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SYMMETRY BREAKING AND THE DECAYS $\psi' \to J/\psi\pi^0(\eta)$ and $J/\psi \to \eta(\eta')\gamma$*

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ABSTRACT

The theoretical mechanisms for the decays $\psi' \to \psi\pi^0$ and $\psi' \to \psi\eta$, which violate SU_2 and SU_3 respectively, are discussed. It is argued that symmetry breaking in the decay amplitudes may be as important as $\pi^0 - \eta - \eta'$ mixing. The $\pi^0 - \eta$ mixing mechanism $\psi' \to \psi\eta \to \psi\pi^0$ leads to $\Gamma(\psi' \to \psi\pi^0) = (3.3 \pm 1.0) \times 10^{-3}\ \Gamma(\psi' \to \psi\eta)$, but this number may be enhanced by a factor as large as 12 by $\pi^0 - \eta'$ mixing and isospin violation in the decay amplitude. The related decays $\psi \to \eta\gamma$ and $\psi \to \eta'\gamma$ are also discussed.

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The isospin violating decay $\psi' \to \psi\pi^0$ has recently been observed [1,2] by the Crystal Ball and MARK II collaborations at SPEAR. The Crystal Ball group [1] find a large branching ratio $B(\psi' \to \psi\pi^0) = (0.08 \pm 0.02 \pm 0.02)\%$. They also find $B(\psi' \to \psi\eta) = (2.06 \pm 0.10 \pm 0.58)\%$, somewhat lower than the old average [3,4] of $(4.2 \pm 0.7)\%$. The ratio $R \equiv B(\psi' \to \psi\pi^0)/B(\psi' \to \psi\eta)$ is less subject to systematic uncertainties. The Crystal Ball result is [1] $R = (39 \pm 10) \times 10^{-3}$. Similar results have been found by the MARK II group [2].

The first theoretical discussion of the $\psi' \to \psi\pi^0$ decay was by Segre and Weyers [5], who suggested that the decay could proceed via $\pi^0 - \eta$ mixing, as shown in Fig. 1a. It was subsequently pointed out by Genz [6] that the η' intermediate state of Fig. 1b might also be important since the $\psi' \to \psi\eta$ transition is SU_3 forbidden while the (virtual) $\psi' \to \psi\eta'$ coupling is SU_3 allowed. In this paper I reconsider the $\psi' \to \psi\pi^0$ and $\psi' \to \psi\eta$ decays, considering not only SU_2 and SU_3 breaking in the states ($\pi^0 - \eta - \eta'$ mixing) but also symmetry breaking in the transition amplitudes into the unmixed states. The related decays $\psi \to \eta\gamma$ and $\psi \to \eta'\gamma$ are also considered.

All of these decays require violation of the OZI rule. I assume that this violation takes place via annihilation of a $c\bar{c}$ pair into a $u\bar{u}$, $d\bar{d}$ or $s\bar{s}$ pair [7], as shown in Fig. 2. It is irrelevant for my purposes whether I regard this annihilation as generating small $c\bar{c}$ components in the π^0, η, and η' wave functions or as inducing direct decays into states consisting of $u\bar{u}$, $d\bar{d}$, and $s\bar{s}$ only. I will take the latter point of view.

First consider $\pi^o - \eta - \eta'$ mixing. In the absence of SU_2 breaking, the three bare states are

$$|\pi_B^o\rangle = \frac{1}{\sqrt{2}}|u\bar{u} - d\bar{d}\rangle$$

$$|\eta_B\rangle = \cos\theta_P|\eta_8\rangle - \sin\theta_P|\eta_o\rangle$$

$$|\eta_B'\rangle = \sin\theta_P|\eta_8\rangle + \cos\theta_P|\eta_o\rangle \quad , \tag{1}$$

where $\theta_P \approx -10.4°$ is the $\eta - \eta'$ mixing angle and η_8 and η_o are SU_3 octet and singlet states. For $\theta_P = -10.4°$, η_B and η_B' are well approximated by [8]

$$|\eta_B\rangle \simeq \frac{1}{\sqrt{2}}|\frac{u\bar{u} + d\bar{d}}{\sqrt{2}}\rangle - \frac{1}{\sqrt{2}}|s\bar{s}\rangle$$

$$|\eta_B'\rangle \simeq \frac{1}{\sqrt{2}}|\frac{u\bar{u} + d\bar{d}}{\sqrt{2}}\rangle + \frac{1}{\sqrt{2}}|s\bar{s}\rangle \quad . \tag{2}$$

When SU_2 breaking is turned on, the physical states are

$$|\pi^o\rangle \simeq |\pi_B^o\rangle + \lambda_{\pi\eta}|\eta_B\rangle + \lambda_{\pi\eta'}|\eta_B'\rangle$$

$$|\eta\rangle \simeq |\eta_B\rangle - \lambda_{\pi\eta}|\pi_B^o\rangle$$

$$|\eta'\rangle \simeq |\eta_B'\rangle - \lambda_{\pi\eta'}|\pi_B^o\rangle \quad , \tag{3}$$

where $\lambda_{\pi\eta} = \mu_{\pi\eta}^2/(\mu_\pi^2 - \mu_\eta^2)$ and $\lambda_{\pi\eta'} = \mu_{\pi\eta'}^2/(\mu_\pi^2 - \mu_{\eta'}^2)$ are the $\pi^o - \eta$ and $\pi^o - \eta'$ mixing angles. One can estimate [9] $\lambda_{\pi\eta} = (0.017 \pm 0.008)$ from the observed $\eta \to 3\pi$ decay rate. However, the $\eta \to 3\pi$ system is very messy, both theoretically and experimentally. An indirect but more reliable estimate of $\lambda_{\pi\eta}$ can be obtained by using the fact that $\pi^o - \eta$ mixing is due almost entirely to a mass difference between the up and down current quarks. The relevant formulas are

$$\mu^2_{\pi\eta_8} = \frac{-2}{\sqrt{3}} (\mu^2_K - \mu^2_\pi) \frac{m_d - m_u}{2m_s}$$

$$\mu^2_{\pi\eta_o} = \sqrt{2}\, \mu^2_{\pi\eta_8} \quad , \tag{4}$$

where m_i is the current quark mass of quark i. The expression for $\mu^2_{\pi\eta_8}$ is based on lowest order SU_3 breaking, and the formula for $\mu^2_{\pi\eta_o}$ expresses the OZI rule ansatz that the π^o is not directly mixed with the $s\bar{s}$ state. From a combined study [10] of the baryon mass splittings (e.g. $M_p - M_n$), $m_{K^+} - m_{K^o}$, the $\eta \to 3\pi$ decay, and $\rho - \omega$ mixing, one has $(m_d - m_u)/2m_s = 0.011 \pm 0.002$, which implies (for $\theta_P = -10.4°$) $\mu^2_{\pi\eta} = -3.6 \times 10^{-3}$ GeV^2 and $\mu^2_{\pi\eta'} = -4.1 \times 10^{-3}$ GeV^2. Hence $\lambda_{\pi\eta} = 0.013 \pm 0.002$ and $\lambda_{\pi\eta'} = 0.0039 \pm 0.0007$.

I now turn to the $\psi' \to \psi\pi^o$ decay. The relative width is

$$R = \frac{\Gamma(\psi' \to \psi\pi^o)}{\Gamma(\psi' \to \psi\eta)} = \left(\frac{p_{\pi^o}}{p_\eta}\right)^3 \lambda^2_{\pi\eta} \Delta^2 = (3.3 \pm 1.0) \times 10^{-3} \Delta^2 \quad , \tag{5}$$

where

$$\Delta = 1 + \frac{\lambda_{\pi\eta'}}{\lambda_{\pi\eta}} \frac{g_{\eta'}}{g_\eta} + \frac{g_B}{\lambda_{\pi\eta} g_\eta} \tag{6}$$

and $(p_{\pi^o}/p_\eta)^3 = 19.4$ is a phase space correction factor. g_η, $g_{\eta'}$, and g_B are the amplitudes for $\psi' \to \psi\eta$, $\psi' \to \psi\eta'$, and $\psi' \to \psi\pi_B$, respectively. g_η vanishes in the SU_3 limit while g_B, $\lambda_{\pi\eta}$, and $\lambda_{\pi\eta'}$ all vanish in the SU_2 limit. The three terms in Δ represent the contributions of $\pi^o - \eta$ mixing, $\pi^o - \eta'$ mixing, and the direct decay of ψ' into ψ and π_B, respectively, and are illustrated in Fig. 1.

Before proceeding, let us consider the decays [11] $\psi \to \eta\gamma$ and $\psi \to \eta'\gamma$. These decays also require the annihilation of a $c\bar{c}$ pair into a $u\bar{u}$, $d\bar{d}$ or $s\bar{s}$ pair. If the dominant decay mechanism involves radiation from a c quark [12], (Fig. 2, but with ψ' and ψ replaced by ψ and γ, respectively), then it is reasonable to assume that the relevant amplitudes $\hat{g}_\eta$ and $\hat{g}_{\eta'}$ are proportional to g_η and $g_{\eta'}$. Then

$$r \equiv \frac{\Gamma(\psi \to \eta'\gamma)}{\Gamma(\psi \to \eta\gamma)} = \left(\frac{p_{\eta'}}{p_\eta}\right)^3 \left(\frac{g_{\eta'}}{g_\eta}\right)^2 = 0.813 \left(\frac{g_{\eta'}}{g_\eta}\right)^2 \quad . \qquad (7)$$

There are other contributions involving radiation from a light quark, but these are presumably smaller by α_s/π. The value usually given [13] for r is 1.8 ± 0.8 but new measurements [1,14] suggest that r may be closer to 5. The Crystal Ball group [1] find r = 5.9 ± 1.5.

One can regard measurements of R and r as a determination of $g_{\eta'}/g_\eta$ and g_B/g_η. However, let us consider several theoretical models for these couplings. The results are summarized in Table 1.

a) Setting $\Delta = 1$ in (5) corresponds to the Segre-Weyers [5] model (only $\pi^o - \eta$ mixing), except for the value of $\lambda_{\pi\eta}$ used [15]. The predicted value of $R = (3.3 \pm 1.0) \times 10^{-3}$ is very small.

b) If all SU_2 and SU_3 breaking is due to $\pi^o - \eta - \eta'$ mixing, then $g_{\eta 8} = 0$, $g_B = 0$, and $g_{\eta'}/g_\eta = -\cot\,\theta_P = 5.45$. This implies a large value of $(23 \pm 7) \times 10^{-3}$ for R but also an unacceptably large value of 24 for r.

c) Genz's model [6] includes $\pi^o - \eta$ and $\pi^o - \eta'$ mixing ($g_B = 0$) but takes $g_{\eta'}/g_\eta$ from r. For the old value of r = (1.8 ± 0.8) this ansatz yields $R = (6.9 \pm 2.3) \times 10^{-3}$. For r = 5 one has a slightly larger value $R = (9.9 \pm 3.0) \times 10^{-3}$.

d) Let us allow for symmetry breaking in the decay amplitude, so that $g_{\eta 8} \neq 0$ and $g_B \neq 0$. Define A_q^i as the amplitude for $c\bar{c}$ to annihilate into a $q\bar{q}$ pair in meson i (see Fig. 2). I assume that up to an overall factor the same amplitudes are responsible for $\psi \to (\eta,\eta')\gamma$. Then

$$g_B \sim \frac{1}{\sqrt{2}} (A_u^\pi - A_d^\pi) \tag{8}$$

$$g_\eta \sim \sqrt{\frac{2}{3}} \cos\theta_P(A_\ell^\eta - A_s^\eta) - \frac{1}{\sqrt{3}} \sin\theta_P(2A_\ell^\eta + A_s^\eta) \sim A_\ell^\eta - \frac{1}{\sqrt{2}} A_s^\eta \tag{9}$$

and

$$g_{\eta'} \sim \frac{1}{\sqrt{3}} \cos\theta_P(2A_\ell^{\eta'} + A_s^{\eta'}) + \sqrt{\frac{2}{3}} \sin\theta_P(A_\ell^{\eta'} - A_s^{\eta'}) \sim A_\ell^{\eta'} + \frac{A_s^{\eta'}}{\sqrt{2}} \quad , \tag{10}$$

where

$$A_\ell^i \equiv (A_u^i + A_d^i)/2 \quad .$$

If I take all of the A's equal, which corresponds to no symmetry breaking in the decay amplitudes, I recover model (b) above.

In order to obtain a rough estimate of the A_q^i, observe that if the η, η', and π^o are treated nonrelativistically, then [16]

$$A_q^i \sim \frac{\alpha_s^2 \, \psi_q^i(0)}{M_q} \quad , \tag{11}$$

where M_q is the constituent mass of q and $\psi_q^i(0)$ is the $q\bar{q}$ wave function at the origin. It is by no means clear that the π^o, η, and η' are really non-relativistic, but (11) should suffice as a rough approximation. It is assumed [16] that the effects of anomalies are phenomenologically incorporated in (11). The dependence of A_q^i on M_q is complicated, in that the running coupling α_s should decrease with increasing M_q, while

$\psi_q^i(0)$ should increase. Two possibilities are [17] A_q^i = constant, which reproduces model (b), and [16] $A_q^i \sim C_i/M_q$.

The latter ansatz has been used successfully [16] by Isgur and by Cohen and Lipkin to describe the $\eta - \eta'$ system [18].

I will assume $A_q^i \sim C_i/M_q$ with C_i independent of i as a simple model of symmetry breaking [19]. I choose for the constituent quark masses M_u, M_d, and M_s the values 327 MeV, 333 MeV, and 550 MeV, respectively, obtained [20] by Copley, Isgur, and Karl from quark models of the baryons and mesons. These values are in reasonable agreement with the current quark mass ratio assumed above (one expects $(m_d - m_u)/m_s \simeq (M_d - M_u)/(M_s - M_u)$). I then obtain $g_{\eta'}/g_\eta \simeq 2.5$, $g_B/g_\eta \simeq 0.022$, and $\Delta^2 = 12$, which implies r = 5.0 and R = $(40 \pm 12) \times 10^{-3}$. In this model the direct decay into π_B is as important as the $\pi^o - \eta$ and $\pi^o - \eta'$ mixing contributions combined.

In conclusion, $\pi^o - \eta$ mixing alone (model (a)) is insufficient to account for a large $\psi' \to \psi\pi^o$ decay width. $\pi^o - \eta$ and $\pi^o - \eta'$ mixing considered together can give a large $\psi' \to \psi\pi^o$ width (model (b)) or a small (1.8 to 5) value for r (model (c)), depending on the assumed value for $g_{\eta'}/g_\eta$, but cannot satisfy both constraints simultaneously. In order to obtain reasonable values for both r and R one must consider SU_2 and SU_3 breaking in the decay amplitudes in addition to $\pi^o - \eta - \eta'$ mixing. A simple symmetry breaking scheme (model (d)) yields the values r = 5, and R = $(40 \pm 12) \times 10^{-3}$, in excellent agreement with the data. An additional check on this model (and its assumption that $C_{\eta'} \simeq C_\eta \simeq C_{\pi^o}$) involves the decays $(\eta', \eta, \pi^o) \to 2\gamma$. Assuming that the constituent quark picture incorporates the effects of anomalies, then a calculation analogous to Eq. (11) yields for the amplitudes the ratios $T_{\eta'}/T_{\eta}/T_{\pi^0} = 1.4/1.0/1.0$ if $\psi^i_q(0)/M_q \sim 1/M_q$, in reasonable agreement with the experimental [4,21] ratios $1.4 \pm 0.3/0.78 \pm 0.06/1$. (The ansatz $\psi^i_q(0)/M_q \sim$ constant is actually better, yielding the ratios 1.5/0.83/1.0.) These decays have recently been considered from a more canonical point of view by M. Chanowitz [22] and H. Goldberg [23].

The decays $\psi \to (\eta, \eta')\gamma$ have been discussed from more formal points of view by H. Goldberg [23] and V. A. Novikov et al. [24]. Novikov et al. argue that the η' has a large gluonium component. This effect is presumably incorporated in the constituent quark model by the annihilation amplitudes A^i_q used here and in the description [16,17] of the η and η'. However, I have found no evidence for the quark wave function of the η' to be either anomalously small at the origin [19] or anomalously large [23].

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15. The large value for $\lambda_{\pi\eta}$ used in Ref. 5 was based on an old and incorrect value for the $\eta \to 3\pi$ width.

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18. N. Isgur, Ref. 16, obtains $\lambda_{\pi\eta} = 0.02$, $\lambda_{\pi\lambda'} = 0.003$ from a constituent quark model. The resulting predictions for R are very similar to those obtained here. Also the ansatz for $\mu^2_{\pi\eta_o}$ in Eq. (4) is modified slightly by symmetry breaking in the annihilation amplitudes, but this effect appears to be small.

19. I. Cohen and H. Lipkin, Ref. 16, assume $C_\eta \neq C_{\eta'}$ due to mixing with radially excited states. If g_B is neglected, the results of $C_{\eta'} < C_\eta$ are very similar to model (c). For $g_B \neq 0$ one can obtain a smaller value for r without significantly reducing R.

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Table I. Several models for $R = \Gamma(\psi' \to \psi\pi^o)/\Gamma(\psi' \to \psi\eta)$ and $r = \Gamma(\psi \to \eta'\gamma)/\Gamma(\psi \to \eta\gamma)$. $\lambda_{\pi\eta} = 0.013 \pm 0.002$ is used throughout. The Crystal Ball group has found $R = (39 \pm 10) \times 10^{-3}$ and $r = 5.9 \pm 1.5$.

Model	$\frac{g_{\eta'}}{g_\eta}$	$\frac{g_B}{g_\eta}$	Δ^2	r	$R \times 10^3$
a) Segre-Weyers	0	0	1	-	3.3 ± 1.0
b) $A^i_q \sim$ constant					
$(\pi^o - \eta - \eta'$ mixing)	5.45	0	6.9	24	23 ± 7
c) Genz	$1.5^{+.3}_{-.4}$	0	2.1 ± 0.3	1.8 ± 0.8	6.9 ± 2.3
	2.5	0	3.0	5	9.9 ± 3.0
d) $A^i_q \sim \frac{1}{M_q}$	2.5	0.022	12	5	40 ± 12

Figure Captions

1. Three contributions to $\psi' \to \psi\pi^o$:

   (a) $\pi^o - \eta$ mixing;

   (b) $\pi^o - \eta'$ mixing;

   (c) A direct isospin violating decay into the unmixed π^o_B state.

2. The amplitude A^i_q for $c\bar{c}$ to annihilate into a $q\bar{q}$ pair in meson i ($i = \eta, \eta', \pi^o_B$).

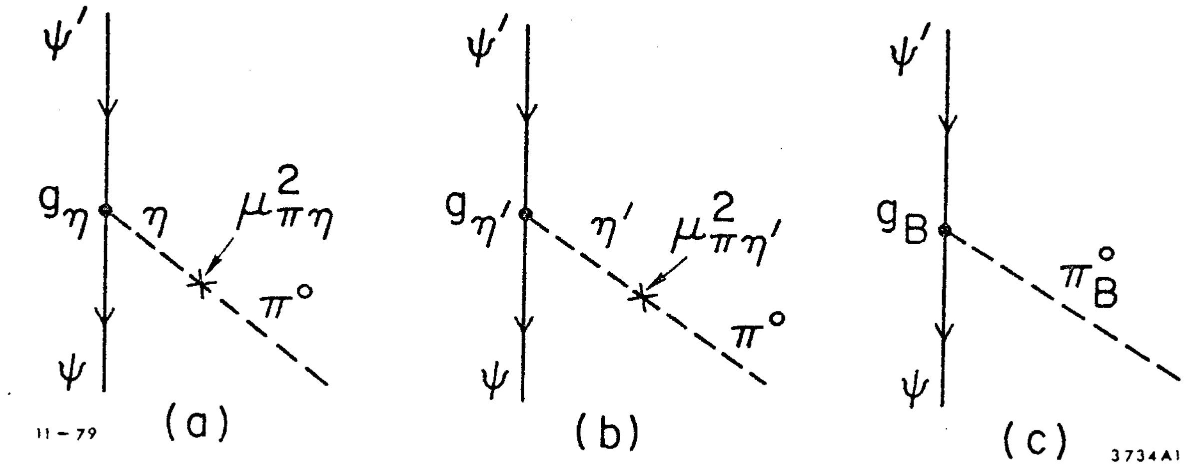
ψ′
g_η
η
$\mu^2_{\pi\eta}$
π°
ψ
11-79
(a)
ψ′
$g_{\eta'}$
η′
$\mu^2_{\pi\eta'}$
π°
ψ
(b)
ψ′
g_B
π°_B
ψ
(c)
3734A1

Fig. 1

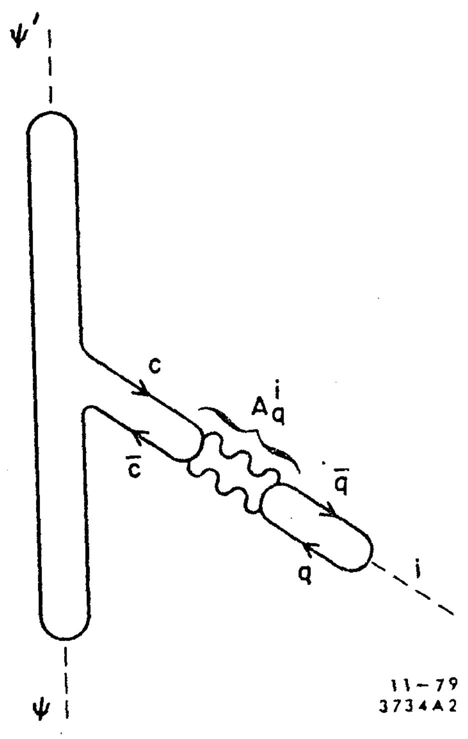

Fig. 2