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COMMENT ON CABIBBO-SUPPRESSED NONLEPTONIC D DECAYS*

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We discuss why an extension of the ideas used to explain the $\Delta I = 1/2$ rule in kaon and hyperon decays does not lead to an analogous large enhancement in the rates of Cabibbo-suppressed nonleptonic D meson decays. The possibility of seeing the contribution of diagrams with a virtual b-quark loop through interference effects is also discussed.

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The analysis of nonleptonic weak decays has proven to be a difficult problem involving complex features of the strong interactions. However, some progress has been made[1,2,3] by using renormalization group techniques to generate, from the standard Hamiltonian in which W-bosons and various heavy quarks appear, an effective field theory involving only "light" (u, d and s for K decays and u, d, s and c for D decays) quarks. It is then hoped that enough features of the strong interactions have been incorporated so that a simple estimate (for example, by inserting the vacuum in all possible ways) of hadronic matrix elements of the operators in the effective Hamiltonian will lead to an approximate understanding of nonleptonic decays.

In the case of kaon (or hyperon) decays, when W-exchange graphs are replaced by effective four-fermion interactions involving only the u, d, s and c quark fields, the QCD corrections enhance the Wilson coefficients of operators with $I = 1/2$ relative to the Wilson coefficients of $I = 3/2$ operators.[4] These operators still have the $(V - A) \times (V - A)$ form typical of W-exchange. However, when the charmed quark is treated as heavy and removed to generate an effective theory involving only u, d and s quarks, operators with the structure $(V - A) \times (V + A)$ appear. Although these operators have small Wilson coefficients, it has been suggested that the matrix elements of such operators are greatly enhanced over the matrix elements of operators with the usual $(V - A) \times (V - A)$ chiral structure.[1] Since the $(V - A) \times (V + A)$ operators are purely $I = 1/2$, a further enhancement of the $\Delta I = 1/2$ amplitudes over $\Delta I = 3/2$ amplitudes occurs. It appears that one can thus qualitatively account for the $\Delta I = 1/2$ rule in nonleptonic kaon and hyperon decays.[5]

A question which naturally arises is whether there is an analogous effect in Cabibbo-suppressed nonleptonic D decays. The effective Hamiltonian relevant for Cabibbo-suppressed nonleptonic D decays is generated by a three step process in which the W-boson, t-quark and b-quark are sequentially removed from explicitly appearing in the theory. The removal of the W-boson and heavy t-quark leads to an adjustment of the coefficients of the operators which appear in the effective Hamiltonian in the absence of strong interactions. In addition, when the b-quark is removed, new operators, which had zero coefficients in the absence of strong interactions, appear due to operator mixing. Some of these new operators have the chiral structure $(V-A)\times(V+A)$. For a typical set of parameters[6] the effective Hamiltonian for Cabibbo-suppressed nonleptonic D decays is:[7]

$$\mathscr{H}_{\text{eff}} = -\frac{G}{2\sqrt{2}}\, s_1 c_1 c_2 \Bigg\{ \left[.70\, \mathscr{O}^{(+)} + 2.03\, \mathscr{O}^{(-)}\right]$$

$$- \left(s_3^2 + \frac{s_2 s_3 c_3 e^{i\delta}}{c_1 c_2}\right) \left[-1.33\, \mathscr{O}_1 + 2.73\, \mathscr{O}_2 + .026\, \mathscr{O}_3\right.$$

$$\left. - .061\, \mathscr{O}_4 + .018\, \mathscr{O}_5 - .074\, \mathscr{O}_6 \right] \Bigg\} + \text{H.c.} \qquad (1)$$

where

$$\mathscr{O}^{(\pm)} = (\bar{c}_\alpha s_\alpha)_{V-A}\, (\bar{s}_\beta u_\beta)_{V-A} \pm (\bar{c}_\alpha u_\alpha)_{V-A}\, (\bar{s}_\beta s_\beta)_{V-A}$$

$$- (\bar{c}_\alpha d_\alpha)_{V-A}\, (\bar{d}_\beta u_\beta)_{V-A} \mp (\bar{c}_\alpha u_\alpha)_{V-A}\, (\bar{d}_\beta d_\beta)_{V-A}$$

$$
\begin{aligned}
\mathscr{O}_1 &= (\bar{c}_\alpha u_\alpha)_{V-A}\ (\bar{s}_\beta s_\beta)_{V-A} \\
\mathscr{O}_2 &= (\bar{c}_\alpha u_\beta)_{V-A}\ (\bar{s}_\beta s_\alpha)_{V-A} \\
\mathscr{O}_3 &= (\bar{c}_\alpha u_\alpha)_{V-A}\left[(\bar{u}_\beta u_\beta)_{V-A} + (\bar{d}_\beta d_\beta)_{V-A} + (\bar{s}_\beta s_\beta)_{V-A} + (\bar{c}_\beta c_\beta)_{V-A}\right] \\
\mathscr{O}_4 &= (\bar{c}_\alpha u_\beta)_{V-A}\left[(\bar{u}_\beta u_\alpha)_{V-A} + (\bar{d}_\beta d_\alpha)_{V-A} + (\bar{s}_\beta s_\alpha)_{V-A} + (\bar{c}_\beta c_\alpha)_{V-A}\right] \qquad (2) \\
\mathscr{O}_5 &= (\bar{c}_\alpha u_\alpha)_{V-A}\left[(\bar{u}_\beta u_\beta)_{V+A} + (\bar{d}_\beta d_\beta)_{V+A} + (\bar{s}_\beta s_\beta)_{V+A} + (\bar{c}_\beta c_\beta)_{V+A}\right] \\
\mathscr{O}_6 &= (\bar{c}_\alpha u_\beta)_{V-A}\left[(\bar{u}_\beta u_\alpha)_{V+A} + (\bar{d}_\beta d_\alpha)_{V+A} + (\bar{s}_\beta s_\alpha)_{V+A} + (\bar{c}_\beta c_\alpha)_{V+A}\right]
\end{aligned}
$$

with the notation

$$(\bar{\psi}\psi)_{V\pm A}\ (\bar{\psi}\psi)_{V\pm A} = \left(\bar{\psi}\,\gamma^\mu\,(1\pm\gamma_5)\psi\right)\left(\bar{\psi}\,\gamma_\mu\,(1\pm\gamma_5)\psi\right) .$$

The indices α and β run over the three colors and when repeated are summed.

Note that the operators $\mathscr{O}_5$ and $\mathscr{O}_6$ have a $(V-A)\times(V+A)$ structure. Along with being Cabibbo-suppressed, their contribution to the effective Hamiltonian is suppressed by an additional angular factor $\left(s_3^2 + s_2 s_3 c_3 e^{i\delta} / c_1 c_2\right)$ which is expected to be small.[8] Thus, even if a sizeable enhancement of the matrix elements of the $(V-A)\times(V+A)$ operators over those of the $(V-A)\times(V-A)$ operators occurs in D decays, we do not expect any large enhancement of the Cabibbo-suppressed decay rates relative to the Cabibbo-allowed decays (where no $(V-A)\times(V+A)$ operators occur in the effective Hamiltonian). This is in qualitative agreement with experiment.[9]

When hadronic matrix elements of the effective Hamiltonian (1) are taken, they should of course be evaluated to all orders in the strong coupling. Among the higher-order corrections to the matrix elements of the usual $(V-A)\times(V-A)$ operators $\mathcal{O}^{(\pm)}$ are those coming from the diagrams of Figure 1 involving virtual d and s quark loops. Diagrams like those in Figure 1 involving a virtual heavy b-quark have been shown to sum up,[1,10] to leading order in logs of m_b, to produce the local $(V-A)\times(V+A)$ operators appearing in Equation (1). When virtual light quarks like d and s are involved in the loop, no such approximation is valid and the contributions of Fig. 1 should not be thought of as giving rise to a local effective operator but rather as QCD corrections to the hadronic matrix elements of the local four-quark operators $\mathcal{O}^{(\pm)}$. Nevertheless, one might wonder how important the contribution from diagrams like those in Figure 1 will be in Cabibbo-suppressed D decays. We feel that it will not be very important for the following reason. If $m_s = m_d$, then, because of the GIM cancellation mechanism, the diagrams of Figure 1 involving an s-loop would exactly cancel those with a d-loop. The typical momenta flowing through the loops in these diagrams is of order m_c. The contribution from the diagrams in Figure 1 is then expected to go something like $\left(m_s^2 - m_d^2\right)/m_c^2 \approx .01$, and indeed explicit calculation shows that the lowest-order diagram in Figure 1 goes like $\ell n\left(m_c^2 + m_s^2 \,/\, m_c^2 + m_d^2\right) \approx \left(m_s^2 - m_d^2\right)/m_c^2$. Hence, we find that the contribution of the diagrams in Figure 1 should be on the order of 1%.

Although the above analysis has lead us to expect no dramatic enhancement in the rates for Cabibbo-suppressed D decays, it is interesting to note that the possibility of seeing the effects of virtual b-quark loops might exist through interference effects in the ratio of $D \to KK$ to $D \to \pi\pi$ decay rates. At the tree level (and to lowest order in s_2 and s_3) the amplitude for $D \to KK$ is proportional to $-s_1c_1$, whereas the amplitude for $D \to \pi\pi$ goes like s_1c_1. It follows that any amplitude which contributes with the same sign in both decays can constructively interfere for one of these decays and destructively interfere for the other. The contributions of the $(V-A)\times(V+A)$ operators in Equation (1) have this property. If we assume that the matrix elements of the $(V-A)\times(V+A)$ operators are enhanced over those of the $(V-A)\times(V-A)$ operators and that all matrix elements are SU(3) symmetric, then neglecting operators whose matrix elements are color suppressed

$$\frac{\Gamma(D\to K^+K^-)}{\Gamma(D\to \pi^+\pi^-)} \approx \left[\frac{-1+(1+A)\left(s_3^2+\dfrac{s_2s_3c_3}{c_1c_2}\cos\delta\right)}{1+A\left(s_3^2+\dfrac{s_2s_3c_3}{c_1c_2}\cos\delta\right)}\right]^2 \tag{3}$$

where[11]

$$\begin{aligned} A &\approx -.03\ \frac{\langle KK \text{ or } \pi\pi\,|\,(V-A)\times(V+A)\,|\,D\rangle}{\langle KK \text{ or } \pi\pi\,|\,(V-A)\times(V-A)\,|\,D\rangle} \\ &\approx .06\ \frac{\langle KK \text{ or } \pi\pi\,|\,(S+P)\times(S-P)\,|\,D\rangle}{\langle KK \text{ or } \pi\pi\,|\,(V-A)\times(V-A)\,|\,D\rangle}\quad . \end{aligned} \tag{4}$$

The second form for A follows from a Fierz transformation of the $(V-A)\times(V+A)$ operators. It is the scalar-pseudoscalar structure of the resulting operators which can lead to an enhancement. In Equation (3) we have assumed $|\sin\delta| \ll 1$, so we have set $e^{i\delta}\approx\cos\delta$. If A is or order unity then sizeable interference effects can take place. For example, if we take[12] $s_3^2+\left(s_2s_3c_3\,/\,c_1c_2\right)\cos\delta = -.1$ and $A\approx 2$ (which corresponds to about the same enhancement as is supposed to take place in K decays) then $\Gamma(D\to KK)/\Gamma(L\to\pi\pi)\approx 2.6$. In view of the recent experimental result[9] $\Gamma(D\to KK)/\Gamma(D\to\pi\pi) = 3.4^{+2.8}_{-1.2}$ this might be viewed as encouraging. However, we must stress that although this large an enhancement of the matrix elements of $(V-A)\times(V+A)$ operators in D decay is not inconceivable, we view it as unlikely. In fact, we expect the enhancement of the matrix elements of the $(V-A)\times(V+A)$ operators in D decay to only be about $m_s/m_c \approx .1$ of what it is in nonleptonic K decays.

In order to see why we expect this, consider the case of nonleptonic K decays. There, the matrix elements of the $(S+P)\times(S-P)$ and $(V-A)\times(V-A)$ operators can be estimated by using current algebra to remove one pion and then approximating the remaining $\pi - K$ matrix element by inserting the vacuum in all possible ways. Relating the P operators to the A operators using the Dirac equation for the quark fields yields the ratio

$$\frac{\langle\pi\pi|\,(S+P)\times(S-P)\,|K\rangle}{\langle\pi\pi|\,(V-A)\times(V-A)\,|K\rangle} \approx \frac{f_k \dfrac{\left(m_\pi^2\, m_k^2\right)}{\left(m_s+m_u\right)\left(m_u+m_d\right)}}{f_k\left(\dfrac{m_k^2}{2}\right)} \tag{5}$$

Numerically with $m_u+m_d \approx 10$ MeV, $m_s+m_u \approx 150$ MeV, this ratio is 30. To show physically where this enhancement is coming from[13] we relate the pion and kaon masses to the current quark masses by

$$\begin{aligned} m_\pi^2 &\approx \left(m_u+m_d\right)\mu \\ m_k^2 &\approx m_s\,\mu \end{aligned} \tag{6}$$

where μ is of order 2 GeV. Inserting this into Equation (5) gives

$$\frac{\langle\pi\pi|\,(S+P)\times(S-P)\,|K\rangle}{\langle\pi\pi|\,(V-A)\times(V-A)\,|K\rangle} \approx \frac{2\mu}{m_s} \tag{7}$$

so the enhancement is coming from the fact that the current algebra strange quark mass is light on the scale of typical hadronic masses. In the case of D decays, the strange quark is replaced by a charm quark and the enhancement is expected to be roughly m_s/m_c times what it is for K decays. Thus, although some enhancement of the matrix elements of $(V-A)\times_-(V+A)$ operators is possible in D decays it is not likely to be large enough to lead to appreciable effects.[14]

NOTE ADDED: After completion of this work a preprint by M. Glück entitled "Cabibbo Suppressed Two-Body Hadronic Decays of D-Mesons" came to our attention. In this paper, the author has failed to realize that "Penguin" diagrams with s and d quark loops do not produce local four-quark operators and has ignored the GIM cancellation between these diagrams. Furthermore, his estimate of hadronic matrix elements is singular in the limit of vanishing current quark masses.

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2. E. Witten, Nucl. Phys. B120, 189 (1977).

3. F. J. Gilman and M. B. Wise, SLAC-PUB-2341 (1979).

4. M. K. Gaillard and B. W. Lee, Phys. Rev. Lett. 33, 108 (1974); G. Altarelli and L. Maiani, Phys. Lett. 52B, 351 (1974)

5. For a review, see M. K. Gaillard, Proceedings of the SLAC Summer Institute on Particle Physics 1978, edited by M. C. Zipf, SLAC Report No. 215, p 397.

6. The effective Hamiltonian is calculated using $M_W = 85$ GeV, $m_t = 15$ GeV, $m_b = 4.5$ GeV and a renormalization point mass of 1.5 GeV. The running coupling constant at these mass scales was calculated using

$$\alpha_s\left(Q^2\right) = \frac{12\pi}{33 - 2N_f} \frac{1}{\log Q^2/\Lambda^2}$$

where $\Lambda = .5$ GeV and $N_f = 6$, 5 and 4 at the mass scale of the top, bottom and charm quark masses respectively. The calculation is fairly insensitive to the value of the top quark mass.

7. This can be derived by a straightforward application of the techniques used in References 1, 2 and 3.

8. J. Ellis, M. K. Gaillard, D. V. Nanopoulos and S. Rudaz, Nucl. Phys. B131, 285 (1977); V. Barger, W. F. Long and S. Pakvasa, Phys. Rev. Lett. 42, 1585 (1979); R. E. Shrock, S. B. Treiman and Ling-Lie Wang, Phys. Rev. Lett. 42, 1589 (1979); L. Wolfenstein, Carnegie-Mellon University Preprint COO-3066 (unpublished) (1979).

9. G. S. Abrams et al., SLAC-PUB-2337 (1979) (unpublished). We thank G. Feldman for a discussion of the errors on the ratio $\Gamma(D \to K\bar{K})/\Gamma(D \to \pi\pi)$.

10. M. B. Wise and E. Witten, SLAC-PUB-2282 (1979) (unpublished).

11. We use the Fierz identity $(V-A) \times (V+A) = -2(S+P) \times (S-P)$.

12. We adopt the convention that all angles θ_j lie in the first quadrant so that their sines and cosines are positive. Then to get an enhancement of the $K\bar{K}$ mode over the $\pi\pi$ mode $\cos\delta$ must be negative. The value $-.1$ for the angular factor corresponds to the choice of angles $\theta_3 = 15^o$, $\theta_2 = 35^o$ and $\delta \approx \pi$.

13. We are grateful to L. Susskind for this argument.

14. For other discussions of the ratio $\Gamma(D \to K\bar{K})/\Gamma(D \to \pi\pi)$ see: G. Kane, SLAC-PUB-2326 (unpublished); M. Suzuki, U. C. Berkeley and LBL preprint (unpublished); V. Barger and S. Pakvasa, Preprint UH-511-341-79.

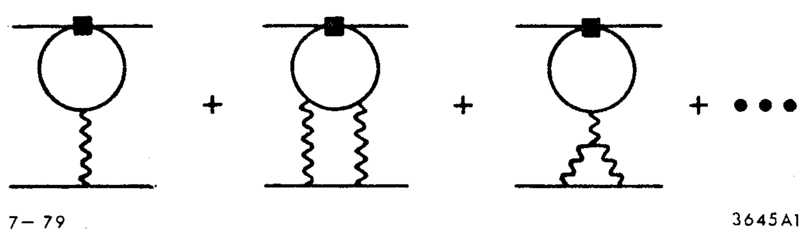

Fig. 1

Some strong interaction corrections to the hadronic matrix elements of $(V-A)\times(V-A)$ four-quark operators $\mathcal{O}^{(\pm)}$.