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INFRARED BEHAVIOR OF THE EFFECTIVE COUPLING IN QUANTUM CHROMODYNAMICS*

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ABSTRACT

We examine the infrared behavior of the effective coupling in quantum chromodynamics (QCD) using: Slavnov-Taylor identities, renormalization group and Schwinger-Dyson equations. We show that the effective coupling ansatzed solution $\bar{g}(q^2/\mu^2, g_R(\mu)) = (\mu^2/q^2)^{\lambda/2} g_R(\mu)$ in the infrared limit $q^2/\mu^2 \to 0$ where μ^2 is the Eculidean subtraction point; $\lambda = (d-2)/2$ where d is the space-time dimension, can satisfy the above equations.

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Some time ago it was discovered that non-Abelian gauge theories had the unique characteristic of controllable short-distance behavior known as asymptotic freedom [1]. This result led to the conjecture of infrared slavery; i.e., the property responsible for the confinement of quarks and the resulting spectrum of color singlet state [2].

In the intervening time period many interesting physical mechanisms have been-proposed in an attempt to prove the infrared slavery conjecture from the QCD Lagrangian. Such attempts have so far been inconclusive [3]. In this letter we present a different viewpoint based on a self-consistent approach [4]. This means that rather than attempting to identify any particular physical mechanism as dominating the QCD vacuum state we use the non-perturbative Schwinger-Dyson equations and Slavnov-Taylor identities of QCD to obtain the self-consistent behavior of the effective coupling in the infrared region [5].

QCD is a relativistic and renormalizable field theory of colored quarks and gluons based on the classical non-Abelian Lagrangian:

$$\mathscr{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \bar{\psi}_\alpha (i\not{D} - m)^{\alpha\beta} \psi_\beta \qquad (1)$$

where $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f_{abc} A^b_\mu A^c_\nu$ and $D^{\alpha\beta}_\mu = \delta^{\alpha\beta}\partial_\mu - igA^a_\mu \frac{\lambda^{\alpha\beta}_a}{2}$ where $A^a_\mu(x)$ is a colored gauge field, $\psi_\alpha(x)$ is a quark field (quark flavor indices are suppressed) α color indices, where repeated indices are summed. λ_a are SU(N) colored matrices $[\lambda^a,\lambda^b] = 2if^{abc}\lambda^c$ with f^{abc} the structure constants of SU(N) and g the universal coupling constant.

As is well known for the quantization of QCD one must add to the Lagrangian in eq. (1) a gauge fixing term $\mathscr{L}_{g.f.}$ and a Faddeev-Popov ghost term $\mathscr{L}_g$. We initially choose $\mathscr{L}_{g.f.} = \frac{1}{2\alpha}(\partial_\mu A^\mu_a)^2$; $\lim \alpha \to 0$, the covariant Landau gauge, for pedagogical simplicity. Several problems however arise with this gauge choice and will later be shown to be resolved by using the axial gauge $\mathscr{L}_{g.f.} = \frac{1}{2\alpha}(n \cdot A)^2$; $\lim \alpha \to 0$. We shall also initially ignore the effect of the quark Lagrangian term which will be treated later self-consistently.

The invariance of the QCD Lagrangian eq. (1) under non-Abelian gauge transformations leads to Slavnov-Taylor identities for the Green's functions of QCD [5]. They are the non-Abelian generalization of Ward-Takahashi identities in local Abelian gauge theories. In the Landau gauge, ghost-gluon and ghost-quark scattering-like kernels which appear in the Slavnov-Taylor identities for the one particle irreducible triple gluon and gluon-quark-antiquark vertices have been shown to vanish for incoming ghost momentum going to zero [3,5]. Using this result as well as an initial approximation of dropping the ghost self-energy term $ib(q^2)$ in the infrared region, the Slavnov-Taylor identities considerably simplify in the infrared region. We obtain Abelian-like Ward identities for the one particle irreducible (IPI) triple gluon (fig. 1(a)) and gluon-quark-antiquark (fig. 1(b)) where internal color symmetry is taken to be unbroken:

$$q_{1\mu_1} \Gamma^{abc}_{\mu_1\mu_2\mu_3}(q_1,q_2,q_3) = g_R\, f_{abc} \left[D^{-1}_{\mu_2\mu_3}(q_3) - D^{-1}_{\mu_2\mu_3}(q_2) \right] \qquad (2)$$

$$q_{1\mu}\, \Gamma^{\mu}_{a}(q_1,q_2,q_3) = g_R\, t^a \left[S^{-1}(q_3) - S^{-1}(q_2) \right] \qquad (3)$$

The IPI ultraviolet renormalized Green's function with n external gluons ${}^R\Gamma^{(n)}_{\mu_1 \ldots \mu_n} = Z_3^{n/2} \Gamma^{(n)}_{\mu_1 \ldots \mu_n}$ satisfies the renormalization group equation in the Landau gauge:

$$\left[\mu \frac{\partial}{\partial\mu} + \beta(g_R) \frac{\partial}{\partial g_R} - n\gamma(g_R)\right]\Gamma^{(n)}_{\mu_1\ldots\mu_n}(q_1,\ldots,q_n,g_R) = 0 \qquad (4)$$

where μ^2 is the Euclidean subtraction point mass; $\beta(g_R) = \mu \frac{\partial g_R}{\partial\mu}$ is the Callan-Symanzik function; $\gamma(g_R)$ the gluon anomalous dimension and $g_R(\mu) = Z_3^{1/2} g$ the renormalized coupling. The renormalization constants Z_3 and Z_1 are defined by normalizing the gluon propagator and triple gluon vertex at the subtraction point μ^2 to their bare amplitudes (zeroeth order perturbation terms). Upon multiplying both left and right hand sides of eq. (2) by the gluon propagators $D_{\mu_2\mu_3}(q_2)$ and $D_{\mu_2\mu_3}(q_3)$ and taking the limit $\alpha \to 0$ we obtain upon substituting the normalized forms the simple Abelian-like relation $Z_1 = Z_3$. Using this relation we find the relation $\beta(g_R) = g_R\gamma(g_R)$. Therefore, only one independent function $\beta(g_R)$ remains in our infrared self-consistency scheme.

The effective coupling $\bar{g}(t,g_R)$ function is next introduced by the defining equation:

$$\left.\frac{\partial\bar{g}(t,g_R)}{\partial t}\right|_{g_R} = \beta(\bar{g}); \qquad \bar{g}(0,g_R) = g_R \qquad (5)$$

where t is a dimensionless variable given implicitly by $t = \int_{g_R}^{\bar{g}} \frac{dx}{\beta(x)}$. More specifically if we take $t = \frac{1}{2} \ln q^2/\mu^2$ we obtain:

$$\left(\mu \frac{\partial}{\partial\mu} + \beta(g_R) \frac{\partial}{\partial g_R}\right)\bar{g} = 0 \qquad . \qquad (6)$$

Using this equation and the relation $g_R\gamma(g_R) = \beta(g_R)$ we obtain a general solution to eq. (3):

$$\Gamma^R_{\mu_1\ldots\mu_n} = g_R^n \sum_{i=1}^{n} F(\bar{g}^{-2}(t_1)\ldots\bar{g}^{-2}(t_n))\, T_i^{\mu_1\ldots\mu_i} \qquad (7)$$

where $T_i^{\mu_1\ldots\mu_n}$ are tensors constructed out of tensor elements $g_{\mu_i\mu_j}$, q_{μ_i}, $q_i \cdot q_j$; $i,j = 1\ldots n$ where for example the gluon propagator solution in the Landau gauge is:

$$D_{\mu_1\mu_2}(q_1) = \frac{\bar{g}^2(q_1^2)}{q_1^2}\frac{1}{g_R^2}\left(g_{\mu_1\mu_2} - \frac{q_{1\mu_1}q_{1\mu_2}}{q_1^2}\right) + \frac{\alpha q_{1\mu_1}q_{1\mu_2}}{q_1^4} \quad ;$$

$$\lim \alpha \to 0 \quad . \qquad (8)$$

In order to construct an infrared effective triple gluon IPI longitudinal vertex $\Gamma^{L\,abc}_{\mu_1\mu_2\mu_3}(q_1\, q_2\, q_3)$ we impose the following constraints: (a) Boson and Lorentz symmetry, (b) renormalization group solution eq. (7), (c) Abelian-like Ward identity eq. (2), and (d) absence of kinematical singularities. The transverse gluon vertex $\Gamma^{T\,abc}_{\mu_1\mu_2\mu_3}(q_1\, q_2\, q_3)$ is similarly determined by the homogeneous version of eq. (2): $q_{1\mu_1}\Gamma^{T\,abc}_{\mu_1\mu_2\mu_3}(q_1\, q_2\, q_3) = 0$. We therefore construct the most general gluon vertex satisfying all of these constraints using the kinematically singularity-free elements $\rho_{ij} \equiv (\bar{g}^{-2}(q_i^2) - \bar{g}^{-2}(q_j^2))/(q_i^2 - q_j^2)$; where $q_i^2/\mu^2 << 1$:

$$\begin{aligned}\Gamma^{L\,abc}_{\mu_1\mu_2\mu_3}(q_1q_2q_3) = g_R^3\, f^{abc} \Big\{ & g_{\mu_1\mu_3}\left[q_{3\mu_2}\bar{g}^{-2}(q_3^2) - q_{1\mu_2}\bar{g}^{-2}(q_1^2)\right] \\ & + \rho_{13}\left[q_1 \cdot q_3\, g_{\mu_1\mu_3} - q_{3\mu_1}q_{1\mu_3}\right] \\ & \times (q_1 - q_3)_{\mu_2} + \text{cyclic permutations} \Big\} \\ & + \frac{1}{\alpha}\bar{\Gamma}^{L\,abc}_{\mu_1\mu_2\mu_3}(q_1q_2q_3) \quad ; \quad \lim \alpha \to 0 \end{aligned} \qquad (9)$$

where

$$q_{1\mu_1}\bar{\Gamma}^{L\,abc}_{\mu_1\mu_2\mu_3}(q_1 q_2 q_3) = g_R^3\, f^{abc}\left(q_{2\mu_2}q_{2\mu_3} - q_{3\mu_2}q_{3\mu_3}\right)$$

and

$$\Gamma^{T\,abc}_{\mu_1\mu_2\mu_3}(q_1 q_2 q_3) = g_R^3\, f^{abc}\Big\{\eta_1\left[\rho_{12}+\rho_{23}+\rho_{31}\right]$$

$$\times\ \left[\bar{g}_{\mu_1\mu_2}\left(q_2\cdot q_3 q_{1\mu_3} - q_1\cdot q_3 q_{2\mu_3}\right) + \frac{1}{3}\left(q_{1\mu_2}q_{2\mu_3}q_{3\mu_1} - q_{1\mu_3}q_{2\mu_1}q_{3\mu_2}\right)\right]$$

$$+\ \left(\eta_2\rho_{13}\rho_{23} + \eta_3\rho_{12}\rho_{12}\right)\left(q_1\cdot q_2 g_{\mu_1\mu_2} - q_{1\mu_2}q_{2\mu_1}\right)$$

$$\times\ \left(q_2\cdot q_3 q_{1\mu_3} - q_1\cdot q_3 q_{2\mu_3}\right)\Big\} + \text{cyclic permutations} \qquad (10)$$

where η_1, η_2, η_3 are unknown constants.* In the limit of any one of the incoming momentum $q_{i\mu_i}$ going to zero with others held fixed $(q_1+q_2+q_3=0)$ we observe $\Gamma_{\mu_1\mu_2\mu_3}(0,-q_3,q_3) = \partial D^{-1}_{\mu_2\mu_3}/\partial q_{3\mu_1}$ and $\Gamma^T \to 0$ and where $\bar{\Gamma}^L \to 0$ by construction. The differential version of the Slavnov-Taylor identity eq. (2) is thus satisfied.

The Schwinger-Dyson equation for the gluon vacuum polarization tensor $\pi^{\mu_3\mu_3'}_{ab}(q_3,\alpha)$ is given in fig. 2 and obeys the transversality condition $q_3^{\mu_3}\pi^{\mu_3\mu_3'}_{ab}(q_3,\alpha) = 0$ where $\pi^{\mu_3\mu_3'}_{ab}(q_3,\alpha) = \delta_{ab}\,\pi(q_3,\alpha)\left(g^{\mu_3\mu_3'}q_3^2 - q_3^{\mu_3}q_3^{\mu_3'}\right)$. Unlike QED the vacuum polarization is gauge dependent. We will initially consider only the terms figs. 2(a) and 2(b), treating the remaining terms later. Using eq. (2) it is then straightforward to show

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* Note that the total triple gluon vertex is normalized at the Euclidean point $q_1^2 = q_2^2 = q_3^2 = \mu^2$, only in so far as the Slavnov-Taylor identity eq. (2) is satisfied.

that the terms figs. 2(a) and 2(b) will obey the transversality condition $q_{3\mu_3}\pi^{\mu_3\mu_3'}(q_3,\alpha) = 0$. This can be seen by either regulating the denominators with a small mass $\delta^2 \to 0$ or by analytically continuing in space-time dimension n where integrals of the form $\int d^n k (k^2)^\beta = 0$. Such a proof is analogous to that given in QED.

After replacing unrenormalized coupling propagators and vertices by their renormalized counterparts according to the standard renormalization prescription, i.e., $g = Z_3^{-1/2} g_R$, $D = Z_3 D_R$, $\Gamma = Z_3^{-1}\Gamma_R$, etc. we obtain a vacuum polarization equation:

$$\delta^{ab}\ \pi_R(q_3^2,\alpha)\left(q_3^2 g^{\mu_3\mu_3'} - q_3^{\mu_3} q_3^{\mu_3'}\right)$$

$$= \frac{g_R}{2!}\int d^d\ q_1\ \Gamma^{ocda}_{\mu_1'\mu_2'\mu_3'}(-q_1,-q_2,-q_3)\ D^{cc'}_{R\mu_1\mu_1'}(q_1)\ D^{dd'}_{R\mu_2\mu_2'}(q_2)$$

$$\times\ \Gamma^{c'd'b}_{R\mu_1\mu_2\mu_3}(q_1,q_2,q_3) \tag{11}$$

where

$$\pi_R(q_3^2,\alpha) \;=\; Z_3^{-1} g_R^2 / \bar{g}^2(q_3^2) - 1 \quad \text{and} \quad Z_3^{-1} < \infty$$

Imposing the weak constraint condition $q_3^2\bar{g}^{-2}(q_3^2) \to 0$ as $q_3^2/\mu^2 \to 0$ one observes that the left-hand side of eq. (11) goes to zero as $q_3 \to 0$. Substituting the limiting forms for the propagator eq. (8) and vertices eq. (9) and eq. (10) as $q_3 \to 0$ into the right-hand side of eq. (11), one obtains a self-consistency condition using dimensional regularization:

$$(q_1^2)^{(d/2)-1}\ \bar{g}^2(q_1^2)\Big|_0^\infty = 0 \tag{12}$$

This constraint follows from the longitudinal vertex solution eq. (9) as the transverse vertex eq. (10) vanishes when any single momentum is taken to zero. A solution to this constraint condition which exhibits singular infrared behavior is:*

$$g^2\left(\frac{\mu^2}{q_1^2}, g_R\right) = \left(\frac{\mu^2}{q_1^2}\right)^{\lambda} g_R^2 \quad ; \quad \lambda = \frac{d-2}{2} \tag{13}$$

Other solutions are also possible, for example $\bar{g}^2 = \bar{e}^{q_1^2/\mu^2} g_R^2$ which is infrared non-singular and a Coulomb-like charge as $q_1^2 \to 0$.

Next we substitute our infrared singular solution (eq. (13)) into the vacuum polarization eq. (11) using eq. (9) and eq. (10) in order to ascertain its self-consistency for small but non-vanishing values of q_3. We find after straightforward although rather tedious algebra and after evaluating Feynman type integrals via the standard n-dimensional regularization techniques the expression:

$$\frac{\mu^2}{q_3^2}\left(\frac{c_0}{n-4} + c_0' + \eta_1 c_1\right) + \eta_2 c_2 + \eta_3 c_3 = -1 \tag{14}$$

where $c_0 \neq 0$, c_0', c_1, c_2, and c_3 are dimensionless constants; $q_3^2/\mu^2 << 1$ and where we have set $d = 4$, i.e., $\lambda = 1$ and have already factored out the transverse tensor $(g^{\mu_3\mu_3'} q_3^2 - q_3^{\mu_3} q_3^{\mu_3'})$ from both sides. Explanatory comments on eq. (14) are appropriate here. In obtaining eq. (14) we have taken the lim $\alpha \to 0$ and subtracted out a term with coefficient $1/\alpha$. The remaining term, i.e., the left-hand side of eq. (14) which is

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* Such a solution is strictly only valid for the region $q^2/\mu^2 << 1$, as we know from the "asymptotic freedom," short distance behavior: $\bar{g}^2(q^2) \sim 1/\ln\,(q^2/\mu^2)$ for $q^2/\mu^2 >> 1$.

independent of α has been shown to remain transverse. The term $\frac{\mu^2}{q_3^2}\frac{c_0}{n-4}$ which arises from the sum of infrared divergent integrals of the form $\int d^n q_1 \frac{1}{(q_1^2)^\alpha}\frac{1}{[(q_1+q_3)^2]^\beta}$; $\alpha+\beta > 2$, originates from the longitudinal vertex eq. (9) contribution to eq. (11). It is clear that eq. (14) cannot be consistent unless the terms in the parenthesis multiplying μ^2/q_3^2 disappear.

Let us suppose that we are able to eliminate the $c_0/(n-4)$ term which leads to an infrared divergent vacuum polarization. It is then possible to adjust the underdetermined transverse parameters η_1, η_2, and η_3, i.e., $\eta_1 = -c_0'/c_1$; $\eta_2 = 0$; $\eta_3 = -1/c_3$; etc., so that eq. (14) is satisfied. Knowledge of the longitudinal vertex solution eq. (9) is therefore not sufficient in order to satisfy the vacuum polarization eq. (11) with solution eq. (13) for $\bar{g}^2$. Using the equations available in this paper one can therefore only show that the ansatzed solution eq. (13) for $\bar{g}^2$ satisfies a necessary condition eq. (12) although not a sufficient condition eq. (14). In order to determine η_i from QCD itself it is necessary to understand the global properties of the total triple gluon vertex. Such information however is contained in the infinite hierarchy of coupled Schwinger-Dyson equations for the triple gluon, quadruple gluon vertex, etc. A truncation procedure for these equations, similar to our analysis of the vacuum polarization function may help determine the η_i parameters.

To understand the origin of the infrared divergent $1/(n-4)$ term let us examine the infrared content of the right-hand side of the vacuum polarization eq. (11). This can be done by looking at the small integration momentum $q_1 \ll q_3$ region, where q_3 is also small but finite:

$$\pi_{\mu_3\mu_3'}(q_3) \approx \frac{1}{2!}\int^{\sigma} d^n q_1 \Gamma^{o}_{\mu_1'\mu_2'\mu_3'}(0,+q_3,-q_3)\, D_{\mu_1\mu_1'}(q_1)$$

$$\times\; D_{\mu_2\mu_2'}(-q_3)\,\Gamma_{\mu_1\mu_2\mu_3}(0,-q_3,q_3)$$

where $q_1 << \sigma << q_3$ restricts the integration strictly to the infrared region. Thus, in order that $\pi_{\mu_3\mu_3'}(q_3)$ contains no infrared divergent $1/(n-4)$ term we find the constraint: $\int_0^{\sigma} d^n q_1\, D_{\mu_1\mu_1'}(q_1) < \infty$, $\lim n \to 4$. Substituting the Landau gauge propagator eq. (8) and $\bar{g}^2$ eq. (13) we observe that this condition is not met, giving rise to a $1/(n-4)$ infrared divergent term in the Landau gauge.

In order to ameliorate this problem let us instead choose the axial gauge $\mathscr{L}_g = \frac{1}{2\alpha}(n \cdot A)^2$ where n is the gauge direction vector, $\lim \alpha \to 0$. In this gauge the Slavnov-Taylor identities eqs. (2) and (3) are exact as Faddeev-Popov ghosts are absent. The full gluon propagator is however more complicated (ignoring color indices):

$$D_{\mu_3\mu_3'}(q_3^2, q_3 \cdot n) = A_{ren}(q_3^2, q_3 \cdot n) P^{\mu_3\mu_3'} + B_{ren}(q_3^2, q_3 \cdot n) g^{T\mu_3\mu_3'}$$

$$+\; \alpha q_3^{\mu_3} q_3^{\mu_3'}/(n \cdot q_3)^2 \qquad \lim \alpha \to 0$$

where

$$P^{\mu_3\mu_3'} \equiv \frac{1}{q_3^2}\left(g^{\mu_3\mu_3'} - \frac{q_3^{\mu_3} n^{\mu_3'} + q_3^{\mu_3'} n^{\mu_3}}{n \cdot q_3} + \frac{n^2 q_3^{\mu_3} q_3^{\mu_3'}}{(n \cdot q_3)^2} \right)$$

and

$$g^{T\mu_3\mu_3'} = \left(g^{\mu_3\mu_3'} - \frac{n^{\mu_3} n^{\mu_3'}}{n^2} \right)$$

It is straightforward to show that by choosing the solution:

$A = \bar{g}^2(q_3^2)/g_R^2$; $B=0$, the Slavnov-Taylor identity eq. (2) is satisfied with the triple gluon vertex eq. (8) and eq. (9) (after dropping the $\bar{\Gamma}^{L\,abc}_{\mu_1\mu_2\mu_3}$ term). In terms of the general vacuum polarization term:

$$\pi^{\mu_3\mu_3'}(q_3) = \pi_1(q_3^2 g^{\mu_3\mu_3'} - q_3^{\mu_3} q_3^{\mu_3'}) + \pi_2\left(q_3^{\mu_3} q_3^{\mu_3'} - q_3^2\left(\frac{q_3^{\mu_3} n^{\mu_3'} + q_3^{\mu_3'} n^{\mu_3}}{n \cdot q_3}\right) + \frac{(q_3^2)^2}{(n \cdot q_3)^2} n^{\mu_3} n^{\mu_3'}\right)$$

our solution corresponds to $\pi_1 = 1/A$; $\pi_2 = 0$. Our propagator also obeys the same renormalization group eq. (4).

Substituting our axial propagator into the infrared convergence criterion $\int_0^\sigma d^n q_1 D^{\mu_1\mu_1'}(q_1) < \infty$ lim $n \to 4$, we observe that this criterion is satisfied due to the vanishing of the angular integral $\int d\Omega_{q_1} P^{\mu_1\mu_1'} = 0$. This can be seen by evaluating the integrals:

$$\int d\Omega_{q_1} (n_{\mu_1} q_{1\mu_1'} + n_{\mu_1'} q_{\mu_1})/n \cdot q_1 = 2n_{\mu_1} n_{\mu_1'}/n^2$$

and

$$\int d\Omega_{q_1}\, q_{1\mu_1} q_{1\mu_1'}/(n \cdot q_1)^2 = -g_{\mu_1\mu_1'} + 2n_{\mu_1} n_{\mu_1'}/n^2$$

The rest of the arguments leading to our self-consistency condition eq. (14) are retained in the axial gauge. The right-hand side of the vacuum polarization equation however does induce a new π_2 term. We conjecture that such an induced term will not give rise to an infrared singular B term propagator which could potentially violate our infrared convergence criterion. The remaining quadruple gluon terms figs. 2(d) and 2(e) may be treated in a similar manner to our I.P.I. triple gluon term through the use of the I.P.I. quadruple gluon Slavnov-Taylor identity, and may be shown to be transverse. The basic conclusions of this letter are not altered by inclusion of these terms in the vacuum polarization equation. Details will be published elsewhere. Finally, returning to our $q_3 \to 0$ consistency condition we find in the axial gauge with our propagator solution the same self-consistency condition as in the Landau gauge, eq. (12).

It is instructive to examine the quark self-energy equation fig. 3(b) in the Landau gauge (albeit its difficulties) and in the infrared region. We can take advantage of the effective coupling's infrared singular behavior by making use of the renormalized version of eq. (3):

$\Gamma^a_{\mu_R}(p,p+q) = g_R t^a \frac{\partial S_R^{-1}(p)}{\partial p_\mu}$, $\lim q \to 0$; where $S = Z_2 S_R$ and $\Gamma^\mu_{aR} = Z_2^{-1}\Gamma^\mu_a$.

We obtain in the case of zero bare mass quarks the equation:

$$\Sigma(p) \approx \kappa C_2(R) \frac{\partial S_R^{-1}}{\partial p_\mu} S_R(p) \gamma_\nu \int_{\delta^2}^{p^2} \frac{\bar{g}^2(q^2)}{q^2}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) \times (q^2)^{(d/2)-1}\, dq^2 \qquad \lim \delta^2 \to 0 \qquad (15)$$

where $C_2(R) = (N^2-1)/2N$, κ is an angular integration constant and δ^2 is an infrared cutoff regulator. Substituting the quark propagator $S_R = i(\not{p} A_R(p^2) + B_R(p^2))$ one obtains uncoupled first order differential equations if we use the approximation $\Sigma(p) \approx S^{-1}(p) = Z_2^{-1} S_R^{-1}$ which ignores the kinetic term $\not{p}$. One easily solves these equations obtaining the solutions $A_R(p^2,\delta^2) = \rho/\beta_1 \log|\log p^2/\delta^2|/(p^2/\delta^2)^{\beta_2/\beta_1}$ and $B_R(p^2) =$ constant, where $\rho = 1/(2\pi)^4 \times Z_2^{-1}/\kappa C_2(R)\mu^2 g_R^2$ and $\beta_1, \beta_2 > 0$ are constants. To understand this result let us compare the electron propagator's infrared behavior in QED: $S_{unren} = Z_2 \not{p}/(p^2)^{1+\gamma}$; $\gamma = \frac{\alpha}{2\pi} + \ldots$. One can define $Z_{2_{I.R.}}(p) \equiv Z_2/(p^2)^{1+\gamma}$ thus absorbing the soft coherent infrared photon cloud into the wave function renormalization leaving a renormalized single particle pole state. By analogy our QCD quark propagator can be rewritten as $S_{unren} = Z_{2_{QCD}}(p) S_R$ where $S_R = Z^{-1}_{2_{QCD}}(p)\not{p}/p^2$ and where $Z^{-1}_{2_{QCD}}(p) = p^2 A_R(p^2,\delta^2)$. One therefore observes that unlike QED the soft coherent gluon cloud cannot be renormalized away but in fact confines as $\delta^2 \to 0$, leading to no on-shell renormalized quark state. The second important property to note is that the solution $B_R(p^2)$ = constant violates Chiral symmetry $[\gamma_5, S^{-1}]_+ \neq 0$ and therefore realizes the PCAC phase. One thus obtains a dynamical Goldstone boson in the axial vertex Γ^5_μ as a consequence of our infrared solution [3,4a]. Next using the product $S_R \Gamma^\mu_R = -i\gamma_\mu (1/\not{p})$ lim $q_\mu \to 0$; $\delta^2 \to 0$ it is straightforward to observe that the quark term fig. 2(f) vanishes in the lim $\delta^2 \to 0$. Inserting an explicit bare quark mass, i.e., $\not{p} \to \not{p} - m$ does not alter this conclusion. We expect the quark propagator in the axial gauge to behave in a similar way to the preceding results.

Similarly the Schwinger-Dyson equation for the ghost self-energy $b_{ren}(k^2)$, fig. 3(a) may be solved in the infrared region $k^2 \to 0$ by making use of the effective coupling's eq. (13) infrared singular behavior and the Landau gauge's simplifying properties for the ghost-ghost-gluon vertex [3]. Solving we obtain $b_{ren}(k^2) \sim \tilde{Z}_3(k^2,\delta^2)$, lim $k^2 \to 0$; lim $\delta^2 \to 0$ where $\tilde{Z}_3(k^2,\delta^2) = \tilde{Z}_3 \ln k^2/\delta^2$. One therefore observes that our initial approximation of dropping $b_{ren}(k^2)$ in the Slavnov-Taylor identities is inconsistent (due to its infrared divergent behavior as $\delta^2 \to 0$), confirming again the lack of a simple self-consistent infrared scheme in the Landau gauge.

The β function in the infrared region is obtained by substituting $\bar{g}^2 = (\mu^2/q^2)^{\lambda} g_R^2$ into eq. (5) to obtain $\beta(g_R(\mu)) = -\frac{\lambda}{2} g_R(\mu)$. Solving for $g_R(\mu)$ we observe that the quantity $g_R(\mu)\mu^{\lambda/2} = g_R(\mu_0)\mu_0^{\lambda/2}$ is a renormalization group invariant where μ_0 is an arbitrary mass point. In particular for $d=4$, $g_R(\mu_0)$ is dimensionless and $\Lambda = g_R^2(\mu_0)\mu_0$ may be identified as a dynamical mass. This may also be seen explicitly by noting that $\Lambda = \mu \times \exp - \int^{g_R} \frac{dg}{\beta(g)}$ is the dimensionally transmuted mass. Note also that by setting $\mu_0 = \Lambda$ we see that Λ is defined as the scale for which $g_R^2(\Lambda) = 1$. Substituting our expression for $g_R(\mu)$ in terms of Λ we may rewrite $\bar{g}^2$ as follows:

$$\bar{g}^2(q^2) = \frac{\mu\Lambda}{q^2} \quad ; \qquad \mu \le \Lambda \qquad \lim q^2/\mu^2 \to 0 \tag{16}$$

Finally let us note that the choice of the Euclidean subtraction point μ is arbitrary and does not affect the values of physical quantities such as hadronic masses. In consequence with this observation some convenient choices for μ are $\mu = \Lambda$ or $\mu = \Lambda/N$. The choice $\mu = \Lambda/N$ can be shown to correspond to the well-known topological expansion of diagrams in the asymptotic $\lim N \to \infty$. Details of these and other results reported in this letter will be published elsewhere.

In conclusion we have shown in this letter that confining infrared behavior of the effective coupling is consistent with the QCD Lagrangian. In particular a self-consistent set of Green's functions in the axial gauge can be constructed. Using these Green's functions one can then calculate the spectrum and scattering amplitudes of this theory.*

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REFERENCES

[1] H. D. Politzer, Phys. Rev. Lett. 30 (1973) 1346; D. Gross and F. Wilczek, Phys. Rev. Lett. 30 (1973) 1343 and G. 't Hooft, unpublished.

[2] H. Fritzsch, M. Gell-Mann and H. Leutwyler, Phys. Lett. 47B (1973) 365; S. Weinberg, Phys. Rev. Lett. 31 (1973) 494; Phys. Rev. D8 (1973) 4482; D. J. Gross and F. Wilczek, Phys. Rev. D8 (1973) 3633.

[3] W. Marciano and H. Pagels, Quantum Chromodynamics, Phys. Reports 36C (1978) 137.

[4] See (a) H. Pagels, Phys. Rev. D16 (1977) 2991; (b) J. S. Ball and F. Zachariasen, Nucl. Phys. B143 (1978) 148 and R. Delbourgo, Preprint "The Gluon Propagator", University of Tasmania, November (1978) for other self-consistent approaches.

[5] A. A. Slavnov, Theor. Math. Phys. 10 (1972) 99; J. C. Taylor, Nucl. Phys. B10 (1971) 99.

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* After completion of this work we received preprints CERN TH 2501, F. Zachariasen and M. Baker, University of Washington RLO-1388-781 which reached some of the conclusions in this letter.

FIGURE CAPTIONS

Fig. 1 (a) Full gluon propagator.

(b) Triple gluon IPI vertex.

(c) Gluon-quark-antiquark IPI vertex.

(d) Four gluon IPI vertex.

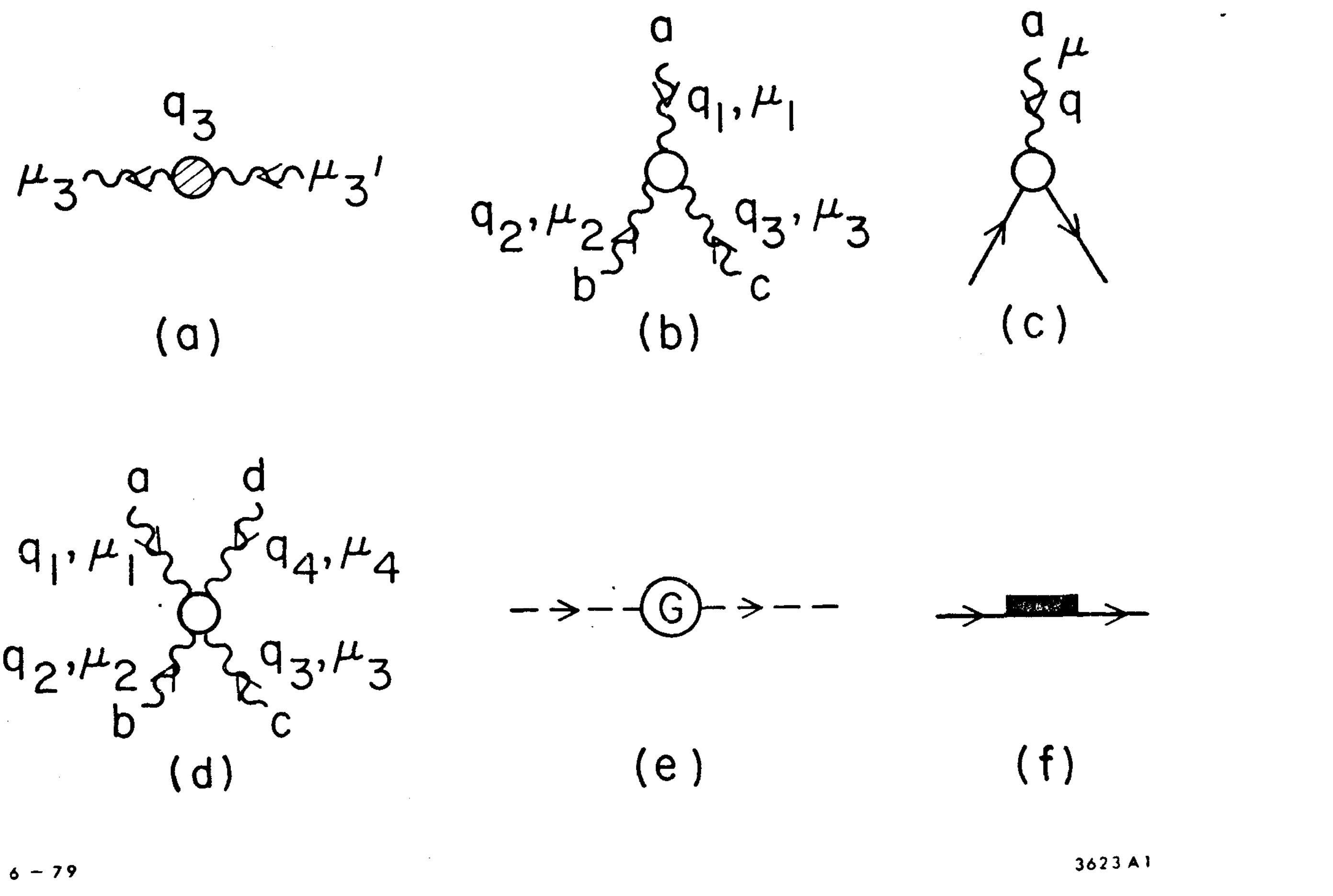
(e) Full ghost propagator.

(f) Full quark propagator.

Fig. 2 (a)-(f) Schwinger-Dyson equations for gluon vacuum polarization tensor.

Fig. 3 (a) Schwinger-Dyson equation for ghost self-energy.

(b) Schwinger-Dyson equation for quark self-energy.



q3
μ3
μ3'
(a)
a
q1, μ1
q2, μ2
q3, μ3
b
c
(b)
a
μ
q
(c)
a
d
q1, μ1
q4, μ4
q2, μ2
q3, μ3
b
c
(d)
G
(e)
(f)
6 – 79
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Fig. 1

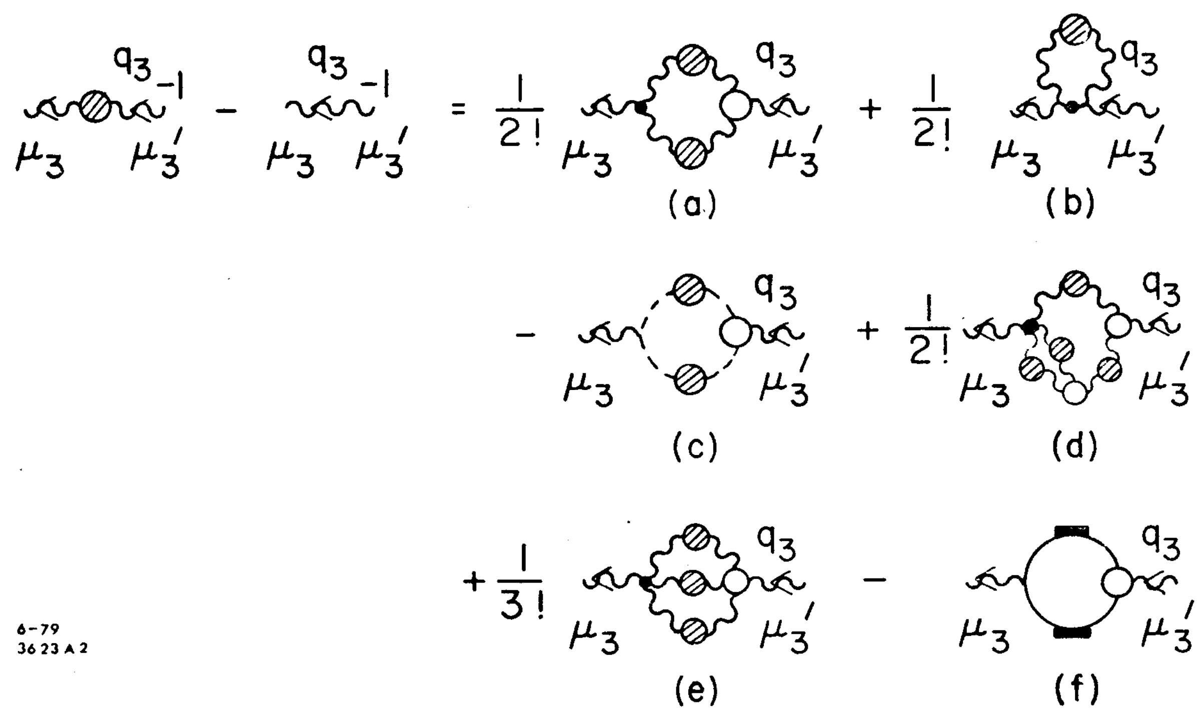
q3 -1
q3 -1
=
1/2!
q3
+ 1/2!
q3
μ3 μ3'
μ3 μ3'
μ3 μ3'
μ3 μ3'
(a)
(b)
−
q3
+ 1/2!
q3
μ3 μ3'
μ3 μ3'
(c)
(d)
+ 1/3!
q3
−
q3
μ3 μ3'
μ3 μ3'
(e)
(f)
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Fig. 2

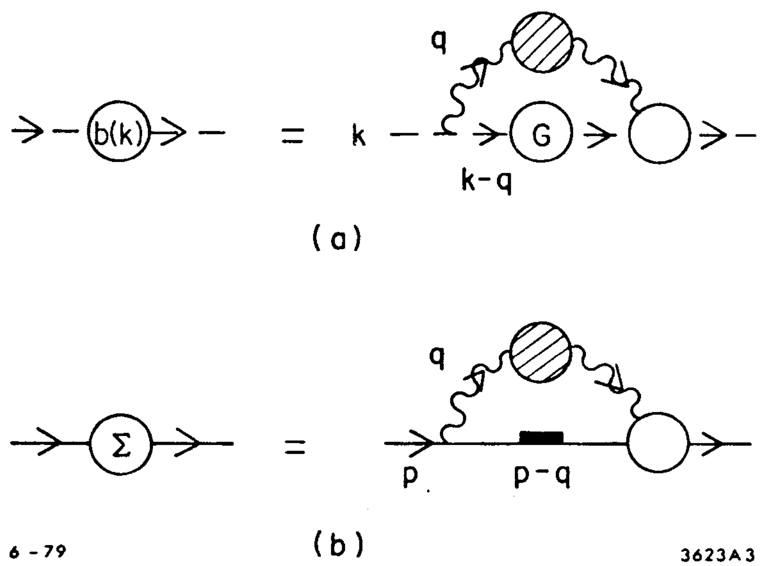

Fig. 3