# angular distributions in the lepton patr production* 

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## Abstract

The angular distributions of high mass lepton pairs are reviewed. It is argued that the detailed study of polar distributions provide the evidence for the substantial contributions to the Drell-Yan process from the higher order QCD effects. It is also pointed out that the first order QCD "Compton" diagrams predict non-trivial azimuthal dependence which could be measured experimentally.
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The high mass lepton pair production is usually described in terms of the Drel1-Yan mechanism ${ }^{1}$ depicted in Fig. 1. In this process it is proposed that a quark from one of the incoming hadrons annihilates with the corresponding antiquark from the second hadron producing a virtual photon of mass $M$, which then decays into a pair of leptons. The process first discussed in the simple parton model was recently also calculated in terms of Quantum Chromodynamics (QCD). For recent reviews of the Drell-Yan mechanism and the available data the reader is referred to Ref. 2.

In the simple quark parton model the two annihilating quarks travelling along the polar axis result, in the rest frame of the virtual photon, in the angular distribution of the form:

$$
\begin{equation*}
W\left(\theta^{*}\right)=1+\cos ^{2} \theta^{*} \tag{1}
\end{equation*}
$$

There are several ways of defining the axes in the rest frame of the lepton pair. The usual choice is the Gottfried-Jackson frame, where $\theta^{*}$ is the angle between the beam and one of the leptons. This reference frame is well suited for testing of the Drell-Yan process when the direction of the quarks coincide with the direction of the beam. The transverse momentum of quarks inside the parent hadrons may, however, introduce additional smearing effects to the angular distributions, as the direction of the beam and of the annihilating quarks will no longer overlap. To deal with this problem Collins and Soper ${ }^{3}$ have proposed a reference frame in which the $z$ axis bisects the angle between the beam and reverse target momentum directions in the lepton rest system. Experimentally, the Collins-Soper frame is found to be very close to the

Gottfried-Jackson reference system. In the $\pi^{-} \mathrm{N}$ interactions at $225 \mathrm{GeV} / \mathrm{c}$ the average angular difference between the two $z$ axes was estimated ${ }^{4}$ to be about $14^{\circ}$.

The non-zero mass of the outgoing leptons introduces small corrections ${ }^{5}$ to the formula (1)

$$
\begin{equation*}
W(\theta)=\left(1-\frac{4 m^{2}}{Q^{2}}\right)\left[\left(1+\frac{4 m^{2}}{Q^{2}}\right)+\left(1-\frac{4 m^{2}}{Q^{2}}\right) \cos ^{2} \theta^{*}\right] \tag{2}
\end{equation*}
$$

where $m$ is the mass of the lepton and $Q$ is the mass of the virtual photon. The correction is negligible for the outgoing electrons and muons but may play an important role in the study of e.g., heavy leptons T .

The general form of the angular distribution for the decay of the virtual photon into a lepton pair may be written ${ }^{6}$ as:

$$
\begin{align*}
W\left(\theta^{*}, \phi\right) & =\frac{3}{8 \pi}\left[\rho_{11}\left(1+\cos ^{2} \theta^{*}\right)+\left(1-2 \rho_{11}\right) \sin ^{2} \theta^{*}\right. \\
& \left.+\rho_{1-1} \sin ^{2} \theta^{*} \cos 2 \phi+2 \operatorname{Re\rho } \rho_{10} \sin ^{2} \theta^{*} \cos \phi\right] \tag{3}
\end{align*}
$$

where the density matrix elements $\rho_{i j}$ depend on the choice of the reference frame and all the variables describing the virtual photon, i.e., $\mathrm{Q}^{2}, \mathrm{x}_{\mathrm{F}}, \mathrm{P}_{\mathrm{T}}$, s . Integration over either polar or azimuthal angle gives:

$$
\begin{align*}
& \mathrm{W}_{1}\left(\theta^{*}\right) \sim 1+\alpha \cos ^{2} \theta^{*}  \tag{4}\\
& \mathrm{~W}_{2}(\phi) \sim 1+\beta \cos 2 \phi \tag{5}
\end{align*}
$$

where both $\alpha$ and $\beta$ may vary between -1 and +1 .
Experimentally, the polar distributions of the lepton pairs were measured by various groups ${ }^{4}, 7$ and found to be well described by the formula (5) with the parameter $\alpha$ compatible with $\alpha \approx 1$ (see Fig, 2).

The Quantum Chromodynamics calculations of the Drell-Yan process predict a significant deviation from purely transverse polarization of the virtual photon, i.e., $\alpha<1$. Two possible origins of these deviations have been discussed recently in the literature. 1. First order in $\alpha_{s}$ "Compton" subprocesses illustrated in Fig. 3, in which a quark interacts with a spin 1 gluon, were shown to lead to a $\sin ^{2} \theta^{*}$ behavior at large $p_{T}$ of the lepton pair. A non-trivial azimuthal dependence is also


Fig. 2. Distribution of $\cos \theta^{*}$ for $\pi^{-} N \rightarrow \mu^{+} \mu^{-} \mathrm{X}$ at $200 \mathrm{GeV} / \mathrm{c}$ with $4<\mathrm{M}<6 \mathrm{GeV}$ and $\mathrm{p}_{\mathrm{T}}<1$ $\mathrm{GeV} / \mathrm{c}$. The two curves correspond to formula (4) description of the data. expected, and the parameter $\beta$ in the formula (5) is related ${ }^{8}$ to $\alpha$ through:

$$
\begin{equation*}
\beta=\frac{1-\alpha}{2(3+\alpha)} \tag{6}
\end{equation*}
$$

The variation of $\alpha$ and $\beta$ as functions of $x_{F}$ of the virtual photon is shown in Fig. 4.


Fig. 3. First order QCD "Compton" diagrams contributing to the Drell-Yan process.
2. Another approach was taken by Berger and Brodsky ${ }^{10,11}$ who introduced the correlations between the valence quarks in the pion calculating the
"higher twist" terms of Fig. 5.
These higher order effects result in the decay angular distribution varying strongly with $Q^{2}$ and $x_{F}$ of the virtual photon:

$$
\begin{aligned}
\mathrm{d} \sigma & \sim(1-x)^{2}\left(1+\cos ^{2} \theta^{*}\right) \\
& +\frac{4}{9} \frac{\left\langle\mathrm{k}_{\mathrm{T}}^{2}\right\rangle}{Q^{2}} \sin ^{2} \theta^{*} \\
& +\frac{2}{3} \sqrt{\frac{4 \mathrm{k}_{T}^{2}}{Q^{2}}}(1-x) \sin ^{2} \theta^{*} \cos \phi
\end{aligned}
$$

At fixed mass the longitudinal polarization of the virtual photon increases as x approaches 1 .


Fig. 4. The $x_{F}$ dependence of $\alpha$ and $\beta$ calculated in Ref. 8.

The theoretical work
stimulated the experimentalists to study the angular distributions in more detail. The data of the Chicago-I11inoisPrinceton collaboration ${ }^{4}$ for the $\pi^{-} \mathrm{N} \rightarrow \mu^{+} \mu^{-} \mathrm{X}$ at $225 \mathrm{GeV} / \mathrm{C}$

Fig. 5. Higher twist diagrams used in calculations of Berger and Brodsky.

(b)

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$1-19 \quad(a)$

) the pion. In each of the intervals the form (4) is a good representation of the data, but the parameter $\alpha$ shows strong $x_{1}$ and $p_{T}$ dependence as seen in Fig. 7. Also shown in Fig. 7 are the predictions due to the two QCD calculations described above. Though first order diagram calculations predict ${ }^{12}$ some $x$ dependence of the parameter $\alpha$, they do not describe the available data in the region of $x$ close to 1 . This may be taken as conclusive evidence that
the "higher twist" effects, which describe the data adequately, contribute substantially to this
kinematical region.
It should be also noted that in the approximation taken by Berger and Brodsky one does not expect strong azimuthal dependence in the angular distributions integrated over $\theta^{*}$ ( $\beta \approx 0$ ). Furthermore, the first order QCD diagrams contribute equally to the dimuons produced by the pion or proton beams, while the "higher twist"


Fig. 6. Angular distribution of dimuons for various intervals of the fractional momentum x of the pion antiquark.
effects appear ${ }^{13}$ to be much reduced in the latter case. Therefore, careful studies of both azimuthal and polar distributions may provide a measurement of the relative contribution of the two QCD effects.


Fig. 7. Dependence of the parameter $\alpha$ on (a) transverse momentum of dimuons and (b) fractional momentum $x$ of the pion antiquark. The dashed lines are the prediction of first-order QCD "Compton" diagrams. Solid line is from the calculations of Berger and Brodsky.

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