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THE QUARK COLOR MAGNETIC MOMENT *

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ABSTRACT

The quark color magnetic moment is examined in the framework of the interaction of a quark with a constant external color magnetic field. Quark and gluon propagators in the external field are evaluated and shown to lead to a a field dependent anomalous magnetic moment $\mu = \frac{g\hbar}{2mc} \frac{3\alpha_s}{4\pi} \ln \frac{gB}{m^2}$ where B is the external field strength. Comparison is made with the behavior of the electron anomalous magnetic moment in strong fields. Possible phenomenological consequences for hadron mass splittings are discussed.

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SECTION I

The magnetic moments of the electron and muon have been experimentally determined and theoretically predicted to great precision,[1] and the agreement of theory and experiment remains one of the most impressive successes of Quantum Electrodynamics (QED). An important feature of this situation is our ability to produce magnetic fields of known intensity and uniformity. If such magnetic fields were not available, one could still get information about magnetic moments from bound state spectroscopy since magnetic fields exist inside atoms and the interaction of spin 1/2 constituents with these fields leads to spin splittings. Indeed, investigation of atomic spectroscopy led to the introduction of the electron magnetic moment[2] and provided the first evidence of the anomalous magnetic moment.[3] However, as the fields are no longer externally controlled, but are determined by the interaction of the bound state, some understanding of the bound state problem is needed before the effect of magnetic moments can be taken into account. Thus, if one did not have the option of studying free electrons in external uniform magnetic fields, the concept of magnetic moments would be of less practical importance, having meaning only in the context of the solution of the bound state problem. It is precisely this situation that is encountered in a non-abelian gauge theory of the strong interactions, Quantum Chromodynamics (QCD).[4] This theory, which describes a color triplet of quarks interacting with an octet of vector mesons is supposed to have the property of confinement, so that free quarks and long range color fields are not observed in nature. The interaction of quarks with the vector field is based on a generalization of QED, so that in particular one can introduce the concept of the quark color magnetic moment. Due to confinement this magnetic moment cannot be directly measured with a controlled external color magnetic field, so to see its effect one must rely on the color magnetic field inside hadrons, which should produce mass differences between hadrons with differing quark spin orientations. In fact, several groups[5] have found qualitative evidence for this effect in the N-Δ, π-ρ, and other mass differences. In addition, Schnitzer[6] has investigated the phenomenological consequences of a color anomalous magnetic moment. However, unlike the case in QED, where the bound state problem is well understood,[7] a fundamental understanding of the bound state problem for hadrons does not yet exist, so that the role of the quark color magnetic moment in hadron spin splittings has still to be quantitatively understood. In this paper we address a related problem, the behavior of a quark in a constant external color magnetic field. We will show that the greater simplicity of this problem allows an exact solution to one loop. Of particular interest is the fact that the graph of Figure 1.b gives rise to an infrared singular anomalous magnetic moment.[8] As other infrared singularities in the perturbative treatment of free quarks and gluons have been shown to cancel in appropriately defined cross-sections,[9] an infinity in a static quantity would be surprising. Our main result is that the interaction energy of the quark with the external field evaluated with the mass operator method[10] is

$$\Delta E = -\frac{gB}{2m}\frac{3\alpha_s}{4\pi}\ln\frac{gB}{m^2} \qquad (1.1)$$

where g is the strong interaction coupling constant renormalized at some scale μ, B is the external field intensity, and m is the quark mass.

The relevant graph is given in Figure 3, where the quark and gluon propagators are evaluated to all orders in the external field, as indicated in Fig. 2a and 2b. This should be compared with the QED case, where the mass operator gives for the interaction energy of an electron with a magnetic field[11]

$$\Delta E = -\frac{eB}{2m}\frac{\alpha}{2\pi}\left(1 + \frac{8}{3}\left(\frac{eB}{m^2}\right)\ln\frac{2eB}{m^2} + \mathcal{O}\left(B^2\right)\right) \tag{1.2}$$

Thus in QED the mass operator can be expanded to first order in the external field, which corresponds to using the diagram in Figure 1.a. However, the QCD mass operator clearly cannot be expanded due to the presence of a logarithm in Eq. (1.1). Using the Feynman diagram of Figure 1.b. and encountering an infrared divergence is the consequence of making an improper expansion of the mass operator, so the associated infinity is a mathematical artifact rather than a physical effect. This same problem arises in QED when the mass operator is expanded to higher order in eB, as first noted by Newton,[12] but due to the weakness of attainable magnetic fields compared to the critical field of 10^{13} gauss such effects are negligible in practice.

In Section II the mass operator formalism is set up and expressions for the quark and gluon propagators derived. In Section III the mass operator is evaluated for a particular quark state and Eq. (1) proved. Section IV contains discussion of the results and some comments about possible phenomenological consequences.

SECTION II

The problem we wish to deal with is an external field problem. We assume that by some external agency a constant color magnetic field has been set up, and treat this field as classical. Fluctuations of the field about this classical value are treated with perturbation theory. We form the field from a particular choice for the vector potentials

$$A_\mu^i = B\,\delta_{i8}\begin{cases} 0 & \mu = 0,1,3 \\ x_1 & \mu = 2 \end{cases} \qquad (2.1)$$

This leads to a constant magnetic field in the 3 direction in ordinary space and the 8 direction in SU(3) space. It is also possible to form such a magnetic field from constant vector potentials with different SU(3) orientations for different space time indices as discussed by Brown and Weisberger.[13] The two cases are distinguished by the former being produced by sources at infinity (a solenoid), while the latter is produced by a nonvanishing finite current. We consider here only the first choice of potentials. By allowing only the 8 direction of the field to be present, many of the non-abelian features of the problem drop out. Firstly, due to $f_{88i} = 0$, graphs of the form of Figure 2c vanish, so that the external field does not couple to itself. In addition, a quark of definite color i (i = 1,2,3) remains that color in this magnetic field, while a non-diagonal orientation of the field in SU(3) space would lead to the quark color constantly changing. Similarly, gluons keep their color unchanged. This leads to a situation like that encountered in the evaluation of the electron magnetic moment when other charged particles are present, such as in the W boson contribution to the electron anomaly.[14] Indeed, that contribution is also afflicted with the same infrared divergence in the limit $M_W \to 0$ that afflicts the quark color magnetic moment. The only complication is that there are several charge arrangements possible. There exists a well developed procedure to handle this situation in the mass operator formalism.[10] Associated with the graph of Figure 3 we have

$$\Delta E = -ig^2 \int dx \int dx' \overline{\psi}_i(x')\gamma^\nu \left(T_a\right)_{ik} S_A^{kj}(x',x)\gamma^\mu \left(T_b\right)_{j\ell} \psi_\ell(x) D_{\mu\nu}^{ab}(x',x) \,. \quad (2.2)$$

In this expression $s_A^{kj}(x',x)$ is the amplitude for a quark color k at position x to propagate in the presence of the external field to position x' with color j, $D_{\mu\nu}^{ab}(x',x)$ is the analogous amplitude for a gluon, and the ψ's are wave functions appropriate to a quark in a constant magnetic field. Due to our choice of the external color field, the quark and gluon propagators are diagonal in color space. The quark propagator is determined by the equation

$$\left(i\gamma_\mu \frac{\partial}{\partial x_\mu}\delta_{jk} - g\gamma_\mu A_a \left(T_a\right)_{jk} - m\delta_{jk}\right) S_A^{k\ell}(x',x) = \delta_{j\ell}\delta^4(x'-x) \qquad (2.3)$$

Because $A_a^\mu T_a$ is diagonal, this matrix equation breaks into three separate equations, with the only distinction between them being the coefficient of A_μ for the particular quark color i, $g\left(T_8\right)_{ii}$. Thus, introducing a covariant derivitive

$$\frac{1}{i}\Pi_a^\mu = \frac{\partial}{\partial x_\mu} - ie_a A^\mu \,,$$

$$e_a = g\left(T_8\right)_{aa} \,, \qquad (2.4)$$

we can write the equation for the propagator of a quark color a as

$$\left(\not{\Pi}_a - m\right) S_a = \mathbb{1} \qquad (2.5)$$

Before the equation the gluon propagators satisfy can be determined, the question of gauge must be settled. Because there is an external magnetic field present, it is most convenient to use a background field gauge,[15] working with quantum fields $Q_\mu = A_\mu - A_\mu^{ext}$. The calculation is by far the simplest in the generalized Feynman gauge, characterized by a gauge fixing term

$$\mathscr{L}_{g.f.} = -\frac{1}{2}\left(\partial_\mu Q_\mu^a + g f_{abc} A_\mu^b Q_\mu^c\right)^2 \qquad (2.6)$$

In that gauge the gluon propagator satisfies

$$\left[\Pi_a^2 g_{\mu\nu} + 2ie_a F_{\mu\nu} \right] D_{\nu\alpha}^a = \mathbb{1} \, g_{\mu\alpha} \qquad (2.7$$

where we have again used the fact that T_8 is diagonal to write separate equations for each color gluon.[16] Knowing that ψ is determined through

$$\left(i\gamma_\mu \frac{\partial}{\partial x_\mu} - e_a \not{A} - m\right) \psi_a = 0 \qquad (2.8)$$

then determines all components of the mass operator. In order to obtain useful forms for the propagators along with relations useful in explicitly evaluating the mass operator, we begin by considering the propagator of a spin zero field of charge e in a constant magnetic field described by a vector potential as in Eq. (2.1). We find that

$$\left(\Pi_e^2 - m^2\right) D(x',x) = \delta^4(x' - x) \qquad (2.9)$$

is formally solved by

$$D(x',x) = -i \int_0^\infty ds \langle x' | e^{is\left(\Pi^2 - m^2 + i\varepsilon\right)} | x\rangle$$

$$\equiv -i \int_0^\infty ds\, e^{-ism^2}\, U(S,S) \quad . \tag{2.10}$$

To explicitly evaluate the last term we introduce a complete set of states that are labelled by p_0, p_3, p_2 and n, where n=0,1,2, ... , satisfying

$$\mathbb{1} = \int \frac{dp_0}{2\pi} \int \frac{dp_3}{2\pi} \int \frac{dp_2}{2\pi} \sum_{n=0}^{\infty} |p_0 p_3 p_2 n\rangle\langle p_0 p_3 p_2 n| \tag{2.11}$$

where

$$\langle p_0 p_3 p_2 n | x\rangle = \left(\frac{\sqrt{\frac{eB}{\pi}}}{2^n n!}\right)^{1/2} H_n(\xi)\, e^{-\frac{1}{2}\xi^2}\, e^{-ip_0 t}\, e^{ip_3 x_3}\, e^{ip_2 x_2} \quad ;$$

$$\xi = \sqrt{eB}\left(x_1 - \frac{p_2}{eB}\right) \quad . \tag{2.12}$$

Introducing the notation

$$\Pi_\parallel^2 = \Pi_3^2 - \Pi_0^{\,2} \quad ;$$

$$\Pi_\perp^2 = \Pi_1^2 + \Pi_2^2 \quad . \tag{2.13}$$

We are now in a position to evaluate the generalized expression

$$U\left(s_1, s_2\right) = \langle x' | e^{-is_1 \Pi_\parallel^2}\, e^{-is_2 \Pi_\perp^2} | x\rangle \quad ;$$

$$= \int \frac{dp_0}{2\pi} \int \frac{dp_3}{2\pi} \int \frac{dp_2}{2\pi} \sum_{n=0}^{\infty} \langle x' | p_o p_3 p_2 n\rangle\langle p_0 p_3 p_2 n | x\rangle$$

$$\times\, e^{-is_1\left(p_3^2 - p_0^2\right)}\, e^{-ieBs_2(2n+1)} \quad . \tag{2.14}$$

Inserting the matrix elements from Eq. (2.12) into Eq. (2.14) and performing the sum over n with the use of Mehler's formula,

$$\sum_{n=0}^{\infty} \frac{1}{n!}\left(\frac{z}{2}\right)^{n} H_{n}(x)\, H_{n}(y) = \frac{1}{\sqrt{1-z^{2}}}\; e^{\dfrac{2\,xyz - z^{2}\left(x^{2}+y^{2}\right)}{1-z^{2}}} \tag{2.15}$$

one is left with three Gaussian integrals. Their evaluation is straightforward and yields

$$U(s_1,s_2) = \frac{-ieB}{16\pi^{2} s_{1}\, \sin\left(eBs_{2}\right)}\; e^{\dfrac{ix_{\parallel}^{2}}{4s_{1}}}\; e^{\dfrac{ix_{\perp}^{2}\, eB}{4\tan\left(eBs_{2}\right)}}\; \Phi_{e}(x',x) \quad ;$$

$$x_{\parallel}^{2} = \left(x_{3}' - x_{3}\right)^{2} - \left(x_{0}' - x_{0}\right)^{2} \quad ;$$

$$x_{\perp}^{2} = \left(x_{1}' - x_{1}\right)^{2} + \left(x_{2}' - x_{2}\right)^{2} \quad ;$$

$$\Phi_{e}(x',x) = e^{\dfrac{ieB\left(x_{1}+x_{1}'\right)\left(x_{2}-x_{2}'\right)}{2}} \quad . \tag{2.16}$$

Later on we will want to use this equation in the other direction, that is start with a product of the exponentials and end with a matrix element of an operator, so we rewrite the above as

$$e^{\dfrac{ix_{\parallel}^{2}}{4w_{1}}}\; e^{\dfrac{ix_{\perp}^{2}}{4w_{2}}}\; \Phi_{e}(x',x) = \frac{16\,{}^{2}\, i\, w_{1}w_{2}}{\sqrt{\Delta}} \left\langle x' \right| e^{-\,iw_{1}\,\Pi_{\parallel}^{2}}\; e^{-i\beta\, \Pi_{\perp}^{2}} \left| x \right\rangle \quad ;$$

$$\Delta = 1 + \left(eBw_{2}\right)^{2} \quad ; \qquad \beta = \frac{1}{eB}\tan^{-1} eBw_{2} \quad . \tag{2.17}$$

We now must determine the quark and gluon propagators in terms of the spin zero propagator. This is particularly simple for the gluon propagator, as the formal solution of Eq. (2.7) is

$$D_{\mu\nu}(x',x) = -i \int_0^\infty ds \left[e^{-2esF} \right]_{\mu\nu} \langle x' | \; e^{is\Pi^2} | x \rangle \quad (2.18)$$

where in the last step we have used the fact that the color field is constant,

$$\left[\Pi_\alpha , \; F_{\mu\nu} \right] = 0 \quad (2.19)$$

Thus outside of an x independent factor, the gluon propagator is proportional to the scalar propagator. The quark case is more complicated, due to the Dirac structure. To solve Eq. (2.5) we rewrite it formally as

$$\begin{aligned} S_A(x',x) &= \langle x' | \left(\not{\Pi} + m \right) \frac{1}{\not{\Pi}\not{\Pi} - m^2} | x \rangle \quad ; \\ &= -i \int_0^\infty ds \; e^{-ism^2} \langle x' | (\not{\Pi} + m) \, e^{is\not{\Pi}\not{\Pi}} | x \rangle \; . \end{aligned} \quad (2.20)$$

Using

$$\begin{aligned} \not{\Pi}\not{\Pi} &= \Pi^2 + \frac{e}{2} \sigma_{\mu\nu} F^{\mu\nu} \quad ; \\ &\equiv \Pi^2 + M \end{aligned} \quad (2.21)$$

This becomes

$$S_A(x',x) = -i \int_0^\infty ds \; e^{-ism^2} \langle x' | \left(\not{\Pi} + m \right) e^{is\Pi^2} | x \rangle \; e^{isM} \quad (2.22)$$

Introducing the explicit form for Π allows us to write, after some manipulations,

$$S_A(x',x) = -\frac{1}{16\pi^2}\int_0^\infty \frac{ds}{s^2}\,\frac{eBs}{\sin(eBs)}\, e^{-ism^2}\left[m-\frac{1}{2s}\right.$$
$$\left.\times\left(\gamma\cdot x_\parallel + \frac{eBs}{\sin(eBs)}\, e^{isM}\,\gamma\cdot x_\perp\right)\right] e^{\frac{ix_\parallel^2}{4s}}\, e^{\frac{i\,eB\,x_\perp^2}{4\tan(eBs)}}$$
$$\times\ \Phi_e(x',x)\ e^{isM} \qquad (2.23)$$

which in the inverse form becomes

$$\gamma\cdot x_\parallel\ e^{\frac{ix_\parallel^2}{4w_1}}\, e^{\frac{ix_\perp^2}{4w_2}}\,\Phi_e(x',x) = \frac{16\pi^2 iw_1w_2}{\sqrt{\Delta}}\ 2w_1\langle x'\left|\gamma\cdot\Pi_\parallel\, e^{-w_1\Pi_\parallel^2}\, e^{-i\beta\,\Pi_\perp^2}\right|x\rangle$$

$$\gamma\cdot x_\perp\ e^{\frac{ix_\parallel^2}{4w_1}}\, e^{\frac{ix_\perp^2}{4w_2}}\,\Phi_e(x',x) = \frac{16\pi^2 iw_1w_2}{\sqrt{\Delta}}\ \frac{2w_2\, e^{-i\beta M}}{\sqrt{\Delta}}$$
$$\times\langle x'\left|\gamma\cdot\Pi_\perp\, e^{-iw_1\Pi_\parallel^2}\, e^{-i\beta\Pi_\perp^2}\right|x\rangle \qquad (2.24)$$

SECTION III

With these expressions we are now ready to evaluate the mass operator. Defining the charges to be e1 for the initial quark, e2 for the intermediate quark, and e3 for the gluon, we have, introducing an integration for each propagator,

$$\Delta E = \frac{ig^2}{(4\pi)^4}\int dx \int dx' \int_0^\infty \frac{ds}{s^2}\int_0^\infty \frac{dt}{t^2}\; e^{-itm^2}\; \frac{z}{\sin z}\; e^{\frac{ix_\parallel^2}{4}\left(\frac{1}{s}+\frac{1}{t}\right)}$$

$$e^{\frac{ix_\perp^2}{4}\left(\frac{1}{t}+\frac{z}{s\tan z}\right)}\;\bar{\psi}(x')\gamma_\mu\left[m - \frac{\gamma\cdot x_\parallel + \gamma\cdot x_\perp}{2t}\right] e^{itM}\,\gamma_\nu \psi(x)\,\Phi_{e_2}(x',x)$$

$$\Phi_{e_3}(x',x)\left[e^{-2e_3 sF}\right]_{\mu\nu} \qquad ;$$

$$z = e_3 Bs.$$

The exact gluon propagator has been kept, but the quark propagator has been approximated by a form in which only the Φ_{e_2} and the ext(itM) terms are kept as modifications to the free propagator, which is valid to first order in the external field. Then, using Eq. (2.17) and Eq. (2.24) we can write this, defining $\psi(x) = \langle n|x\rangle$, as

$$\Delta E = \frac{\alpha_s}{4\pi}\int_0^\infty \frac{ds}{s^2}\int_0^\infty \frac{dt}{t^2}\int dx \int dx'\; e^{-itm^2}\;\frac{z}{\sin z}\;\frac{ww_2}{\sqrt{\Delta}}\;\langle n|x'\rangle\,\langle x'|\gamma_\mu$$

$$\left(m - \frac{w}{t}\gamma\cdot\Pi_\parallel - \frac{w_2}{t\sqrt{\Delta}}\,e^{-i\beta M}\gamma\cdot\Pi_\perp\right) e^{-itM}\gamma_\nu\, e^{-iw\Pi_\parallel^2}\, e^{-i\beta\,\Pi_\perp^2}$$

$$|x\rangle\langle x|n\rangle\left[e^{-2e_3 sF}\right]_{\mu\nu}$$

$$\frac{1}{w} = \frac{1}{s}+\frac{1}{t} \quad ; \qquad \frac{1}{w_2} = \frac{1}{t}+\frac{z}{s\tan z} \tag{3.2}$$

But now completeness allows us to perform the x and x' integrations trivially, and we are left with the expression

$$\Delta E = \frac{\alpha_s}{4\pi} \int_0^\infty \frac{ds}{s^2} \int_0^\infty \frac{dt}{t^2} e^{-itm^2} \frac{z}{\sin z} \frac{ww_2}{\sqrt{\Delta}} \langle n \left| \gamma_\mu \left(m - \frac{w}{t} (\gamma \cdot \Pi)_\parallel - \frac{w_2}{t\sqrt{\Delta}} e^{-i\beta M} (\gamma \cdot \Pi)_\perp \right) e^{itM} \gamma_\nu e^{-iw \Pi_\parallel^2} e^{-i\beta \Pi_\perp^2} \right| n \rangle \left[e^{-2e_3 sF} \right]_{\mu\nu} \tag{3.3}$$

It is important to note that the Π in this expression refers to a charge e1 particle, so that it can be applied onto the states n . This follows from the Φ's in Eq. (3.1) combining into Φ_{e_1} due to $e_1 = e_2 + e_3$. The mass operator is particularly simple when evaluated in the ground state of the system, where n=0 and the spin is parallel to the magnetic field, for then

$$\begin{aligned}
\Pi_\parallel^2 \, |n\rangle &= -m^2 \, |n\rangle \\
\Pi_\perp^2 \, |n\rangle &= M \, |n\rangle = e_1 B \, |n\rangle \\
\gamma \cdot \Pi_\parallel \, |n\rangle &= -m \, |n\rangle \\
\gamma \cdot \Pi_\perp \, |n\rangle &= 0
\end{aligned} \tag{3.4}$$

In that case we end up with the expression, after transforming $s \to -is$,

$$\Delta E = \frac{\alpha m}{2\pi} \int_0^\infty \frac{ds}{s^2} \int_0^\infty \frac{dt}{t^2} e^{-m^2(t-w)} \frac{z}{\sinh z} \frac{w_2 w}{\sqrt{\Delta}} e^{e_3 B\beta} \left(1 + e^{-2e_3 Bs} \left(1 - \frac{w}{t} \right) \right) \tag{3.5}$$

where we have dropped a term corresponding to Fig. 1.a. For $B = 0$, this gives the usual ultraviolet divergent self energy correction. A straightforward expansion in B, using the fact that

$$\frac{z}{\sinh z} = 1 + \mathcal{O}\left(B^2\right)$$

$$\Delta = 1 + \mathcal{O}\left(B^2\right)$$

$$\beta = w + \mathcal{O}\left(B^2\right) \tag{3.6}$$

then yields the expression

$$\Delta E = \frac{-e_3 B\alpha m}{2\pi} \int_0^\infty ds \int_0^1 u\,du\; e^{-m^2 s \frac{(1-u)^2}{u}} (1-u) \quad ;$$

$$u \equiv \frac{s}{s+t} \quad . \tag{3.7}$$

Performing the s integration exposes a logarithmic divergence at $u = 1$, which is the same divergence as encountered in the Feynman diagram evaluation. In the region $u \sim 1$, we are able to set

$$\frac{w_2}{\sqrt{\Delta}} = w \quad ;$$

$$\beta = w \quad . \tag{3.8}$$

However, due to the divergence encountered above, although the term $z/\sinh(z)$ is formally of $\mathcal{O}\left(B^2\right)$, we must leave it in.

Then Eq. (3.7) is replaced by

$$\Delta E = \frac{-e_3 B\alpha m}{2\pi} \int_0^\infty ds \frac{z}{\sinh z} \int_0^1 udu\, e^{-m^2 s \frac{(1-u)^2}{u}} (1-u) \tag{3.9}$$

At high s, 1/sinh (z) behaves as exp(-eBs), and has the effect of softening the $u \to 1$ behavior of the integrand. Explicit integrations gives, up to constants irrelevant to our discussion

$$\Delta E = \frac{-e_3 B\alpha}{2\pi m} \ell n \frac{gB}{m^2} \quad . \tag{3.10}$$

Summing over all charge configurations and pulling out the matrix element of T_8 then leads to the result in (1.1)

SECTION IV

The result (3.10) has a counterpart in the very high field correction to the electron mass[11]

$$\Delta E = \frac{\alpha}{4\pi} m \left(\ell n \frac{2eB}{m^2} \right)^2 \tag{4.1}$$

This illustrates the fact that the strong field limit of QED, which requires $eB \gg m^2$, is always obtained in QCD, due to the relevant limit being $gB \gg \lambda^2$, which is always satisfied for gluons of mass $\lambda = 0$. Similar behavior is encountered in the effective Lagrangians of the two theories in intense constant fields[17]

$$\mathscr{L}_{QED} = -\frac{1}{2} B^2 + \frac{(eB)^2}{24\pi^2} \ell n \frac{eB}{m^2} \tag{4.2a}$$

$$\mathscr{L}_{QCD} = -\frac{1}{2} B^2 - \frac{11(gB)^2}{48\pi^2} \ell n \frac{gB}{\mu^2} \tag{4.2b}$$

Due to the weakness of practicably attainable magnetic fields in QED, however, the typical features of the strong field limit are not observable. In QCD, on the other hand, this behavior is seen in the lowest order of perturbation theory and for any field strength. The ideal place to see this effect is in the spin splittings of hadrons. However, it must be recognized that, outside of the fact that the color magnetic field inside hadrons is certainly not constant, there is also another scale in the problem that is set by confinement, namely the radius of the hadron. This finite radius will act by itself to cut off the infinity in color magnetic moment, so two cutoff effects exist in hadrons. The confinement scale is of the order of an inverse Fermi, 200 MeV. As spin splittings, which characterize the magnetic field present, are also of this order, both effects will in general have to be taken into account. However, the cutoff coming from the magnetic field has the unusual feature of field dependence. If one replaces the constant field in (2.1) with a dipole field, we see that the anomaly depends on the separation of the quarks, a feature that has been discussed by Schnitzer.[11] The ideal testing ground for these ideas is the behavior of very massive quark anti-quark systems, where one may argue that a Coulomb potential determines the properties of at least the lowest states. Then a calculation of radiative corrections to hyperfine splittings could be carried out using standard QED techniques, with the effect of the moment being position dependent giving rise to deviations from a positronium-like spectrum. This question is presently under investigation.

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Figure Captions

1.a. Diagram leading to a finite contribution to the quark color anomalous magnetic moment.

1.b. Diagram leading to an infrared singularity in the quark color anomalous magnetic moment.

2.a. Graphical expansion of the quark propagator in the external field.

2.b. Graphical expansion of the gluon propagator in the external field.

2.c. Example of a graph not contributing due to color orientation of the external field.

3. The QCD mass operator.

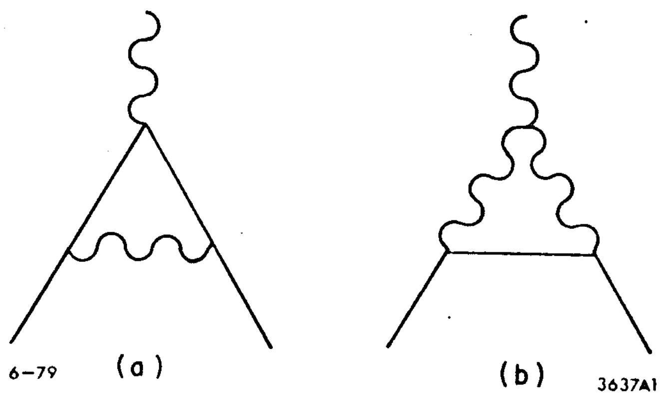

Fig. 1

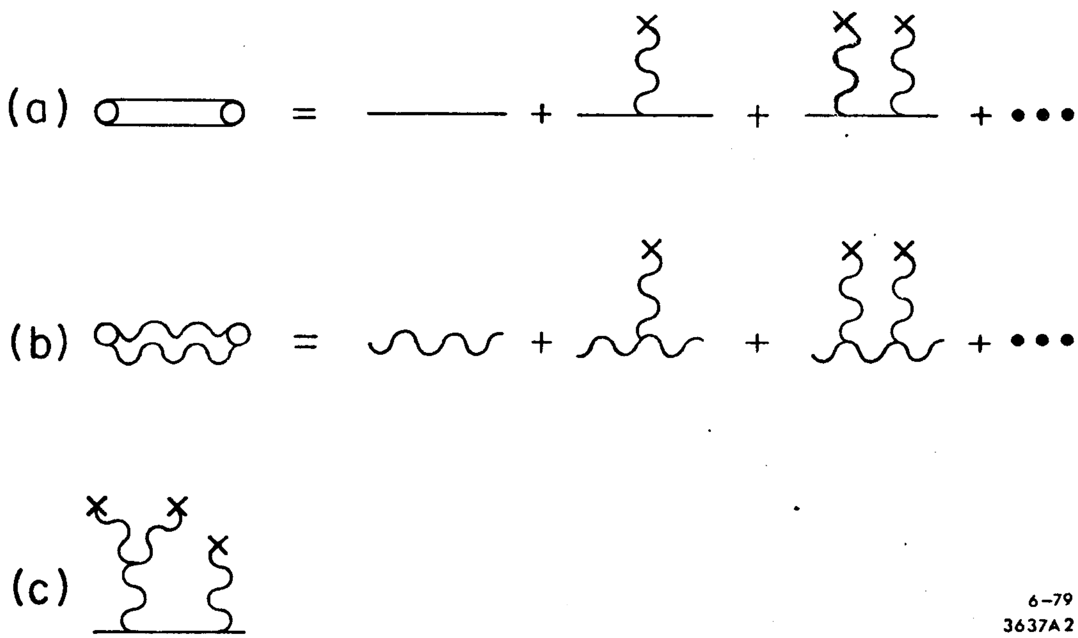

Fig. 2

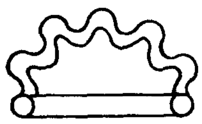

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Fig. 3