PARTICLE ACCELERATION WITH TE_{011} CAVITY

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Summary

Presently, the maximum effective electric field, E_a, in an accelerating structure is limited by the available peak rf power. If the structure is superconducting, the peak power requirements are reduced by several orders of magnitude but E_a is limited by electron loading caused by surface electric fields. A right circular cylinder TE_{011} cavity has no surface electric fields and therefore, E_a is not limited by electron loading but can reach a much higher value which is limited by magnetic breakdown. A particle traversing the cavity along a chord of a cylinder wall circle gains a net voltage. This paper calculates the shunt resistance of a TE_{011} structure. It concludes that for the same E_a, a 4.2°K TE_{011} structure at 500 MHz requires 1000 times less peak power and four times less average power then a nonsuperconducting TM_{010} structure, and can sustain an E_a several times greater than can be presently sustained by a superconducting TM_{010} structure. Acceleration by a TE_{111} right circular cavity is also considered.

Cavity Fields

The TE_{011} cavity fields are:

$$H_z = HJ_0(3.832r/a)\sin(\pi z/L)$$
$$E_\phi = H\eta(\lambda_c/\lambda)\,J_1(3.832r/a)\sin(\pi z/L)$$
$$H_r = H(\lambda_c/\lambda_g)\,J_1(3.832r/a)\cos(\pi z/L)$$

a is the radius and L is the length of the cavity. The relative field values are shown in Fig. 1. Using $PQ = \omega U$

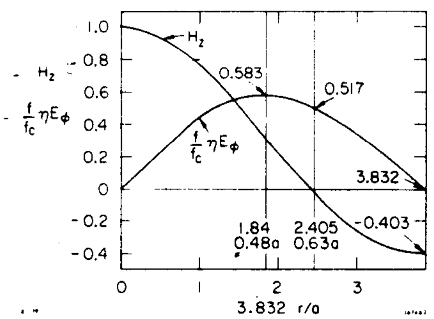

Fig. 1. Relative field values along a radius at $z = L/2$.

and integrating E^2 or H^2 over the volume of the cavity to obtain U we have

$$H = H_{zm} = 2\sqrt{2}\,(D/L)/\pi\eta\lambda\,(D/\lambda)^{5/2}(PQ)^{1/2}; \quad D = 2a$$

To obtain Q it is necessary to integrate the square of magnetic field over the cavity surface. It is given by:

$$QR_s = 722[1+0.168(D/L)^2]^{1.5}/[1+0.168(D/L)^3]$$

For copper $R_s = 2.61 \times 10^{-4} f^{1/2}$. R_s is the surface resistance in ohms, f is the frequency in MHz. Thus, for a given power dissipated in the cavity, the value of the maximum electric field, $E_{\phi m}$, and E_ϕ at any point in the cavity can be obtained.

TE_{011} Cavity Shunt Resistance

After traversing a path tangent to the E_ϕ circle, located at the middle of the cavity axis, as shown in Figs. 2 and 3, a charged particle gains a voltage, V, given by:

$$V = 2\int_0^{x_0} E_x\,dx$$

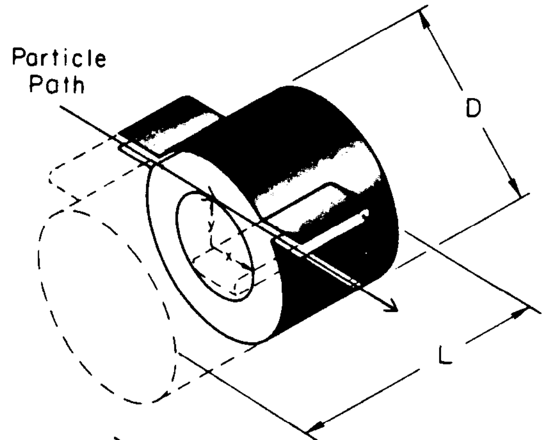

Fig. 2. Electric field circle and beam path in a TE_{011} cavity.

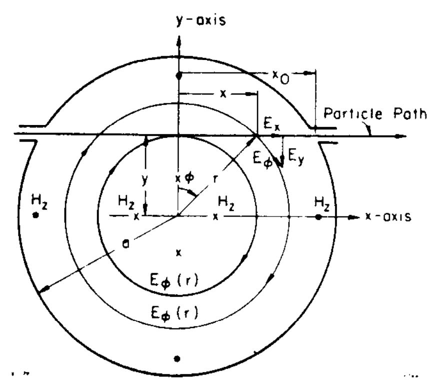

Fig. 3. Coordinate system and field lines at $z = L/2$.

where E_x is the amplitude of the instantaneous electric field, in the x direction, as seen by a particle which arrives at the y-axis when the electric field is maximum. $E_x = E_{\phi m}$ at $y = 0.48a$ and $x = 0$. E_x decreases from its maximum value for three reasons:

1. The space factor S_f, the amplitude of E_ϕ decreases as r deviates from r = 0.48a.
2. The direction factor D_f, the x component of E_ϕ decreases as we move away from the y-axis.

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3. The time factor, T_f, E_ϕ decreases with time as we move away from the y-axis. Thus,

$$E_x = E_{\phi m} S_f D_f T_f$$

where

$$S_f = J_1(3.832r/a)/J_1(1.84)$$
$$D_f = \cos\phi = y/r$$
$$T_f = \cos(2\pi x/\lambda)$$
$$r = (x^2 + y^2)^{\frac{1}{2}}$$

The value of E_x as a function of x/a at $y = 0.48a$ with $D/L = 1$ is plotted in Fig. 4. After integrating $E_x dx$

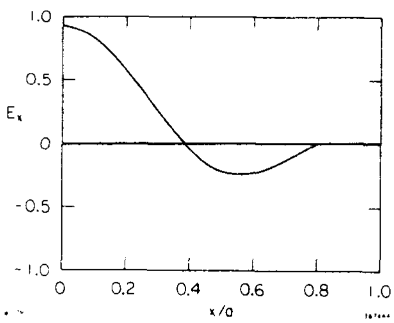

Fig. 4. Accelerating electric field, E_x, vs normalized distance, x/a.

numerically we obtain the voltage of the particle path for 1 W dissipated in the cavity, and hence its shunt resistance. The ratio of $\lambda/2$ long TM_{010} cavity shunt resistance to a TE_{011} cavity shunt resistance, which we define as the degradation factor, D_f, of a TE_{011} cavity, is 39:1. Nevertheless, a superconducting TE_{011} cavity requires less rf and line power than a copper TM_{010} cavity.

The rf and line power ratios which we define as the rf and line power improvement factors, respectively, are:

$$I_{frf} = S_{if}/D_f, \quad I_{f\ell p} = I_{rf}/E_{\ell rf} R_{eff}$$

where S_{if} is the superconducting (surface resistance) improvement factor, $E_{\ell rf}$ is line to rf power klystron efficiency, R_{eff} is the refrigeration factor. For Niobium[1] at 4.2°K: $S_{if} = 10^9 f^{-1.5}$, $R_{eff} = 350$. Assume $E_{\ell rf} = 0.7$. A plot of the rf and line power improvement factors are shown in Fig. 5.

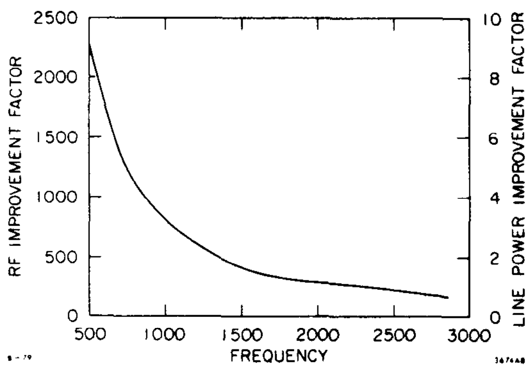

Fig. 5. RF and line power improvement factors vs frequency.

Why not use superconducting TM_{010} cavities? One reason is that at frequencies less than 1 GHz, the effective accelerating field of a superconducting TE_{011} cavity, 14 MeV/m, is greater than can be sustained by a superconducting TM_{010} cavity. The electric field lines are generated by induction and do not end on charges. (The hang-loose mode.) Thus, the problem of multipactoring, which so far resisted solution, is bypassed. True, the beam hole creates a surface electric field. But the field is localized, and is smaller than the TM_{010} surface fields. But even if we are not limited by the maximum effective electric field, the TE_{011} cavity might be preferable because of its following unique characteristics:

1. Because there are no axial currents, and no currents between cylinder wall and end plates, it can be put together without resorting to exotic methods such as electron beam welding.
2. The simple topology facilitates chemical processing and coating the surface with exotic superconductors.
3. The surface can be protected by an oxide coating so that exposure to air will not deteriorate the cavity. But, in a TM_{010} cavity, the electric field lines are normal to the surface and pass through the oxide layer and cause prohibitive losses.

For an Nb cavity at 4.2°K, the maximum accelerating gradient is 13.7 MV/m, the maximum accelerating potential, at 500 MHz, is 8 MV, for an rf input of 3 kW and line power input of 1 MW. For the same 8 MV, a copper cavity requires 7 MW rf and 10 MW line power. If Nb_3Sb is used, the rf and line powers are further reduced by the improvement factor of Nb_3Sb compared to Nb. At Wuppertal[2] an improvement factor of 50 has been achieved. It is possible that with TE_{011} cavities we can come closer to the theoretical value of 200.

Accelerating Structure

Several cavities can be placed in tandem to form an accelerating structure as shown in Fig. 6. Because

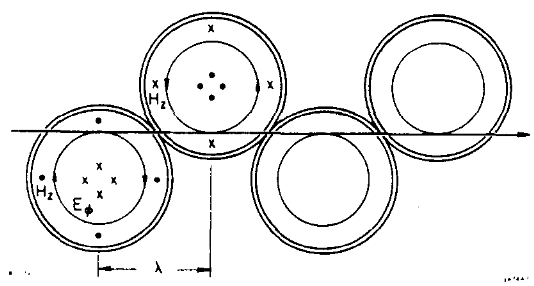

Fig. 6. Coupled TE_{011} cavities accelerating structure.

the length along the particle path of an optimized cavity is approximately λ, the per unit length degradation factor, the ratio of the per unit length shunt resistances of the TM_{010} over a TE_{011} cavity, is twice as large as D_f for a single cavity. Thus, for structures the values of the improvement factors given in Fig. 5 should be halved. The cavities can be parallel coupled with a TEM line similarly to the CESR[3] structure.

Perturbation of Cavity Geometry

It is hoped that the holes for the beam passage and for coupling into and between cavities, will not perturb the TE_{011} mode sufficiently to negate its advantages. We can tamper with the geometry, put drift tubes inside the cavity to shield the particles from the E field near the wall, or make the cavity elliptical. Both perturbations will increase the nonsuperconducting shunt resistance and the maximum gradient but will cause E lines to terminate on the surface and negate the main purpose for using the TE_{011} mode. If we carry these perturbations to their logical extreme we end up with a ridged cylindrical TE_{111} cavity, shown in Figs. 7 and 8, or

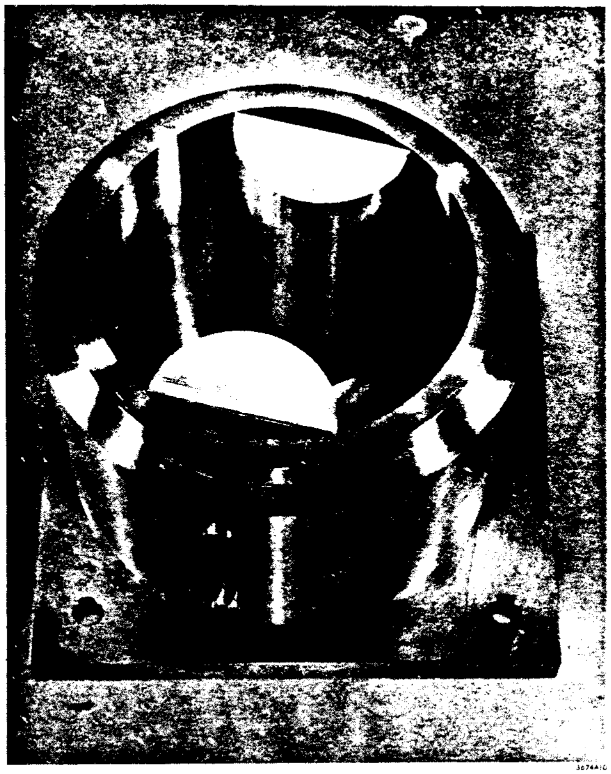

Fig. 7. TE_{111} cavity with half-cylinder ridge.

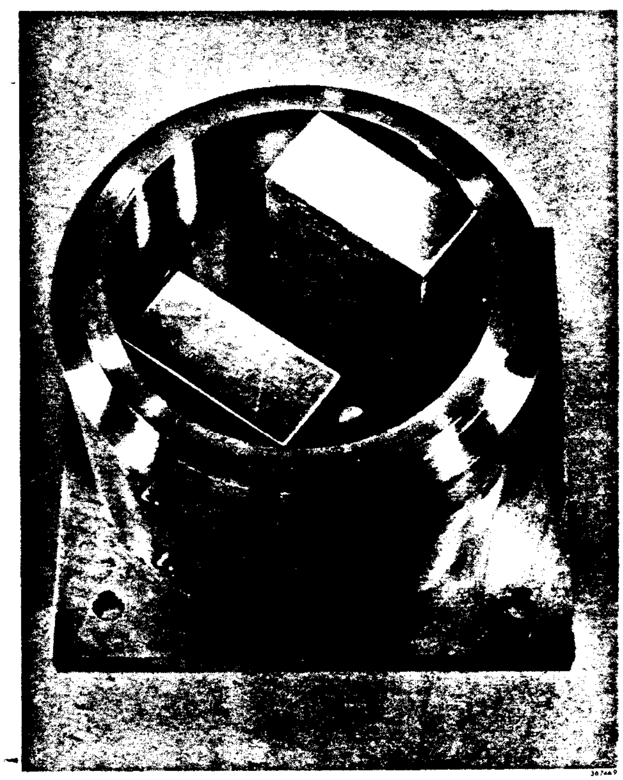

Fig. 8. TE_{111} cavity with rectangular ridge.

with an elliptical TE_{111} cavity. The ratio of surface area and volume of a TE_{111} to a TM_{010} cavity, and hence their shunt resistance and R/Q are comparable. Therefore, even though a TE_{111} cavity does not have the main advantages of the TE_{011} cavity, i.e., no E field terminating on the surface and no current between cylinder and end plates, still, it should be considered for particle acceleration because it offers the following advantages over a TM_{010} cavity:

1. Asymmetrical transverse cross section - the width can be made arbitrarily long (at the expense of shunt resistance).
2. As with the Muffin-Tin[4] structure, there are no cavity wall currents across the plane of the maximum E field. Also, there is no current across the plane $z = L/2$. Therefore, blind holes can be machined in two half-blocks which can be linked together to form a TE_{111} structure as shown in Fig. 9.
3. No E fields terminate on a flat surface. Generally, a TM_{010} structure cell is circular in the transverse direction and rectangular in the beam axis direction. To prevent multipactor, Wuppertal[2] made the cell also circular in the beam axis direction, resulting in a spherical cell. The TE_{111} cell goes one step further and makes the transverse cross section rectangular. Even though there is a flat plane, there are no E lines terminating on it because of the nature of the mode, and therefore, the TE_{111} cavity should resist multipactoring as does a spherical cavity.
4. Because of cylindrical geometry, the TE_{111} structure is easy to machine and make mechanically rigid. Also, tubes can be drilled across the cylinder wall and end plates to achieve good cooling of the cavity material near the beam aperture.
5. Easier removal of higher order modes, and lower higher order modes shunt resistance.

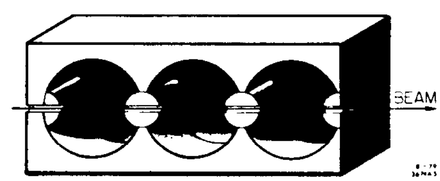

Fig. 9. Accelerating structure formed by ridged cylindrical TE_{111} cavities in tandem.

Other Types of Geometric Alterations

One reason for the low TE_{011} cavity shunt resistance and low maximum field gradient is that only a fraction of the electric field circle is used for acceleration. One way to get around this is to place a toroidal conductor along the E circle with a gap for the beam path. This will substitute over a large fraction of the E circle conduction current for displacement current. The complete E circle voltage will not be recovered because the cavity will become smaller, and the unloaded Q will decrease. This modification makes this cavity the same as the Argonne cavity[5] except without the twist.

A way to make use of the high Q and large energy storage capacity of a TE_{011} cavity and still have an effective high shunt resistance and field gradient is to place a saphire gas filled tube similar to the one described by Birx and Scalapino,[6] along the E field circle with a gap at the beam path. When the field in the cavity reaches a given value, the gas breaks down and the tube becomes a conductor. Because the magnetic field threading the tube does not change instantaneously, the full E ring potential should appear across the gap. In effect, the storage and accelerating cavities are one and the same. If the above postulates are indeed true, then for one watt input into a copper cavity, the accelerating potential across the cavity is $(\pi D/2)\, E_{\phi m}$ = 5000 volts. The effective shunt resistance is 25 megohm, about the same as for a TM_{010} cavity. For a niobium cavity at 4.2°K, at 500 MHz, the maximum voltage limited by magnetic breakdown in 90 MV and the maximum gradient is 100 MV/m.

Conclusion

It was shown that the TE_{011} and TE_{111} cavities have possibilities for particle acceleration and merit further study.

Acknowledgment

The use of nonsuperconducting TE_{011} cavities for particle acceleration was suggested by W.K.H. Panofsky and linking them together as in Fig. 6 was suggested by H. Hogg. I am grateful to P. Wilson, R. Miller, and J. Weaver for valuable discussions.

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