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DIBARYON PRODUCTION IN ELECTRON-DEUTERON SCATTERING*

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Abstract

Estimates are presented for the cross sections for production of dibaryon resonances in electron-deuteron scattering. Two different models, with these resonances as two-body and three-body excited states, are analysed.

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1. Introduction

Recent reports about the possible existence of p-p and p-n dibaryon resonances have created considerable theoretical and experimental interest.[1] If confirmed, they will have a great influence in our understanding of nuclear forces. It is therefore important to discuss new reactions in which these resonances could be produced, in order to get a better knowledge of their properties. In this note we want to present some estimates of the cross sections for the production of these dibaryon resonances in electron-deuteron scattering ($ed \rightarrow eD^*$).

We will investigate in some detail two possibilities. First we will consider D^* as an excited state of two nucleons. Then, as a second and more interesting possibility, we will consider it as a three-body (pion-nucleon-nucleon) bound state. This last case has been in part motivated by a simple analysis done by MacGregor.[2] He found that the masses of these resonances follow a straight line, when plotted against their angular momentum in the form $\ell(\ell+1)$ (see Fig. 1). If we believe that this is due to a difference in rotational energy, we can extrapolate the straight line and find the $\ell=0$ and $\ell=1$ states. The masses come out to be 2060 and 2020 MeV respectively. This result is given in Fig. 1, and we see that the mass of the $\ell=0$ resonance is much closer to be the sum of the masses of two protons plus a pion than just two protons. Furthermore, the analysis of the experimental results for p-p scattering gives an elasticity of ≈25%, which indicates the presence of additional channels. These could turn out to be three-body bound states.

In the models presented here the constituents will be considered as scalar particles, which for our purposes is a good approximation. More detailed calculations should include these spin effects.

2. Two-Body Bound State

In this section we will consider D^* as a two-body bound state. For low values of $|\vec{q}|$ (momentum transfer) we can use a non-relativistic approximation and write [3]

$$\frac{d\sigma_n}{d\Omega} = \frac{4e^4|\vec{k}'|^3}{|\vec{k}|\ |\vec{q}|^4} \left| \int \sum_a^N F_a(\vec{q}^{\,2})\ e^{i\vec{q}\cdot\vec{r}_a}\ \psi_f^*\ \psi_0\ dV_1 \ldots dV_N \right|^2 , \tag{1}$$

for the effective inelastic scattering cross section. Here ψ_0 and ψ_f are the initial and final wave functions for the system under consideration (in our case the deuteron). $F_a(\vec{q}^{\,2})$ is the nucleon form factor, N is the number of nucleons, and $\vec{k}$ and $\vec{k}'$ are the momenta of the incident electron before and after the collision.

The experimentally determined dibaryon resonances are given in Fig. 1 ($^1G_4$, $^3F_3$, $^1D_2$). Since the spacing is approximately uniform we can put the $\ell = 1$ state at 2010 (MeV), and the $\ell = 0$ state (deuteron) at 1880 (MeV). We will assume a potential of the harmonic oscillator type, for which the wave functions are given by [3]

$$\psi_{n\ell m} = A_n\ e^{-\frac{M\omega}{2} r^2}\ r^n\ Y_{\ell m}(\theta,\varphi) \quad , \tag{2}$$

for $n = \ell$ (n is the principal quantum number, and ℓ is the angular momentum, with projection m). After normalization we get:

$$A_n^2 = \frac{2(2M\omega)^{n+1}}{(2n+1)!!}\sqrt{\frac{M\omega}{\pi}} \quad . \tag{3}$$

Here ω is a parameter, and M is the reduced mass. If we consider the case $m=0$ for simplicity, the integrations can be performed explicitly, and we find

$$\frac{d\sigma_n}{d\Omega} = \frac{4e^4|\vec{k}'|^3}{|\vec{k}|\ |\vec{q}|^4}\,|F_p(\vec{q}^{\,2})|^2\,\frac{(2\ell+1)\ |\vec{q}|^{2\ell}}{(2\ell+1)!!(8M\omega)^\ell}\,e^{-\frac{|\vec{q}|^2}{8M\omega}} \quad . \tag{4}$$

Then it is possible to identify the resonance production form factor as

$$F_{R2}^2(q^2) = F_p^2(q^2)\,\frac{(2\ell+1)(-q^2)^\ell}{(2\ell+1)!!(8M\omega)^\ell}\,e^{\frac{q^2}{8M\omega}} \quad , \tag{5}$$

which for $\ell=0$ corresponds to the deuteron form factor. Graphs for $F_{R2}^2(q^2)$ are shown in Fig. 2. We have chosen a value of ω ($\simeq$32.5 [MeV]) which gives reasonable fits of this expression to the experimental data for the deuteron form factor,[4] for $-q^2 \lesssim 0.7\ \mathrm{GeV}^2$. It is interesting to note that this value of ω is not the one that we would get from the spectrum of states, the difference coming presumably from spin and tensor forces. For larger q^2, the short-range nature of the potential enters and one expects a power law fall off with q^2.[5]

3. Three-body Bound State

For this case will use a different approach from the nonrelativistic one of the previous section. The method consists in enhancing every partial wave amplitude for $\gamma^* + d \to \pi + d$ with a final state interactions factor for the production of the particular resonance under consideration. Actually one should include more than just the deuteron ground state in the intermediate (nucleon-nucleon) states. However, in order to estimate the magnitude of the transition matrix element, only this state will be retained.

We start with the expression for the structure function νW_2 in terms of the total cross section σ_T and σ_L for the photoabsorption of transverse and longitudinal photons

$$\nu W_2 = \frac{\nu + \frac{q^2}{2M_d}}{4\pi^2\alpha} \frac{\nu q^2}{q^2 - \nu^2} \left(\sigma_T + \sigma_L\right) \quad . \qquad (6)$$

We want to find the contribution to this expression that comes from the production of a particular three-body (pion-nucleon-nucleon) resonance. In the low q^2 region one can approximate

$$\begin{aligned} \sigma_T(q^2) &\simeq F_p^2(q^2)\ \sigma_T(0) \\ \sigma_L(q^2) &<< \sigma_T(q^2) \quad , \end{aligned} \qquad (7)$$

and then use photoproduction data, which is more readily available to normalize the amplitudes. We expect Eq. (7) to hold reasonably well in models based on an impulse approximation scheme.

As was mentioned before, we will enhance every partial wave for pion-photoproduction off deuterons with a certain final state interactions factor. For this purpose we need a parametrization of these data,[7] and so we write (see Ref. 8):

$$M \simeq \frac{20}{7} s F_d(t) \quad ,$$

where (for low t):

$$F_d(t) = \frac{1}{4}\left(\frac{-1}{1-4t} + \frac{5}{1-20t}\right) \qquad (8)$$

is the form factor for the deuteron. Here s and t are Mandelstam variables for the γ-d system. In terms of the amplitude M, the spin-averaged cross section for photoproduction of pions off deuterons is (in the CM of the γ-d system):

$$\left(\frac{d\sigma}{d\Omega}\right)_{CM} = \frac{|\vec{p}_{\pi}^{\,CM}|}{|\vec{p}_{\gamma}^{\,CM}|} \frac{|M|^2}{(8\pi)^2 s} \quad . \tag{9}$$

The partial waves corresponding to M are:

$$\begin{aligned} M_{\ell} &= \frac{1}{2}\int_{-1}^{1} dz\, M(s,z)\, P_{\ell}(z) \\ &= \frac{5s}{56pp'}\Big(Q_{\ell}(b) - Q_{\ell}(a)\Big) \quad , \end{aligned} \tag{10}$$

where

$$p \equiv |\vec{p}_{\gamma}^{\,CM}| = \frac{(s - M_d^2)}{2\sqrt{s}}$$

$$p' \equiv |\vec{p}_{\pi}^{\,CM}| = \frac{1}{2}\left[\frac{\left(s - (M_d + m_{\pi})^2\right)\left(s - (M_d - m_{\pi})^2\right)}{s}\right]^{\frac{1}{2}} \quad ,$$

and

$$a = \frac{1}{2pp'}\left[\frac{1}{4} + 2\left(\sqrt{p^2 + M_d^2}\,\sqrt{p'^2 + M_d^2} - M_d^2\right)\right]$$

$$b = \frac{1}{2pp'}\left[\frac{1}{20} + 2\left(\sqrt{p^2 + M_d^2}\,\sqrt{p'^2 + M_d^2} - M_d^2\right)\right]$$

(P_{ℓ} and Q_{ℓ} are Legendre polynomials of the first and second kind respectively). This means that $p \approx p'$. Then the Born partial amplitude M_{ℓ} has a cut in the real axis of the p^2 complex plane, from $p^2 = -\frac{1}{16}$ to $p^2 = -\frac{1}{80}$. We can approximate this cut by a pole at $p^2 \approx -\frac{1}{25}$, which corresponds to a $\mathrm{Re}(s) \equiv s_0 = 2p^2 + M_d^2$. This means that a good approximation (see Ref. 6) for the enhancement factor is:

$$F_\ell = \frac{D(s_0)}{D(s)} \quad , \tag{11}$$

where[9]

$$D(s) = \frac{1}{s}\left(s - s_R + i\Gamma\left(s - (M_d + m_\pi)^2\right)^{\frac{1}{2}}\right)$$

and

$$s_R = M_R^2$$

$$\gamma_R = \Gamma\left[\frac{s_R - (M_d + m_\pi)^2}{s_R}\right]^{\frac{1}{2}} \quad .$$

Here M_R and γ_R are the mass and full-width at half maximum of the resonance under consideration. We have included the requirement that $D(s) \to 1$ for $s \to \infty$. Then the total cross section for the photoproduction of a particular resonance is going to be

$$\sigma_T \approx (4\pi)\,\frac{p'}{p}\,|F_\ell|^2\,\frac{|M_\ell|^2}{(8\pi)^2 s} \quad . \tag{12}$$

Finally:

$$\left(\nu W_2\right)_R = \frac{\nu + \frac{q^2}{2M_d}}{4\pi^2\alpha}\;\frac{\nu q^2}{q^2 - \nu^2}\;F_p^2(q^2)\;\frac{|M_\ell F_\ell|^2}{4\pi s}\;\frac{p'}{p} \tag{13}$$

where

$$s = W^2 = M_d^2 + 2M_d\nu + q^2$$

This result has been plotted in Fig. 3, for different values of q^2. Compared with experimental data,[10] these estimates are smaller than the rest of the inelastic channels that contribute in that region by about one or two orders of magnitude. This fact makes the testing of this model a rather difficult experimental problem. However, since the width of these resonances is much smaller than the width of the nucleon resonances that are present in this q^2 range, it may still be possible to observe at least the $\ell = 2$ state (see Ref. 10) with very precise data.

For this three-body model we have computed numerical results for the $\ell = 2,3,4$ resonances only, which are the ones whose existence has been experimentally reported. In the case of the $\ell = 0$ and $\ell = 1$ resonances, we need to know their masses and widths in order to evaluate the corresponding cross sections.

One can also calculate the corresponding transition form factor, using

$$\nu W_2 = \left(M_R^2 - M_d^2 - q^2\right) F_{R3}^2(q^2)\ \delta\left(W^2 - M_R^2\right)$$

or (14)

$$\int dx\ \nu W_2 = x_R\ F_{R3}^2(q^2) \quad ,$$

where

$$x = \frac{q^2}{q^2 + M_d^2 - W^2} \quad ,$$

and the integration is over the resonance peak. Then for the cases given in Fig. 3, we get the results presented in Table I. If we compare these values with the calculation for the two-body case (F_{R2}^2) given in Section 2, we see that the present ones are clearly smaller by approximately one order of magnitude.

4. Conclusions

The results presented in this paper should help to clarify the nature of the dibaryon resonances. It is interesting to note that the two possibilities presented in Sections 2 and 3 give significantly different predictions, with the pion-nucleon-nucleon model being smaller than the nucleon-nucleon case. This difference is bigger than the uncertainty in our results due to the approximations we have made, which means that experimental measurements should be able to distinguish between the two possibilities. We should remark, however, that we expect for the three-body model that the actual magnitude is going to be larger than our estimate, because we have not included other intermediate states (different from the deuteron) that could contribute to the transition matrix element. In any event, present experimental results[10] do not rule out any of the two models.

Further theoretical analysis of these resonances and their production in different reactions should especially consider the pion-nucleon-nucleon model, which if established will certainly have great importance in theories of few body systems in nuclear physics.

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TABLE I

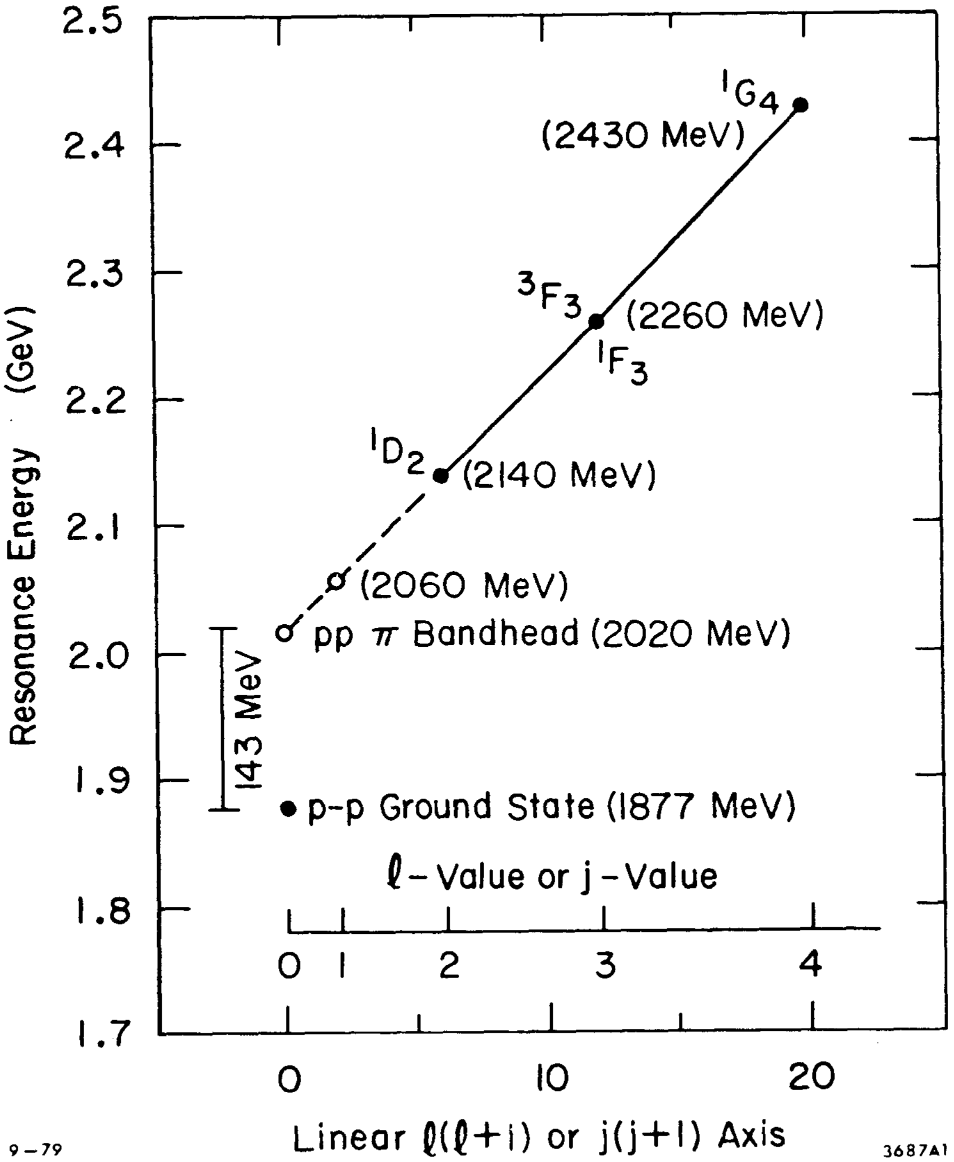
ℓ	$-q^2(\mathrm{GeV})^2$	$F^2_{R3}(q^2)$	$F^2_{R2}(q^2)$
2	0.3	0.146×10^{-2}	0.042
	0.5	0.078×10^{-2}	0.011
	0.7	0.046×10^{-2}	0.002
3	0.5	0.051×10^{-2}	0.009
4	0.5	0.080×10^{-2}	0.005

Figure Captions

1. Extrapolation used in ref. 2 in order to find the $\ell = 0$ and $\ell = 1$ resonances.

2. Transition form factors (squared), calculated using a (nucleon-nucleon) two-body bound state model for the resonances.

3. Contribution of the resonances to the structure function νW_2 for different ℓ and q^2 values, calculated using a (pion-nucleon-nucleon) three-body bound state model.



2.5
2.4
2.3
2.2
2.1
2.0
1.9
1.8
1.7
Resonance Energy (GeV)
1G4
(2430 MeV)
3F3
(2260 MeV)
1F3
1D2
(2140 MeV)
(2060 MeV)
pp π Bandhead (2020 MeV)
143 MeV
p-p Ground State (1877 MeV)
ℓ-Value or j-Value
0 1 2 3 4
0
10
20
Linear ℓ(ℓ+1) or j(j+1) Axis
9-79
3687A1

Fig. 1

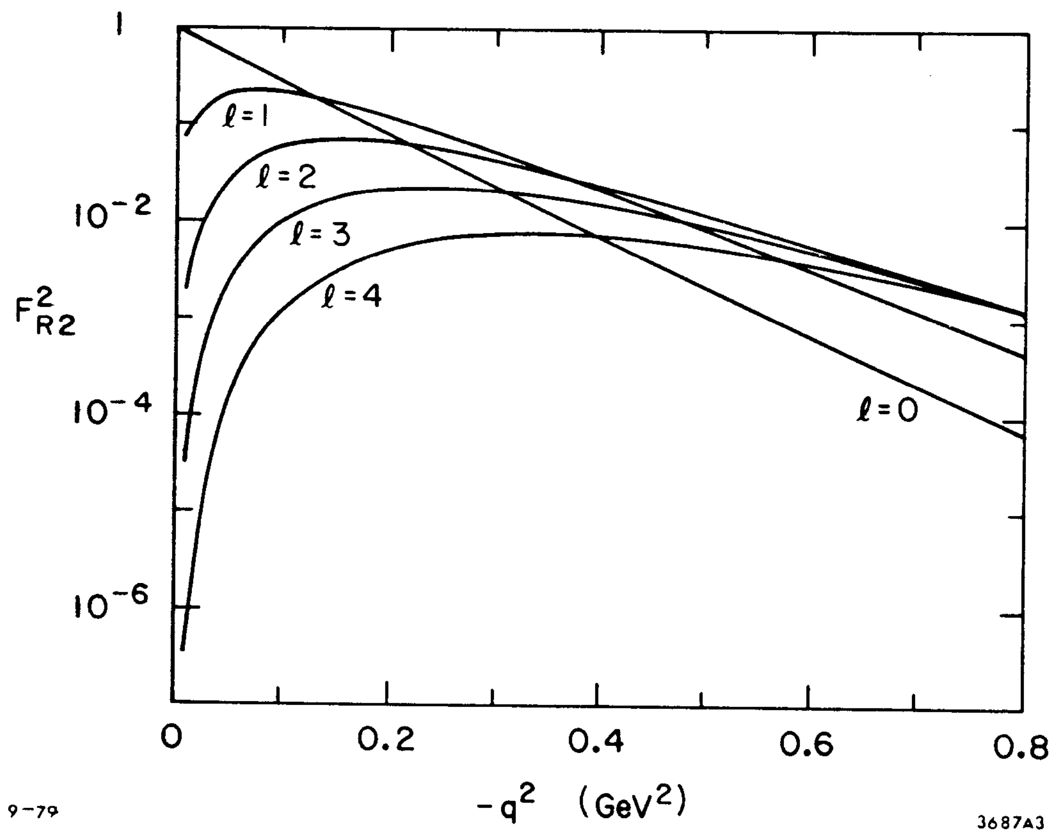

1
10^{-2}
F^2_{R2}
10^{-4}
10^{-6}
$\ell = 1$
$\ell = 2$
$\ell = 3$
$\ell = 4$
$\ell = 0$
0
0.2
0.4
0.6
0.8
$-q^2$ (GeV^2)
9-79
3687A3

Fig. 2

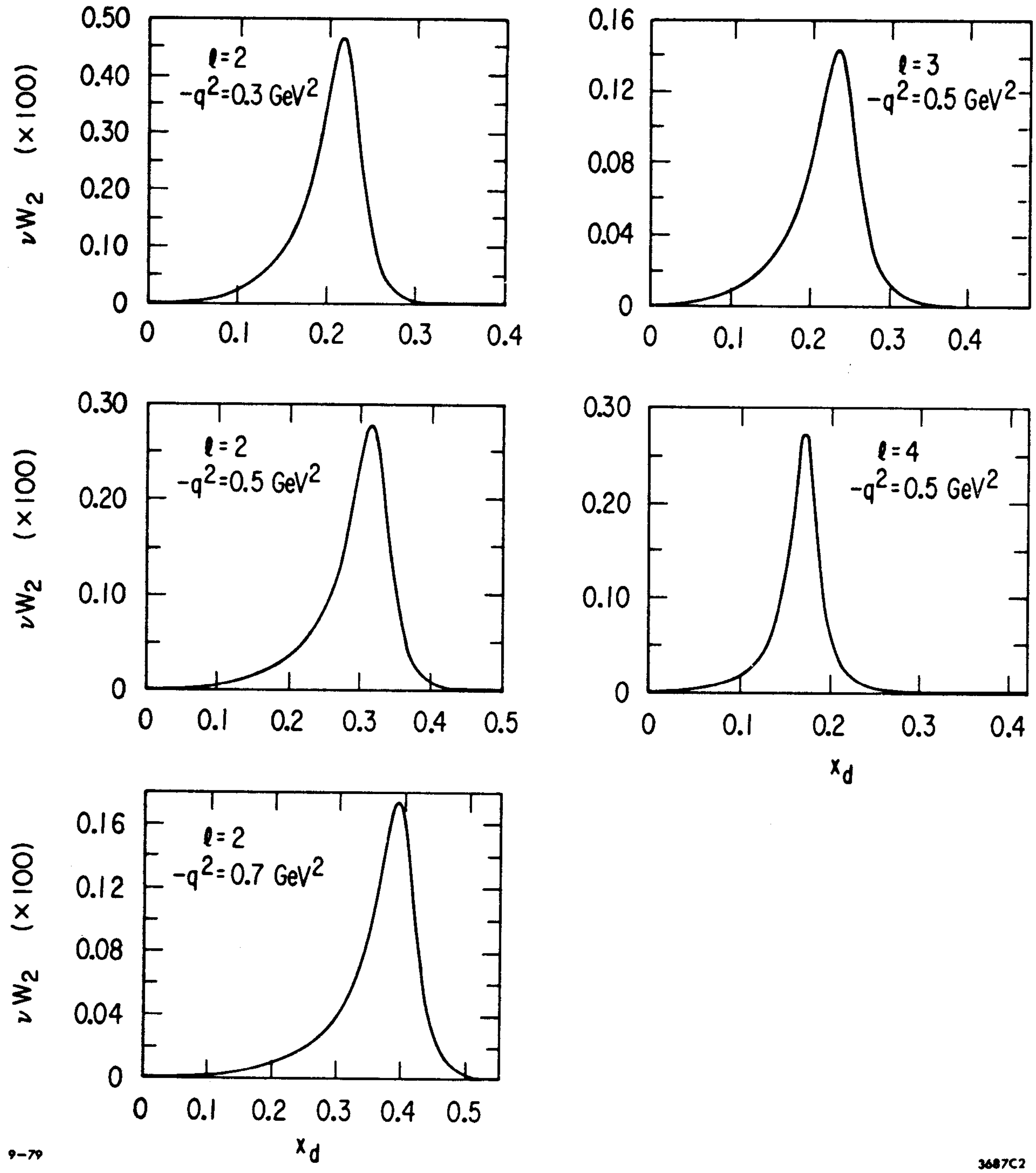

0.50
0.40
0.30
0.20
0.10
0
νW2 (×100)
ℓ=2
−q2=0.3 GeV2
0
0.1
0.2
0.3
0.4
0.16
0.12
0.08
0.04
0
ℓ=3
−q2=0.5 GeV2
0
0.1
0.2
0.3
0.4
0.30
0.20
0.10
0
νW2 (×100)
ℓ=2
−q2=0.5 GeV2
0
0.1
0.2
0.3
0.4
0.5
0.30
0.20
0.10
0
ℓ=4
−q2=0.5 GeV2
0
0.1
0.2
0.3
0.4
xd
0.16
0.12
0.08
0.04
0
νW2 (×100)
ℓ=2
−q2=0.7 GeV2
0
0.1
0.2
0.3
0.4
0.5
xd
9–79
3687C2

Fig. 3