\phi_{i'j'} \tag{B.5}$$

Using $\tilde{U}^{(+)} = U^{(-)+}$ given in (3.2), it becomes

$$\phi_{ij}' = U_{ii'}\phi_{i'j'}\tilde{U}_{j'j} = U_{ii'}^{(+)}\phi_{i'j'}U_{j'j}^{(-)+}$$
$$= \left[U^{(+)}\,\Phi\,U^{(-)+}\right]_{ij}$$

This is just the SU(3) parts of (3.1) with Φ^+ instead of Φ.

The use of the helicity mixed representation provides the following conveniences: It describes both ψ_L of $\underline{3}$ representation and ψ_R of $\underline{3}^*$ representation in a unified formula simultaneously and then most of formulas can be expressed in the matrix form simply. It reflects the left-right symmetry naturally and concisely and gives the close connection between ψ_L and ψ_R, especially the mechanism to get mass after spontaneous symmetry breaking in a simple way.

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TABLE I

Case	a	α
I	1 3 4 6 8	2 5 7
II	1 3 5 7 8	2 4 6
III	2 3 4 7 8	1 5 6
IV	2 3 5 6 8	1 4 7
V	1 2 4 5	3 6 7 8
VI	1 2 6 7	3 4 5 8
VII	4 5 6 7	1 2 3 8

TABLE II

	γ	Z_1	Z_2	W^-	W^+	V^-	V^+	U^{--}	U^{++}
Q	0	0	0	-1	1	-1	1	-2	2
S_w	0	0	0	0	0	1	-1	1	-1

	χ^o	χ^-	χ^+	ϕ^o	ϕ_1^o	$\phi_1'^o$	ϕ_2^-	ϕ_2^+	ϕ_3^-
Q	0	-1	1	0	0	0	-1	1	-1
S_w	0	1	-1	0	0	0	0	0	1

	ϕ_3^+	ϕ_{23}^o	$\phi_{23}'^o$	ϕ_{13}^-	ϕ_{13}^+	ϕ_{12}^-	ϕ_{12}^+	ϕ_{11}^{o*}	ϕ_{11}^o
Q	1	0	0	-1	1	-1	1	0	0
S_w	-1	0	0	0	0	1	-1	1	-1

	ϕ_{22}^{--}	ϕ_{22}^{++}	ϕ_{33}^{--}	ϕ_{33}^{++}
Q	-2	2	-2	2
S_w	1	-1	1	-1

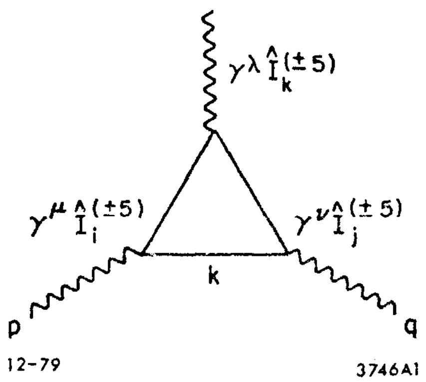
$\gamma^\lambda \hat{I}_k^{(\pm 5)}$
$\gamma^\mu \hat{I}_i^{(\pm 5)}$
$\gamma^\nu \hat{I}_j^{(\pm 5)}$
k
p
q
12-79
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Fig. 1