# an estimate of $J / \psi$ production in b decays* 

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#### Abstract

The branching ratio for the decay $B \rightarrow J / \psi+$ hadrons is estimated from a calculation of the quark decay $b \rightarrow J / \psi+s$.


* Work supported in part by the Department of Energy under contract DE-AC03-76SF00515 and the National Science and Engineering Research Council of Canada.

Recently H. Fritzsch has suggested [1,2] that it may be possible to see $B$ mesons in $p p$ collisions through the decay $B \rightarrow J / \psi K \pi$. To reach this conclusion he argues for a strongly energy dependent production cross section [2]

$$
\begin{equation*}
\frac{\sigma(\overline{\mathrm{B}} \mathrm{~B})[400 \mathrm{GeV}]}{\sigma(\overline{\mathrm{B}} \mathrm{~B})[150 \mathrm{GeV}]} \approx 30 \tag{1}
\end{equation*}
$$

a sizeable branching ratio for the production of $J / \psi$ in $B$ meson decay [1]

$$
\begin{equation*}
\frac{\Gamma(B \rightarrow J / \psi+X)}{\Gamma_{B}} \approx 3-5 \% \tag{2}
\end{equation*}
$$

and that the direct production of a $J / \psi$ in $B$ decay is dominated [1] by the mode $B \rightarrow J / \psi K \pi$.

Fritzsch used somewhat qualitative methods to derive eq. (2). In view of the recent experimental evidence for $B$ production in $\pi p$ collisions [3] a more quantitative estimate is warranted. In this note the branching ratio for $B$ decay to $J / \psi+$ hadrons is estimated by a calculation of the quark decay $b \rightarrow J / \psi+s$ [4]. The results are compatible with Fritzsch's conclusions if a cancellation which occurs in the lowest order evaluation of matrix elements is not taken seriously.

In the absence of strong interactions the effective Hamiltonian for the decay $B \rightarrow J / \psi+$ hadrons has the form [5]

$$
\begin{equation*}
\mathscr{H}_{\mathrm{eff}}=\frac{\mathrm{G}_{\mathrm{F}}}{\sqrt{2}} c_{2}\left(\mathrm{~s}_{2} e^{i \delta}+c_{2} s_{3}\right)\left(\bar{s}_{\alpha} b_{\beta}\right)_{V-A}\left(\bar{c}_{\beta} c_{\alpha}\right)_{V-A}+h \cdot c \tag{3}
\end{equation*}
$$

where

$$
\begin{equation*}
(\overline{\mathrm{q} q})_{V-A}(\overline{\mathrm{q} q})_{V-A} \equiv\left(\overline{\mathrm{q}} \gamma_{\mu}\left(1-\gamma_{5}\right) q\right)\left(\overline{\mathrm{q}} \gamma^{\mu}\left(1-\gamma_{5}\right) q\right) . \tag{4}
\end{equation*}
$$

The color indices $\alpha$ and $\beta$ are summed over $\{1,2,3\}$ when repeated. The
width for direct production of $J / \psi$ in $B$ decay is estimated by assuming the light quark in the $B$ meson acts as a spectator, and calculating the rate for $b \rightarrow J / \psi+s$ using the contribution to the matrix element of $\mathscr{H}_{\text {eff }}$ shown in fig. 1. The strange and spectator quarks are assumed to dress themselves into hadrons with unit probability. The calculation of $\Gamma(b \rightarrow J / \psi+s)$ is similar to that for $\Gamma(\tau \rightarrow \rho \nu)[6,7]$ and when the mass of the strange quark is neglected

$$
\begin{align*}
\Gamma(b \rightarrow J / \psi+s) & =\frac{G_{F}^{2}}{12 \pi^{2}}|R(0)|^{2}\left(\frac{m_{J / \psi}}{m_{b}}\right)\left(\frac{m_{b}^{2}-m_{J}^{2} / \psi}{2 m_{b}}\right)^{2} \\
& \times\left[2+\frac{m_{b}^{2}}{m_{J / \psi}^{2}}\right]\left|c_{2} s_{2} e^{i \delta}+c_{2}^{2} s_{3}\right|^{2} . \tag{5}
\end{align*}
$$

In eq. (5), $R(0)$ is the radial wave function of the $J / \psi$ meson (in its rest frame) evaluated at the origin. The first and second terms in the square brackets of eq. (5) originate from the contributions of the transverse and longitudinal polarizations of the $J / \psi$ respectively. Estimating the total width for $B$ decay by the width for $b$-quark decay yields [8]

$$
\begin{equation*}
\Gamma_{B} \approx \frac{5 G_{F}^{2}}{192 \pi^{3}} m_{b}^{5}\left|s_{2} e^{i \delta}+s_{3} c_{2}\right|^{2} F\left(\frac{m_{c}}{m_{b}}\right) \tag{6}
\end{equation*}
$$

when the contribution from Cabibbo suppressed modes and final states with more than one heavy particle are neglected. The function

$$
\begin{equation*}
F(x)=1-8 x^{2}+8 x^{6}-x^{8}-24 x^{4} \ln x \tag{7}
\end{equation*}
$$

With $\mathrm{m}_{\mathrm{c}}=1.5 \mathrm{GeV}$ and $\mathrm{m}_{\mathrm{b}}=4.5 \mathrm{GeV}, \mathrm{F}\left(\mathrm{m}_{\mathrm{c}} / \mathrm{m}_{\mathrm{b}}\right) \approx 0.45$. Combining eqs. (5) and (6), using the above values for the charm and bottom quark masses and [9] $|R(0)|^{2} \approx 0.5 \mathrm{GeV}^{3}$, leads to a branching ratio for direct $J / \psi$
production in $B$ decay of about $2 \%$.
A B meson can also decay to $J / \psi$ indirectly through an intermediate $x$ (in the absence of strong interaction corrections to the matrix element only the ${ }^{3} \mathrm{P}_{1}$ state contributes) or $\psi^{\prime}$ state, however the indirect production through a X will involve a photon in the final state. The branching ratio for $B \rightarrow \psi^{\prime}(3684)$ + hadrons can be estimated from eqs. (5) and (6). Using [9] $|R(0)|^{2} \approx 0.3 \mathrm{GeV}^{3}$, the same quark masses as before, and replacing $m_{J / \psi}$ by $m_{\psi}$, in eq. (5), gives a $0.6 \%$ branching ratio. The branching ratio for indirect production of $J / \psi$ through $\psi^{\prime}$ (3684) is about half this and is negligible compared with direct production.

So far strong interaction effects (which I shall assume are described by Quantum Chromodynamics) have been neglected. In the presence of strong interactions the effective Hamiltonian in eq. (3) must be modified. The appropriate effective Hamiltonian is calculated by a two step process in which the $W$-boson and $t$-quark are successively treated as heavy and removed from explicitly appearing in the theory $[10,11,12]$. The resulting effective Hamiltonian has six terms, four of which arise from Penguin-type diagrams [10] and have small Wilson coefficients. Neglecting these "Penguin" contributions

$$
\begin{align*}
\mathscr{H}_{e f f} & \approx \frac{G_{F}}{\sqrt{2}} c_{2}\left(s_{2} e^{i \delta}+c_{2} s_{3}\right)\left\{\frac{\left(f_{+}+f_{-}\right)}{2}\left(\bar{s}_{\alpha} b_{\beta}\right)_{V-A}\left(\bar{c}_{\beta} c_{\alpha}\right)_{V-A}\right. \\
& \left.+\frac{\left(f_{+}-f_{-}\right)}{2}\left(\bar{s}_{\alpha} b_{\alpha}\right)_{V-A}\left(\bar{c}_{\beta} c_{\beta}\right)_{V-A}\right\}+h . c . \tag{8}
\end{align*}
$$

In the leading logarithmic approximation [13]

$$
\begin{equation*}
f_{+}=\left[\frac{\alpha_{s}\left(M_{W}^{2}\right)}{\alpha_{s}\left(m_{t}^{2}\right)}\right]^{6 / 21}\left[\frac{\alpha_{s}\left(m_{t}^{2}\right)}{\alpha_{s}\left(\mu^{2}\right)}\right]^{6 / 23} \tag{9a}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{f}_{-}=1 / \mathrm{f}_{+}^{2} \tag{9b}
\end{equation*}
$$

The renormalization point $\mu$ should be chosen at the typical mass scale for the decay. At this scale the running coupling constant is small and the matrix element of the effective Hamiltonian in eq. (8) can be estimated by the lowest order diagram in fig. 1. The net result of using the effective Hamiltonian in eq. (8), instead of that in eq. (3), is to correct eq. (5) by a factor $\eta_{1}^{2}$ and eq. (6) by a factor $\eta_{2}^{2}$ where

$$
\begin{equation*}
n_{1}^{2}=\left(2 f_{+}-f_{-}\right)^{2} \tag{10a}
\end{equation*}
$$

and

$$
\begin{equation*}
\eta_{2}^{2}=\left(2+2 f_{+}^{2}+f_{-}^{2}\right) / 5 \tag{10b}
\end{equation*}
$$

While the factor $\eta_{2}^{2}$ is close to unity for reasonable values of the running coupling constant and renormalization point mass, the factor $\eta_{1}^{2}$ is significantly smaller than unity [14]. The smallness of $\eta_{1}^{2}$ is a direct consequence of the fact that the $\bar{c} c$ pair attached to the fourfermion vertex in fig. 1 must be in a color singlet, causing a cancellation between the matrix elements of the two terms in the effective Hamiltonian. Higher order contributions to the matrix element of $\mathscr{H}_{\text {eff }}$, where a gluon is attached to either the c or $\bar{c}$ quark line in fig. 1, for example, might partially destroy this cancellation. In $D^{\circ}$ decays a cancellation with a similar origin leads to the prediction [15] of a small value for the ratio $\Gamma\left(D^{0} \rightarrow \bar{K}^{\circ} \pi^{0}\right) / \Gamma\left(D^{0} \rightarrow K^{-} \pi^{+}\right)$, however experimentally [16] the ratio is not small. This leads one to believe that higher order contributions to the matrix elements also destroy,
at least partially, the cancellation which results in the smallness of $\eta_{1}^{2}$. Hence it is perhaps not unreasonable that the branching ratio for direct $J / \psi$ production be as large as $2 \%$ (i.e., its value calculated in the absence of strong interactions). Fritzsch estimates [1] that half of the branching ratio in eq. (2) (i.e., 1.5-2.5\%) is due to direct J/ $\psi$ production.

Finally, it is worth noting that if the same techniques are applied to charm decays one predicts that [17] as much as $3 \%$ of the nonleptonic Cabibbo suppressed $D$ decays could be of the form $D \rightarrow \phi+X$, for example. However, the uncertainties involved in such an estimate are very great since the charm quark is probably not heavy enough for a lowest order in perturbation theory estimate of the $c \rightarrow \phi+u$ matrix element to be reliable.

## Acknowledgements

This work was supported in part by the Department of Energy under contract DE-AC03-76SF00515 and the National Science and Engineering Research Council of Canada. I thank R. Cahn, J. Fllis and F. Gilman for useful discussions.

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14. Using $M_{W}=85 \mathrm{GeV}, \mathrm{m}_{\mathrm{t}}=30 \mathrm{GeV}, \mu=4.5 \mathrm{GeV}$ and parametrizing the running coupling constant by

$$
\alpha_{s}\left(Q^{2}\right)=\frac{12 \pi}{33-2 N_{f}} \frac{1}{\log \left(Q^{2} / \Lambda^{2}\right)}
$$

where $\Lambda^{2}=0.1 \mathrm{GeV}^{2}$ and $\mathrm{N}_{\mathrm{f}}=6$ and 5 at the mass scales of the top and bottom quarks respectively, gives: $f_{+} \approx 0.84, \mathrm{f}_{-} \approx 1.4$, $\eta_{1}^{2} \approx 0.08, \eta_{2}^{2} \approx 1.1$.
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Figure Caption

Fig. 1. $b \rightarrow J / \psi+s$ matrix element. The square represents a fourfermion vertex.


Fig. 1

