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A SOLUTION TO THE STRONG CP-PROBLEM IN THE SU(5) MODEL*

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ABSTRACT

The possibility of naturally suppressing strong CP-violation in the grand unified SU(5) model is examined. We show that a recent solution to the quark-lepton mass ratio problem by Georgi and Jarlskog can be extended in such a way as to yield a zero strong CP violating parameter θ at the tree level and a small value for it coming from higher order effects, along with reasonable mass ratios and mixing angles. Several phenomenological implications of the model are noted.

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With the discovery of instantons [1] the possibility arose that the strong interactions may violate CP-invariance [2], thereby possibly giving rise to large CP violating effects which have not been observed. The best bound on such strong CP violation, characterized by a parameter θ, comes from limits on the electric dipole moment of the neutron [3] which yields $\theta < 10^{-8}$ [4]. The problem is then to find a reason for the occurrence of such a small number. One possibility is to eliminate θ completely from the theory by requiring an additional symmetry [5] which allows θ to be "rotated" away. However, it was shown that the realization of this symmetry leads to phenomenologically unacceptable consequences: either there should exist a very light particle [6] which has not been observed [7], or a massless quark which is in conflict with the present view of chiral symmetry of hadrons [8]. A second alternative is to use the fact that in a theory with spontaneous CP breakdown (due to the Higgs mechanism) the value of θ is given by

$$\theta = \arg (\det M) \qquad (1)$$

where M is the quark mass matrix, and construct a theory for the electroweak interactions in which arg (det M) = 0 naturally (but with a complex mass matrix in order to have <u>some</u> CP violation in the gauge interactions) and in which θ arises only as a small higher order effect. Such a model was first discussed in ref. [9] and subsequently by several authors in the context of the gauge models based on $SU(2)_L \otimes SU(2)_R \otimes U(1)$ group [10] and $SU(2)_L \otimes U(1)$ group [11,12].

In recent years, grand unified theories are receiving considerable attention: they are attractive because they promise to describe all interactions (except gravity) with a single coupling constant. The simplest such model is the SU(5) theory of Georgi and Glashow [13]. The question then arises whether it is possible to combine the absence of strong CP violation with these theories. Since SU(5) contains $SU(2)_L \otimes U(1)$ as its electroweak gauge group, one might try to take over the results of the papers in ref. [11]. However, a straightforward application of the models of ref. [11] requires that only identical (i.e., same transformation properties under the gauge group) Higgs fields give mass to the fermions. For SU(5), this means only 5-dimensional Higgs fields or only 45-dimensional ones. But this leads to the unsatisfactory fermion mass relations

$$\frac{m_e}{m_d} = \frac{m_\mu}{m_s} = \frac{m_\tau}{m_b} \tag{2}$$

So, other solutions must be sought.

An answer to the problem of mass ratios is to assume that Higgs fields transforming like the 5- as well as 45-dimensional [14] representations of SU(5) give masses to fermions. Georgi and Jarlskog [15] have constructed a model in which these Higgs fields enter in such a way into the Yukawa coupling as to give the relations

$$m_b = m_\tau \quad ; \quad m_d = 3m_e \quad ; \quad m_s = \frac{1}{3} m_\mu \tag{3}$$

at a mass scale of 10^{16} GeV, and which are expected to renormalize to the currently preferred values at low energies [16]. As we now proceed to show, the result of ref. [15] can be amended in such a way as to yield a small, calculable θ.

To start with, we consider a theory invariant under $SU(5) \otimes CP \otimes D$, where SU(5) is a local symmetry and D is a discrete symmetry whose action will be displayed below. CP invariance requires all coupling constants to be real. The fermions come in 5-dimensional representations χ_a and antisymmetric 10-dimensional representation ψ_a, where a labels the "generations" (e.g., a = 1 corresponds to u,d,e,ν; a = 2 to c,s,μ,ν', etc.) and we take three generations. The latin indices i,j,... will be used to denote SU(5) indices. The particle content of χ and ψ is given by

$$\chi_a \equiv \begin{pmatrix} n_1 \\ n_2 \\ n_3 \\ E^+ \\ E^o \end{pmatrix}_{a,R} \quad ; \quad \psi_a \equiv \begin{pmatrix} 0 & p_{c3} & -p_{c2} & p_1 & n_1 \\ & 0 & p_{c1} & p_2 & n_2 \\ & & 0 & p_3 & n_3 \\ & & & 0 & E^+ \\ & & & & 0 \end{pmatrix}_{a,L} \qquad (4)$$

where the subscripts 1,2,3 are color indices, p_c stands for the charge conjugate field of p (i.e., $p_c = C\tilde{\bar{p}}$); p and n correspond to fields with electric charge +2/3 and -1/3 respectively.

As usual, SU(5) is broken down to $SU(3)_C \otimes SU(2)_L \otimes U(1)$ symmetry via the Higgs fields transforming as 24-dimensional representation [17]. The breaking of the remaining symmetry as well as fermion mass generation is achieved by the Higgs bosons φ_a (a = 1,2,3) belonging to 5-dimensional representation and Σ belonging to 45-dimensional representation. Now, we proceed with the construction of the Yukawa couplings which is of interest here. (The gauge couplings remain as usual.) The general gauge invariant Yukawa couplings of fermions and Higgs bosons in SU(5) are given as follows:

$$
\begin{array}{lcl}
\bar{\psi}\,\chi\,\varphi \text{ coupling} & : & \bar{\psi}^{ij}_L\ \chi^i_R\ \varphi^j \\
\bar{\psi}\,\chi\,\Sigma \text{ coupling} & : & \bar{\psi}^{ij}_L\ \chi^k_R\ \Sigma^{ij}_R \\
\tilde{\psi}\,\psi\,\varphi \text{ coupling} & : & \tilde{\psi}^{ij}\ C^{-1}\ \psi^{k\ell}\ \varphi^m\ \varepsilon_{ijk\ell m} \\
\tilde{\psi}\,\psi\,\Sigma \text{ coupling} & : & \tilde{\psi}^{ij}\ C^{-1}\ \psi^{k\ell}\ \Sigma^{mn}_k\ \varepsilon_{ij\ell mn}
\end{array}
\qquad (5)
$$

where C is the Dirac charge conjugation matrix; ~ stands for the transpose of ψ and $\varepsilon_{ijk\ell m}$ is the totally antisymmetric tensor of SU(5). Henceforth in writing Yukawa couplings, we will drop SU(5) indices and will write them symbolically as in the left-hand column of eq. (5). In order to restrict the form of the Yukawa couplings and hence the fermion mass matrices, we have introduced the following discrete symmetry, D;

$$
\begin{array}{lll}
\psi_1 \to e^{i\pi/4}\psi_1 & ; \quad \chi_1 \to e^{5\pi i/4}\chi_1 & ; \quad \varphi_1 \to e^{-3\pi i/4}\varphi_1 \\
\psi_2 \to e^{i\pi/2}\psi_2 & ; \quad \chi_2 \to \chi_2 & ; \quad \varphi_2 \to e^{i\pi/4}\varphi_2 \\
\psi_3 \to e^{-i\pi/8}\psi_3 & ; \quad \chi_3 \to e^{-3\pi i/8}\chi_3 & ; \quad \varphi_3 \to e^{5\pi i/8}\varphi_3 \\
 & & \quad\ \Sigma \to i\Sigma
\end{array}
\qquad (6)
$$

The Yukawa coupling now is given by

$$
\begin{aligned}
\mathscr{L}_Y = {} & A\,\bar{\psi}_2\,\chi_1\,\varphi_1 + A'\,\bar{\psi}_1\,\chi_2\,\varphi_2 + G\,\bar{\psi}_1\,\chi_3\,\varphi_3 \\
& + D\,\bar{\psi}_2\,\chi_2\Sigma + E\,\bar{\psi}_3\,\chi_3\,\varphi_1 \\
& + B\left(\tilde{\psi}_1\psi_2 + \tilde{\psi}_2\psi_1\right)\varphi_1 \\
& + C\left(\tilde{\psi}_2\psi_3 + \tilde{\psi}_3\psi_2\right)\varphi_3 \\
& + F\,\tilde{\psi}_3\,\psi_3\,\varphi_2 + \text{h.c.}
\end{aligned}
\qquad (7)
$$

It is possible to show[F1] that, the vacuum expectation value (vev) of Σ, $\langle\Sigma_4^{45}\rangle = -3\kappa$ can be made real by an SU(5) rotation leaving the vev of φ's in general complex: i.e., $\langle\varphi_a^5\rangle \equiv v_a e^{i\alpha_a}$, $a = 1,2,3$. We notice [14,15] that $\langle\Sigma_4^{45}\rangle = -3\langle\Sigma_i^{i5}\rangle$ where $i = 1,2,3$ denotes the color index (no sum over i). The resulting fermions mass matrices are

$$M_u^\dagger = \begin{pmatrix} 0 & Bv_1e^{i\alpha_1} & 0 \\ Bv_1e^{i\alpha_1} & 0 & Cv_3e^{i\alpha_3} \\ 0 & Cv_3e^{i\alpha_3} & Fv_2e^{i\alpha_2} \end{pmatrix} \qquad (8a)$$

$$M_d = \begin{pmatrix} 0 & Av_1e^{i\alpha_1} & 0 \\ A'v_2e^{i\alpha_2} & D\kappa & 0 \\ Gv_3e^{i\alpha_3} & 0 & Ev_1e^{i\alpha_1} \end{pmatrix} \qquad (8b)$$

$$M_{lepton} = \begin{pmatrix} 0 & Av_1e^{i\alpha_1} & 0 \\ A'v_2e^{i\alpha_2} & -3D\kappa & 0 \\ Gv_3e^{i\alpha_3} & 0 & Ev_1e^{i\alpha_1} \end{pmatrix} \qquad (8c)$$

Note that eq. (8a) involves $M_u^\dagger$ rather M_u. It follows from eqs. (8a) and (8b) that

$$\text{Arg (Det } M) = \text{Arg (Det } M_u \cdot \text{Det } M_d) = 0 \qquad (9)$$

Thus, $\theta = 0$ naturally at the tree level, and is finite and computable as a result of higher order effects. We will come back to this point later. Let us first comment on the fermion masses and mixings in this model. Note that, if we drop all the phases α_i and set $G = 0$, we get the fermion mass matrices obtained by Georgi and Jarlskog [15]. As they have discussed, this leads to the mass relations given in eq. (3) and also leads to interesting mixing angles that we discuss below. We do this in steps: first, we show that nontrivial CP-phase survive arbitrary phase redefinition of the fermions in the theory. Then, we cast the weak charged current in terms of the generalized Cabibbo matrix [18] to discuss phenomenological implications of the model.

To proceed, let us work in the approximation in which $G \approx 0$, $A \approx A'$ and $V_1 \approx V_2 \approx V_3$. Then, if we make the following phase redefinition of the fields:

$$p'_{1L} = p_{1L} \quad ; \quad p'_{2L} = e^{i(\alpha_1 + \alpha_2 - 2\alpha_3)} p_{2L} \quad ; \quad p'_{3L} = e^{i(\alpha_1 - \alpha_3)} p_{3L}$$
$$p'_{1L} = e^{i(2\alpha_1 + \alpha_2 - 2\alpha_3)} p_{1R} \quad ; \quad p'_{2R} = e^{i\alpha_1} p_{2R} \quad ; \quad p'_{3R} = e^{i(\alpha_1 + \alpha_2 - \alpha_3)} p_{3R} \tag{10}$$

and

$$n'_{1L} = n_{1L} \quad ; \quad n'_{2L} = e^{-i\alpha_1} n_{2L} \quad ; \quad n'_{3L} = n_{3L}$$
$$n'_{1R} = e^{-i(\alpha_1 + \alpha_2)} n_{1R} \quad ; \quad n'_{2R} = e^{-i\alpha_1} n_{2R} \quad ; \quad n'_{3R} = e^{-i\alpha_1} n_{3R} \tag{11}$$

the mass matrices M_u and M_d become real but the weak charged current in terms of the unrotated fields p'_i and n'_i looks like

$$J_\mu = \left(\bar{p}'_{1L}, \bar{p}'_{2L}, \bar{p}'_{3L}\right)\gamma_\mu \begin{pmatrix} 1 & & \\ & e^{i\alpha} & \\ & & e^{i\beta} \end{pmatrix} \begin{pmatrix} n'_{1L} \\ n'_{2L} \\ n'_{3L} \end{pmatrix} \tag{12}$$

where $\alpha = 2\alpha_1 + \alpha_2 - 2\alpha_3$; $\beta = \alpha_1 - \alpha_3$. Now we can diagonalize the real symmetric mass matrices by means of the following real orthogonal matrices:

$$\begin{pmatrix} d \\ s \\ b \end{pmatrix} = \begin{pmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ +\sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} n_1' \\ n_2' \\ n_3' \end{pmatrix} \tag{13}$$

$$\begin{pmatrix} u \\ c \\ t \end{pmatrix} = \begin{pmatrix} 1 & +\phi_1 & -\phi_3 \\ -\phi_1 & 1 & +\phi_2 \\ 0 & -\phi_2 & 1 \end{pmatrix} \begin{pmatrix} p_1' \\ p_2' \\ p_3' \end{pmatrix} \tag{14}$$

where[F2] [19]

$$\theta_1 \approx \sqrt{\frac{m_d}{m_s}} \quad ; \quad \phi_1 \approx \sqrt{\frac{m_u}{m_c}} \quad ; \quad \phi_2 \approx -\sqrt{\frac{m_c}{m_t}} \quad ; \quad \phi_3 \approx \sqrt{\frac{m_u}{m_t}}$$

and we have ignored terms proportional to higher orders in the small quantities ϕ_i. The left-handed charged current is then given by

$$J_{\mu L} = \left(\bar{u}_L\ \bar{c}_L\ \bar{t}_L\right)\gamma_\mu \begin{pmatrix} 1 & \theta_2 + \phi_1 e^{i\alpha} & -\phi_3 e^{i\beta} \\ -\phi_1 - \theta_2 e^{i\alpha} & e^{i\alpha} & +\phi_2 e^{i\beta} \\ 0 & -\phi_2 e^{i\alpha} & e^{i\beta} \end{pmatrix} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} \tag{15}$$

No further phase redefinition of the physical fields can make this matrix totally real. We therefore conclude that CP violation in the gauge sector in this model is genuine inspite of the restricted form of the mass matrices.

An immediate phenomenological implication of the mixing matrix in eq. (15) that $\bar{b}c$ and $\bar{t}s$ mixings are the same, i.e.,

$$\frac{(b \to c + X)}{(t \to s + X)} \simeq 1 \tag{16}$$

Further, within these approximations, one would expect,

$$\frac{\Gamma(b \to c+X)}{\Gamma(b \to u+X)} \simeq \frac{\phi_2^2}{\phi_3^2} \approx \frac{m_c}{m_u} \simeq (2\text{-}3) \times 10^2 \qquad (17)$$

where we choose $m_c \approx 1.5$ GeV and $m_u \approx 5$-7 MeV.

After these phenomenological remarks, we turn to the one loop corrections to the strong CP violating parameter θ. First, we note that since the photon and Z-couplings to fermions are diagonal, they do not contribute to θ up to one-loop level. Furthermore, since $W^{\pm}$ always couples to left-handed currents, it too does not contribute to the mass matrix up to this order. Let us now consider the effect of the remaining superheavy gauge bosons X and Y. A look at their couplings to fermions [20] makes it clear that, even though they involve both left as well as right handed chirality states of fermions, at the one loop level, they also do not contribute to the quark mass matrix. Thus, the only non-vanishing contribution at the one loop level comes from graphs with one Higgs boson in the intermediate state. These graphs generally involve mixing between two Higgs bosons and make contributions to θ typically of order, $\theta_{1\text{-loop}} \approx h^2\Delta^2/m_H^2$, where h is the strength of the Yukawa coupling ($h \approx gm_q/M_W$); Δ is the mixing parameter between Higgs bosons and m_H is the Higgs boson mass. If $\Delta^2 \approx 10^{-1}$ GeV^2 and $m_H \approx 100$ GeV, one obtains $\theta \approx 10^{-10}$.

In summary, we have considered an SU(5) grand unified model in which strong CP violation is naturally absent at the tree level with one-loop induced finite effects expected to be small. The model yields an acceptable quark-lepton mass spectrum at present energies as well as a satisfactory set of quark mixing angles.

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FOOTNOTES

F1. We have verified this statement using the Higgs potential allowed by the symmetry group of our theory with the addition of billinear terms of the form $\varphi_a\varphi_b$ to it. Some of these latter terms, however, give a soft breaking of this discrete symmetry.

F2. We wish to point out that the approximate relations between mixing angles and mass ratios are obtained only in the approximations stated in the text (such as $G \approx 0$, $A \approx A'$, $v_1 \approx v_2 \approx v_3$, etc.) and are therefore not predictions of the theory. These approximations enable us to extract the physics of model in a succinct fashion. We also point out that, if we keep a non-zero G, the required phase redefinitions required to get a real mass matrix are the same as in eqs. (11) and (12), except that we must have $n'_{3L} = n_{3L}e^{i(\alpha_3 - \alpha_1 - \alpha_2)}$ and $n'_{3R} = n_{3R}e^{i(\alpha_3 - \alpha_2 - 2\alpha_1)}$. That would change β in eq. (12) to $\beta = 2\alpha_1 - 2\alpha_3 + \alpha_2$.

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