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ERRATUM

A SIMPLE MODEL OF THE GROUND STATE OF QUANTUM CHROMODYNAMICS*

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Please replace Table II with the following corrected Table II.

TABLE II. Hadron Mass Spectrum for Two Values of "b"

CASE I	Particle	π	ρ	N	Δ
$b = .65$	M/A	0	1.84	2.16	2.86
$\bar{x} = 2.21$	M(GeV)	0	.80	.94	1.25
$\Lambda = .436$ GeV	R(GeV^{-1})	3.9	4.7	5.2	5.5
$B^{1/4} = .145$ GeV $= .332\ \Lambda$	α_s	1.5	1.7	1.9	2.0

CASE II	Particle	π	ρ	N	Δ
$b = 1.04$	M/A	0	3.01	4.21	5.09
$\bar{x} = 2.49$	M(GeV)	0	.67	.94	1.14
$\Lambda = .223$ GeV	R(GeV^{-1})	3.4	4.3	5.0	5.3
$B^{1/4} = .144$ GeV $= .645\ \Lambda$	α_s	.83	.97	1.1	1.1

NOTE: Ground state masses of hadrons composed of "bare" massless up and down quarks, for two values of "b". The momentum spread of the valence quarks is parameterized by $\bar{x}$ and this is adjusted to make $m_\pi = 0$. Λ is taken to fit the nucleon mass, .94 GeV.

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I am indebted to Carleton DeTar and Dale Izatt for bringing my attention to the errors in the original of the table.

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ABSTRACT

A proposal for the form of the ground state wave function of quantum chromodynamics is made. It is shown to lead to the phenomenology of the MIT Bag Model. The parameters of this model are related to the fundamental scale parameter of QCD.

I. INTRODUCTION

There have been many ideas about the ground state wave function of quantum chromodynamics. One common feature has been a state which involves some sort of condensed phase with color magnetic properties.[1] Here I shall suggest a version of this which is extremely simple, but which leads to a quantitative model, in this case, the "static" bag model with some additional features, not the least of which is a relationship between the ad-hoc bag constant "B" and the scale of the running coupling constant of QCD.

The discussion will proceed in the following order. First, I will review the main features of the static bag model with particular reference to those aspects which are universal for all hadrons. Next, I will make a suggestion of how the strong coupling regime of QCD may be handled quantitatively, and show how the MIT Bag Model evolves from it. I will then discuss how the large N (equals number of quark colors) limit appears. I will briefly allude to the inclusion of light quarks. Finally, I will, also briefly, discuss how the model works phenomenologically with a few examples, and make the conclusions.

II. STATIC BAG

The bag[2,3] has provided a reasonably simple and successful model of hadron structure. Colored quark constituents are confined; all hadrons are composite, color singlet states. These results are obtained in a natural way by the enforcement of simple boundary conditions on the constituent wave functions at the surface of the bag. In addition, the confinement is associated with a term in the energy of the hadron of the form BV, where V is the volume occupied by the valence quark wave functions and $B \sim 55$ MeV/f^3 or $B^{1/4} = .145$ GeV. B is the same for all particles. Finally, the interior of the bag is assumed to be described by the perturbative QCD vacuum, that is, quarks interact within a bag by ordinary, perturbative QCD.

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In the more elaborate version of the bag model[3] used to compute the spectrum of light quark hadrons, an additional term of the form $-Z/R$ was added to the energy on an ad hod basis (R is the radius of the bag). As for the volume term, this term was taken to be the same for all hadrons, and the parameter Z determined by a fit to the spectrum, which gave $Z \sim 1.8$. It was subsequently realized that a contribution to the mass of this form is associated with a center of mass effect.[4] Since the bag is a localized hadron, it is not a momentum eigenstate. If it is assumed that the cost of localization is associated with the total momentum of the valence quarks, a correction to the mass equal to $-Z/R$ with $Z \sim .75$ and independent of the number of quarks is obtained. Thus, if this is removed from the Z/R term a universal contribution to the energy of all hadrons which is equal to

$$E^{o} = BV - Z'/R \qquad (2.1)$$

with $Z' \sim 1$, and $B^{1/4} = .145$ GeV, has been found to yield the best phenomenology. The energy E^{o} can be thought of as an "inside" vacuum energy. One may note that the principal effect of the Z'/R term has been to lower the energy difference between the vacuum inside a bag, and the true vacuum outside. At the same time, the confinement "pressure"

$$p^{o} = \partial E^{o}/\partial V = B + Z'/4\pi R^{4} \qquad (2.2)$$

has been left about the same. In practice, for a typical state where $R = 5\ \mathrm{GeV}^{-1}$, we find a substantial cancellation in the energy; $(4\pi/3)BR^3 = .23$ GeV, whereas, $-Z'/R = -.20$ GeV. At the same time the integrated pressures, $4\pi R^2 p$, are $4\pi BR^2 = .14\ \mathrm{GeV}^2$, and $Z'/R^2 = .04\ \mathrm{GeV}^2$, so Z'/R^2 is a small part of the total.

III. A MODEL OF THE QCD VACUUM

Here I shall try to relate some of these bag ideas to a proposal for the form of the ground state wave function of QCD. I shall show that the ad hoc "inside" vacuum energy is related to a simple ansatz for the form of this wave function. I will derive an improved form for this energy, and obtain an expression for the constant B.

QCD involves no parameters, only a scale Λ, which gives the variation of the running coupling constant in arbitrary units. Hence $B^{1/4}$ should come out proportional to Λ, with a specified numerical coefficient.

Several authors have suggested ideas about the ground state of QCD.[1] The proposal most closely related to the one which will be developed here is that of a Bose condensate with color magnetic properties. I will suggest a model for such a condensate which permits one to make quantitative and manifestly gauge invariant calculations.

In an asymptotically free theory, for momentum changes which are large on the scale Λ, the interactions are weak. For changes small in comparison to Λ, the interactions are strong. It is reasonable that these separate regions be treated by distinct approximations. It is obvious that the weak coupling domain should be treated perturbatively. The strong interaction is presumably associated with confinement. Confinement should mean that it costs a lot of energy to separate colors over distances long in comparison to $1/\Lambda$.

One would then believe that in QCD ground state wave functional colors are not so separated, that is, the magnitude of the wave functional should be negligible if it corresponds to a configuration of separated colors. This suggests the use of boundary conditions, viewed as a strong coupling approximation, to define a trial wave functional in which color separation is absolutely forbidden. The use of gauge invariant boundary conditions to handle the strong coupling features, seems particularly useful in a non-Abelian gauge theory. At the same time, perturbation theory should be used in the domain of its validity. Thus, start with the fields in a box with volume V, which is subdivided into N smaller equal boxes with volume $V_0 = V/N$. One may take the small boxes, small enough to permit the use of perturbative QCD.[5] That is, each small box contains a perturbative vacuum. On the walls of the small boxes, color confining boundary conditions are imposed;

$$\hat{n} \cdot \vec{E}_a = 0, \quad \hat{n} \times \vec{B}_a = 0 \quad . \tag{3.1}$$

These are gauge invariant boundary conditions which specify a complete set of operators for each box. That is, together with the use of perturbation theory, we have defined a vacuum wave function for each small box, and hence also for the large box. I shall briefly discuss the addition of quarks to the boxes in section V. Their introduction can be made in a straightforward way. The imposition of color confining boundary conditions means that color does not flow between boxes. Color separation in the large box is ruled out over distances longer than the scale defined by the size of the small boxes. This would be expressed by the absence of correlations in color density fluctuations in distinct boxes. The boundary conditions (3.1) imply that a thin layer of induced color magnetic poles (and currents) lie "between" the boxes (that is, on the surfaces). The effect of this on the energy of the boxes is expressed in the dependence on the size of the box of the perturbative energy associated with each box. Let E_0 and V_0 be the energy and volume, respectively, of the small boxes. The total energy density is then,

$$E/V = E_0 / V_0 \quad . \tag{3.2}$$

The field energy associated with each box may be calculated using the following method.

Consider the small boxes. Each is surrounded by several neighbors. If one takes quasi-spherical Wigner-Seitz boxes, each would have fourteen neighbors. No matter how the large box is subdivided, each small box will be surrounded by many neighbors each containing a perturbative vacuum. As the walls are moved, the total number of modes in the large box is left unchanged. We assume that one may approximate the energy of each small box by calculating the effect on the zero point energy stored in the perturbative vacuum of a shell enclosing a sphere with radius R in empty space. One now has the classic, Casimir[6] stress problem first estimated for a sphere by

Boyer.[7] The term in the energy which depends on the radius R was obtained recently[8,9] with great accuracy with the result,

$$E_{QED} = a_{QED}/R \ , \qquad a_{QED} = .04618 \ .$$

Although E was calculated in QED for a conducting sphere ($n \cdot E = 0$, $n \times B = 0$), the symmetry of the free Maxwell equations under the interchange $E \to B$, $B \to -E$, means that the energy with the boundary conditions (3.1) is the same. Since there are eight vector fields in QCD, the magnitude will be eight times larger, that is

$$E_{QCD} = a/R \ , \qquad a = .3694 \ . \tag{3.3}$$

It is important to discuss briefly the physical basis of this result. In the case of the Casimir stress the computation is made by introducing a cut off on the frequency of the fields. This cut off may be imagined to be of the order of the plasma frequency ω_p associated with the material of the metal boundaries. The metal excludes fields with lower frequencies, that is, the boundary condition is maintained by the dynamics of the metal. Since the dependence of the energy of the shell on its radius is insensitive to the cut off, as long as $1/R \ll \omega_p$, the stress on the shell can be computed with the aid of the boundary condition. The boundary condition expresses the effect of the strong coupling between the field fluctuations and the induced currents in the metal. It is important to stress that the finite result (3.3) depends upon a cancellation between cut off dependent terms which came from short wave length fluctuations localized near the inside and outside of the spherical shell. That is, the energy (3.3) should be regarded as being closely associated with the surface carrying the induced charges and currents.

In the QCD problem one may have a similar picture. The difference is that in the QCD case, the boundary conditions correspond to a screening by induced color magnetic poles and currents; these sources are present in the non-Abelian gauge fields. They will be produced automatically if the ground state can lower its energy with them present. The boundary dependent terms represent the field energy associated with these sources. However, one may see that if there is only the term indicated in (3.3), then

$$E/V = E^o/V^o = a/(4\pi/3)R^4 \ . \tag{3.4}$$

The lowest energy comes with one large box, i.e., $R = \infty$.

It is in the first order that the effects of the non-Abelian gauge theory within each box become apparent. It has been emphasized by several people that the Feynman diagrams indicated in Fig. 1 lead to an attractive interaction with gluons in a color singlet state. This suggests a pairing instability in the QCD vacuum and the

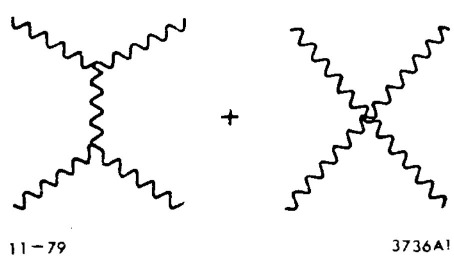

Fig. 1. Diagrams which lead to an attractive interaction between colorless pairs of gluons.

formation of a condensate with color screening properties. In the present case, all color singlet states in the vacuum will be paired in each box. (In lowest order we have a "pairing" effect, in higher orders we get multiple gluon effects). Since the gluons attract each other, when they are confined in a box the attraction should be enhanced, that is, increase as $R \to 0$. Thus, one expects a finite term of the form

$$- (b/R)\alpha_s \quad , \tag{3.5}$$

with $b > 0$. This energy corresponds to the sum of Feynman diagrams indicated in Fig. 2.

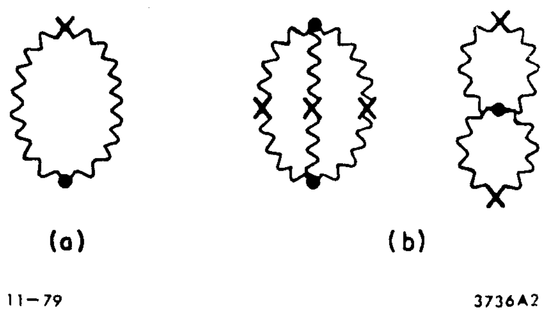

Fig. 2. (a) Diagram corresponding to the term (3.4) in the energy. The x indicates the vector field propagator for all space, with the boundary condition (3.1) imposed on the surface of a sphere. (b) Diagrams corresponding to the energy in the next order.

On including the higher order effects which transform the perturbative coupling constant into a running coupling constant (3.5) becomes

$$- (b/R)\ \alpha_s\ (\Lambda R) \tag{3.6}$$

where Λ is the basic scale of QCD. Unfortunately, the diagrams in Fig. 2b are difficult to evaluate, and so at present the numerical value of b has not been obtained. Thus, the energy per unit volume, taken to second order is,

$$\frac{E}{V} = \frac{1}{\frac{4\pi}{3} R^3}\left(\frac{a}{R} - \frac{b}{R}\alpha_s(R\Lambda)\right) + \rho_0 \tag{3.7}$$

with

$$a = .3694 \qquad b = ?\ (>0) \tag{3.8}$$

and where ρ_0 is the (divergent) boundary independent vacuum energy. As a consequence of asymptotic freedom, as $R \to 0$, $E/V \sim +1/R^4$. However, as R increases, $\alpha_s(\Lambda R)$ also increases and when R is such that $\alpha_s > a/b$, the vacuum subdivided into boxes has an energy density below ρ_0. As $R \to \infty$, the boundary dependent term vanishes, so (3.7) has a minimum. Thus, the subdivided vacuum has its lowest energy for "boxes" with a size given by the solution of

$$\frac{\partial}{\partial R}\left(\frac{1}{R^4}\left\{a - b\,\alpha_s\,(R)\right\}\right)_{R = R_0} = 0 \quad . \tag{3.9}$$

Boxes with this size shall be called "empty bags".

It is now possible to consider the excited states of this system, that is, the hadron spectrum. For convenience let

$$e(R) = \frac{1}{R}\left\{a - \alpha_s(R\Lambda)b\right\} \quad . \tag{3.10}$$

One may now imagine an excitation of this ground state which corresponds to locally exciting the system in one of the boxes. It will be convenient to call the valence energy $e_V(R)$. The box with the excited modes is taken to have a size R. For simplicity, the excited box is assumed to have the same shape as the empty bags. At the same time, assume that the remaining empty bags change their size to R_0'. With V as the total volume, the number of empty bags will be $\left(V - V(R)\right) / \left(V(R_0')\right)$, and the total energy is therefore

$$E = [\rho_0 V(R) + e(R) + e_V(R)] + \frac{V - V(R)}{V(R_0')} [\rho_0 V(R_0') + e(R_0')]$$

$$= \rho_0 V + e(R) + e_V(R) + \frac{V - V(R)}{V(R_0')} e(R_0') \tag{3.11}$$

The minimum occurs with $R_0' = R_0$, as before, and at the minimum with respect to R of

$$E_{BAG}(R) = e(R) + e_V(R) - V(R) \frac{e(R_0)}{V(R_0)} \quad . \tag{3.12}$$

This is recognized as the static bag model with

$$B = \frac{-e(R_0)}{V(R_0)} = - \frac{1}{\frac{4\pi}{3} R_0^4} \left(a - \alpha_s(R_0) b\right) \quad . \tag{3.13}$$

Since $e(R_0)$ at the minimum given by (3.9) is negative, B, is of course positive.

Using this simple model for the vacuum, an effective "inside" vacuum energy equal to

$$B \frac{4\pi}{3} R^3 + \frac{1}{R} \left(a - \alpha_s(R) b\right) = E(R) \tag{3.14}$$

has been obtained. The empty bags with fixed sizes outside provide the pressure B, and (3.13) together with (3.9) determines B in terms of the scale parameter of QCD. The model has also provided an extra term which has roughly the same phenomenological effect as the ad hoc $-Z/R$ term. This is because for $R > R_0$, the second term in (3.14) is negative, and this acts to reduce the cost of "drilling" the hole in the vacuum occupied by the valence particles. Indeed, not only does (3.14) vanish when $R = R_0$, but its derivative is also zero at $R = R_0$, that is, the "inside" vacuum energy E(R) takes the form

$$\frac{1}{2} E''(R_0) \ (R - R_0)^2 \quad .$$

near R_0. Thus for low energy excitations the cost of enlarging the bag is small. The enlarged bag associated with a given hadron is close to the size of the empty bags found in the vacuum. The "inside" and "outside" vacua are not very different. This explains qualitatively why large renormalization effects on quark operators are absent. The fact that the vacuum is filled with such bags also explains why it was consistent to assume that the only cost for localizing a bag is that associated with the total momentum of the valence particles. These features of earlier work were the principal motivation for the proposal made here.

In summary, I have shown that a vacuum densely filled with bubbles of perturbative vacua, with the pairing effect of the many colored gluons in each computed in lowest non-trivial order, has a minimum energy at a finite size R_0 of the order $1/\Lambda$. In such a vacuum, color density correlations between neighboring bubbles are absent. Neighboring bubbles are screened by a surface layer of induced color magnetic poles and currents.

IV. LARGE N

Because the behavior of non-Abelian gauge theories for large N (order of color group) has been of continuing interest,[10] it is appropriate to study the model of the ground state which has been proposed here in this limit. Vacuum energy is of order N^2. Thus, the constants a and b are of order N^2. Consequently the radius of the empty vacuum bags given by the solution of (3.9) is independent of N, for large N. Since a color singlet gluon field (or meson) exitation with just two valence gluons (or a quark and antiquark) can be made, the "valence" term is of order zero in N, so the energy (3.12) is dominated by the "inside" vacuum term which is of order N^2 and has its minimum ($=0$) at R_0. The "glueball" (or meson) mass is thus independent of N. For baryons which are color singlets, the valence term is of order N, so the "inside" vacuum term still dominates. The minimum radius is asymptotically equal to R_0 and the mass is of order N. These results are consistent with what is known about the spectrum for large N. It should be remarked that as N increases a ground state in which the condensed empty bags organize themselves to spontaneously break translation invariance (and also Lorentz invariance) might not be unexpected since the repulsion between empty bags increases as N^2. We are, of course, optimistically assuming that for $N = 3$, when translation invariance is restored to our ansatz, the symmetry will not be spontaneously broken.

V. THE ADDITION OF LIGHT QUARKS TO THE VACUUM

Quarks whose "bare" (QCD independent) masses are small in comparison to the scale Λ, also can produce long range separation of color in the ground state. In practice only the up down and, perhaps marginally, the strange quarks have such bare masses. One would thus also expect a surface boundary condition of the color confining form,

$$\hat{n} \cdot \bar{q}_a \vec{\gamma} q_b = 0 \tag{5.1}$$

on the light quark wave functions. That is, the trial ground state for the light quarks will consist of filled Dirac seas of light quarks in each small box. The complete sets of wave functions associated with each box are defined by solutions of the free Dirac equation with a linear boundary condition which implies (5.1) on the surface of each small box.

Although for massless quarks, the boundary condition (5.1) and free Dirac equation are chirally symmetric, the realization of (5.1) in a linear form requires the breaking of chiral symmetry. The most general linear boundary condition which implies (5.1) is

$$-i\,\gamma\cdot\hat{n}\,q_a = e^{\frac{i}{2}\omega_\alpha\lambda_\alpha\gamma_5}\,q_a \qquad (5.2)$$

where λ_α are the flavor generators together with the singlet generator and ω_α is arbitrary. Since there is no particular reason also to break flavor symmetry, one may take $\omega_\alpha = 0$ (also for the U(1) component). Chiral symmetry is now broken in order to prevent the high energy cost of color separation in the ground state. Since the energy of the light quarks is independent of which of the boundary conditions is chosen, we have the signal of a spontaneously[11] broken chiral symmetry.

One can now include in the vacuum energy, the energy of the light quarks. The Feynman diagrams are pictured in Fig. 3.

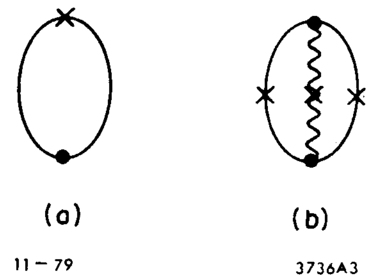

Fig. 3. Diagrams for the quark contributions to a and b. The x indicates a quark propagator for all space with (5.2) imposed on the surface of a sphere.

The additional contribution to "a" of massless up, down and strange quarks is

$$a_{\text{Quarks}} \simeq \frac{2}{64} \times \underset{\text{color}}{3} \times \underset{\text{flavor}}{3} = .28 \qquad (5.3)$$

The number 2/64 is based upon an approximation[12] equivalent to one made for the vector fields,[8] which gives $a_{QED} = 3/64$. The total value of "a" from both gluons and light quarks is then

$$a_{\text{Total}} = .37 + .28 = .65 \quad .$$

As for the gluons, one expects that the second order term will be attractive, so the quark contribution to "b" will increase it.

VI. PHENOMENOLOGICAL APPLICATION - THE SPECTRUM OF THE UP AND DOWN QUARK STATES, π, ρ, N, Δ

Because the constant "b" has not been calculated, a detailed quantitative comparison of this vacuum model with reality is not possible. However, a comparison of the predictions of the model with the observed mass sprectrum can be made for values of "b" which are such that $a - \alpha_s b$ becomes negative in the region where $\alpha_s \sim 1$. Since "a" is approximately .65, there are no free parameters other than "b". The energy of the valence quarks has not been computed beyond order α_s so to be consistent one should use a "lowest order" form for α_s,

$$\alpha_s = \frac{1}{\frac{9}{2\pi} \ln \left(\frac{1}{R\Lambda} + 1\right)} \quad . \tag{6.1}$$

In (6.1) the liberty has been taken to fix a form which is not singular at $R \sim 1/\Lambda$, since if propagators associated with the boxes are used there is no infrared singularity in the energy for finite values of R. The nature of the singularity at $R = \infty$ is irrelevant in this application since $R\Lambda$ will be of order 1.

With (6.1) chosen for $\alpha_s(R\Lambda)$, in Table I the parameters $B^{1/4}/\Lambda$ and $(R_0\Lambda)$, as determined by (3.13) and (3.9), are given for several values of "b". The value of "a" is taken to be .65. α_s for the corresponding values of $R_0\Lambda$ is also shown.

TABLE I. "Empty Bag" Parameters for Various Values of "b"

b/a	b	$R_0\Lambda$	$B^{1/4}/\Lambda$	$R_0B^{1/4}$	$\alpha_s(R_0\Lambda)$
.6	.39	2.56	.177	.453	2.12
.8	.52	1.78	.250	.445	1.57
1	.65	1.32	.332	.438	1.24
1.2	.78	1.01	.424	.428	1.01
1.4	.91	.80	.528	.422	.86
1.6	1.04	.65	.645	.419	.75

The scale R corresponding to the bag size, and the momentum transfer q used in perturbative QCD are not necessarily related in the simple form $q = 1/R$, so Λ cannot be directly related to the corresponding QCD scale. They presumably are related within a factor of 2 or 3. To judge which of the values in Table I corresponds most closely to previously obtained results using the bag model, one may determine Λ by the requirement that $\alpha_s = 2$ when $R \sim 5\ \text{GeV}^{-1}$ which is what was obtained previously.[3] In this case, $\Lambda \sim .5$ GeV. Since, in Ref. (3) $B^{1/4} = .145$ GeV, one may see that the case of $b = .65$ agrees well ($B^{1/4} = .165$ GeV) with the earlier work. One should also note that with $\Lambda \sim .5$ GeV, the empty bag size R_0 is such that $1/R_0 \approx .38$ GeV. $1/R_0$ should be compared with the "primordial" transverse momentum observed in strong interactions, since the quark constituents of hadrons are made in "empty bags" with the size R_0.

To make a crude but more accurate assessment using previous bag model results which also includes a "center of mass" correction one may use the formula

$$E_{BAG}^2 = M^2 + \langle P_{cm}^2 \rangle \tag{6.2}$$

with

$$\langle P_{cm}^2 \rangle = n(\bar{x}/R)^2 \tag{6.3}$$

and with

$$E_{BAG} = n\,\frac{2.04}{R} + \left[\frac{4\pi}{3}\,BR^3 + \left(a - \alpha_s(R\Lambda)b\right)\frac{1}{R}\right] + \mu\,\frac{\alpha_s(R\Lambda)}{R} \,. \tag{6.4}$$

Here, n = number of quarks. To roughly estimate the center of mass effect we have included the term $n(\bar{x}/R)^2 = \langle P_{cm}^2 \rangle$. We fit $\bar{x}$ so that $m_\pi \sim 0$. For consistency, $\bar{x}/R$ should be close to the momentum of a valence quark, that is, 2.04/R. We then use the same value for all other states. μ is determined by the free quark wave functions[3] and the color-spin matrix elements in the various states, $\mu_\pi = -.70$, $\mu_\rho = .70/3$, $\mu_N = -.70/2$, $\mu_\Delta = .70/2$. We determine the minimum M^2 as a function of R. The results for two different values of "b" are given in Table II. We again see that $b \sim .65$ corresponds well with the previous bag model calculations (and the observed masses).

TABLE II. Hadron Mass Spectrum for Two Values of "b"

CASE I	Particle	π	ρ	N	Δ
$b = .65$	M/Λ	0	1.50	1.85	2.50
$\bar{x} = 1.43$	M(GeV)	0	.76	.94	1.27
$\Lambda = .510$ GeV	$R(GeV^{-1})$	3.8	4.3	4.7	4.8
$B^{1/4} = .168$ GeV	α_s	1.7	1.8	2.0	2.1
$= .332\ \Lambda$					
CASE II	**Particle**	π	ρ	N	Δ
$b = 1.04$	M/Λ	0	2.45	3.67	4.48
$\bar{x} = 1.81$	M(GeV)	0	.63	.94	1.15
$\Lambda = .256$ GeV	$R(GeV^{-1})$	3.5	4.0	4.6	4.7
$B^{1/4} = .165$ GeV	α_s	.94	1.0	1.14	1.16
$= .645\ \Lambda$					

Note: Ground state masses of hadrons composed of "bare" massless up and down quarks, for two values of "b". The momentum spread of the valence quarks is parameterized by $\bar{x}$ and this is adjusted to make $m_\pi = 0$. Λ is taken to fit the nucleon mass, .94 GeV.

VII. CONCLUSIONS

I have indicated how the attractive interaction between colored particles can lead to a vacuum densely filled with bubbles of perturbative vacua consisting of paired quanta. The bubbles are separated by walls of induced color magnetic poles and currents which screen long range color density correlations. Naturally, only the long wave length fluctuations will actually experience this pairing, but the energy of the ground state is insensitive to the short scale fluctuations. We have shown how this vacuum state leads directly to the static bag model phenomenology. It should be needless to remark on all the difficiencies which exist in this treatment. Although the vacuum wave function is manifestly gauge invariant, it is not translation invariant. It is also not Lorentz invariant. We have indicated how chiral symmetry may be spontaneously broken, but we have not given a complete treatment of this symmetry breaking. Since α_s must be of order unity to accommodate the large spin-spin interaction observed in the mass spectrum, the calculation must be extended to higher orders of α_s.

In spite of these problems, I believe that this picture provides a simple, intuitive, and quantitative basis for the development of more detailed phenomenologies for the theory of hadron structure.

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REFERENCES

1. S. Mandelstam, Phys. Reports 23C, 245 (1976); R. Fukuda, T. Kugo, Prog. Theor. Phys. 60, 565 (1978); C. G. Callen, Jr., R. A. Dashen, D. J. Gross, Phys. Rev. D19, 1826 (1979); H. B. Nielsen, P. Olesen, NBI-HE-79-17 (preprint) (1979); H. B. Nielsen, M. Ninomiya, Nucl. Phys. (to be published); C. B. Thorn, Phys. Rev. D19, 639 (1979).
2. A. Chodos et al., Phys. Rev. D9, 3471 (1974); K. Johnson, Acta Phys. Pol. B6, 865 (1975); R. Friedberg, T. D. Lee, Phys. Rev. D18, 2623 (1978); T. D. Lee, Phys. Rev. D19, 1802 (1979); C. Detar, UUHEP-79-6 (University of Utah) (1979); R. L. Jaffe, MIT-CTP-814 (1979).
3. T. A. deGrand et al., Phys. Rev. D12, 2060 (1975).
4. J. F. Donoghue, K. Johnson, MIT-CTP-802 (1979).
5. J. D. Bjorken, SLAC-PUB-2372 (1979).
6. H. B. G. Casimir, Proc. Kon. Ned. Akad. Wetensch. 51, 793 (1948).
7. T. H. Boyer, Phys. Rev. 174, 1764 (1968).
8. K. A. Milton, L. L. DeRaad, J. Schwinger, Ann. Phys. [N.Y.] 115, 388 (1978).
9. R. Balian , B. Duplantier, Ann. Phys. [N.Y.] 112, 165 (1978). This reference gives methods which will allow one to calculate the vacuum energy for boxes with arbitrary shapes.
10. G. 't Hooft, Nucl. Phys. B72, 461 (1974); R. Brower, R. Giles, C. B. Thorn, Phys. Rev. 18, 484 (1978).
11. Y. Nambu, G. Jona-Lasinio, Phys. Rev. 122, 345 (1961).
12. Details will be published later.