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THE IMPACT OF HIGHER-TWIST TERMS ON THE ANALYSIS OF SCALING VIOLATION*

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ABSTRACT

A critical discussion is given of analyses of scaling violation in deep-inelastic scattering in the context of QCD. Several possible approaches are examined. Higher-twist contributions are defined, and it is shown that they can have a crucial impact on tests of QCD. Higher-twist terms can dramatically affect $R = \sigma_L/\sigma_T$. QCD may be harder to test than previously realized.

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Since Quantum Chromodynamics (QCD) was proposed as the theory of the strong interactions, there have been a number of claims about tests of QCD using deep-inelastic scattering data. It has been suggested that QCD has been proven by certain data. It has also been suggested that some data present serious problems for QCD. It has been suggested that particular tests can distinguish QCD from other theories (though no other theories have been proposed).

I believe that most of these claims are at best naive. Almost all analyses of scaling violations in deep-inelastic scattering ignore the critical role of higher-twist (order $1/Q^2$) corrections. Some analyses follow procedures which bias the results, and others use or weight very low Q^2 data. Still others use procedures which do not take full advantage of the available data. In this talk I discuss the impact of such problems on tests of QCD.

I will also discuss an analysis[1,2,3] by Larry Abbott, Bill Atwood and myself[1,2] of data from experiments for $eN \to eX$, $\mu N \to \mu X$ and $\nu N \to \mu N$ ($X \equiv$ anything). The electron data[4] includes all high Q^2 data ever taken at SLAC by the SLAC-MIT collaboration. The muon data[5] is from Fermilab and was taken by Anderson _et al_. The neutrino data used were taken at CERN by the BEBC[6] and CDHS[7] collaborations.

Let me begin with a brief review. For neutrino scattering in the lab frame the cross section is given by

$$\frac{d^2\sigma}{dE'\,d\Omega'} = \frac{G^2E'^2}{2\pi^2}\left[\left(\frac{2}{M}\sin^2\frac{\theta}{2}\right)F_1 + \left(\frac{1}{\nu}\cos^2\frac{\theta}{2}\right)F_2 \mp \left(\frac{E+E'}{\nu M}\sin^2\frac{\theta}{2}\right)F_3\right] \tag{1}$$

where (−) and (+) refer to neutrinos and antineutrinos respectively and $\nu \equiv E - E'$ (E' is the outgoing lepton energy). The structure functions F_i are functions of Q^2 and $x = Q^2/2m\nu$. F_3 arises from VA interference terms (it is a parity-violating piece) and therefore is not present in eN or μN scattering.

The structure functions can be written in terms of the distributions of u,d,c and s quarks as shown in Tables I and II (for electron, muon and neutrino scattering). The symbols u,d,... are to be read as u(x),d(x),... in the tables. Note that u and d refer to the distributions of u and d quarks in the proton, but in a neutron u is the distribution of d quarks and d is the distribution of u quarks.

In QCD the "emission" of gluons and of quark-antiquark pairs during the scattering process leads to a scaling violation which is evidenced by the presence of inverse powers of $\ell n\, Q^2/\Lambda^2$. There are two basic approaches[1-3] which have been used to study the deep-inelastic scattering data to seek evidence for the QCD scaling violation: (1) to observe the Q^2 dependence of $F_i(x,Q^2)$ and (2) to observe the Q^2 dependence of the moments of $F_i(x,Q^2)$ where the moments are given at large Q^2 by

$$M_i(N\,,\,Q^2) \;=\; \int_0^1 x^{N-2}\, F_i(x\,,\,Q^2)\, dx \qquad (2)$$

where $F_i = F_2$ or $F_i = xF_3$.

Before discussing the advantages and disadvantages of these two approaches, it is important to note that there are different types of corrections to the simple, leading order QCD calculations. Here I will describe three QCD corrections. They are target-mass corrections, higher-twist effects and corrections of higher order in α_s.

As Fig. 1 shows clearly, when a particle is accelerated at a massive target, there are corrections to be made. Fortunately the Nachtmann ξ formalism[8] can account fully for target-mass effects. It does not, of course, account for the final-state kinematics; no input requires, for example, that the invariant hadron mass $W \geq m_p$. One also finds in the ξ formalism that $F_i(x,Q^2)$ cannot approach zero at $x = 1$ for all Q^2 (see Fig. 2). This may be acceptable when one considers that the ξ formalism seems to account[9,1] in a "local duality" sense for elastic scattering and resonance production (in $F_i(x,Q^2)$ and in moments). However, local

TABLE I

The quark distribution content of $F_2(x,Q^2)$ in electron-proton and electron-neutron scattering. Each distribution $(u,\bar{u},d,\bar{d},s,\bar{s},c,\bar{c})$ is a function of x. $F_2(x,Q^2)$ is exactly same for muon scattering. NS and S refer to non-singlet and singlet. eN refers to scattering on an isoscalar target. u and d refer to scattering off u and d quarks in protons. In neutrons u refers to scattering off d quarks and vice versa.

$$F_2^{ep} = x\left[\frac{4}{9}(u+\bar{u}) + \frac{1}{9}(d+\bar{d}) + \frac{4}{9}(c+\bar{c}) + \frac{1}{9}(s+\bar{s})\right]$$

$$= \frac{5}{18}F_2^S + \frac{3}{18}F_2^{NS}$$

$$F_2^{en} = x\left[\frac{1}{9}(u+\bar{u}) + \frac{4}{9}(d+\bar{d}) + \frac{4}{9}(c+\bar{c}) + \frac{1}{9}(s+\bar{s})\right]$$

$$= \frac{5}{18}F_2^S - \frac{3}{18}F_2^{NS} + \frac{1}{3}x\left[(c+\bar{c}) - (s+\bar{s})\right]$$

$$F_2^{ep} - F_2^{en} = \frac{1}{3}x\left[(u+\bar{u}) - (d+\bar{d})\right]$$

$$= \frac{1}{3}F_2^{NS} - \frac{1}{3}x\left[(c+\bar{c}) - (s+\bar{s})\right]$$

$$F_2^{eN} = x\left[\frac{5}{18}(u+\bar{u}) + \frac{5}{18}(d+\bar{d}) + \frac{4}{9}(c+\bar{c}) + \frac{1}{9}(s+\bar{s})\right]$$

$$= \frac{5}{18}F_2^S + \frac{3}{18}x\left[(c+\bar{c}) - (s+\bar{s})\right]$$

$$F_2^S \equiv x\left[(u+\bar{u}) + (d+\bar{d}) + (c+\bar{c}) + (s+\bar{s})\right]$$

$$F_2^{NS} \equiv x\left[(u+\bar{u}) - (d+\bar{d}) + (c+\bar{c}) - (s+\bar{s})\right]$$

TABLE II

The quark distribution contents of $F_2(x,Q^2)$ and $xF_3(x,Q^2)$ in neutrino-proton and neutrino-neutron scattering. Each distribution is a function of x. νN refers to scattering on an isoscalar target. u and d refer to scattering off u and d quarks in protons. In neutrons u refers to scattering off d quarks and vice versa.

$$F_2^{\nu p} = 2x[\bar{u} + d + \bar{c} + s]$$

$$F_2^{\nu n} = 2x[u + \bar{d} + \bar{c} + s]$$

$$F_2^{\bar{\nu} p} = 2x[u + \bar{d} + c + \bar{s}]$$

$$F_2^{\bar{\nu} n} = 2x[\bar{u} + d + c + \bar{s}]$$

Using $c = \bar{c}$ and $s = \bar{s}$:

$$F_2^{\nu p} = F_2^{\bar{\nu} n} \qquad F_2^{\nu n} = F_2^{\bar{\nu} p} \qquad F_2^{\nu N} = F_2^{\bar{\nu} N}$$

$$F_2^{\nu N} = x[(u+\bar{u}) + (d+\bar{d}) + (c+\bar{c}) + (s+\bar{s})]$$

$$F_2^{\nu p} - F_2^{\nu n} = -\left(F_2^{\bar{\nu} p} - F_2^{\bar{\nu} n}\right) = -2x[(u-\bar{u}) - (d-\bar{d})]$$

$$xF_3^{\nu p} = 2x[-\bar{u} + d - \bar{c} + s]$$

$$xF_3^{\nu n} = 2x[u - \bar{d} - \bar{c} + s]$$

$$xF_3^{\bar{\nu} p} = 2x[u - \bar{d} + c - \bar{s}]$$

$$xF_3^{\bar{\nu} n} = 2x[-\bar{u} + d + c - \bar{s}]$$

$$xF_3^{\nu N} = x[(u-\bar{u}) + (d-\bar{d}) - (c+\bar{c}) + (s+\bar{s})]$$

$$xF_3^{\bar{\nu} N} = x[(u-\bar{u}) + (d-\bar{d}) + (c+\bar{c}) - (s+\bar{s})]$$

$$xF_3^{\nu p} - F_3^{\nu n} = -\left(xF_3^{\bar{\nu} p} - xF_3^{\bar{\nu} n}\right) = -2x[(u+\bar{u}) - (d+\bar{d})]$$

$$\left(F_2^{ep} - F_2^{en}\right) = -\frac{1}{6}\left(xF_3^{\nu p} - xF_3^{\nu n}\right)$$

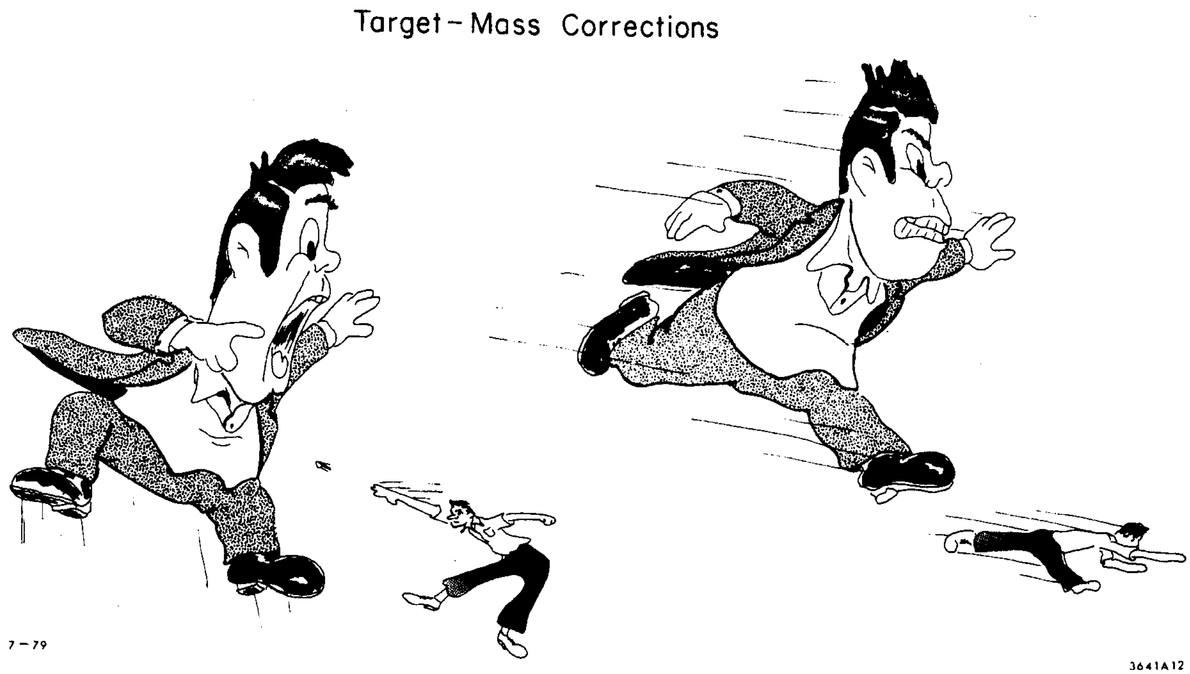

Fig. 1. When a particle is accelerated at a massive target, there are corrections to be made (artist, Sylvia A. Harris).

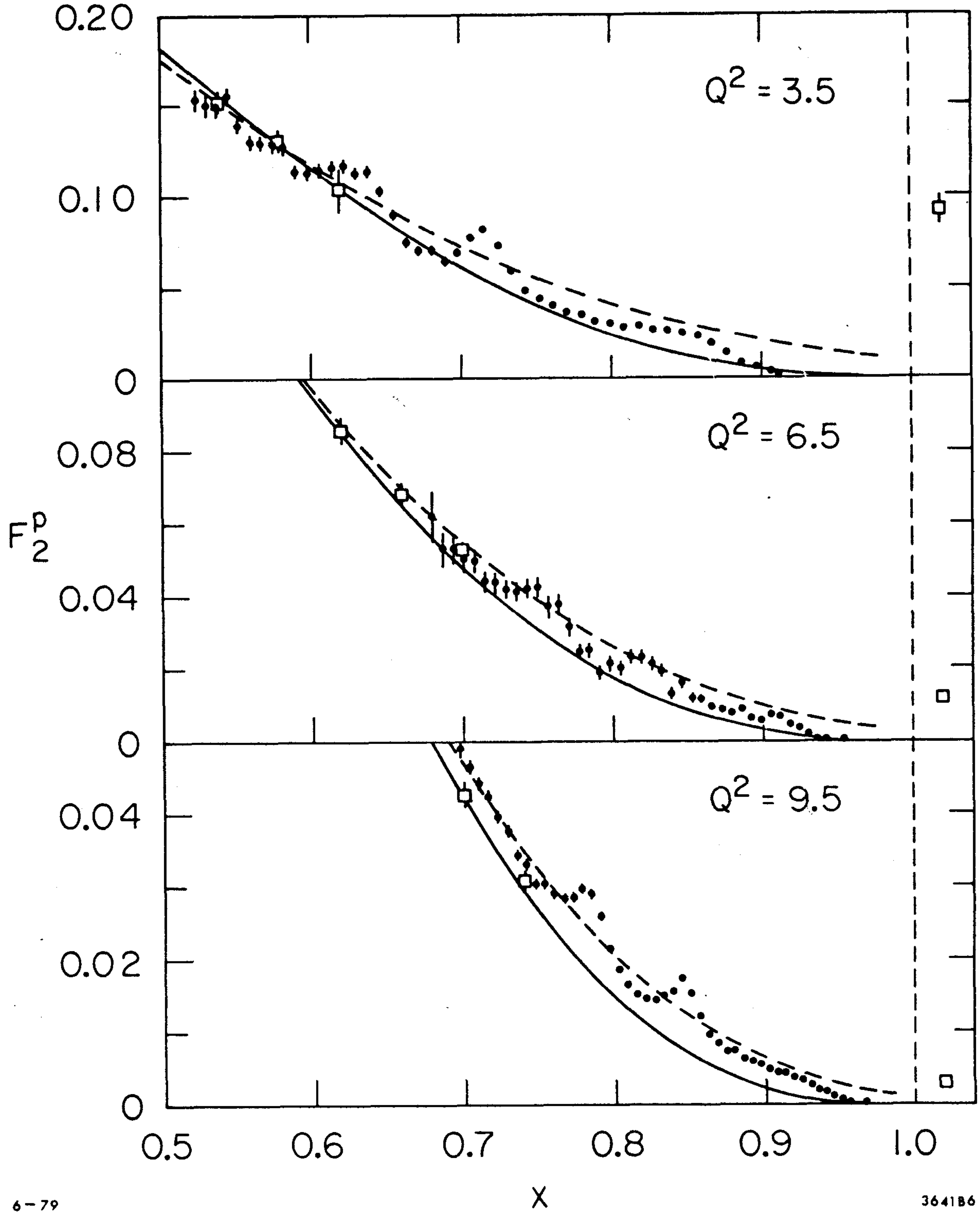

Fig. 2. $F_2(x,Q^2)$ for the proton. Elastics are shown in extra bins from x = 1 to 1.04 where the areas under the data points in these bins are equal to the area under the elastic spike at x = 1 in the original data. The solid (dashed) curve is the $x(\xi)$ scaling prediction of QCD. All data are from SLAC-MIT (Ref. 4). The square points have W > 2 GeV and are a compilation of all SLAC data. The dots indicate some of the data in the resonance region.

duality is a separate assumption and strict tests or QCD should probably avoid that assumption (by using large Q^2 and W).

There are a large variety of coherent phenomena such as transverse moment effects, resonance production, diquark scattering, and elastic scattering which are $1/Q^2$, $1/Q^4$,... corrections to the simple QCD predictions. These corrections are called higher-twist corrections where "twist" is defined as the dimension minus the spin of operators in the operator-product expansion. Although higher-twist terms describe coherent phenomena, there is no coherent explanation for the use of the word "twist". Figure 3 shows an example of a higher-twist effect: on the left, there are three independent bicyclists (quarks) which is the lowest twist case; on the right, two of the bicyclists (quarks) are on a tandem bicycle (diquark) which is a higher-twist contribution. It is not possible (at present) to calculate rigorously higher-twist terms in QCD. One can attempt to make models for significant higher-twist contributions and such attempts are in progress. But these models are not (necessarily) QCD and should only be considered as giving guidance in estimating higher-twist effects.

Alternatively, one can just parameterize these higher-twist terms. It is frequently assumed that these terms take the following forms which follow from quark-counting arguments:[10]

$$F_2(x\,,Q^2) \approx C(1-x)^a \left[1 + \frac{\mu_1^2}{Q_0^2(1-x)} + \frac{\mu_2^4}{Q_0^4(1-x)^2} + \ldots\right] \quad (3)$$

$$M_3(N\,,Q^2) \approx \frac{K_N}{\left(\ln Q^2/\Lambda^2\right)^{d_N}} \left[1 + \frac{\mu_3^2\,N}{Q^2} + \frac{\mu_4^4\,N^2}{Q^4} + \ldots\right] \quad (4)$$

Equations (3) and (4) indicate that higher-twist contributions should be largest at large x and large N. It should be emphasized that every theory must contain such order $1/Q^2$ corrections; they are not unique to QCD.

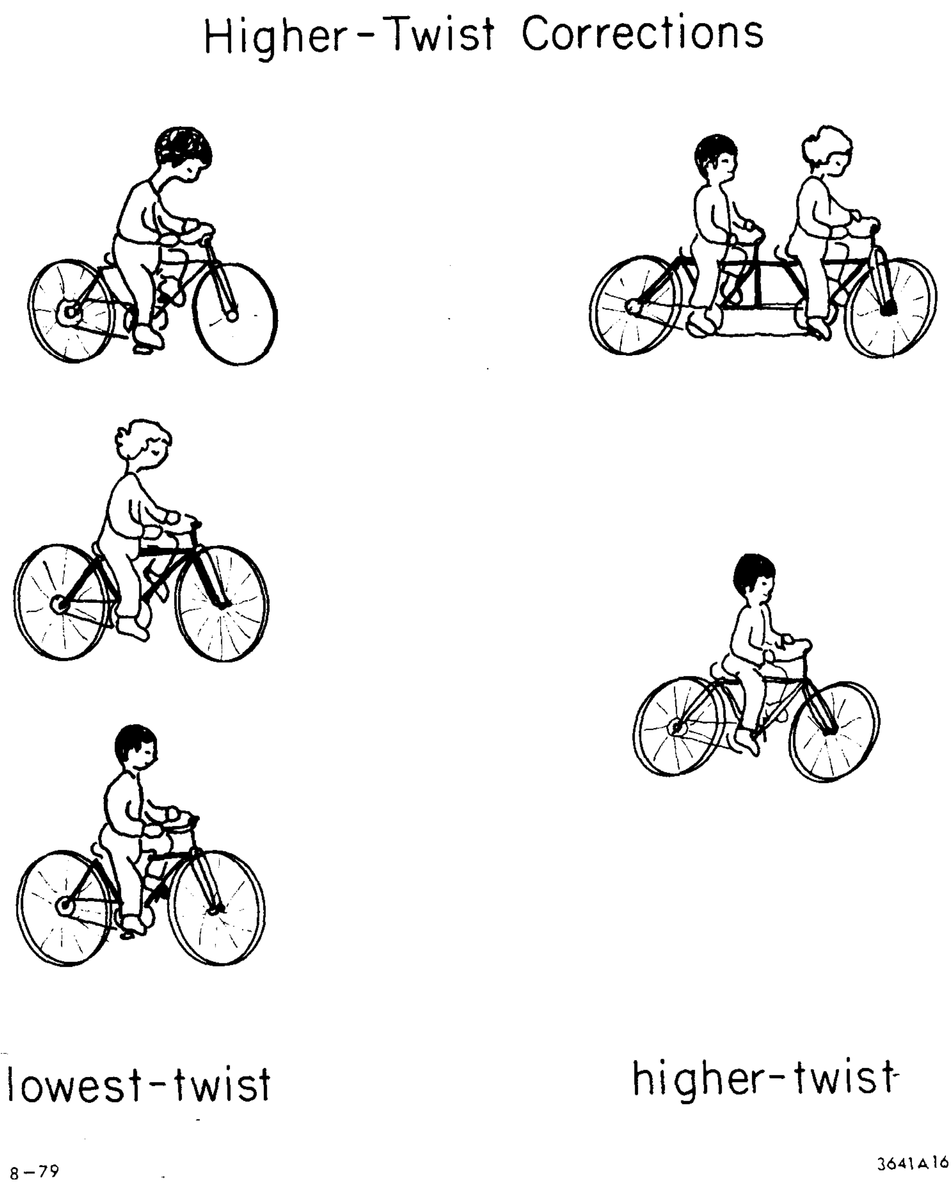

Fig. 3. Three independent bicyclists (quarks) with no transverse momentum which is the lowest twist case. Two of the bicyclists (quarks) on a tandem bicycle (diquark) which is a higher-twist correction (artist, Sylvia A. Harris).

Finally, there are corrections to the simple QCD predictions which come from terms of higher-order in α_s. Figure 4 shows an experimentalist finding more than one gluon in his basket. Such corrections have been computed by Floratos, Ross and Sachrajda[11] and by Bardeen, Buras, Duke and Muta.[12] For the moments of xF_3, they found

$$M_3(N,Q^2) = \frac{K_N}{\left(\ell n\ Q^2/\Lambda^2\right)^{d_N}} \left[1 + \frac{A_N + B_N\ \ell n\ \ell n\ Q^2/\Lambda^2}{\ell n\ Q^2/\Lambda^2} + \ldots\right] \tag{5}$$

where B_N like d_N are known.

Note, however, that the substitution

$$\Lambda^2 \rightarrow \Lambda^2 e^a \tag{6}$$

(where a is any number) is equivalent to a change in the term A_N since

$$\frac{1}{\left(\ell n\ \frac{Q^2}{\Lambda^2 e^a}\right)^{d_N}} \approx \frac{1}{\left(\ell n\ \frac{Q^2}{\Lambda^2}\right)^{d_N}} \left(1 + \frac{a d_N}{\ell n\ \frac{Q^2}{\Lambda^2}}\right) \tag{7}$$

So if one use $\Lambda_a^2 = \Lambda^2 e^a$, then $A_N^a = A_N - a d_N$. In particular we see that in first-order calculations Λ can be multiplied by any number, since it is compensated by a (neglected) second-order correction.[13] Therefore, Λ is meaningless in first-order (each specific quantity in a specific process can have different Λ). In second-order, Λ is meaningful only in the context of a specification of A_N in a given renormalization scheme. α_s is also ambiguous without such specification of scheme and parameters in second-order.

Different choices of A_N and Λ leave varying amounts to higher-order (3rd, 4th, ...) corrections. Moshe[14] has recently argued that for $Q^2 \lesssim 5\ \mathrm{GeV}^2$, the 3rd-order term in the moments will be very large (in the $\overline{\mathrm{MS}}$ scheme) even though the second-order term is small. This is another reason why it is best to avoid low Q^2 data.

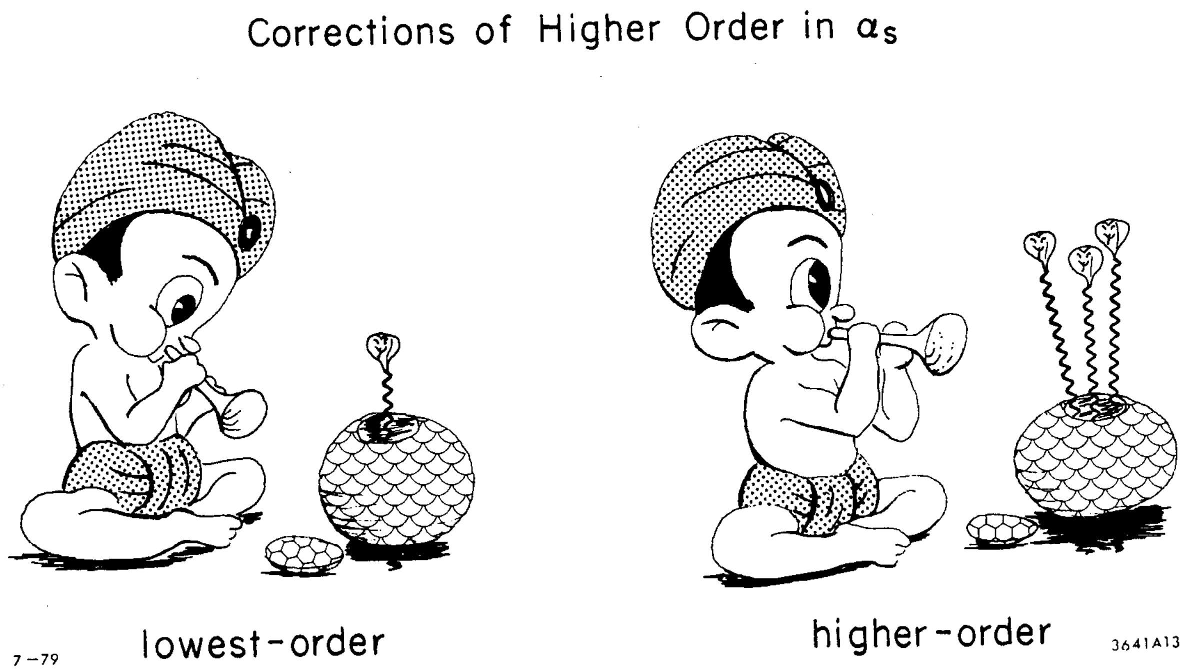

Fig. 4. An experimentalist finding more than one gluon in his basket (artist, Sylvia A. Harris).

Let me now return to a discussion of the two approaches to studying deep inelastic scattering data. After considering the QCD predictions in each approach, the advantages and disadvantages of each approach will be discussed. As is well-known QCD predicts that the structure functions shrink as Q^2 increases, i.e., they become more sharply peaked at small x. To leading order in $1/\ln (Q^2/\Lambda^2)$, this behavior is described by the Altarelli-Parisi equations.[15] For xF_3 one has

$$Q^2 \frac{\partial}{\partial Q^2} xF_3(x,Q^2) = \frac{\alpha_s}{2\pi} \int_x^1 dw \left(\frac{x}{w^2}\right) wF_3(w,Q^2) \; P_{q\to q}\left(\frac{x}{w}\right) \tag{8}$$

$P_{q\to q}\left(\frac{x}{w}\right)$ is related to the probability for a quark of momentum fraction x to arise from a quark of momentum fraction w, when probing with momentum Q^2 (see Fig. 5). This differential equation describes the evolution of $xF_3(x,Q_0^2)$ to other values of Q^2.

The equation for xF_3 is relatively simple since xF_3 is a flavor non-singlet (see Table II). $F_2^{ep} - F_2^{en}$ is also a flavor non-singlet although its quark content is different than that of xF_3. It obeys the same evolution equation. However, the evolution equations for F_2^{ep}, $F_2^{\nu p}$, etc. are more complicated since F_2^{NS}, F_2^{S} (the non-singlet and singlet parts of F_2) and $xG(x,Q^2)$ (the gluon distribution) each obey a different evolution equation. Furthermore, the equations for F_2^S and xG are coupled.

The shrinkage of the F_i is not a feature unique to QCD. The radiation of gluons plus momentum conservation are essentially the source of shrinkage in any theory. QCD predicts, however, a particular form of shrinkage.

To leading order in $1/\ln (Q^2/\Lambda^2)$ the expression for the moments, $M_3(N,Q^2)$, of $xF_3(x,Q^2)$ is quite simple

$$M_3(N,Q^2) = \frac{K_N}{\left(\ln Q^2/\Lambda^2\right)^{d_N}} \tag{9}$$

where K_N are free parameters and d_N are proportional to the anomalous

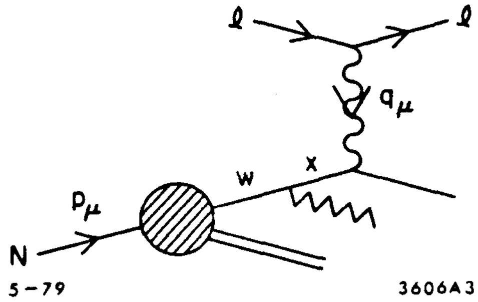

Fig. 5. An example of a current striking a quark of momentum fraction x which arises from a quark of momentum fraction w after gluon radiation.

dimensions γ_0^N

$$d_N = \gamma_0^N / 2\beta_0 \tag{10}$$

$$\gamma_0^N = \frac{8}{3}\left(1 - \frac{2}{N(N+1)} + 4\sum_{j=2}^{N} \frac{1}{j}\right) \tag{11}$$

$$\beta_0 = 11 - \frac{2}{3} N_f \tag{12}$$

where N_f is the effective number of quark flavors. One naively expects that in other theories, moments would involve powers of Q^2 not $\ln Q^2/\Lambda^2$.

Let us begin the discussion of the relative advantages and disadvantages of the direct use of $F_i(x,Q^2)$ versus the use of moments by considering $F_i(x,Q^2)$. To use the evolution equations one must choose the form of $F_i(x,Q^2)$ at some $Q^2 = Q_0^2$. However, it is not even necessary to have any data at that Q_0^2. The evolution equations give the resulting $F_i(x,Q^2)$ at all Q^2 and a comparison can be made with the data at all Q^2. The parameters, such as Λ and the k_i in

$$F_2(x,Q_0^2) = k_1(1-x)^{k_2}(1+k_3x) \quad , \tag{13}$$

are adjusted until the best fit is obtained. If necessary, additional parameters can be added to the x dependence. Note that Eq. (13) is the form of F_2 only at $Q^2 = Q_0^2$; after evolving to other Q^2, this form is modified.

In most experiments, the range in x for which there are statistically significant data changes radically as Q^2 increases. As a result it is crucial that the x-dependence be fit at all Q^2 rather than only at some Q_0^2. In other words, the determination of Λ and k_i should be done simultaneously. This procedure makes the best use of all available data and insures that the correct values of Λ and k_i are obtained.

As an example of the results of this procedure, I show in Fig. 6, the predicted shapes of the valence and sea quark distributions with parameters determined by fitting CDHS data.[7]

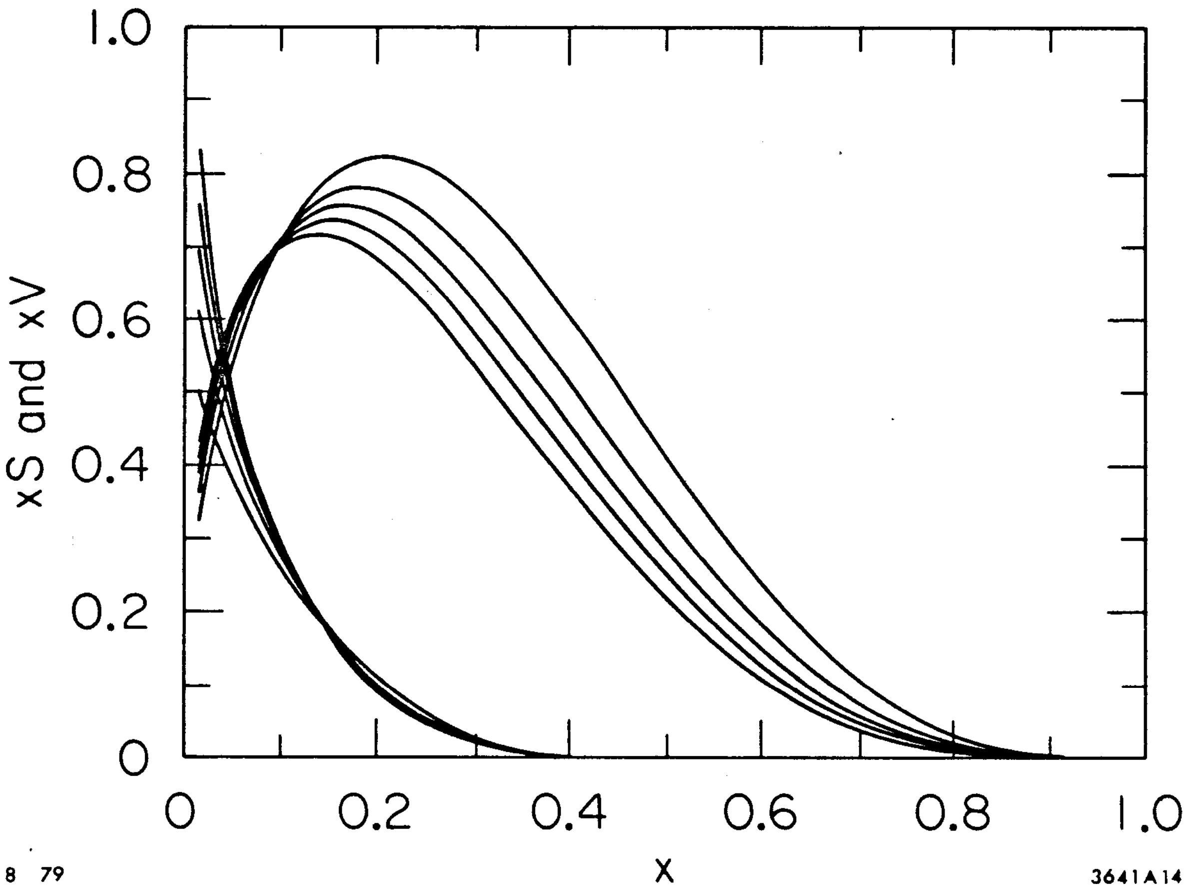

Fig. 6. The sea distribution [$xS(x) = 1/2(F_2 - xF_3)$] and the valence distribution [$xV(x) = 1/2(F_2 + xF_3) - xS(x)$] predicted by QCD. The parameters (as in Eq. (13)) are determined by fitting the CDHS data (Ref. 7). The valence curves from top to bottom (at $x = 0.4$) and the sea curves from bottom to top (at $x \approx 0$) refer to $Q^2 = 4$, 10.5, 25.4, 56.4, 152.4 GeV^2.

The direct use of $F_i(x,Q^2)$ gives one a clear visual interpretation of scaling violation. It also allows examination of the impact of exclusive channels, see Fig. 2. Of course, perturbative QCD will never reproduce resonances or the elastic peak except in the "local duality" sense (discussed above and in Refs. 9 and 1). It is clearly important, therefore, when fitting theory for $F_i(x,Q^2)$ to data (i.e. - calculating χ^2) to exclude all data with hadron invariant mass $W \lesssim 2$ GeV.

The use of moments to analyze scaling violations provides very clean predictions for Q^2 dependence which do not depend on any assumptions about the x-dependence of $F_i(x,Q^2)$. Furthermore, the next-to-leading-order-in-α_s corrections[11,12] have been calculated for moments so that Λ can be defined unambiguously. However, moments have some very serious shortcomings. In order to calculate moments, one must have data over the entire x range (especially at high x) for each Q^2 value. Otherwise one must extrapolate into unmeasured regions and place an unnecessary uncertainty into the results. The moments with $N \geq 4$ are dominated by high x where most experiments have the poorest statistics; full advantage is not taken of the low x data. Successive moments ($N = 5,6,7,8,\ldots$) are similar integrals over the same data and do not provide much independent information (care must be taken with correlations).

The high x region contains resonance production and elastic scattering unless Q^2 is large. The big impact of the $W < 2$ region on moments is shown in Fig. 7. It has been argued[9] that use of the Nachtmann ξ formalism[8] compensates for the presence of such terms. Figure 8 shows the difference between Nachtmann moments and ordinary moments as a function of Q^2. As discussed earlier, other higher twist (order $1/Q^2$) corrections are expected, in general, to be largest at high x. Since these corrections cannot be calculated and (also discussed later) can confuse the analysis, I think it best to require large Q^2 ($Q^2 > 10$ or 20 GeV^2) when using moments to test QCD. Alternatively one might attempt

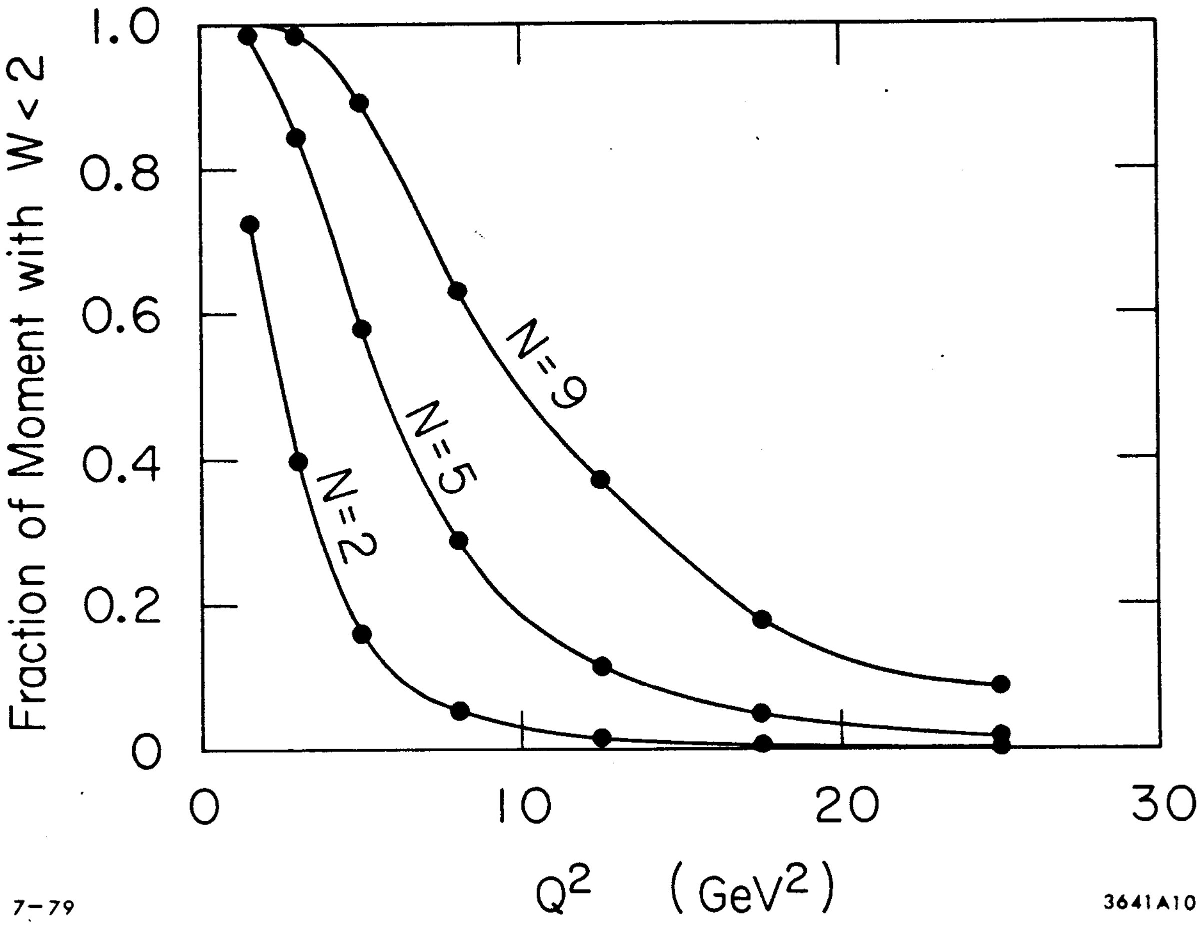

Fig. 7. The fraction of the Nachtmann moments (for N = 2,5,9) which come from the resonance region (W > 2 GeV). The contributions at relatively large Q^2 are quite significant. The data are from Ref. 4 with error bars not shown.

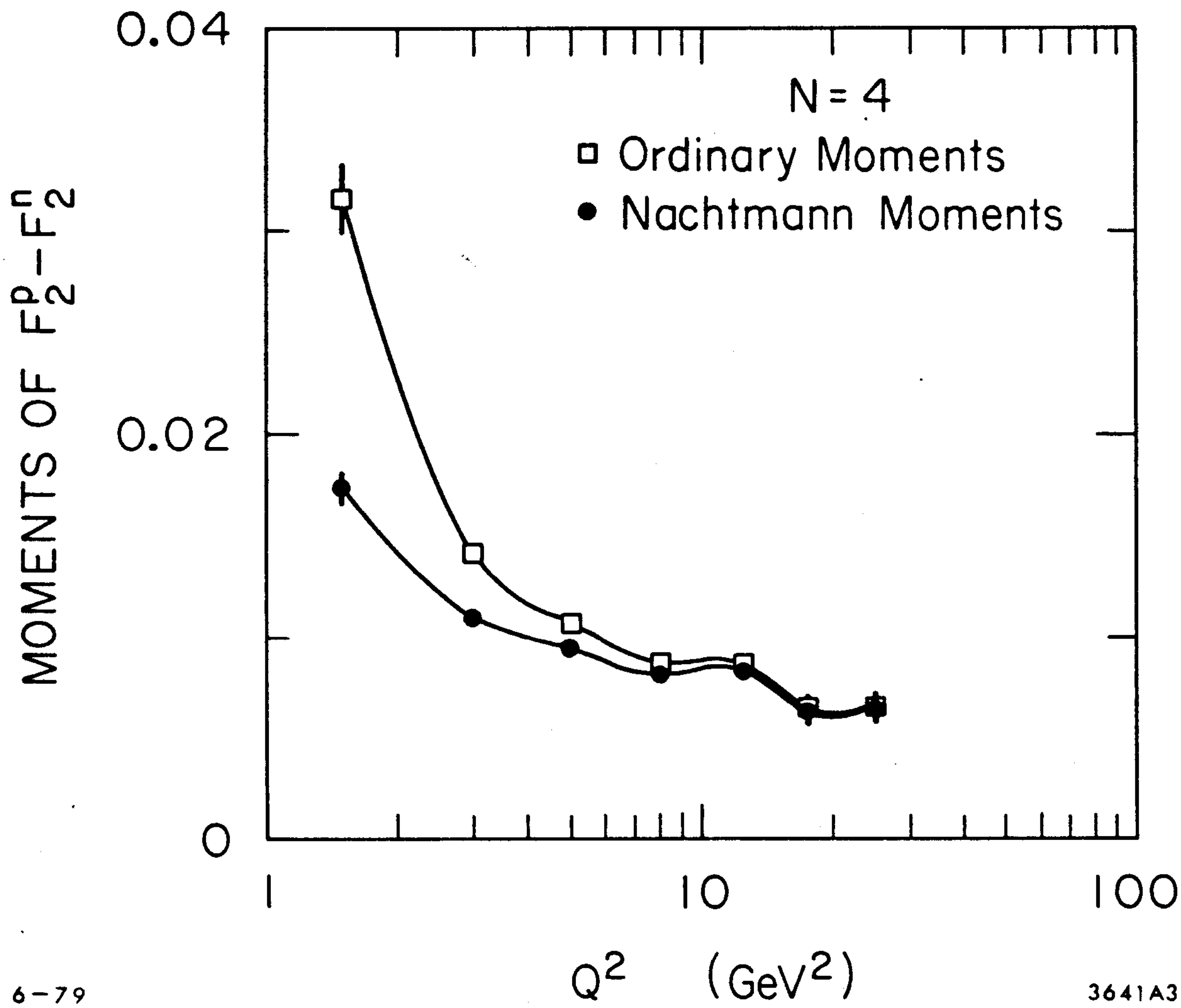

Fig. 8. A comparison of ordinary moments and Nachtmann moments from the SLAC data of Ref. 4. Curves drawn connecting the data points to help guide the eye. Target-mass effects appear to be large for $Q^2 \lesssim 3$ GeV^2.

to measure them experimentally; but that could prove quite difficult. It is certainly unreasonable to use $Q^2 \lesssim 3\ \mathrm{GeV}^2$ and $W < 2$ GeV.

I would like to comment on what I think is a very poor way to test QCD with moments: the scheme in which Λ's extracted for each moment are compared (the Λ_N scheme[12,16]). When Λ is extracted from data, the low Q^2 data are weighted exponentially. Thus the analysis is based on the data points for which one trusts QCD the least. Furthermore, moments weight high x where higher twist effects will be largest and where there are statistically poor (if any) data. Since most moments measure the same high x data repeatedly as N increases, the correlations among the data points shown are not clear. The scheme should not be used to test for 2nd-order-in-α_s effects, since if such effects are large enough to measure reliably (and at low Q^2 they are large), then perturbation theory is breaking down anyway (see especially Ref. 14). I believe that experimentalists (and others) should not consider the Λ_N scheme.

For the remainder of my talk I would like to address the problems discussed above in the context of five questions:

(1) Is there scaling violation?

(2) Is QCD consistent with all data?

(3) Could higher-twist terms alone account for all data?

(4) Are hypothetical alternative theories ruled out?

(5) Can Λ and other parameters be determined with present data?

The first question is easy to answer. With the BEBC[6] and CDHS[7] data the probability for perfect scaling is about 10^{-3} for $Q^2 > 3\ \mathrm{GeV}^2$. For the SLAC data the probability is less than 10^{-10} for $Q^2 > 5\ \mathrm{GeV}^2$ and $W > 2$ GeV (systematic errors included). There is scaling violation.

The answer to the second question is that QCD is completely consistent with all deep-inelastic data with one apparent exception. QCD fits all data for $F_2^{\nu N}$, F_2^{ep}, $F_2^{ep} - F_2^{en}$ and $xF_3^{\nu N}$ as shown with solid lines in Figs. 9, 10, 11 and 12. However, for R where

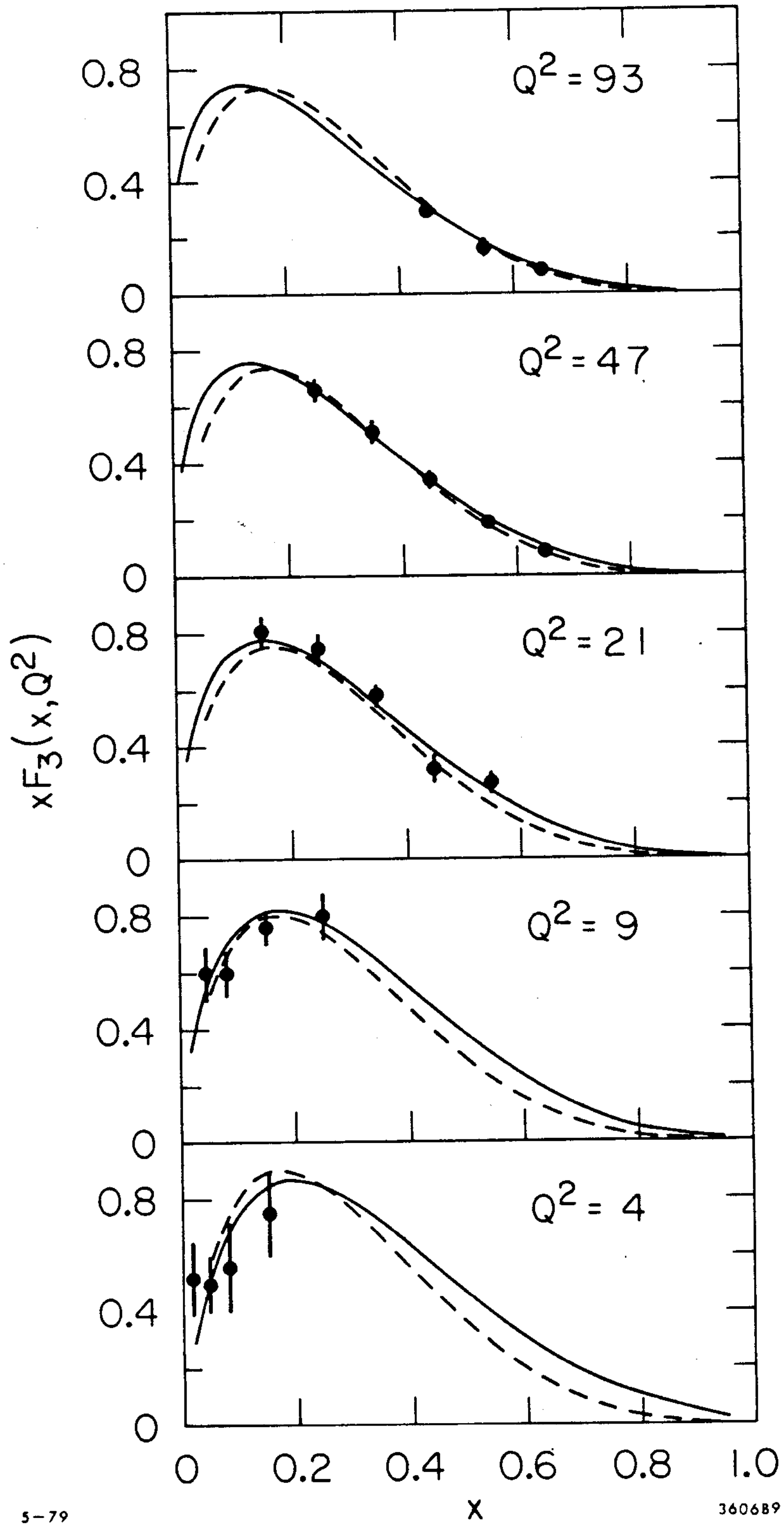

Fig. 9. $xF_3(x,Q^2)$ at various Q^2 values. The solid curves are the QCD predictions; the dashed curves are described in the text. The CDHS data are from Ref. 7.

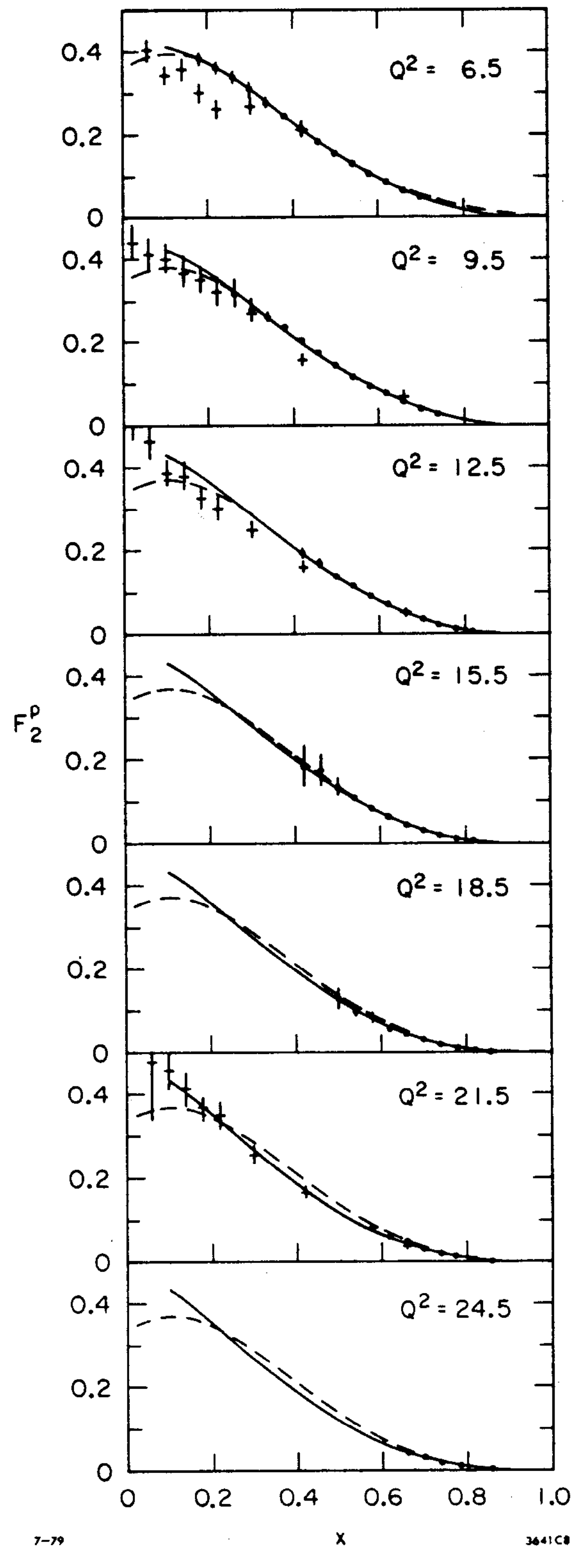

Fig. 10. $F_2(x,Q^2)$ on protons at various Q^2 values. The solid curves are the QCD predictions; the dashed curves are described in the text. The fit was done using only SLAC-MIT data,[4] but CHIO data[5] are also shown.

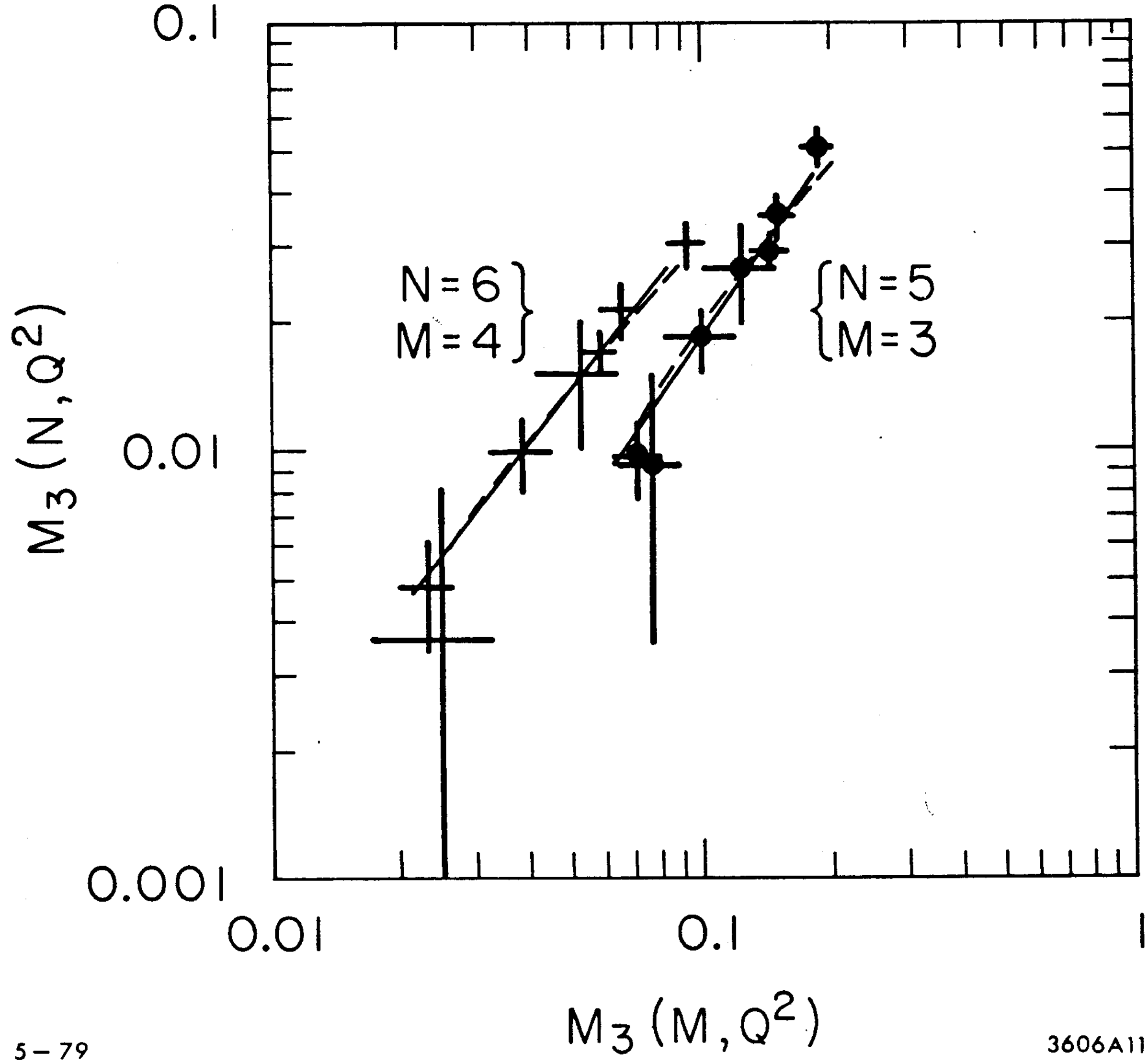

Fig. 11. The data (Ref. 6) for $M_3(N,Q^2)$ are plotted versus the data for $M_3(M,Q^2)$ on a log-log scale. The solid curves are the predictions of leading order QCD; the dashed curves are described in the text. This plot does not indicate the strong correlations between $M_3(N,Q^2)$ and $M_3(M,Q^2)$.

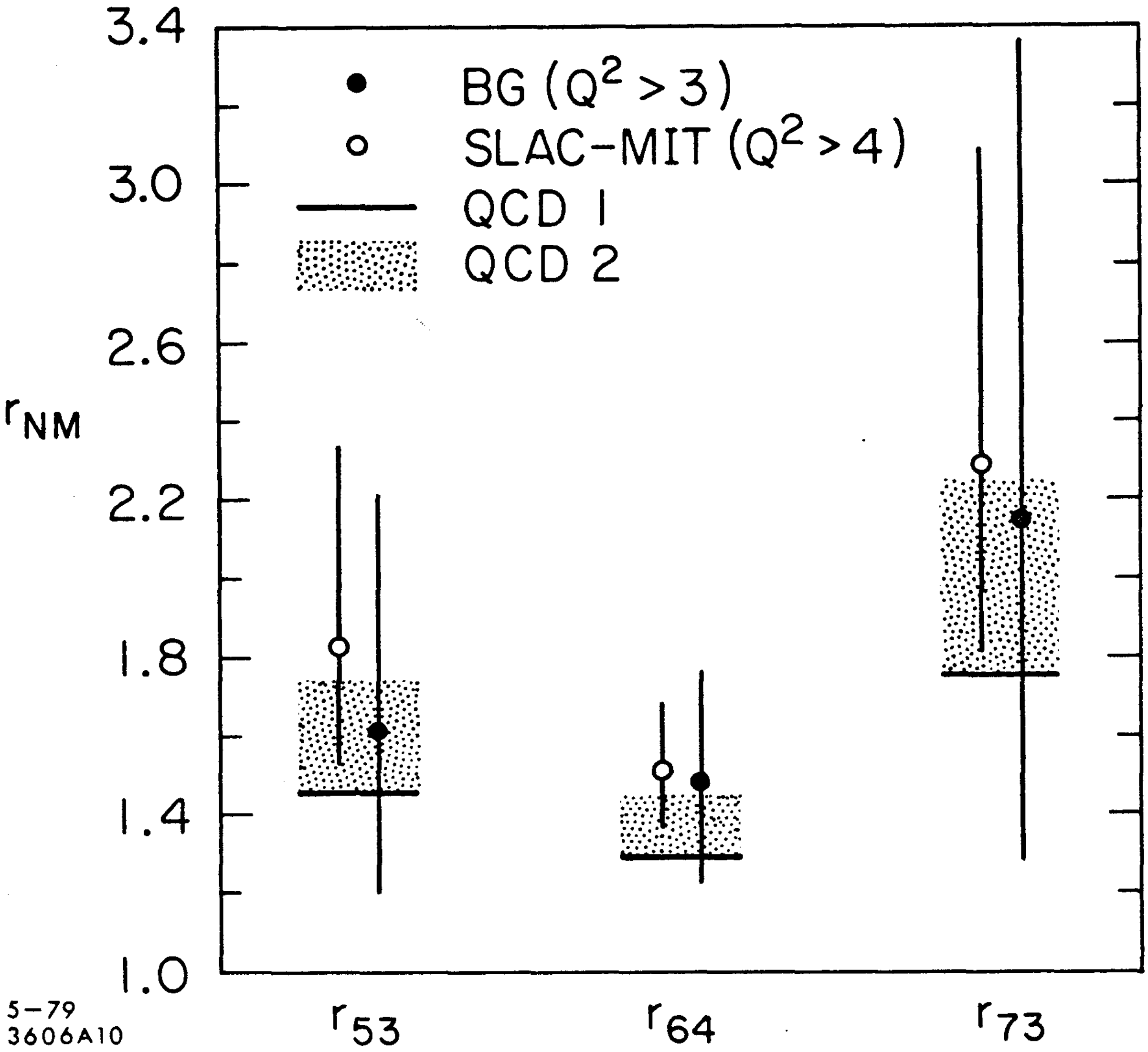

Fig. 12. Values of $r_{NM} \equiv d_N/d_M$ for various combinations of N and M from the SLAC-MIT data[4] and the BEBC-Gargamelle data[6].

$$R \equiv \sigma_L / \sigma_T \tag{14}$$

the data[4] are $R \simeq .21 \pm .1$ with no evident Q^2 or x dependence and QCD predicts $R \approx 0$ at high x and $R = .05 - .1$ (for $x \approx 0.3$). R will be discussed further in a moment.

To address the question of how well the data are fit by higher-twist alone, we used[1,2] parameterizations (from quark-counting arguments[10]) such as:

$$F_2(x,Q^2) = C(1-x)^a(1+bx)\left[1+\frac{\mu_1^2}{Q^2(1-x)}+\frac{\mu_2^4}{Q^4(1-x)^2}\right] \tag{15}$$

$$M_3(N,Q^2) = K_N\left[1+\frac{\mu_3^2\,N}{Q^2}+\frac{\mu_4^4\,N^2}{Q^4}\right] \tag{16}$$

(one can set μ_1 or μ_2 and μ_3 or μ_4 equal to zero). The evolution equations are, of course, not used. Although higher-twist terms must be present in QCD (as in all theories), they have normally been ignored. In this case I am (for sake of argument only) ignoring QCD instead. Looking again at Figs. 9, 10 and 11, but this time at the dashed lines, one sees that higher-twist terms alone can in fact fit the data remarkably well.

What I found to be surprising is that the famous ratio d_N/d_M of anomalous dimensions (Fig. 12) obtained from $\ln M_N$ versus $\ln M_M$ plots, is obtained from higher-twist terms alone almost independent of the values of μ_3 and μ_4 as long as

$$0 < \mu_3, \mu_4 \lesssim 1 \text{ GeV}^2 \tag{17}$$

(the Q^2 dependence of the moments is, of course, determined by μ_3 and μ_4). This result occurs because Eq. 16 gives $d_N/d_M \approx N/M$ which is very similar to the data and QCD.

How do higher-twist terms fare in fitting $R = \sigma_L/\sigma_T$? It is necessary to make a specific model in order to calculate R. Abbott, Berger, Blankenbecler and Kane[17] have made a model assuming diquark scattering to be a dominant higher-twist contribution. As seen in Fig. 13 they obtained reasonable agreement with the data, certainly better than QCD.

If both lowest-twist QCD and higher-twist terms alone can fit the data separately, then clearly so can a mixture. In fact, we cannot tell from present data how much of the observed scaling violation is due to each. Neither can we yet calculate higher-twist contributions although I expect and hope we will see a considerable theoretical effort at making models of these contributions in the next few years. Until we know more, any time QCD does not work, we can try to fix it with higher-twist effects. For example, Abbott, Atwood and I have,[2] following the model of Ref. 17, added higher-twist terms to QCD in R with the results shown in Figs. 14 and 15 (Fig. 15 shows a comparison with lowest-twist QCD).

While there are no alternative theories to QCD, one can ask, for example, whether power-law scaling violations are ruled out. The answer (at least at present) is <u>no</u>. As with QCD, one must allow for higher-twist terms. An extreme example: if one uses the BEBC data with $Q^2 > 1$ GeV^2 and assumes the moments take the form

$$M_3(N,Q^2) = \frac{C_N}{(Q^2)^{\epsilon}} \tag{18}$$

then the probability is 10^{-5} (i.e., it is "ruled out"). But if one assumes

$$M_3(N,Q^2) = \frac{C_N}{(Q^2)^{\epsilon}}\left[1 + \frac{aN}{Q^2} + \frac{bN^2}{Q^4}\right] \tag{19}$$

then an excellent fit to the data results when a and b are very small, $a = .008$ and $b = .07$! Similarly, one may believe that hypothetical theories with scalar gluons can fit the data, if higher-twist terms are included.

Finally, can Λ (and other parameters) be determined with present data? The answer is not very well until we can determine the magnitude of higher-twist terms either theoretically or experimentally. One can

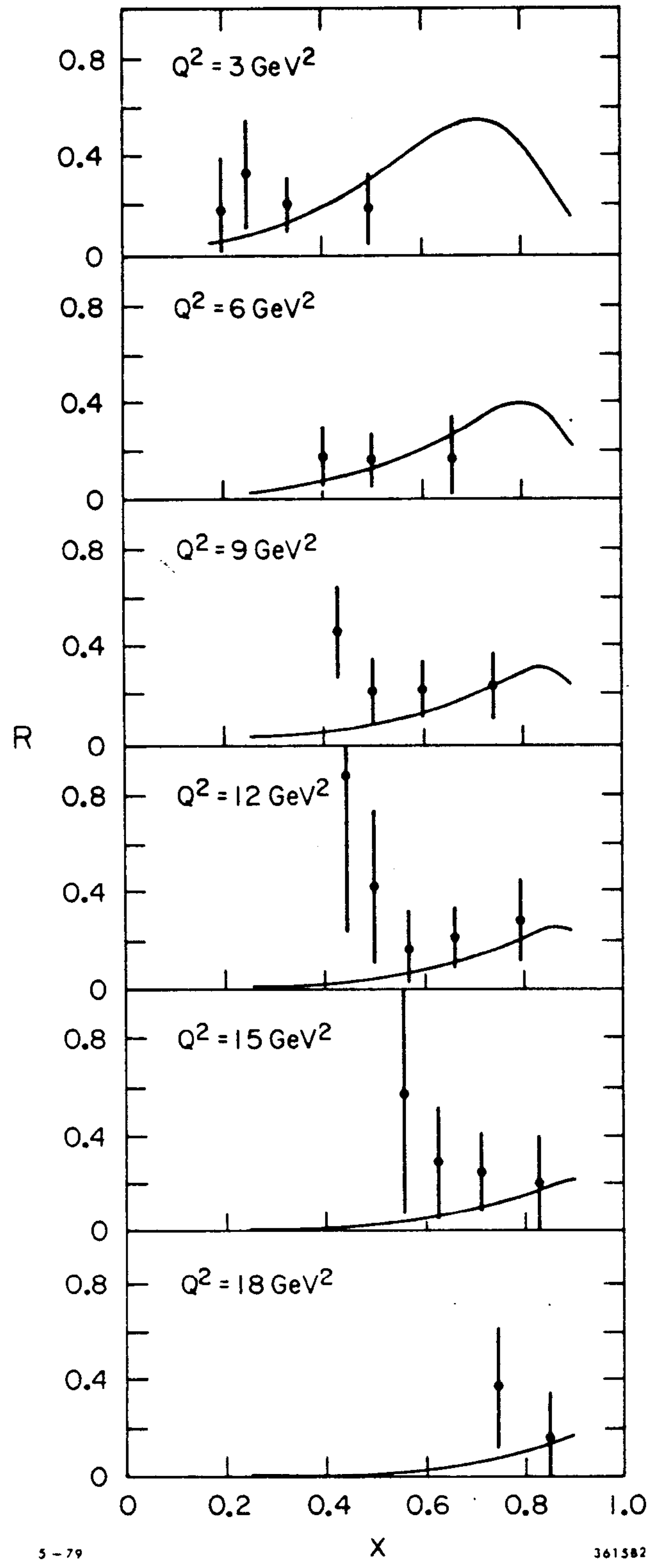

Fig. 13. $R \equiv \sigma_L/\sigma_T$ at various Q^2 values. The data are a compilation of all SLAC-MIT data[4]; the error bars are mostly systematic. The curves are from Ref. 17 and are the result of a model in which scaling violation comes from diquark scattering.

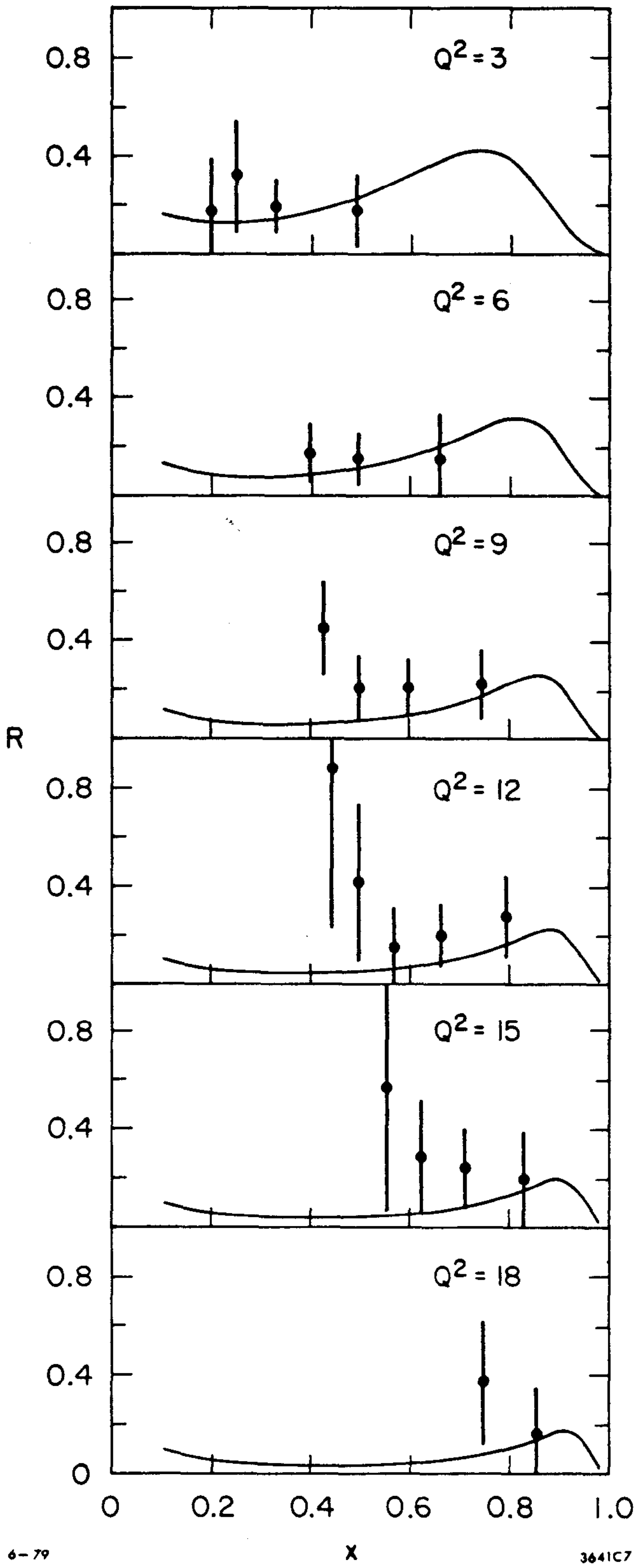

Fig. 14. $R \equiv \sigma_L/\sigma_T$ at various Q^2 values. The data are a compilation of all SLAC-MIT data[4]; the error bars are mostly systematic. The curves are from Ref. 2 and show the results of QCD when a higher-twist contribution from diquark scattering (using the model of Ref. 17) is added.

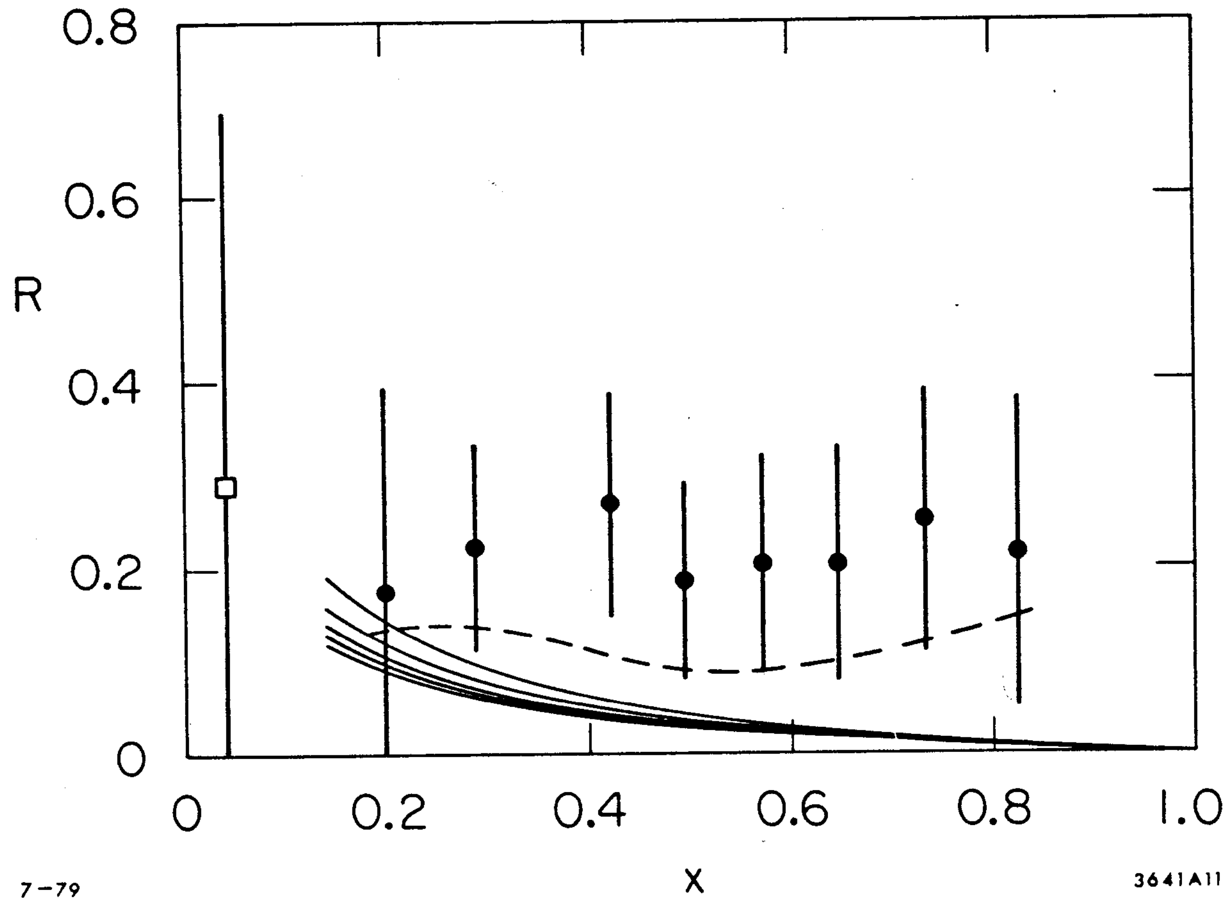

Fig. 15. $R \equiv \sigma_L/\sigma_T$ versus x. Since the SLAC-MIT data[4] show no evidence of Q^2 dependence, in this figure all data have been combined; the errors bars are mostly systematic. The solid curves show QCD with no higher-twist contributions for the Q^2 values covered by the data. The dashed curve is QCD plus diquark model (Ref. 17) of higher-twist; the curve reflects the average Q^2 of the data points through which it is drawn.

modify QCD with terms such as

$$1 + \frac{\mu_1^2}{(1-x)Q^2} \tag{20}$$

or

$$1 + \frac{\mu_2^2}{(1-x)^2 Q^4} \tag{21}$$

and then obtain the resulting Λ_1 or Λ_2 from the SLAC F_2^{ep} data;[4] the result are shown in Fig. 16. It is plausible that Λ may be reduced by as much as a factor of 2 or by as little as a few percent when the correct higher-twist terms are added.

In examining F_2 it is necessary to make assumptions about the power P_G, of (1-x) in the gluon distribution; this assumption can affect the value of Λ obtained. Some have suggested that P_G can be extracted from the present data. We find that if all parameters are allowed to vary freely that P_G cannot be determined. For example, with the CDHS data[7] for F_2 we find (where k_i are defined as in Eq. (13))

P_G	χ^2	χ^2/d.o.f.	Λ	k_1	k_2	k_3
4	37.8	.82	.401	1.36	2.70	1.95
5	38.4	.83	.349	1.34	2.81	2.22
7	39.6	.86	.307	1.34	2.90	2.43

One can also see that the sensitivity of Λ to the value of P_G is not too great.

My conclusions are:

(1) QCD has not been contradicted by the data, but neither has it been confirmed.

(2) Higher-twist effects could be crucial in understanding scaling violation. Λ may be smaller than we have previously assumed.

(3) Those planning new experiments should Monte Carlo their expected data assuming say Λ = .5 GeV and μ_1 = .5 GeV and see if analysis

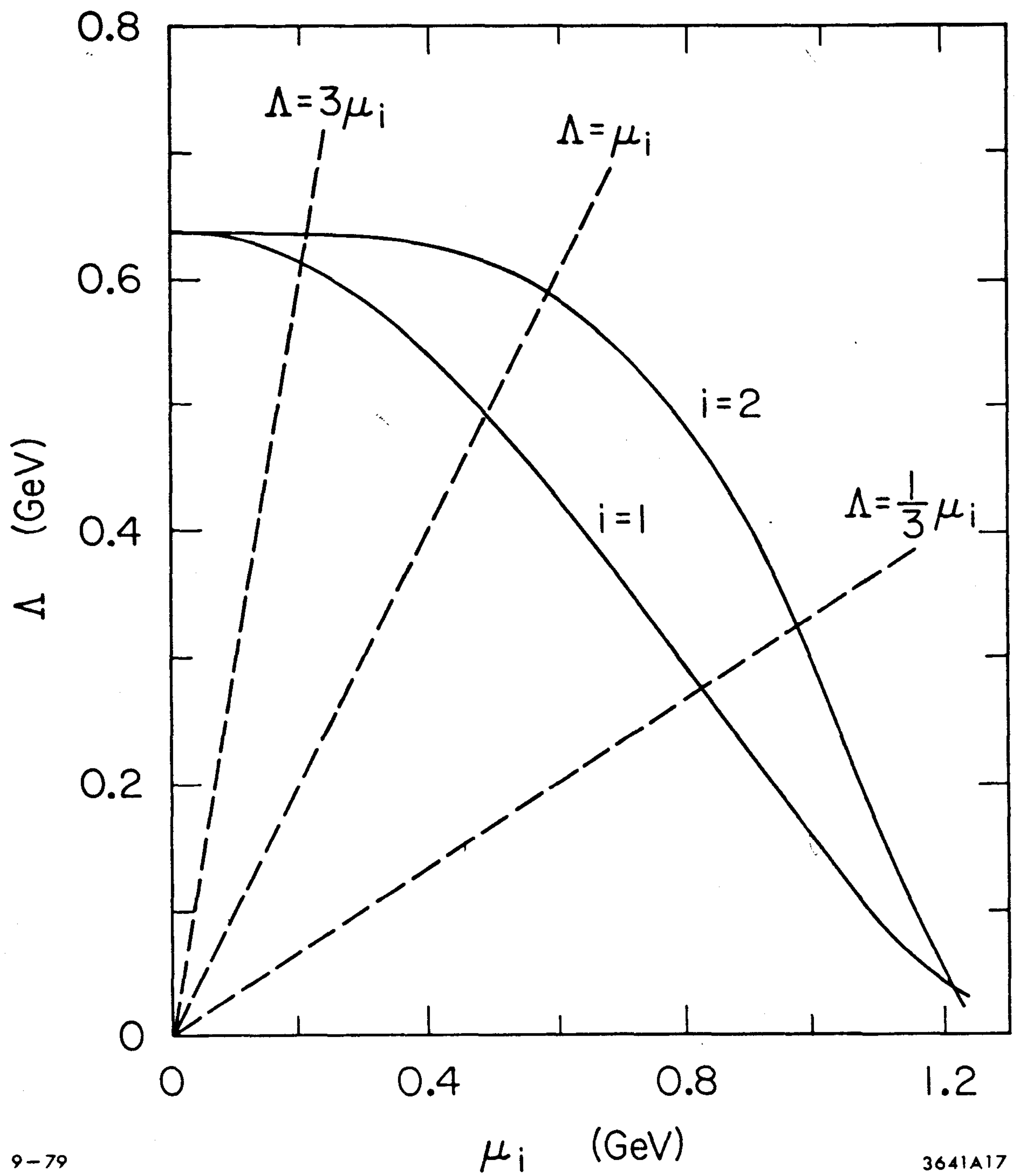

Fig. 16. The value of Λ obtained when higher-twist contributions have been assumed. μ_1 and μ_2 indicate the magnitude of the higher-twist terms where the two forms considered are shown in Eqs. (20) and (21). I thank H. Georgi for suggesting this plot.

programs can separate the logarithmic behavior from the $1/Q^2$ behavior.

(4) Do not use the Λ_N scheme for testing QCD.

(5) Regretfully, QCD may be harder to test than we realized.

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