SLAC-PUB-2376
August 1979
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GROUP WEIGHT AND VANISHING GRAPHS *

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ABSTRACT

Various properties of the group weight of Feynman graphs in non-Abelian gauge theories are discussed. Infinitely many skeleton graphs with vanishing weight are exhibited for every compact Lie group. The $1/N^2$ dependence of the topological expansion is related to an $1/N^2$ expansion in some channels with the exchange of definite quantum numbers.

(Submitted to Phys. Rev. D)

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*This work was supported by the Department of Energy under contract number DE-AC03-76SF00515.

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I. Introduction

In the perturbative analysis of gauge theories it is convenient to represent every Feynman graph G as the product of a weight factor W_G depending on the gauge group, times a momentum integral. An efficient graphic method to compute W_G for most simple Lie groups has been described.[1] We make use of that method to discuss further properties of the weight factor.[2]

In Section II we show that the weight factors of a SU(N) gauge theory are polynomials in N^2 (apart possibly from a factor N) and we relate this property to the topological expansion of the scattering amplitude in the channels where definite quantum numbers are exchanged. In Section III we observe that, at first sight surprisingly, infinitely many skeleton graphs have vanishing weight in every non-Abelian gauge theory. This feature is typical of non-Abelian gauge theories and may be of help in the analysis of perturbation theory although a rough estimate suggests that the number of non-vanishing graphs grows with the perturbative order much faster than the number of the vanishing ones.

In Section IV we derive the projection operators corresponding to exchanged states with definite quantum numbers in gluon-gluon elastic scattering in SU(N) gauge theory.

II. Planarity and Topological Expansion

In this section we discuss the dependence on N of the weight W_G of an arbitrary graph in the SU(N) gauge theory.[3-8]

At order v in the coupling constant g, the graph has v trilinear vertices (in the usual way the four-gluon vertex is replaced by couples of three-gluon vertices)[1] some of which being three-gluon vertices v_g, the others being quark-quark-gluon v_q, $v = v_g + v_q$. The Feynman graph has $p = p_g + p_q$ propagators (we do not count the external lines), that is p_g gluon and p_q quark propagators. If the graph has n external lines, $n = 3v - 2p$, the group factor W_G is a tensor of rank n. The graphical method described by Cvitanovic is an efficient way to express the generic tensor W_G as a linear combination of a complete set of independent tensors having the same rank, the basis tensors, which are associated to graphs without internal gluons.[1]

In the SU(N) gauge theory the evaluation of the group factor W_G for any graph only involves the two steps: (a) to re-express the three-gluon vertices in terms of the fundamental representation (see Fig. 1):

$$if_{ijk} = 2\,\mathrm{Tr}\left(T_i T_j T_k - T_k T_j T_i \right) \tag{2.1}$$

(b) to replace all internal gluon lines with gluon projection operators (see Fig. 2):

$$2\,\left(T_i\right)^a_b\,\left(T_i\right)^c_d = \delta^a_d\,\delta^c_b - \frac{1}{N}\,\delta^a_b\,\delta^c_d \tag{2.2}$$

W_G is then expressed as the sum of $2^{v_g+p_g}$ "double line" graphs. As an example, Fig. 3(a) shows a graph at order g^{10} in perturbation theory, with six three-gluon vertices and nine internal gluon propagators. In Fig. 3(b) there is one of the 2^{15} "double line" graphs obtained after steps (a) and (b). In the "double line" graphs may appear index loops, i.e., fermion loops unconnected to the rest of the graph and to external lines, each contributing a factor N, which are called windows. There are also index paths called boundaries which are attached to the external lines. There are no boundaries for graphs where the external sources are all color singlets. Each boundary represents a tensor with rank equal to the number of external lines attached to the index path. For example, the "double line" configuration in Fig. 3(b) has one window and one boundary, as shown in Fig. 3(c).

Basic notions are those of planarity and degree of non-planarity. It is very convenient to "complete" the graph G by adding one more vertex, called P_∞, where all external lines of G are incident (see Fig. 4). One can now "draw" the completed graph on a sphere with h handles so that lines intersect only at vertices (embedding). The minimum h_m for which the embedding is possible is a characterization of the degree of non-planarity of the graph. The graph is planar iff $h_m = 0$.[9] The graph in Fig. 3(a) has $h_m = 1$, that is it may be embedded on a torus. The completed graph embedded on a sphere with h_m handles may be regarded as a polyhedron whose edges are the lines of the graph. Then the Euler formula relating the number of vertices V, of edges P and faces F holds:

$$V - P + F = 2 - 2h_m \tag{2.3}$$

which in terms of the original graph, is:

$$(v+1) - (p+n) + (f+n-1) = 2 - 2h_m \tag{2.4}$$

where $f \equiv F - n + 1$.

The multitude of "double line" graphs originating from a Feynman graph has a different number of faces and handles but the same number of

$$f + 2h = p - v + 2 = \frac{v}{2} - \frac{n}{2} + 2 \tag{2.5}$$

Indeed the number of faces ranges between the maximum $f_M = 2 - v + p - 2h_m$ and the minimum $f_m = 0$ (if f_M is even) or $f_m = 1$ (if f_M is odd).

The quark loops in the original graph do not contribute to the number of faces of the "double line" graph. Then

$$f - q = b + w \qquad (2.6)$$

where q, b, w represent the number of quark loops, boundaries and windows.

The group theoretic weight W_G can then be expressed:

$$W_G = g^v \sum N^w c_m T^{(m)}$$

where $T^{(m)}$ is one of the basis tensor, the summation extends over all the "double line" graphs and the coefficient c_m counts the multiplicity of the configuration with w windows and the factors of 2 and (-1/N) arising from steps (a) and (b).

By use of (2.5) and (2.6) the power N^w can be rewritten:

$$W_G = g^v \sum c_m N^{f-q-b} T^{(m)}$$

$$= (g\sqrt{N})^v N^{-q} N^{1-\frac{n}{2}} \sum c_m N^{1-b} N^{-2h} T^{(m)} \qquad (2.7)$$

The coefficient c_w may contain a dependence on N only for those "double line" configurations which arise from the singlet subtraction term in step (b):

$$2 \left(T_i\right)^a_b \left(T_i\right)^c_d \longrightarrow -\frac{1}{N} \delta^a_b \delta^c_d$$

For the first such replacement, the "double line" configuration has v-2 vertices and p-3 propagators, then its invariant f + 2h is a positive integer with different parity from the original graph or any configuration where instead the replacement

$$2\left(T_i\right)^a_b \left(T_i\right)^c_d \rightarrow \delta^a_d \ \delta^c_b \tag{2.8}$$

has been made.

Therefore (1/N) times the value of the double line configuration where the singlet subtraction term has been used, has the same parity of the power of N as the graph with the replacement (2.8) everywhere and both can be written in the form (2.7) with coefficients c_m now independent on N. The same argument holds for multiple use of the singlet subtraction terms and Eq. (2.7) holds for the general graph, with coefficient c_m independent on N.

It is then clear that if one is interested in processes with fixed number of boundaries, for instance processes with color singlet sources only ($b = 0$) then Eq. (2.7) arranges the contribution of each graph in decreasing powers of N^2. Or one may sum over the perturbation series and take the limit $N \rightarrow \infty$ with $g^2 N = \gamma^2$ fixed and one would obtain that the contribution to amplitudes with fixed number of boundaries and quark loops are arranged in decreasing powers of N^2 and are associated with increasing degree of non-planarity.[4-8]

A simple remark may shorten the computation of W_G. Step (b) (Eq. (2.2)) may be substituted by the simpler replacement (2.8) when i) the gluon connects two three-gluon vertices or ii) the gluon connects one three-gluon vertex with one quark-quark-gluon vertex.

Proofs are straightforward:

$$(i)^2 f_{jik} f_{klm} = -4\mathrm{Tr}\left(T_j T_i T_k - T_k T_i T_j\right) \mathrm{Tr}\left(T_k T_l T_m - T_m T_l T_k\right)$$

$$= -2\mathrm{Tr}\left(T_j T_i T_l T_m - T_j T_i T_m T_l - T_i T_j T_l T_m + T_i T_j T_m T_l\right) \qquad (2.9)$$

For the point (ii) we have:

$$i\left(T_k\right)_a^b f_{kij} = 2i \left(T_k\right)_a^b \mathrm{Tr}\left(T_k T_i T_j - T_j T_i T_k\right)$$

$$= i \; [T_i, T_j]_a^b \qquad (2.10)$$

Therefore, in all Feynman graphs where there are no gluons which directly connect quark lines, the number of "double line" graphs originating from a a simple graph is reduced to 2^{v_g}. Of course this happens in a pure SU(N) gauge theory (without quarks). Then one finds that for two-point functions and three-point functions, where there is just one basic tensor (respectively δ_{ab} and f_{abc}), the group weight W_G of the generic Feynman graph is

generic Feynman graph is

$$W_G = \delta_{ab}\left(Ng^2\right)^s \sum_{P=0}^{[s/2]} c_P\left(N^2\right)^{-P} \quad \text{at order } g^{2s} \qquad (2.11)$$

$$W_G = f_{abc}\, g\left(Ng^2\right)^s \sum_{P=0}^{[s/2]} c_P\left(N^2\right)^{-P} \text{at order } g^{2s+1} \qquad (2.12)$$

where the leading coefficient c_0 is different from zero if and only if the graph is planar.[10]

As it is shown in Section IV, for the 4-point function one has six basic tensors, three of which (A,B,C) have one boundary and three (D,E,F) have two boundaries. At order g^{2s+2} one finds

$$W_G = g^2(g^2N)^s \left\{ A\sum_0^{[s/2]} a_P(N^2)^{-P} + B\sum_0^{[s/2]} b_P(N^2)^{-P} + C\sum_0^{[s/2]} c_P(N^2)^{-P} \right.$$

$$\left. + \frac{1}{N}\left[D\sum_0^{[s/2]} d_P(N^2)^{-P} + E\sum_0^{[s/2]} e_P(N^2)^{-P} + F\sum_0^{[s/2]} f_P(N^2)^{-P} \right] \right\} \qquad (2.13)$$

higher n-point functions have weights W_G expressed in the same form after one has taken care of the N factors associated with the number of boundaries of the basic tensors.

As one can see from (4.3)-(4.8) in Section IV, the first four projection operators do not mix basis tensors with different boundaries or they mix them with the proper pure factor N. Therefore the gluon-gluon elastic scattering amplitude in those channels will be a polynomial in N^2, apart from an overall normalization independent on the order in perturbation theory, while the scattering amplitude in the channels associated with the last two projectors loses the simpler dependence on N^2.

III. Vanishing Graphs

It is easy to show that in non-Abelian gauge theories there are infinitely many skeleton graphs with vanishing weight. They are identified in an obvious way by only using the anitsymmetry property of the three-gluon vertex so that the results of this section hold for any compact Lie group. It is convenient to restrict first to a pure gauge non-Abelian theory. From the graphical rules[1] it is obvious that a graph containing a subgraph with vanishing weight will also have vanishing weight.

Furthermore, since there is a single independent tensor of rank two and a single one of rank three, we may restrict to skeleton graphs with vanishing weight. In fact each such skeleton will produce vanishing graphs if arbitrary self-energy or vertex insertions are made. While it may be difficult to give necessary and sufficient conditions for the vanishing of the weight of a skeleton graph in a general non-Abelian theory, our remarks select a large class of vanishing graphs.

Let us consider the weight $T^G_{\sigma_1\ldots\sigma_k\,\tau_1\ldots\tau_m}$ of a graph G(see Fig. 5). It may be obtained by partial saturation of the weights of the subgraphs G_1 and G_2

$$T^G_{\sigma_1\ldots\sigma_k\,\tau_1\ldots\tau_m} = T^{G_1}_{\sigma_1\ldots\sigma_k\,\alpha_1\ldots\alpha_n}\,T^{G_2}_{\alpha_1\ldots\alpha_n\,\tau_1\ldots\tau_m}$$

A simple sufficient condition for the vanishing of T^G is that T^{G_1} and T^{G_2} are respectively symmetric and antisymmetric in two corresponding saturated α indices. In particular a vanishing weight is obtained for any three-gluon diagram which is the product of a three-gluon vertex times a four-gluon tensor symmetric in the two saturated indices. Because of the nature of the three-gluon vertex every planar, or non-planar, four-leg graph with a plane of symmetry through two of the external lines, is associated with a tensor W_G symmetric in the couple of indices (say α,β) of the external gluons not lying in the symmetry plane. The lowest order[11] examples of such symmetric skeleton graphs are shown in Fig. 6a-9a.

By convolution with the bare three-gluon vertex $f_{\tau\alpha\beta}$ (or equivalently with any three-gluon Green function $\Gamma_{\tau\alpha\beta}$) one obtains a vanishing graph (see Figs. 6b-9b).

This procedure suggests a very rough estimate of the number of such symmetric four-leg skeletons. At large order n in the coupling constant, the number of symmetric skeletons with only the two external vertices lying in the symmetry plane is roughly $x\left(\frac{n}{2}\right)$, where $x(m) \sim m!$ is the number of skeletons (which in this level of estimate are as many as the generic graphs)[12] at order m. Therefore the ratio R of the vanishing skeletons versus the non-vanishing ones, for the three-point function, would be, at order n

$$R \sim \frac{\left(\frac{n}{2}\right)!}{n!}$$

This estimate neglects the facts that: a) there are symmetric skeletons with vertices on the symmetry plane, b) not all symmetric four-point skeletons lead to three-point skeletons (see for example Fig. 7b or 9b), c) depending on the specific gauge group of the theory, there are non-symmetric four-point skeletons having a symmetric tensor.[13] While (a) and (c) would increase the estimated ratio R, (b) would decrease it. It seems however that none of these points can substantially change the very rough previous estimate.

We can now consider a non-Abelian gauge theory with a multiplet of fermions transforming as the fundamental representation of the group. Again one may look for four-point gluon graphs with a symmetry plane, as in Fig. 10a. When convoluted with the three-gluon vertex, they originate vanishing graphs, as in Fig. 10b. Since the reflection around the symmetry plane must preserve also the direction in the fermion path, one expects that only special ways of replacing a gluon path with a fermion path in the vanishing gluon graphs will still give a vanishing graph.

One may note however that even in some cases where the reflection around the symmetry plane does not preserve the direction of the loop, one may still produce a vanishing graph because of additional properties of weights depending on the gauge group. For instance in the SU(N) case, it is easy to check that the tensor $T_{\alpha\gamma\beta\delta}$ which is the weight of the graph in Fig. 11a is a symmetric tensor in (α,β), although the graph has no symmetry plane through (γ,δ). Therefore the three-point graph in Fig. 11b has vanishing weight.[14]

We also remark that for any given graph with three external lines and vanishing weight one can obtain a vanishing "vacuum" graph by "completing" the former with one more coupling of the type f_{abc} or $(\lambda_a)^b_c$. Next by stereographic projection (as it was mentioned in the definition of planarity in Section II) from another inequivalent vertex of the "vacuum" graph, one may obtain a new vanishing three-point graph. For instance, in this way one shows that the vanishing graph in Fig. 12 is related to that in Fig. 11b.

We finally mention that the study of graphs with definite symmetry properties can be pursued in an algebraic way[15] through the study of the spectral properties of the adjacency matrix. It is amusing to notice that the two lowest order vanishing graphs, already exhibited in Ref. 1, when completed by one more trigluon vertex, are just the first representatives of a peculiar class of graphs, sometimes calles cages.[16] We checked that all the five cages with trilinear vertices, exhibited in Ref. 16, have vanishing weight due to the mechanism previously described.

IV. Basis Tensors and Projectors

In order to discuss the cases where the 1/N expansion is actually a $1/N^2$ expansion (see Section II) and for completeness reasons, we write here the SU(N) tensor basis for processes with $r = 4$ external gluons (no external quarks) and the linear combinations of the basis tensors which are associated to the exchange of definite quantum numbers. The set of all distinct traces over r T_i matrices form a natural tensor basis, but the tensors of rank r, $r \geq N$ so obtained are not linearly independent.[17] Actually in a pure SU(N) gauge theory by the rules (2.1) and (2.2) complete and independent bases are obtained by symmetrizing (or antisymmetrizing) traces of products of T_i matrices, which are graphically fermion loops of even (odd) length. By Furry theorem, this is also true if the Lagrangian contains fermion fields.[18] Therefore, for $r = 4$ one has six instead of nine basis tensors, i.e.:

$$
\begin{aligned}
A &= \frac{1}{2}\left[\mathrm{Tr}\left(T_a T_d T_c T_b\right) + \mathrm{Tr}\left(T_a T_b T_c T_d\right)\right] \\
B &= \frac{1}{2}\left[\mathrm{Tr}\left(T_a T_b T_d T_c\right) + \mathrm{Tr}\left(T_a T_c T_d T_b\right)\right] \\
C &= \frac{1}{2}\left[\mathrm{Tr}\left(T_a T_d T_b T_c\right) + \mathrm{Tr}\left(T_a T_c T_b T_d\right)\right] \\
D &= \delta_{ab}\,\delta_{cd}, \quad E = \delta_{ac}\,\delta_{bd}, \quad F = \delta_{ad}\,\delta_{bc}
\end{aligned}
\tag{4.1}
$$

They are shown in Fig. 13.

One may define a product of basis tensors as a convolution in the "vertical" channel (that is $KL = H$ means $H_{acbd} = K_{actu} L_{tubd}$). Since the fermion loop is symmetrized, this product is commutative, D acts as the identity and one easily finds:

$$A^2 = \frac{1}{32}(D+F) - \frac{1}{4N}(B+C) + \frac{1}{16N^2}E$$

$$AB = AC = \frac{-1}{4N}(B+C) + \frac{1}{16N^2}E$$

$$AF = A, \quad BF = C, \quad CF = B$$

$$BE = CE = \frac{1}{4}\left(N - \frac{1}{N}\right)E$$

$$AE = \frac{-1}{4N}E, \quad FE = E, \quad F^2 = D \tag{4.2}$$

$$B^2 = C^2 = \frac{1}{8}\left[NB - \frac{2}{N}(B+C) + \left(\frac{1}{2N^2} + \frac{1}{4}\right)E\right]$$

$$BC = \frac{1}{8}\left[NC - \frac{2}{N}(B+C) + \left(\frac{1}{2N^2} + \frac{1}{4}\right)E\right]$$

$$E^2 = \left(N^2 - 1\right)E$$

The linear combinations of the basis tensors that are mutually orthogonal projection operators and that represent the exchange of a state with definite quantum numbers in the "vertical" channel are here labelled with the dimension of the irreducible representations in the decomposition of the product $\left(N^2 - 1\right) \otimes \left(N^2 - 1\right)$ and by the symmetry (or antisymmetry) property in the exchange of the indices a and c (or b and d).[19]

$$P_{1,S} = \left(\frac{1}{N^2 - 1}\right)E \qquad \text{(pomeron channel)} \tag{4.3}$$

$$P_{N^2-1,A} = \frac{4}{N}(B - C) \qquad \text{(antisymm. adjoint channel)} \tag{4.4}$$

$$P_{N^2-1,S} = \frac{4N}{N^2-4}\left(B + C - \frac{1}{2N} E\right) \qquad \text{(symm. adjoint channel)} \tag{4.5}$$

$$P_{\frac{(N^2-4)(N^2-1)}{4} + \overline{\frac{(N^2-4)(N^2-1)}{4}},A} = \frac{-4}{N}(B-C) + \frac{1}{2}(D-F) \tag{4.6}$$

$$P_{\frac{N^2(N-1)(N+3)}{4},S} = 2A - \frac{2}{N+2}(B+C) + \frac{1}{4}(D+F) + \frac{1}{2(N+1)(N+2)} E \tag{4.7}$$

$$P_{\frac{N^2(N-3)(N+1)}{4},S} = -2A - \frac{2}{N-2}(B+C) + \frac{1}{4}(D+F) + \frac{1}{2(N-1)(N-2)} E \tag{4.8}$$

for $N = 3$ the representation $\frac{N^2(N-3)(N+1)}{4}$ is not present and indeed the last projection operator vanishes because of the relation[20] (valid only in SU(3))

$$8(A + B + C) = D + E + F$$

In $N = 2$, more relations exist (see for instance Ref. 1) and one is left with only three channels associated with $P_{1,S}$, $P_{3,A}$, $P_{5,S}$.

Acknowledgment

We thank the SLAC Theory Group for their hospitality. This work was supported by the Department of Energy under contract number DE-AC03-76SF00515.

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2. Part of the matter in Section II and III was briefly described by one of us, G. Cicuta, SLAC-PUB-2328 (1979).
3. The subject of this section has already been discussed in the literature[4-8] usually starting from a "double line" Feynman graph, which would correspond in Cvitanovic and our approach to various contribution to the same weight, and/or by restricting the study to the color singlet external sources. Therefore, although this section overlaps with known literature, it may be useful. Furthermore, it does not seem that previous analyses were carried up to equations of the type of (2.13) or the conclusion of Section II.
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8. Many papers of the topological expansion have appeared in the past few years. Many of them are quoted in G. Chew and C. Rosenzweig, Physics Reports C41, 263 (1978) and E. Witten, Harvard preprint HUTP 79/A007 (1979).
9. This definition of planarity appears in §4 of Nakanishi's book, Graph Theory and Feynman Integrals, Gordon and Breach, 1971. It has the benefit of relating the notion of planarity of Feynman graphs to

planarity in graph theory, then making possible the use of Kuratowski theorems. This definition also agrees with the common use in the literature about topological expansion although usually it is not written.

10. All sums in Eq. (2.11) - (2.13) extend up to $\left[\frac{s}{2}\right]$, the entire part of $\frac{s}{2}$.

11. We recall again that within the graphic method[1] it is convenient to express four-gluon couplings in terms of trigluon couplings. Then the occurrence of vanishing graphs with some four couplings results from the cancellation among different graphs with trilinear couplings only.

12. P. Cvitanovic, B. Lautrup, R. Pearson, Phys. Rev. D18, 1939 (1978).

13. For instance, by looking at the basis tensor, Fig. 13, it is easy to see that every four-gluon graph whose weight is a tensor symmetric in one couple of indices, will also be symmetric in the other couple even though the graph may not have such symmetry.

14. This vanishing graph is a subgraph of a sixth order graph which was found to have a vanishing weight in a study of the quark form factor by J. Carazzone, E. Poggio, H. Quinn, Phys. Rev. D11, 2286 (1975), their Figs. 12b and 13.

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17. Tensor basis for all simple Lie groups are described in Ref. 1. The projection operators in SU(3) for gluon-gluon scattering appear in P. Yeung, Phys. Rev. D13, 2306 (1976) with some misprints.

18. We thank Michael Dine for a discussion on this point.

19. We thank Franco Buccella for a discussion on this point.

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Figure Captions

1. Graphical representation of Eq. (2.1).

2. Graphical representation of Eq. (2.2).

3. (a) A graph at order g^{10} in perturbation theory.

   (b) One of the 2^{15} "double line" contributions to the group weight of the graph in Fig. 3(a).

   (c) The same contribution as in Fig. 3(b), now exhibiting boundaries and windows.

4. The graph of Fig. 3(a) is here completed by adding one more vertex P_∞ in order to exhibit its degree of non-planarity.

5. A generic graph G whose weight is considered as a convolution of the weights of the subgraphs G_1 and G_2.

6(a), 7(a), 8(a), 9(a). The lowest order graphs with four external lines that have a symmetry plane through the external lines (γ,δ).

6(b), 7(b), 8(b), 9(b). The corresponding vanishing graphs obtained by convolution of the graphs in Figs. 6(a), 7(a), 8(a) and 9(a) with a three-gluon vertex $f_{\tau\alpha\beta}$.

10. (a) A graph with four external gluons and one fermion loop, which has a symmetry plane through the lines (γ,δ).

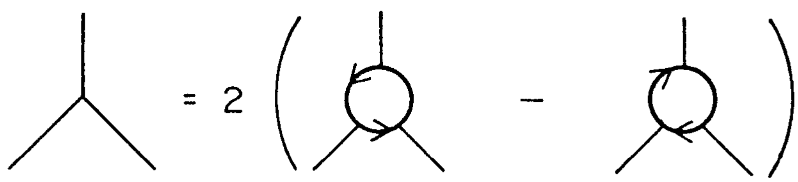
    (b) A vanishing graph obtained by convolution of the graph in Fig. 10(a) with the three-gluon vertex.

11. (a) A graph which does not have a symmetry plane through the lines (γ,δ) but whose weight is a tensor symmetric in the indices (α,β).

    (b) A vanishing graph obtained by convolution of the graph in Fig. 11(a) with the three-gluon vertex.

12. This three-leg graph may be proved to vanish by first completing the vanishing graph of Fig. 11(b) and next by "stereographic projecting" from a three-gluon vertex.

13. Basis tensors for gluon-gluon scattering in SU(N).



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3664A1

Fig. 1

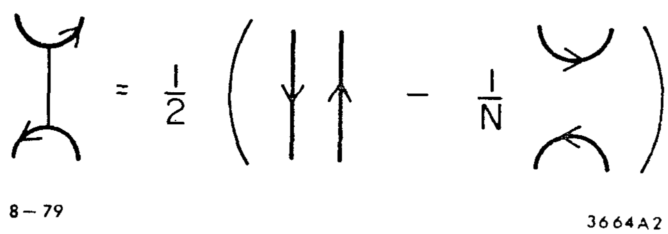
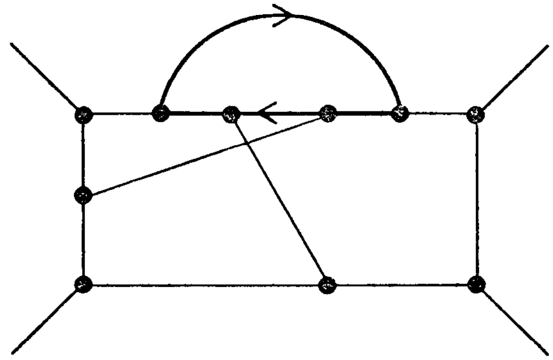
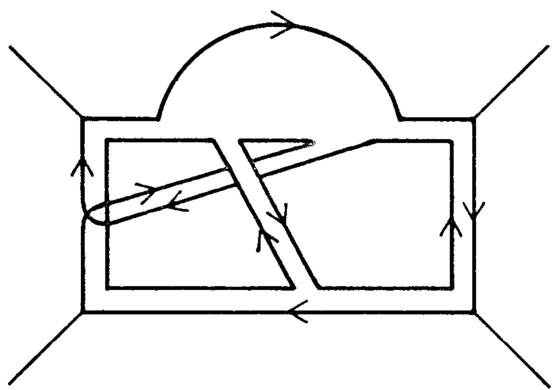


Fig. 2



(a)



(b)

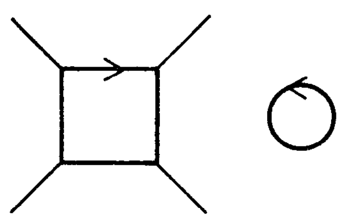


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Fig. 3

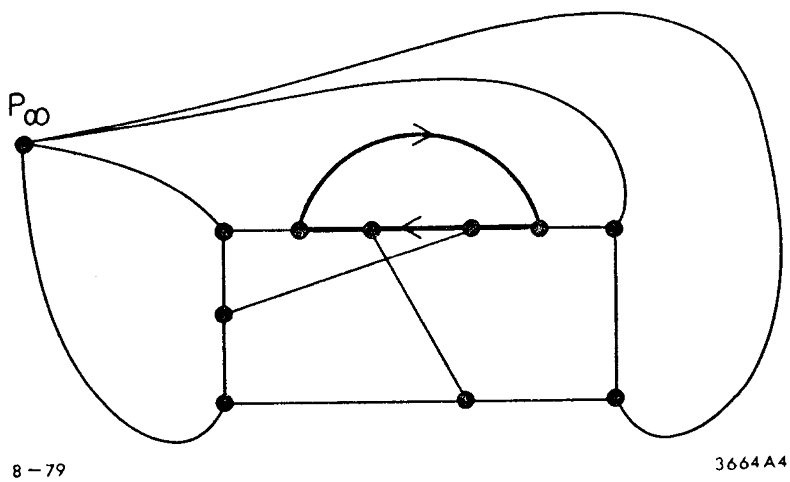

Fig. 4

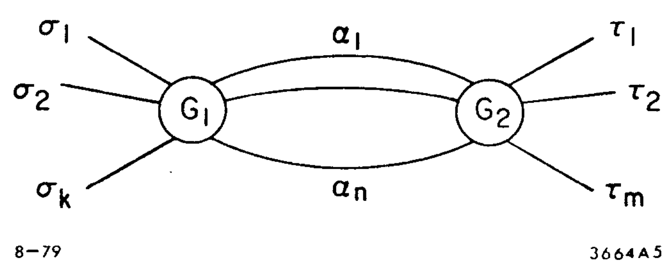
σ_1
σ_2
σ_k
G_1
α_1
α_n
G_2
τ_1
τ_2
τ_m
8-79
3664A5

Fig. 5

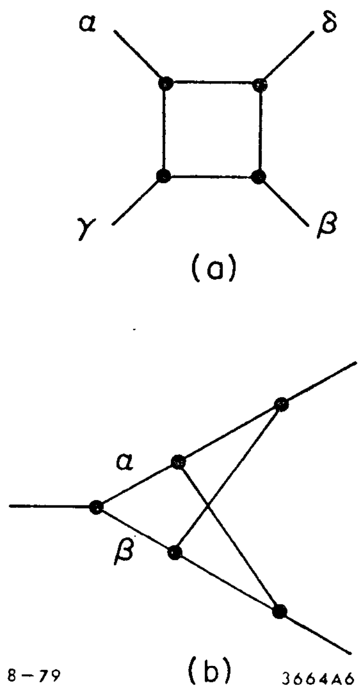

Fig. 6

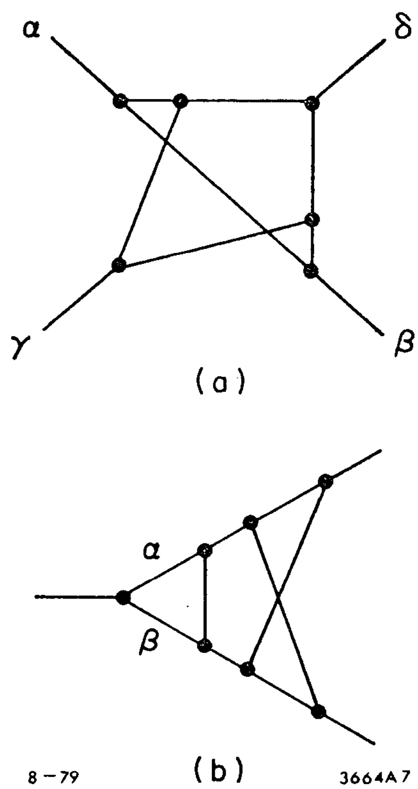

Fig. 7

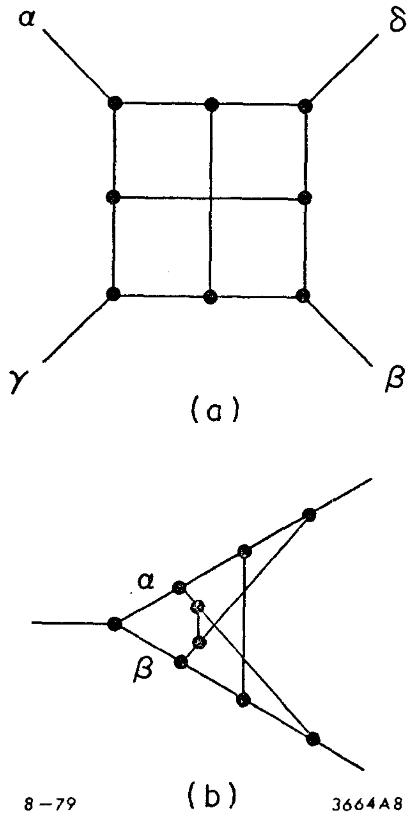

Fig. 8

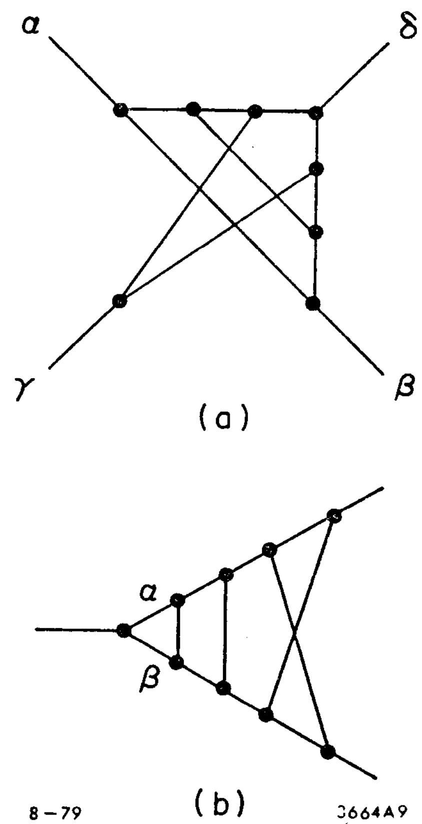

Fig. 9

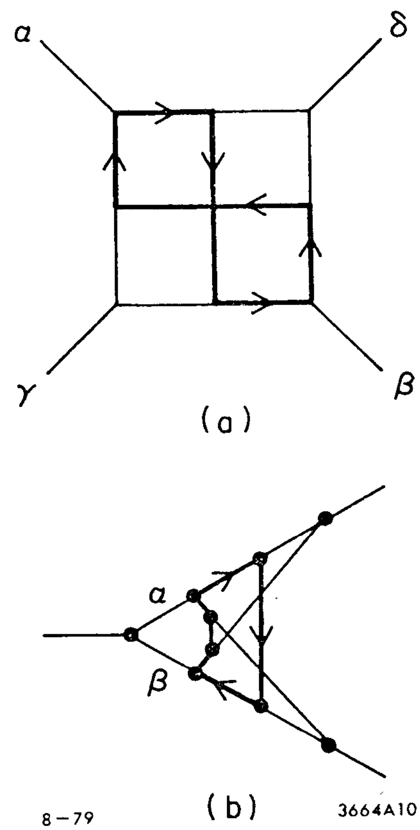

Fig. 10

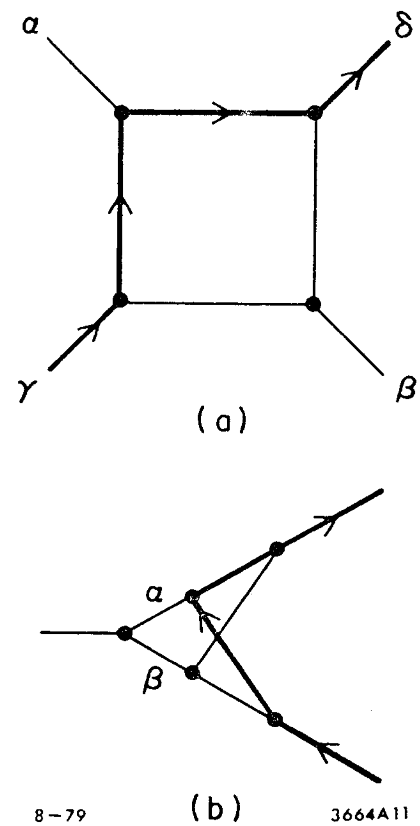

Fig. 11

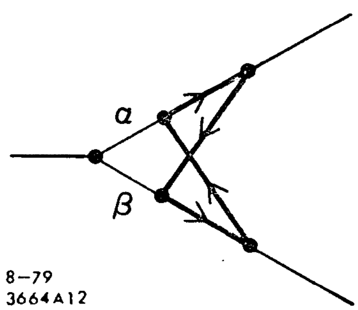

Fig. 12

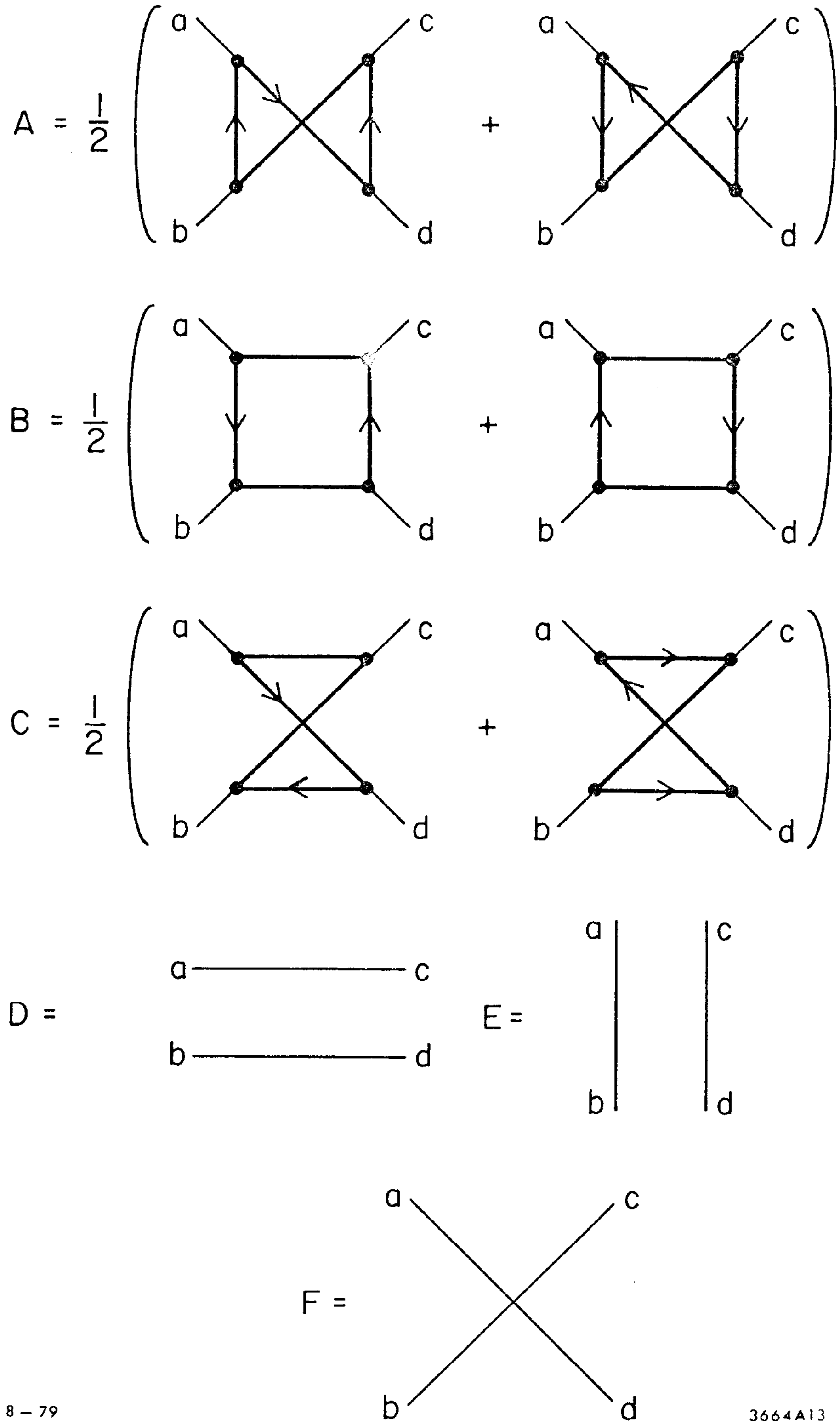

Fig. 13