SLAC-PUB-2364
July 1979
(T)

WHY ISOSPIN IS NOT DYNAMICALLY BROKEN *

P. Sikivie

Stanford Linear Accelerator Center

Stanford University, Stanford, California 94305

ABSTRACT

Isospin invariance, P and CP could well have been dynamically broken along with chiral invariance by the quark-antiquark condensate $\left(\langle\bar{q}q\rangle_0 \neq 0\right)$. We show, however, that under appropriate circumstances, the conservation of isospin, P and CP is "natural".

(Submitted to Phys. Rev. Lett.)

*This work was supported by the Department of Energy under contract number DE-AC03-76-SF00515.

Recent ideas about the dynamical breaking of the Weinberg-Salam $SU(2) \otimes U(1)$ gauge symmetry by a "technicolor condensate"[1] have raised the question whether we properly understand the usual quark-antiquark condensate whose existence has been postulated to explain the breaking of chiral isospin symmetry in the Nambu-Goldstone fashion and the origin of PCAC. Surely, if we do not clearly understand the symmetry properties of the comparitively simple quark-antiquark condensate, we will be at a loss to predict those of the hypothetical technicolor condensates. We propose here a simple technique which should help in this question and which does work properly for the color condensate.

First let us point out that it is not so obvious why the usual quark-antiquark condensate does not break isospin invariance. We will assume as usual that there are two massless u and d quarks whose strong interactions are described by QCD. Along with their (current) masses, we neglect the weak and electromagnetic interactions of these quarks. They are indeed negligible in first approximation in comparison with the effects of the strong interactions in the infra-red region. They do play a crucial role however to which we will come back later. The QCD Lagrangian is now $U_L(2) \otimes U_R(2)$ invariant, except that there is a possible breaking of the $U_A(1)$ axial invariance due to the Adler-Bell-Jackiw anomaly[2] and instanton physics.[3] The arguments as to why isospin invariance is not dynamically broken can be made both in Case I, where the axial U(1) is broken, and in case II, where it is not. Both cases will be discussed separately below.

We will assume that a quark-antiquark condensate is indeed the reason behind the breaking of chiral isospin symmetry and the successes of the PCAC hypothesis. There are eight color singlet, Lorentz scalar quark-antiquark combinations that could take vacuum expectation values.

These form a matrix:

$$\phi = \left\langle \begin{pmatrix} u_R^\dagger u_L & d_R^\dagger u_L \\ u_R^\dagger d_L & d_R^\dagger d_L \end{pmatrix} \right\rangle_0 \qquad (1)$$

$$= \sigma + i\eta + (\vec{\varepsilon} + i\vec{\pi}) \cdot \vec{\tau}$$

where σ, $\vec{\pi}$, η and $\vec{\varepsilon}$ are the vacuum expectation values of eight hermitean effective scalar fields with isospin and parity properties $I^P = 0^+, 1^-, 0^-$, and 1^+, respectively. They are all even under charge conjugation C. ϕ transforms as the (2,2) representation of $U_L(2) \otimes U_R(2)$:

$$\phi \rightarrow \mathcal{U}_L \, \phi \, \mathcal{U}_R^\dagger \qquad (2)$$

Under $SU_L(2) \otimes SU_R(2) \sim SO(4)$, ϕ breaks up into two 4-vectors: $(\sigma,\vec{\pi})$ and $(\eta,\vec{\varepsilon})$. Let us assume that ϕ has an arbitrary direction in group space. By a $U_L(2) \otimes U_R(2)$ transformation, it can always be put into the form:

$$\phi \rightarrow \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} = \frac{a+b}{2} + \frac{a-b}{2}\tau_3 \qquad (3)$$

where a and b are real and positive. A general expectation value thus breaks $U_L(2) \otimes U_R(2)$ down to $U_V(1) \otimes U_V(1)$. Only when $a = b$, is $SU_V(2) \otimes U_V(1)$ the invariance group of the condensate. If $U_A(1)$ is broken by instanton physics (or some other strong interactions physics), then we are only allowed to use $SU_L(2) \otimes SU_R(2) \otimes U_V(1)$ to align the condensate. It can then be put into the form:

$$\phi \rightarrow \begin{pmatrix} a + ic & 0 \\ 0 & b + ic \end{pmatrix} = \frac{a+b}{2} + ic + \frac{a-b}{2}\tau_3 \qquad (4)$$

where a, b and c are real. The invariance group of the condensate is again $U_V(1) \otimes U_V(1)$. Isospin is broken unless $a = b$. Also P and CP are broken unless $c=0$. Let us now go on to show that although an arbitrary condensate would break isospin invariance, it is nevertheless "natural" to expect that the actual condensate does conserve isospin. By "natural", we mean that there is a continuous region of finite volume in some relevant parameter space where the condensate will be exactly isospin invariant. By contrast, in the space of parameters (a,b) and (a,b,c) introduced in Eqs. (3) and (4), the region in which isospin is conserved exactly has zero measure.

Let us first consider case I, where the axial U(1) is not broken by any strong interaction physics such as instantons. Since the strong interactions are then $U_L(2) \otimes U_R(2)$ invariant, the energy V of the condensate as a function of ϕ must be $U_L(2) \otimes U_R(2)$ invariant. In other words, it can only be a function of the two independent $U_L(2) \otimes U_R(2)$ invariants $C_1 = \mathrm{Tr}(\phi^\dagger\phi)$ and $C_2 = \mathrm{Tr}(\phi^\dagger\phi)^2$ that one can build out of ϕ, albeit an a-priori arbitrary function of these invariants. The vacuum minimizes $V(\phi) = V(C_1,C_2)$ and therefore satisfies the extremum equations:

$$\begin{aligned} \frac{\partial V}{\partial \phi^\dagger} &= \frac{\partial V}{\partial C_1}\,\phi + 2\,\frac{\partial V}{\partial C_2}\,\phi\,\phi^\dagger\,\phi \\ &= \frac{\partial V}{\partial C_1}\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} + 2\,\frac{\partial V}{\partial C_2}\begin{pmatrix} a^3 & 0 \\ 0 & b^3 \end{pmatrix} = 0 \end{aligned} \qquad (5)$$

There are in general four types of solution to Eq. (5): i. $(a,b) = (0,0)$; ii. $(a,b) = (0,x)$ or $(x,0)$; iii. $(a,b) = (x,x)$, where $x = \left(-\frac{\partial V}{\partial C_1}\Big/ 2\,\frac{\partial V}{\partial C_2}\right)^{1/2}$; and iv. a and b solve $\frac{\partial V}{\partial C_1} = \frac{\partial V}{\partial C_2} = 0$. Only the type iii extremum has $SU_V(2) \otimes U_V(1)$ as its invariance group. We study in detail an effective potential which is an arbitrary polynomial of degree ≤ 4:

$$V(\phi) = -\frac{\mu^2}{2}\,\mathrm{Tr}(\phi^\dagger\phi) + \frac{\lambda}{4}\,\mathrm{Tr}(\phi^\dagger\phi)^2 + \frac{\gamma}{4}\left(\mathrm{Tr}(\phi^\dagger\phi)\right)^2 \qquad (6)$$

It is bounded from below provided $\lambda+\gamma > 0$. It has each of the four types of extrema except the last one. In Table I, we give the conditions for each of these extrema to be the absolute minimum. As promised, there is a whole region of parameter space $(\mu^2 > 0,\ \lambda > 0)$ where the vacuum conserves isospin while breaking chiral invariance. This remains true when one considers effective potentials more complicated than the one in Eq. (6).[4] The $U_L(2) \otimes U_R(2)$ symmetric effective potential has however the U(1) problem[5] associated with the η-mass, the η being a Goldstone boson in this case.

Let us thus go on to the case where the axial U(1) is broken by instantons or some other strong interaction physics. Since we neglect the small current masses of the u and d quarks, we can set the θ parameter[6] of QCD equal to zero. The strong interactions now conserve $SU_L(2) \otimes SU_R(2) \otimes U_V(1)$, P and CP. The energy V of the quark-antiquark condensate as a function of $\vec{v} = (\sigma,\vec{\pi})$ and $\vec{\omega} = (\eta,\vec{\varepsilon})$ must have these symmetries. V is then an a-priori arbitrary function of the three independent $SU_L(2) \otimes SU_R(2) \otimes U_V(1)$, P and CP invariants $C_1 = \vec{v}\cdot\vec{v}$, $C_2 = \vec{\omega}\cdot\vec{\omega}$ and $C_3 = (\vec{v}\cdot\vec{\omega})^2$ that one can build out of $\vec{v}$ and $\vec{\omega}$. The vacuum satisfies the extrema equations

$$\begin{aligned}\frac{\partial V}{\partial \vec{v}} &= 2\,\frac{\partial V}{\partial C_1}\,\vec{v} + 2\,\frac{\partial V}{\partial C_3}\,(\vec{v}\cdot\vec{\omega})\,\vec{\omega} = 0\\ \frac{\partial V}{\partial \vec{\omega}} &= 2\,\frac{\partial V}{\partial C_2}\,\vec{\omega} + 2\,\frac{\partial V}{\partial C_3}\,(\vec{v}\cdot\vec{\omega})\,\vec{v} = 0\end{aligned} \qquad (7)$$

which have six types of solutions: i. $\vec{v} = \vec{\omega} = 0$; ii. $\vec{v} = 0,\ \vec{\omega} \neq 0$

$\left(|\vec{\omega}|\right.$ solves $\left.\frac{\partial V}{\partial C_2} = 0\right)$; iii. $v \neq 0$, $\vec{\omega} = 0$ $\left(|\vec{v}|\right.$ solves $\left.\frac{\partial V}{\partial C_1} = 0\right)$; iv. $\vec{v} \neq 0$, $\vec{\omega} \neq 0$, $\vec{v} \perp \vec{\omega}$ $\left(|\vec{v}|\right.$ and $|\vec{\omega}|$ solve $\left.\frac{\partial V}{\partial C_1} = \frac{\partial V}{\partial C_2} = 0\right)$; v. $\vec{v} \neq 0$, $\vec{\omega} \neq 0$, $\vec{v} \parallel \vec{\omega}$; and vi. $\vec{v} \neq 0$, $\vec{\omega} \neq 0$, $\vec{v} \cdot \vec{\omega} \neq 0$, where $|\vec{v}|$, $|\vec{\omega}|$ and $\vec{v} \cdot \vec{\omega}$ solve $\frac{\partial V}{\partial C_1} = \frac{\partial V}{\partial C_2} = \frac{\partial V}{\partial C_3} = 0$. Only the type iii extremum conserves $SU_V(2) \otimes U_V(1)$ along with P and CP, while breaking chiral invariance. We again study an effective potential which is an arbitrary polynomial of degree ≤ 4:

$$V(\vec{v},\vec{\omega}) = -\frac{\mu_1^2}{2}(\vec{v}\cdot\vec{v}) - \frac{\mu_2^2}{2}(\vec{\omega}\cdot\vec{\omega}) + \frac{\lambda_1}{4}(\vec{v}\cdot\vec{v})^2 + \frac{\lambda_2}{4}(\vec{\omega}\cdot\vec{\omega})^2 + \frac{\lambda}{2}(\vec{v}\cdot\vec{v})(\vec{\omega}\cdot\vec{\omega}) + \frac{\gamma}{4}(\vec{v}\cdot\vec{\omega})^2. \quad (8)$$

It is bounded from below provided $\lambda_1 > 0$, $\lambda_2 > 0$ and $\lambda \pm \gamma > -\sqrt{\lambda_1\lambda_2}$. This potential has each of the six types of extrema except the last one. In Table II, we give the (mutually exclusive) conditions for each of these extrema to be the absolute minimum. There is a whole region of parameter space $\left(\mu_1^2 > 0,\ \lambda\mu_1^2 - \lambda_1\mu_2^2 > 0,\ (\lambda+\gamma)\mu_1^2 - \lambda_1\mu_2^2 > 0\right)$ where the vacuum conserves isospin, P and CP while breaking chiral invariance. This remains true for effective potentials more complicated than the one in Eq. (8).[4]

We note that usually the condition for strong P and CP conservation is stated to be that with the phase convention on the quark fields where all the quark masses are real and positive, the θ parameter of QCD is equal to zero or π. But there obviously is a second condition: with the phase convention where $\theta = 0$ or π, the quark-antiquark condensate must be P and CP even. We have showed that this condition along with isospin conservation can be satisfied "naturally."

Before we turn on the weak and electromagnetic interactions or the current masses of the quarks, it is pointless to ask how the $SU(2) \otimes U_V(1)$ (assuming we are in the right region of parameter space) invariance group of the condensate is oriented in $U_L(2) \otimes U_R(2)$, since we have truly no way to distinguish u_L from d_L or u_R from d_R. But as soon as we turn on, say, electromagnetism, we can distinguish u from d by their electric charge and the question arises whether SU(2) is properly oriented in $SU_L(2) \otimes SU_R(2)$.

We can introduce the weak and electromagnetic interactions and the current masses of the quarks simultaneously by turning on the full Weinberg-Salam gauge theory.[7] One proceeds as follows: first, one neglects the Yukawa interactions versus the Higgs potential of the Weinberg-Salam theory and the effective potential of the strong interactions (it is easy to justify this). At that point, the relative direction between the Higgs vacuum expectation value and the quark-antiquark condensate is arbitrary. Next we turn on the Yukawa interactions which couple these two directions:

$$K_1\left(u_L^\dagger, d_L^\dagger\right)\begin{pmatrix}\Phi^+\\ \Phi^o\end{pmatrix} d_R + K_2 u_R^\dagger\left(-d_L, u_L\right)\begin{pmatrix}\Phi^+\\ \Phi^o\end{pmatrix} + \text{h.c.} \rightarrow \left(K_1+K_2\right)\mathcal{V}^\dagger\Phi + \left(K_1-K_2\right)\mathcal{W}^\dagger\Phi + \text{h.c.} \tag{10}$$

where $\Phi = \begin{pmatrix}\Phi^+\\ \Phi^o\end{pmatrix}$ is the Higgs doublet of the Weinberg-Salam theory

and $$\mathcal{V} = \begin{pmatrix}\pi_2 + i\pi_1\\ \sigma - i\pi_3\end{pmatrix}, \quad \mathcal{W} = \begin{pmatrix}\varepsilon_1 - i\varepsilon_2\\ -\varepsilon_3 + i\eta\end{pmatrix}.$$

If we are in the right region of the parameter space of the strong effective potential, then $\mathcal{W}=0$ while the norm of $\mathcal{V}$ has some definite value. The effect of the Yukawa interaction is to align $\mathcal{V}$ with $\langle\Phi\rangle_0$. This automatically insures that the condensate is properly oriented with respect to the weak and electromagnetic interactions and the small current masses: $m_u = K_2\langle\Phi^o\rangle$, $m_d = \left(K_1\langle\Phi^o\rangle\right)^*$.

In conclusion, it is heartening to realize that the symmetry properties of a condensate produced by dynamical symmetry breaking can to a large extent be analyzed without having to perform actual dynamical calculations, which have turned out to be quite difficult in the cases of interest, but which will of course ultimately be necessary.

Acknowledgments

I am grateful to L. Susskind for raising the question this paper addresses itself to, and for his criticism and encouragement. I thank the Aspen Center for Physics for its hospitality while part of this work was being done. Finally, I acknowledge useful conversations with M.A.B. Bèg, J. D. Bjorken, E. Farhi, M. Einhorn, H. Pagels, H. Quinn and M. Wise. This work was supported by the Department of Energy under contract number DE-AC03-76SF00515.

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4. For example, if we construct a polynomial, with arbitrary parameters, which includes all possible terms up to a given degree (≥ 4), there always exists a finite region of parameter space where the type iii extremum is the absolute minimum for both cases I and II.

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Region of Parameter Space	Absolute Minimum	Invariance Group	Number of Goldstone Bosons
$\mu^2 < 0$	type <u>i</u>	$U_L(2) \otimes U_R(2)$	0
$\mu^2 > 0,\ \lambda < 0$	type <u>ii</u>	$U_V(1) \otimes U_V(1) \otimes U_A(1)$	5
$\mu^2 > 0,\ \lambda > 0$	type <u>iii</u>	$SU_V(2) \otimes U_V(1)$	$4(\vec{\pi},\eta)$
(empty)	type <u>iv</u>	$U_V(1) \otimes U_V(1)$	6

Table I: Vacuum symmetry properties as a function of the region of parameter space for the effective potential of Eq. (6).

Region of Parameter Space	Absolute Minimum	Invariance Group	Number of Goldstone Bosons	P and CP
$\mu_1^2 < 0,\ \mu_2^2 < 0$	type <u>i</u>	$SU_L(2) \otimes SU_R(2) \otimes U_V(1)$	0	C^*
$\mu_2^2 > 0,\ \lambda\mu_2^2 - \lambda_2\mu_1^2 > 0,$ $(\lambda+\gamma)\mu_2^2 - \lambda_2\mu_1^2 > 0$	type <u>ii</u>	$SU_V(2) \otimes U_V(1)$	$3(\vec{\varepsilon})$	$\mathcal{B}^\dagger$
$\mu_1^2 > 0,\ \lambda\mu_1^2 - \lambda_1\mu_2^2 > 0,$ $(\lambda+\gamma)\mu_1^2 - \lambda_1\mu_2^2 > 0$	type <u>iii</u>	$SU_V(2) \otimes U_V(1)$	$3(\vec{\pi})$	C^*
$\gamma > 0,\ \lambda_1\lambda_2 - \lambda^2 > 0,$ $\lambda_2\mu_1^2 - \lambda\mu_2^2 > 0,\ \lambda_1\mu_2^2 - \lambda\mu_1^2 > 0$	type <u>iv</u>	$U_V(1) \otimes U_V(1)$	5	C^*
$\gamma < 0,\ \lambda_1\lambda_2 - (\lambda+\gamma)^2 > 0,$ $\lambda_2\mu_1^2 - (\lambda+\gamma)\mu_2^2 > 0,$ $\lambda_1\mu_2^2 - (\lambda+\gamma)\mu_1^2 > 0$	type <u>v</u>	$SU_V(2) \otimes U_V(1)$	3	$\mathcal{B}^\dagger$
(empty)	type <u>vi</u>	$U_V(1) \otimes U_V(1)$	5	$\mathcal{B}^\dagger$

<u>Table II</u>: Vacuum symmetry properties as a function of the region of parameter space for the effective potential of Eq. (8).

*Conserved, †Broken