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HIGHER-TWIST EFFECTS IN DEEP-INELASTIC SCATTERING *

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ABSTRACT

Non-scaling, non-factorizing $1/Q^2$ contributions to cross sections are derived for semi-inclusive deep-inelastic scattering, $\ell N \to \ell' \pi X$. These higher twist terms are dominant at large $z = p_\pi \cdot p_N / Q \cdot p_N$. They provide unusual (1-y) terms in the cross section, as well as asymmetries in the azimuthal angle dependence. Calculations are also presented for the quark to pion fragmentation function $D_\pi(z,Q^2)$.

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In the parton model and in the conventional quantum chromodynamics (QCD) approach to processes such as $\ell N \to \ell' \pi X$, it is customary first to isolate a basic pointlike constituent scattering process. The overall cross section is then expressed as a product of three incoherent probabilistic factors, representing (i) the density of "free" on-shell constituents of the hadrons in the initial state, (ii) the constituent to constituent scattering cross section, and (iii) the probability that the "free" on-shell final constituent fragments into the observed final state hadrons. The work described here is motivated by a desire to go beyond this simple approach, and to deal with the fact that constituents are not free, but are always bound in hadronic wave functions and are often considerably off-shell. At large $z = p_\pi \cdot p_N / Q \cdot p_N$, the quark p_a in Fig. 1(a) is pulled far off-shell since $p_a^2 \propto (1-z)^{-1}$. Accordingly, bound state effects not normally considered should grow in relevance as $z \to 1$, and the standard quark-parton model assumption of on-shell constituents becomes increasingly questionable. One of the consequences of <u>on-shell</u> behavior is the dominance of the cross sections associated with transversely polarized virtual photons and W's. When spin-1/2 constituents are far off-shell, however, the <u>longitudinally polarized</u> cross sections may take over, resulting in substantial changes in, e.g., observable angular distributions. Such an effect was predicted[1] in $\pi^- N \to \mu\bar{\mu} X$ at large x_F and later observed in the data.[2] Here I will report entirely analogous predictions for $\ell N \to \ell' \pi X$, where ℓ= e, μ or ν. My focus is on physical effects associated with the structure of the final pion. In the lowest order QCD perturbative Feynman diagrams which I will use, the pion appears explicitly as a $q\bar{q}$ system, in a definite spin state. Cross sections and polarization properties are predicted to depend in a detailed way on the internal dynamics of the pion.

For $\ell N \to \ell' \pi X$, as sketched in Fig. 1(a), it is convenient to let the momentum of the exchanged γ^* (or W in the case of νN) define the positive longitudinal $(\hat{z})$ direction:

$$Q = p_\ell - p_{\ell'} \tag{1}$$

The final pion has transverse momentum $\vec{p}_T$ relative to the direction of Q, and it carries the fraction z of the energy of the initial exchanged quantum. In terms of four-vectors, $z = p_\pi \cdot p_N / Q \cdot p_N$. To describe the lepton kinematics, it is conventional to use the variable $y = Q \cdot p_N / p_\ell \cdot p_N$. The initial and final lepton momenta are chosen to lie in the $(\hat{x},\hat{z})$ plane, both having positive $\hat{x}$ components of momentum. The angle ϕ is the azimuthal angle of p_T relative to this $\hat{x}$ direction. The initial quark p_b from the initial nucleon in Fig. 1(a) is assumed to be on-shell and to carry longitudinal momentum fraction x of the nucleon's momentum, and no transverse momentum $(p_b = xp_N)$. The intermediate quark p_a in the subprocess $Q + xp_N \to \pi + q$ is then off-shell and timelike, with

$$p_a^2 = p_T^2 \Big/ \big(z(1-z)\big) \ . \tag{2}$$

Note that as $z \to 1$, $p_a^2 \to \infty$. The quark, pion and nucleon masses have been neglected in the derivation of Eq. (2) and will be neglected throughout. Note that here $x = \left(p_a^2 - Q^2\right) \Big/ \left(2Q \cdot p_N\right)$ and $Q^2 < 0$.

In the limit of large z, it is sufficient to consider only the $q\bar{q}$ component of the pion wave function. Higher Fock state components ($q\bar{q}g$, $q\bar{q}q\bar{q}$, etc.) yield contributions to the cross-section which decrease as $z \to 1$ more rapidly than the terms I retain.[3] Moreover, at large z, where $p_a^2 \to \infty$ in Fig. 1, one may use first order QCD perturbation theory (i.e., single gluon exchange) to describe the large momentum, far off-shell behavior of the $q\bar{q}$ wave function. This approximation is illustrated in

Fig. 1(b). It corresponds to the first iteration of the Bethe-Salpeter kernel. Higher iterations[3] provide logarithmic $(\log Q^2)$ corrections to the first order results presented here. The same model of bound state effects provides a successful description of form factors[4] and of the cross-section[1,2] for $\pi^- N \to \mu\bar{\mu} X$.

Focusing here solely on effects associated with the pion bound state, I ignore the incident nucleon structure and treat the initial quark from the incident nucleon as free. Relying on the discussion above, one may draw the two lowest-order diagrams shown in Fig. 2. Both diagrams in Fig. 2 are required by gauge invariance, although in a physical (axial) gauge, the scaling contribution as $Q^2 \to \infty$ can be identified solely with Fig. 2(a). The pion's momentum p is portioned equally between the constituent $\bar{d}$ and u. If this simplifying approximation is discarded, a modest change occurs in the prediction of the relative size of the transversely polarized and longitudinally polarized components of the final cross section. The squared four momentum carried by the gluon in Fig. 2, $k^2 = (p_1 + \frac{1}{2}p)^2 = \frac{1}{2}\, p_a^2$, also becomes large as $z \to 1$.

For $\mu p \to \mu' \pi X$ or for $ep \to e' \pi X$, the invariant amplitude corresponding to Fig. 2 is

$$\mathcal{M} \propto \bar{u}(p_{\ell'})\gamma_\mu u(p_\ell)\, \frac{1}{Q^2}\, \frac{\alpha_s(k^2)}{k^2}\, \psi_\pi(\vec{0}) \sum_\lambda \bar{u}(p_1)\gamma_\alpha u_\lambda(p/2)\bar{v}_{-\lambda}(p/2)$$

$$\left[\gamma^\alpha \frac{1}{\not{p}_a} \gamma^\mu + \gamma^\mu \frac{1}{\not{p}_c}\gamma^\alpha\right] u(p_b) \ . \tag{3}$$

For $\nu p \to \mu^- \pi X$, γ_μ is replaced by $\gamma_\mu(1+\gamma_5)$, and the factor $1/Q^2$ is removed. The equality $\sum_\lambda u_\lambda \bar{v}_{-\lambda} = \frac{1}{2}\not{p}\gamma_5$ specifies that the $\bar{u}d$ bound state is a pseudoscalar. The factor $\psi_\pi(\vec{r} = \vec{0})$ in Eq. (3) represents an integration over the unspecified soft momenta in the pion wave function.

Using the amplitude in Eq. (3), one may obtain an explicit expression for the cross-sections for $ep \to e'\pi X$ or $\mu p \to \mu'\pi X$. For $p_T^2 << |Q^2|$, and large z, I derive

$$\frac{z\, d\sigma}{dz\, dy\, d\phi\, dp_T^2} \propto \int G_{q/N}(x)\, dx \frac{1}{y\, p_T^4\, Q^2}\, \psi_\pi^2(\vec{0}) \left\{ (1-z)^2 \left[\frac{1+(1-y)^2}{2}\right] \right.$$

$$\left. + \frac{2}{3}(1-z)\,[1+(1-y)]\,(1-y)^{\frac{1}{2}} \cos\phi \left(\frac{p_T^2}{-Q^2}\right)^{\frac{1}{2}} + \frac{4}{9}(1-y)\frac{p_T^2}{-Q^2} \right\} \quad (4a)$$

In Eq. (4a), $G_{q/N}$ is the quark structure function of the nucleon. The antiquark content of the nucleon is ignored here. However, the results may be generalized easily to include both the quark and antiquark content. To obtain Eq. (4a), an expansion in inverse powers of Q^2 was performed. Terms have been dropped which are down in magnitude by powers of (1-z) or of p_T^2/Q^2 relative to those in the curly brackets in Eq. 4(a). Thus Eq. (4a) is accurate in two $|Q^2| \to \infty$ limits: (i) the fixed z, $Q^2 \to \infty$ "Bjorken limit", and (ii) the fixed $(1-z)\cdot|Q^2|$ limit, with $(1-z)\cdot|Q^2| >> p_T^2$. The omitted terms must be restored at very small Q^2 or for z very near 1. The cross section also contains terms proportional to cos 2ϕ; however, the coefficients of these terms are of order $(1-z)\, p_T^2/Q^2$. They are therefore negligible in relation to the terms I retain, which are proportional to $(1-z)^2$, $(1-z)(p_T^2/-Q^2)^{\frac{1}{2}}$ or p_T^2/Q^2.

Since the initial quark $p_b \equiv xp_N$ is assumed to carry no transverse momentum, and because gluonic radiation effects[5] are ignored, the transverse momentum $\vec{p}_T$ is exactly the transverse momentum of the fragmenting quark with respect to p_π. In this calculation, the dependence of the cross-section on p_T is derived explicitly; it is not an arbitrarily assigned "intrinsic" or "primordial" p_T spectrum.

For $\nu N \to \mu^- \pi^+ X$, the expression in curly brackets in Eq. (4a) is replaced by

$$\left\{(1-z)^2 + \frac{2}{3}(1-z)(1-y)^{\frac{1}{2}} \cos\phi \left(\frac{4p_T^2}{-Q^2}\right)^{\frac{1}{2}} + \frac{4}{9}(1-y)\left(\frac{p_T^2}{-Q^2}\right)\right\}. \tag{4b}$$

For $\bar{\nu} N \to \mu^+ \pi^- X$, the term in the curly brackets in Eq. (4a) becomes

$$\left\{(1-y)^2 (1-z)^2 + \frac{2}{3}(1-z)(1-y)^{3/2} \cos\phi \left(\frac{4p_T^2}{-Q^2}\right)^{\frac{1}{2}} + \frac{4}{9}(1-y)\left(\frac{p_T^2}{-Q^2}\right)\right\}. \tag{4c}$$

Several predictions are embodied in Eqs. (4), involving the correlated behavior of all the variables z, y, p_T^2 and ϕ.

(i) In the fixed z, $Q^2 \to \infty$ limit, the cross sections in Eqs. (4) attain the usual scale invariant form expected in the parton model, with $d\sigma/dy \propto [1+(1-y)^2]$ for $\mu N \to \mu' \pi X$, $d\sigma/dy \propto (1-y)^2$ for $\bar{\nu} N \to \mu^+ \pi X$ and $d\sigma/dy$ independent of y for $\nu N \to \mu^- \pi X$. However, important departures from the parton model are predicted at finite Q^2, as described below.

(ii) According to Eqs. (4), the distribution $d\sigma/dp_T^2$ is expected to decrease as p_T^{-4} (for $p_T^2 \ll Q^2$), except at large z, where a less rapid p_T^{-2} behavior sets in. Thus, the mean $\langle p_T^2 \rangle$ should grow at large z, a "seagull" type of effect. This "jet broadening" phenomenon is distinct from that associated with gluonic radiation diagrams.[5] The two effects may be distinguished in the data by their different dependences on z and Q^2. As Eq. (4a) stands, it would appear that the cross section diverges as $\vec{p}_T \to 0$. However, a finite answer should be obtained once finite masses are restored and the full confining properties implied by $\psi_\pi(0)$ are implemented explicitly.

(iii) In the limit $Q^2 \to \infty$, the y and z dependences in Eqs. (4) are separable. In this limit, an asymptotic quark to pion fragmentation function may be extracted from Eqs. (4). For $z > 0.5$ and $Q^2 \to \infty$,

$$D_{\pi/q}\left(z,Q^2\right) \longrightarrow (1-z)^2 \quad , \tag{5}$$

Moreover, after averaging Eqs. (4) over y, ϕ and p_T^2, one may identify a significant non-scaling term in $D_{\pi/q}\left(z,Q^2\right)$:

$$D_{\pi/q}\left(z,Q^2\right) \propto (1-z)^2 + \frac{c\langle p_T^2\rangle}{-Q^2} \quad . \tag{6}$$

The constant c is process-dependent; for μN, νN and $\bar{\nu}N$, $c = 1/3$, $2/9$, $2/3$. The extra non-scaling term in Eq. (6) is independent of z, and is especially relevant at large z where the scaling term vanishes rapidly.[6] The form of Eq. (6) is similar to the popular phenomenological form proposed by Feynman and Field,[7] except for the important difference here that the constant term (independent of z) falls as $1/Q^2$.

(iv) For modest values of Q^2 <u>or for large z</u>, a significant <u>non-factorizing</u>, non-scaling term is present in Eqs. (4). Averaging over ϕ, I obtain:

For $ep \to e'\pi X$ or $\mu p \to \mu'\pi X$,

$$\sigma\left(z,y,Q^2\right) \propto (1-z)^2 \left[\frac{1+(1-y)^2}{2}\right] + \frac{4}{9}\,(1-y)\left(\frac{p_T^2}{-Q^2}\right) . \tag{7a}$$

For $\nu p \to \mu^- \pi^+ X$,

$$\sigma\left(z,y,Q^2\right) \propto (1-z)^2 + \frac{4}{9}\,(1-y)\left(\frac{p_T^2}{-Q^2}\right) . \tag{7b}$$

For $\bar{\nu}p \to \mu^+\pi^- X$,

$$\sigma\left(z,y,Q^2\right) \propto (1-z)^2 \ (1-y)^2 + \frac{4}{9} \ (1-y) \left(\frac{p_T^2}{-Q^2}\right) . \qquad (7c)$$

These expressions imply that at large z, the distributions in y should be very different from "normal." In the limit $z \to 1$ at fixed Q^2, or in the limit $Q^2 \to \infty$ with $Q^2(1-z)$ fixed, the terms which dominate are the unusual, higher-twist terms $\propto (1-y)$. Thus in either of these limits, the cross section $d\sigma/dy$ is predicted to have an unusual dependence, varying as $(1-y)$ for <u>all three</u> processes: $\mu N \to \mu' \pi X$, $\nu N \to \mu^- \pi^+ X$ and $\bar{\nu} N \to \mu^+ \pi^- X$. Likewise, selections on y can lead to considerably different expectations for the distribution in z. The term proportional to $\frac{4}{9}\ (1-y)\left(p_T^2 / -Q^2\right)$ is analog of the $\frac{4}{9}\ \sin^2\theta\left(k_T^2 / Q^2\right)$ term predicted [1] in $\pi^- N \to \mu\bar{\mu} X$, and corresponds to a longitudinally polarized γ^* or W. It would be valuable to ascertain whether its contribution is as significant in deep-inelastic scattering as it appears to be [2] in $\pi^- N \to \mu\bar{\mu} X$.

(v) The ϕ distributions in Eqs. (4) lead to positive asymmetries $\langle\cos\phi\rangle$ which grow as $1/(1-z)$ for intermediate values of z. The complete behavior of $d\sigma/d\phi$ may be predicted only after inclusion of QCD gluonic radiation effects [5] and kinematic effects associated with transverse momentum fluctuations [8] of the initial quark. Since these appear to provide negative values of $\langle\cos\phi\rangle$, the ϕ effects predicted in Eqs. (4) may be somewhat washed out. The new feature of Eqs. (4) is the growth of of $\langle\cos\phi\rangle$, proportional to $1/(1-z)$.

(vi) In the computations described here, only those hadron structure effects associated with the final pion are considered in $\ell N \to \ell' \pi X$.

In a more complete investigation, off-shell effects associated with the initial quark from the incident nucleon should also be treated. These will be especially relevant for $x > 0.5$ and should lead to the prediction of correlations in the x and z dependences of the cross section.

The most significant aspect of Eqs. (4) is the prediction that $d\sigma/dy \propto (1-y)$ at large z. This term arises because the integer-spin nature of the pion plays a dominant role in the subamplitude $Qq \to (q\bar{q})_\pi q$ at large z. The $(1-y)/Q^2$ term is a higher-twist effect in that more than the minimum twist $Qq \to q$ amplitude is required to produce it. In $\pi^- N \to \mu\bar{\mu} X$ an analogous higher-twist term[1] yields significant effects[2] for values of Q^2 as large as ~ 20 GeV^2. Confirmation of the presence of the $(1-y)$ term in $\ell N \to \ell' \pi X$, with the magnitude expected, would provide another important indication that bound state and high-twist effects can be computed in a QCD framework. The use of Eqs. (4) should also permit the extension of QCD fits to data into a region of low and intermediate Q^2 where the asymptotic terms alone are insufficient.[9]

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6. The parameter $\langle p_T^2 \rangle$ in Eq. (6) is probably larger than, and may not be equated directly to the value measured experimentally. Here it effectively includes mass terms which were omitted when the pion, quark and other masses were set to zero.

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Figure Captions

1. (a) Sketch of $\ell N \to \ell' \pi X$; Q labels the exchanged γ^* or W. The intermediate quark labeled p_a is off-shell and timelike. The initial quark from the incident nucleon carries four momentum $p_b = xp_N$.

   (b) On the left is a diagram showing the disassociation of an off-shell virtual quark into a pion plus X. At large p_a^2, its behavior may be represented by the single gluon exchange diagram sketched on the right, in which the quark lines marked with crosses (x) are essentially on-shell. The unshaded oval in the diagram on the right-hand side of Fig. 1(b) represents the unspecified small momentum behavior of the pion wave function, represented in this paper simply by the wave function at the origin, $\psi_\pi(\vec{r} = 0)$.

2. Gauge-invariant set of diagrams for $\ell N \to \ell' \pi X$; k labels the four-momentum of the exchanged gluon. Solid lines, except for the ones labeled π, ℓ and ℓ', are quark lines.

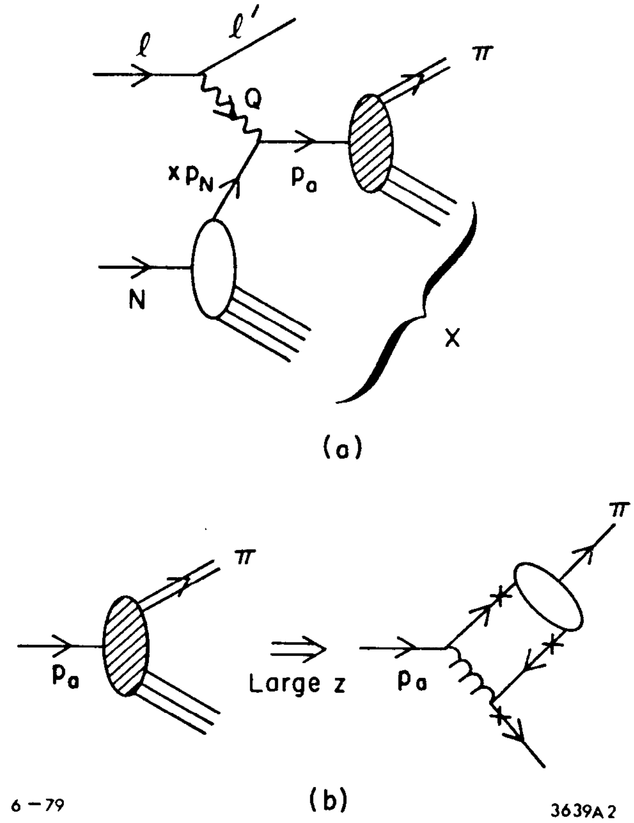
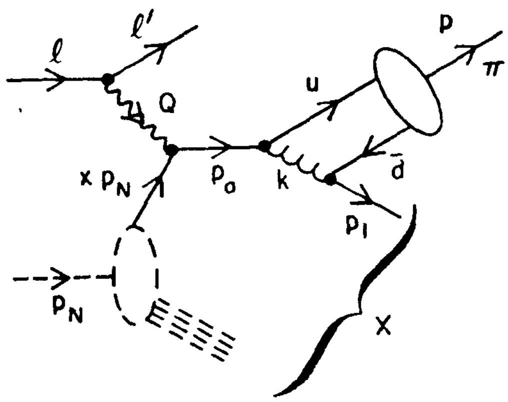


Fig. 1

$\ell N \to \ell' \pi X$



(a)

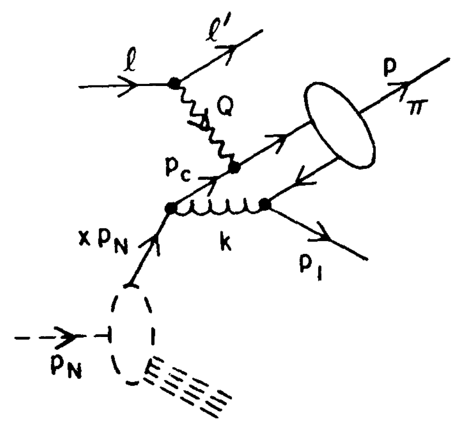


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Fig. 2