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Erratum and Addendum

EXCLUSIVE PROCESSES IN QUANTUM CHROMODYNAMICS:
THE FORM FACTORS OF BARYONS AT LARGE MOMENTUM TRANSFER*

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The expression for T_1 in Eq. (6) is missing one term. The correct result is

$$T_1 = T_3(1\leftrightarrow 3) = \frac{1}{x_2x_3(1-x_3)}\,\frac{1}{y_2y_3(1-y_1)} - \frac{1}{x_3(1-x_1)^2}\,\frac{1}{y_3(1-y_1)^2} - \frac{1}{x_2(1-x_1)^2 y_2(1-y_1)^2}\ ;$$

the lowest anomalous dimension term in Eq. (7) and (8) is then $(-e_{-||})$ not $(e_{||}-e_{-||})$. This correction only makes minor modifications in the prediction for $G_M{}^p(Q^2)$ for typical initial wave function conditions. Figure 2' illustrates the predictions for $Q^4G_M{}^p(Q^2)$ assuming an initial wavefunction condition $\phi(x_i,\lambda) \propto \delta(x_1-1/3)\delta(x_2-1/3)$ with $\lambda^2 = 2\ \mathrm{GeV}^2$ and various QCD scale parameters $\Lambda^2 = 1$, 0.1, 0.01, and 0.001 GeV^2. The ratio $G_M{}^p(Q^2)/G_M{}^n(Q^2)$ is a sensitive measure of the nucleon wave function. For the initial condition $\phi(x_i,\lambda) \propto \delta(x_1-1/3)\delta(x_2-1/3)$, the ratio $-G_M{}^p(Q^2)/G_M{}^n(Q^2) \cong 1$ at $Q^2 = \lambda^2$, and decreases asymptotically to zero as

$$(\log Q^2/\Lambda^2)^{\gamma_0-\gamma_3} = (\log Q^2/\Lambda^2)^{-32/9\beta}.$$

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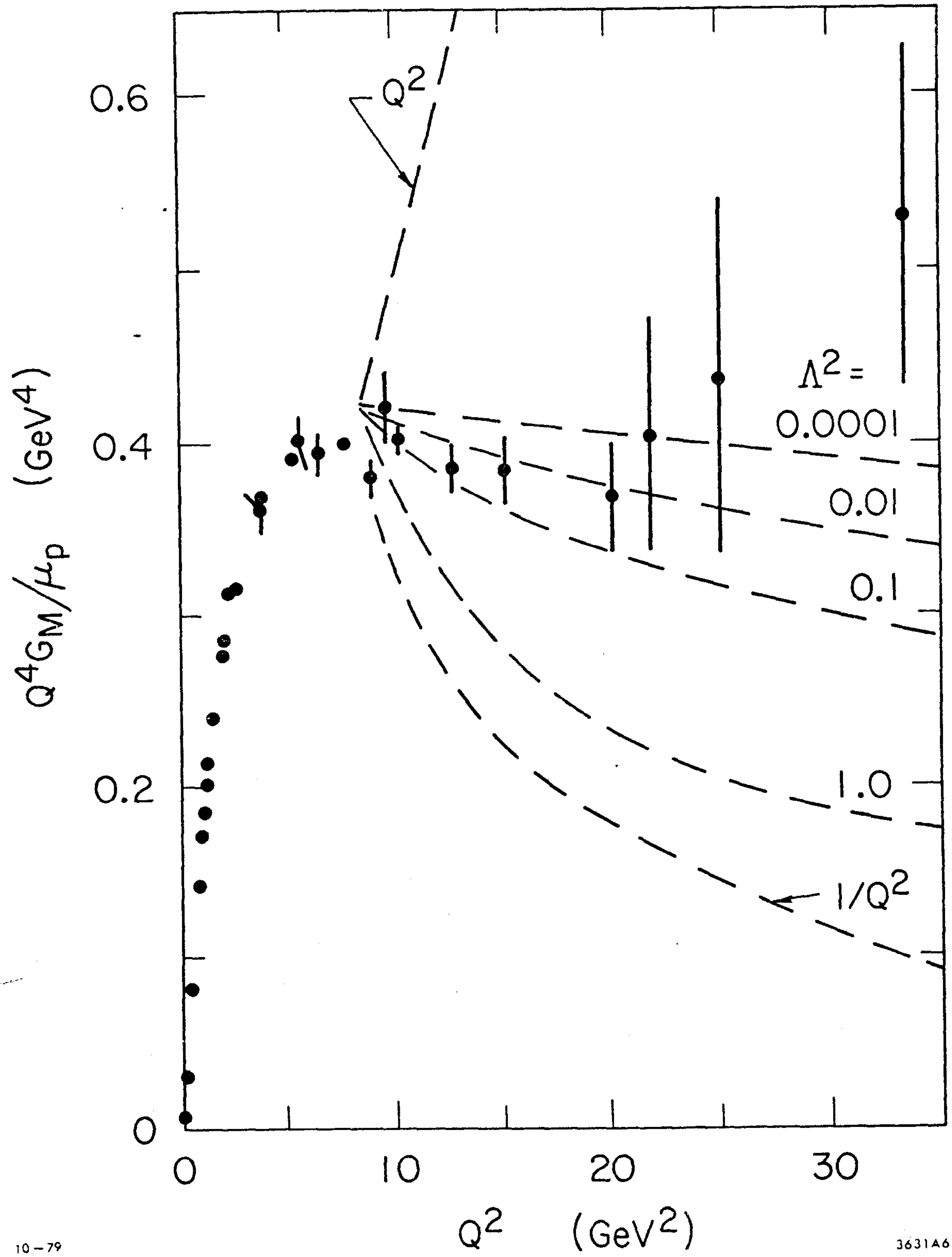

Fig. 2. Prediction for $Q^4G_M^p(Q^2)$ for various QCD scale parameters Λ^2 (in GeV^2). The data are from Ref. 8. The initial wavefunction is taken as $\phi(x,\lambda) \propto \delta(x_1 - 1/3)\delta(x_2 - 1/3)$ at $\lambda^2 = 2\ GeV^2$. The factor $(1 + m_\rho^2/Q^2)^{-2}$ is included in the prediction as a representative of mass effects.

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EXCLUSIVE PROCESSES IN QUANTUM CHROMODYNAMICS: THE FORM FACTORS OF BARYONS AT LARGE MOMENTUM TRANSFER*

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ABSTRACT

The form factors of baryons at large momentum transfer are computed in quantum chromodynamics to leading order in $\alpha_s(Q^2)$ and m^2/Q^2. Form factors for processes in which the baryon helicity is changed or in which the initial or final baryon has helicity greater than one are suppressed by factors of m/Q. We also give QCD predictions for general exclusive scattering processes at large momentum transfer.

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In this letter we present a new analysis of exclusive processes involving baryons produced at large transverse momentum. This analysis is an extension of our earlier work on meson form factors in quantum chromodynamics (QCD).[1,2] Here we will describe QCD predictions for the electromagnetic form factors of baryons, for ratios of form factors, and for transition form factors (e.g., $\gamma^* p \to \Delta$), all at large Q^2. We will also outline the analysis of other large momentum transfer exclusive processes in QCD.

The analysis of baryon form factors in QCD is in essence identical to that for mesons.[1] Leading terms (in $1/Q^2$) involve only the three-quark component of the baryon's wave function (in light-cone gauge, $A^+ = 0$). When the leading logarithms in each order of perturbation theory (i.e., $(\alpha_s \log Q^2)^n$) are summed, the form factor has the form ($-q^2 \equiv Q^2$):

$$F_B(Q^2) = \int_0^1 [dx_i] \int_0^1 [dy_i]\; \phi^\dagger(x_i,Q)\; T_B(x_i,y_i,Q)\; \phi(y_i,Q) \quad . \qquad (1)$$

Here $\left[C_B = (n_{color} + 1)/2n_{color} = 2/3, \quad \alpha_s = 4\pi/\beta \log Q^2/\Lambda^2, \quad \beta = 11 - (2/3)n_{flavor}\right]$

$$T_B = \left(\frac{C_B\ \alpha_s(Q^2)}{Q^2}\right)^2 f(x_i\,,\,y_i)$$

is the minimally-connected amplitude for $\gamma^* 3q \to 3q$ (Fig. 1(a)),[3] and the symbol for symmetric integration over the constituents' longitudinal momenta $\left(x_i \equiv (k^o + k^3)_i/(p_B^o + p_B^3);\ \sum_{i=1}^{3} x_i = 1\right)$ is

$$[dx_i] \equiv dx_1\ dx_2\ dx_3\ \delta\left(1 - \sum_i x_i\right) \quad .$$

The effective wave function $\phi(x_i,Q)$ is the three-body qqq Fock state wave function integrated over transverse momenta $|k_\perp^{(i)}|^2 < Q^2$ $[C_F = (n_c^2-1)/2n_c = 4/3]$:

$$\phi(x_i,Q) = \left(\log\frac{Q^2}{\Lambda^2}\right)^{-(3/2)C_F/\beta}\int_0^Q \prod_{i=1}^{3}\left(\frac{d^2k_\perp^{(i)}}{16\pi^3}\right)16\pi^3\,\delta^2\Big(\sum_i k_\perp^{(i)}\Big)\psi(x_i,k_\perp^{(i)})$$

$$\equiv x_1\,x_2\,x_3\,\tilde{\phi}(x_i,Q) \qquad (2)$$

Only baryon states with $L_z = 0$ contribute to the leading power. The factor $(\log Q^2)^{-(3/2)C_F/\beta}$ is due to vertex and fermion self-energy corrections in T_B which are more conveniently associated with ϕ rather than T_B. As in the meson case, the leading behavior of ϕ for large Q^2 is determined (in $A^+ = 0$ gauge) by planar ladder diagrams with the transverse momenta in successive loops strongly ordered $\lambda^2 << (k_\perp^1)^2 << (k_\perp^2)^2 << \ldots << Q^2$. Three- and four-gluon couplings play no role in this order (other than in standard vertex renormalization) since they destroy the strong ordering. Consequently, defining $\xi = \log\log Q^2/\Lambda^2$, we can derive an evolution equation for $\tilde{\phi}(x_i,Q)$ (relating it to $\tilde{\phi}(x_i,\lambda)$ for some $\lambda < Q$):

$$x_1x_2x_3\left\{\frac{\partial}{\partial\xi}\tilde{\phi}(x_i,Q) + \frac{3}{2}\frac{C_F}{\beta}\tilde{\phi}(x_i,Q)\right\} = \int_0^1 [dy_i]\;V(x_i,y_i)\;\tilde{\phi}(y_i,Q) \qquad (3)$$

where

$$V(x_i,y_i) = x_1x_2x_3\,\frac{2C_B}{\beta}\sum_{i\neq j}\theta(x_i-y_i)\,\delta(x_k-y_k)\,\frac{y_i}{x_i}\left(\frac{\delta_{h_i\bar{h}_j}}{x_i+x_j} - \frac{\Delta}{y_i-x_i}\right)$$

$$= V(y_i\,,\,x_i) \qquad\qquad (\Delta\tilde{\phi} \equiv \tilde{\phi}(y_i,Q) - \tilde{\phi}(x_i,Q))$$

is the interaction between each pair of quarks due to exchange of a single gluon (Fig. 1(b)). The Kronecker delta $\delta_{h_i \bar{h}_j}$ is 1(0) when quark helicities are anti-parallel (parallel). As in the meson case, the infrared singularity at $y_i = x_i$ is cancelled because the baryon is a color singlet. (In detail, the cancellation is due to self energy corrections on the external quark legs.)

Any solution of the evolution equation can be expressed in terms of the eigenfunctions of V

$$\phi(x_i,Q) = x_1 x_2 x_3 \sum_{n=0}^{\infty} a_n \tilde{\phi}_n(x_i)\, e^{-\gamma_n \xi}$$

$$\left(\frac{3}{2}\frac{C_F}{\beta} - \gamma_n\right) x_1 x_2 x_3\, \tilde{\phi}_n = V\, \tilde{\phi}_n \qquad (4)$$

The coefficients a_n may be determined from the soft wavefunction:[4]

$$a_n \left(\log \frac{\lambda^2}{\Lambda^2}\right)^{-\gamma_n} = \int_0^1 [dx_i]\, \tilde{\phi}_n(x_i)\, \phi(x_i,\lambda)$$

The leading eigenvalues γ_n and eigenfunctions $\tilde{\phi}_n(x_i)$ for helicity 1/2 and 3/2 baryons are given in Table I. (See Ref. 2 for further details.) In practical applications it is usually simpler to integrate the evolution numerically (beginning with $\phi(x_i,\lambda)$ at $\xi = \log\log \lambda^2/\Lambda^2$) as opposed to using expansion (4). However, from Eq. (4) and Table I, we can find the asymptotic wave function for very large Q^2:

$$\phi(x_i,Q) \to C\, x_1 x_2 x_3 \begin{cases} (\log Q^2/\Lambda^2)^{-2/3\beta} & |h| = 1/2 \\ (\log Q^2/\Lambda^2)^{-2/\beta} & |h| = 3/2 \end{cases} \qquad (5)$$

where C is determined by the qqq wave function at the origin, and h is the total helicity. Since asymptotically ϕ is symmetric under interchange of the x_i's, Fermi statistics demands that the corresponding flavor-helicity wave functions must be completely symmetric under particle exchange -- i.e., identical to those assumed in the symmetric SU(6) quark model.[6]

The magnetic form factor $G_M(Q^2)$ for nucleons is given by Eq. (1), where T_B is computed from the sum of all minimally connected diagrams for $\gamma^* 3q \to 3q$ (see Fig. 1(a)). We find $(h_1 = h_3 = -h_2 = h)$ [7]

$$T_B = 64\pi^2 \left(\frac{C_B\, \alpha_s(Q^2)}{Q^2} \right)^2 \left\{ \sum_{j=1}^{3} e_j\, T_j(x_i, y_i) + (x_i \leftrightarrow y_i) \right\} \tag{6}$$

where

$$T_1 = T_3(1 \leftrightarrow 3) = \frac{1}{x_2 x_3 (1-x_3)} \frac{1}{y_2 y_3 (1-y_1)} - \frac{1}{x_3(1-x_1)^2} \frac{1}{y_3 (1-y_1)^2} - \frac{1}{x_2 (1-x_1)^2\, y_2 (1-y_1)^2} \quad ;$$

$$T_2 = -\frac{1}{x_1 x_3 (1-x_1)} \frac{1}{y_1 y_3 (1-y_3)}$$

and e_j is the electromagnetic charge (in units of e) of particle j. Convoluting with wave function (4), we obtain the QCD prediction for the large Q^2 behavior of G_M:

$$G_M(Q^2) = \frac{32\pi^2}{9} \frac{\alpha_s^2(Q^2)}{Q^4} \sum_{n,m} b_{n,m} \left(\log \frac{Q^2}{\Lambda^2} \right)^{-\gamma_n - \gamma_m} \left[1 + O(\alpha_s(Q^2), m/Q) \right] \tag{7}$$

For very large Q^2, the $n = m = 0$ term dominates and we find

$$G_M(Q^2) \to \frac{32\pi^2}{9} C^2 \frac{\alpha_s^2(Q^2)}{Q^4} \left(\log \frac{Q^2}{\Lambda^2} \right)^{-4/3\beta} (-e_{-\parallel}) \tag{8}$$

where $e_\parallel$ $(e_{-\parallel})$ is the mean total charge of quarks with helicity parallel (anti-parallel) to the nucleon's helicity (in the fully symmetric flavor-helicity wave function). For protons and neutrons we have

$$e^p_{\parallel} = 1 \qquad e^p_{-\parallel} = 0 \qquad e^n_{\parallel} = -e^n_{-\parallel} = -1/3 \tag{9}$$

The constants C are generally unknown for baryons; however, by isospin symmetry $C_p = C_n$ and thus QCD predicts the ratio of form factors as $Q^2 \to \infty$. The ratio $G_M^p(Q^2)/G_M^n(Q^2)$ is a sensitive measure of the nucleon wave function. For the initial condition $\phi(x_i,\lambda) \propto \delta(x_1 - 1/3)\delta(x_2 - 1/3)$, the ratio $-G_M^p(Q^2)/G_M^n(Q^2) \cong 1$ at $Q^2 = \lambda^2$, and decreases asymptotically to zero as

$$(\log Q^2/\Lambda^2)^{\gamma_0-\gamma_3} = (\log Q^2/\Lambda^2)^{-32/9\beta} \quad . \tag{10}$$

Both the sign and magnitude of the ratio are non-trivial consequences of QCD; they depend upon the detailed behavior of T_B and $\phi(x_i,Q)$ as $Q^2 \to \infty$. For comparison, note that in a theory with scalar or pseudo-scalar gluons, diagrams in which the struck quark has anti-parallel helicity vanish. Thus scalar QCD predicts a ratio $G_M^n/G_M^p \to e^n_{\parallel}/e^p_{\parallel} = -1/3$.

The predictions for $G_M(Q^2)$ in the subasymptotic domain depend on the n,m $\neq$ 0 terms in Eqs. (4) and (7). Figure 2 illustrates the predictions for $Q^4G_M^p(Q^2)$ assuming an initial wave function condition $\phi(x_i,\lambda) \propto \delta(x_1 - 1/3)\delta(x_2 - 1/3)$ with $\lambda^2 = 2$ GeV^2 and various QCD scale parameters Λ^2 = 1, 0.1, 0.01, and 0.001 GeV^2. Due primarily to the factors of α_s in Eq. (6), the theoretical curves fall faster than the data[8] -- though not as fast as a full power of $1/Q^2$. Non-leading terms could well be important for $Q^2 \lesssim 25$ GeV^2. These corrections can and in fact must be computed before a definitive comparison with the data is made.[9]

As is the case for mesons (see Footnote 6 of Ref. 1), form factors for processes in which the baryon's helicity is changed ($\Delta h \neq 0$), or in which the initial or final baryon has $h > 1$, are suppressed by factors of m/Q, where m is an effective quark mass. (Crossing and the $\Delta h = 0$ rule imply that form factors for particles with opposite helicity dominate for q^2 timelike.) Thus the helicity-flip nucleon form factor is predicted to fall roughly as $F_2 \sim mM/Q^6$, and the elastic ep and en cross sections become $(-t = Q^2 \to \infty)$

$$\frac{d\sigma}{dt} \to \frac{4\pi\alpha^2}{t^2}\left[\frac{s^2+u^2}{2s^2}\right] G_M^2(-t) \tag{11}$$

Cross sections for transitions such as $ep \to e\Delta$ ($|h_\Delta| = 1/2$) are also given by Eq. (11) (with G_M as in (7) and (8) but with $C^2 \to C_p C_\Delta$ in the latter); quark charges are still those given in (9). Cross sections with $|h_\Delta| = 3/2$ are suppressed (by m^2/t). The reaction $e^+e^- \to \Delta^+\Delta^-$ is dominated by baryons with $|h_\Delta| = 1/2$; the cross section for production of $|h_\Delta| = 3/2$ pairs or deltas with $|h_\Delta| = 3/2$ and $1/2$ is suppressed. Most of these predictions test the vector nature of the gluon. For example, transitions $ep \to e\Delta$ ($|h_\Delta| = 3/2$) are not suppressed in scalar QCD.

The techniques outlined above for studying asymptotic form factors can clearly be extended to the computation of any exclusive process involving large transverse momentum exchange between color singlets. Thus the fixed angle amplitude for a process $AB \to CD$ is (to leading order in $\alpha_s(p_\perp^2)$)

$$\mathcal{M}_{AB\to CD} = \int_0^1 \prod_{i=A,B,C,D} [dx_i]\ \phi_C^\dagger(x_c,p_\perp)\phi_D^\dagger(x_d,p_\perp)T_H(x_i,p_\perp^2)\phi_A(x_a,p_\perp)\phi_B(x_b,p_\perp)$$

$$p_\perp^2 = \frac{tu}{s} \qquad s \to \infty \qquad (12)$$

where the momentum transfer between constituents occurs through a single hard scattering amplitude T_H (with all internal legs off-shell by $\sim p_\perp^2$). The wave functions ϕ_A , ϕ_B , ... are just those described above and in Ref. 1. The amplitude T_H falls as $(1/p_\perp)^{n-4}$ where n is the total number of constituents, in agreement with dimensional counting rules.[3] For $p_\perp$ sufficiently large, the wave functions tend to their asymptotic form (Eq. (5) for baryons) and the cross section becomes:

$$\frac{d\sigma}{dt}\,(AB\to CD) \to \left(\frac{\alpha_s(p_\perp^2)}{p_\perp^2}\right)^{n-2} \left(\log\frac{p_\perp^2}{\Lambda^2}\right)^{-2\sum_i \gamma_i} f(\theta_{cm}) \qquad (13)$$

where for mesons $\gamma_i = 0,\ -4/3\beta$ (for $|h| = 0,1$) and for baryons $\gamma_i = -2/3\beta$, $-2/\beta$ (for $|h| = 1/2,\ 3/2$). The normalization is, in principle, fixed by form factor data. Contributions due to the pinch singularities discussed by Landshoff[10] are suppressed by Sudakov form factors.[11] Consequently, these contributions fall faster than any power of t and can be neglected relative to (13) except possibly when $s >> |t|$.

It should be emphasized that the specific integral power Q^{-4} predicted for G_M in Eq. (7) reflects both the scale invariance of the internal quark-quark interactions, and the fact that the minimal spin 1/2 color singlet wave function contains 3 quarks. Thus both the dynamics and symmetry properties of QCD are directly tested. Furthermore, the spin dependence of quark-quark interactions can be tested at short distances by studying the helicity dependence of elastic and transition form factors. We also note that it should be possible to relate the normalization and structure of the wave function $\phi(x,\lambda)$ at large distances to wave functions used in the study of baryon spectroscopy.

ACKNOWLEDGEMENTS

We would like to thank Y. Frishman, H. Lipkin, and C. Sachrajda for helpful discussions.

FOOTNOTES AND REFERENCES

1. G. P. Lepage and S. J. Brodsky, SLAC-PUB-2343 (June, 1979). This material is also discussed in S. J. Brodsky and G. P. Lepage, talk presented at the Workshop on Current Topics on High Energy Physics, Cal Tech (February, 1979), SLAC-PUB-2294.

2. G. P. Lepage and S. J. Brodsky, in preparation.

3. The power law fall-off of T_B is consistent with dimensional counting rules. S. J. Brodsky and G. R. Farrar, Phys. Rev. Lett. 31, 1153 (1973); Phys. Rev. D11, 1309 (1975); V. A. Matveev, R. M. Muradyan and A. V. Tavkheldzie, Lett. Nuovo Cimento 7, 719 (1973). See also S. J. Brodsky and B. T. Chertok, Phys. Rev. D14, 3003 (1976); Phys. Rev. Lett. 37, 269 (1976), and A. I. Vainshtain and V. I. Zakharov, Phys. Lett. 72B, 368 (1978).

4. Since V is symmetric under the interchange $x \leftrightarrow y$, the eigenvalues γ_n are real and the eigenfunctions are orthogonal with respect to weight $x_1x_2x_3$. Convergence of the expansion (4) is assured by the boundary conditions satisfied by bound state wave functions describing composite particles (see Ref. 1). These conditions also insure that F_B is dominated by short distance phenomena and consequently that the "leading log" approximation is justified for large Q^2.

5. The anomalous dimensions γ_n in this table have been verified by M. Peskin using a different method (private communication).

6. Note that we are not assuming SU(6) symmetry. The fact that the coordinate space wave functions becomes symmetric at short distances is a dynamical consequence of the theory.

7. T_B is Lorentz- and gauge-invariant. It is most easily computed in the Breit frame ($\vec{p}\,' = -\vec{p}$). This method is used by E. M. Levin, Yu. M. Shabelsky, V. M. Shekter and A. N. Solomin, Leningrad preprint 444, October 1978.

8. M. D. Mestayer, SLAC-Report No. 214 (1978), and references therein.

9. As is well understood, higher order corrections can to a large extent be absorbed into a redefinition of Λ^2. Choosing $\Lambda^2 = 0.00015$ GeV^2 in Eq. (7) reproduces the solid curve in Fig. 2(b).

10. P. V. Landshoff, Phys. Rev. D10, 1024 (1974).

11. G. P. Lepage, S. J. Brodsky, Y. Frishman and C. Sachrajda, in preparation. See also the discussion in S. J. Brodsky and G. P. Lepage (Ref. 1), and references therein.

Table I. Eigensolutions of the evolution Eq. (3) for $|h| = 1/2$ ($\tilde{\phi}^{\uparrow\downarrow\uparrow}$) and $|h| = 3/2$ ($\tilde{\phi}^{\uparrow\uparrow\uparrow}$) baryons.[5] A procedure for systematically determining all $\tilde{\phi}_n$ is given in Ref. 2.

	b_n	N	$a_{00}^{(n)}$	$a_{10}^{(n)}$	$a_{01}^{(n)}$	$a_{20}^{(n)}$	$a_{11}^{(n)}$	$a_{02}^{(n)}$
$\tilde{\phi}_n^{\uparrow\downarrow\uparrow}$	-1	120	1					
	2/3	1260		1	-1			
	1	420	2	-3	-3			
	5/3	756	2	-7	-7	8	4	8
	7/3	34020		1	-1	-4/3		4/3
	5/2	1944	2	-7	-7	14/3	14	14/3
$\tilde{\phi}_n^{\uparrow\uparrow\uparrow}$	0	120	1					
	3/2	420	1	-3				
	3/2	420	1		-3			
	7/3	5760	1	-7/2	-7/2	7/2	7/2	7/2
	17/6	3024	1	-7/2	-7/2	2	8	2
	17/6	34020		1	-1	-4/3		4/3

$$\gamma_n = \left(2b_n C_B + \frac{3}{2}\, C_F\right)\Big/\beta \qquad\qquad \tilde{\phi}_n = \sqrt{N} \sum_{i,j} a_{ij}^{(n)}\, x_1^i\, x_3^j$$

FIGURE CAPTIONS

Fig. 1. (a) Representative diagrams constituting T_B for baryon form factors. The arrows indicate the quark helicity.

(b) The one-gluon interaction in Eq. (3).

Fig. 2. Prediction for $Q^4 G_M^p(Q^2)$ for various QCD scale parameters Λ^2 (in GeV^2). The data are from Ref. 8. The initial wave function is taken as $\phi(x,\lambda) \propto \delta(x_1 - 1/3)\delta(x_2 - 1/3)$ at $\lambda^2 = 2\ GeV^2$. The factor $(1 + m_\rho^2 / Q^2)^{-2}$ is included in the prediction as a representative of mass effects.

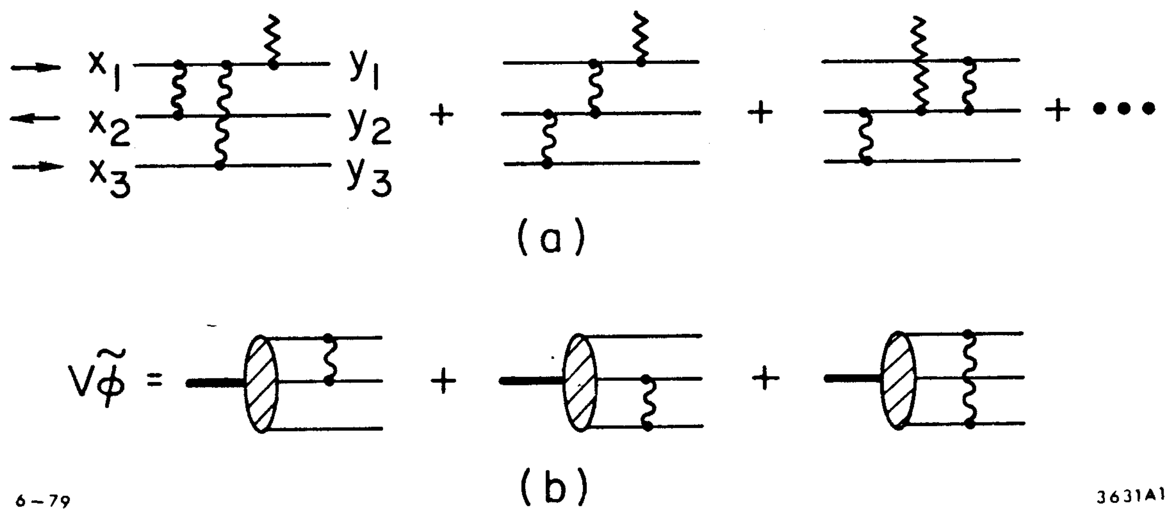
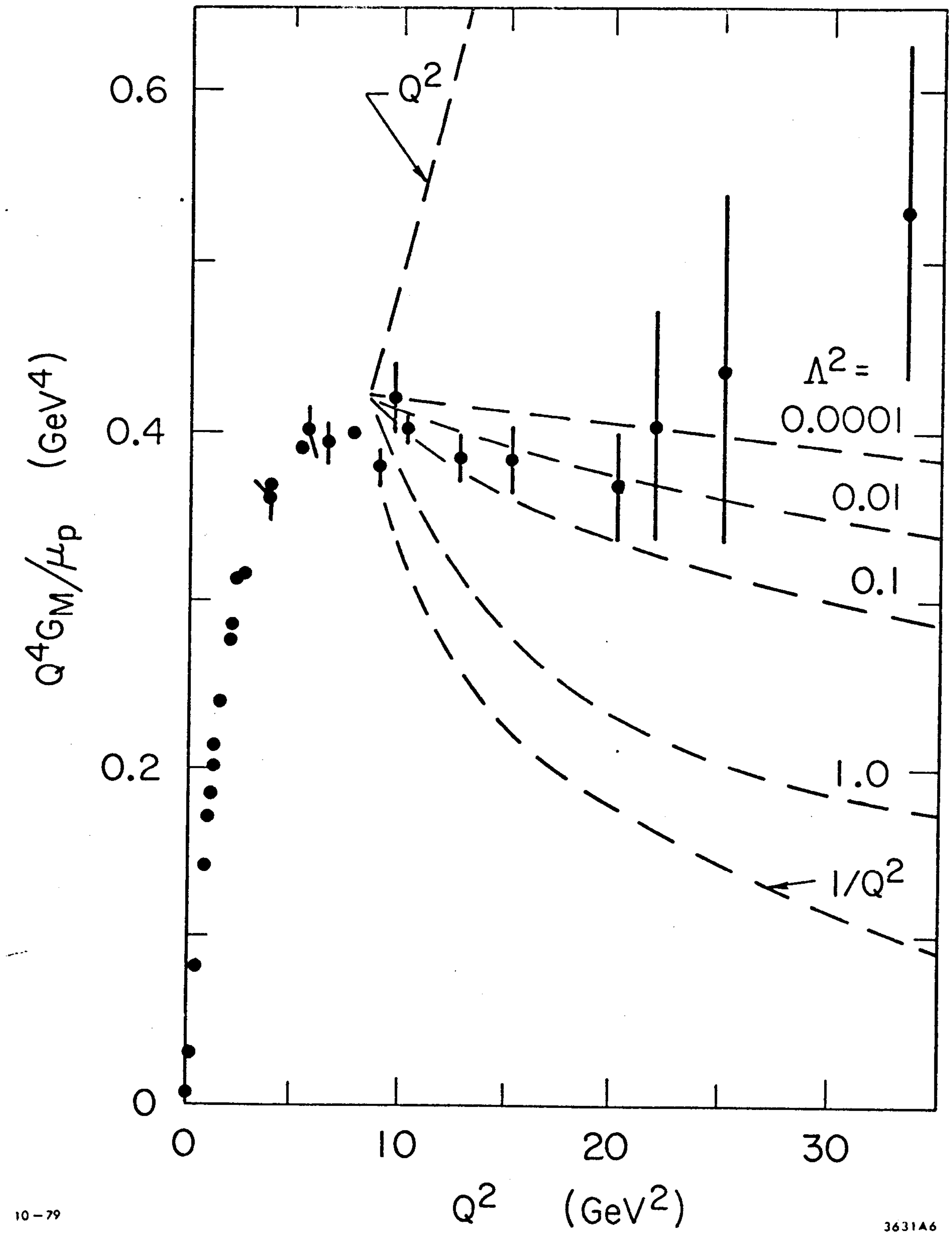


Fig. 1



Q^2
$\Lambda^2 =$
0.0001
0.01
0.1
1.0
$1/Q^2$
0.6
0.4
0.2
0
$Q^4 G_M/\mu_p$ (GeV^4)
0
10
20
30
Q^2 (GeV^2)
10-79
3631A6

Fig. 2