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A TREATMENT OF SPIN ONE POLARIZATION*

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When a particle is produced or decays, the spin states of the particle give much information on the reactions. The treatment of the polarization of spin 1/2 particle in a manifestly covariant way is well-known. For the higher spin particles, however, the helicity formalism of Jacob and Wick [1] is usually used in discussing the production or decay of higher spin particles.

Our approach is to extend the treatment of spin 1/2 case in a manifestly covariant way to spin 1. From the projection operator, one can obtain the density matrix in a straightforward way when the transition amplitudes of reactions are given explicitly.

When a particle with spin, say c, is produced and decays as

$$\begin{array}{l} ab \rightarrow c\ d \\ \qquad\ \ \lfloor\!\!\rightarrow ef \end{array} \tag{1}$$

one can obtain two density matrices ρ^{prod} $(\equiv \rho)$ and ρ^{decay} from the explicit transition amplitudes for the corresponding processes.
The angular distribution becomes then

$$I(\theta,\zeta) \propto \operatorname{Tr}\left(\rho\ \rho^{\text{decay}}\right) \tag{2}$$

where (θ, ζ) are polar coordinates of one of the decay products.
If another particle, d, also decays, three density matrices should be considered in order to discuss the angular distribution. The spin-1/2 case is well-known and the spin-3/2 case is considered by our group [2,3]

For spin 1, the proca vector $\mathscr{E}_\mu$ has the following projection operator

$$\mathscr{E}_\mu \mathscr{E}_\nu^* = \frac{1}{3}\left[I_{\mu\nu} + \frac{3\left(k_1 + k_2\right)}{2(1+|k|)m} \varepsilon_{\mu\nu\lambda\tau} P_\lambda \eta_\tau - \frac{k_1 k_2}{(1+|k|)} \eta_{\mu\nu} \right] \qquad (3)$$

where η_μ denotes a four-vector corresponding to the polarization direction $\vec{s}$

$$\vec{\eta} = \vec{s} + \frac{\vec{p}(\vec{p}\cdot\vec{s})}{m(E+m)} , \qquad (4a)$$

$$\eta_4 = i\,\frac{\vec{p}\cdot\vec{s}}{m} . \qquad (4b)$$

and $I_{\mu\nu}$, $\eta_{\mu\nu}$ are defined as

$$I_{\mu\nu} = \delta_{\mu\nu} + \frac{P_\mu P_\nu}{m^2} \qquad (5)$$

$$\eta_{\mu\nu} = 3\,\eta_\mu \eta_\nu - I_{\mu\nu} \qquad (6)$$

In eq. (3), k implies the spin states of spin 1 particle and $k_1 = k_2 = \pm 1$ for $k = \pm 1$ and $k_1 = -k_2 = 1$ for $k = 0$.

One obtains the square of the transition amplitude for any reaction with a spin 1 particle as

$$|\langle f|T|i\rangle|^2 \propto \left[A + \frac{k_1 + k_2}{(1+|k|)} B_\mu \eta_\mu + \frac{k_1 k_2}{(1+|k|)} C_{\mu\nu} \eta_{\mu\nu} \right] . \qquad (7)$$

A, B_μ and $C_{\mu\nu}$ can be obtained from explicit forms of the transition amplitude, and they contain form factors and momenta of particles involved in the reaction. Then the density matrix becomes

$$\rho = \frac{1}{3}\left[I + \frac{B_i S_i}{A} + \frac{3}{2A} C_{ij} S_{ij} \right] , \tag{8}$$

where I is the 3×3 unit matrix, S_1, S_2, S_3 the 3×3 spin 1 matrices and S_{ij} the 3×3 traceless matrices of the symmetric tensor

$$S_{ij} = S_i S_j + S_j S_i - \frac{4}{3} I \delta_{ij} \quad . \tag{9}$$

Since the C_{ij} in eq. (8) are composed of vector components of momenta, C_{ij} can be written as

$$C_{ij} = C_i^A C_j^B \quad . \tag{10}$$

Using the explicit forms of S_i and S_{ij}, density matrix elements in eq. (8) can be expressed as

$$3A\, \rho_{1,1} = A + B_3 + \frac{1}{2}\left(3C_3^A C_3^B - \vec{C}^A \cdot \vec{C}^B\right),$$

$$3A\, \rho_{0,0} = A - \left(3C_3^A C_3^B - \vec{C}^A \cdot \vec{C}^B\right) ,$$

$$3A\, \rho_{-1,-1} = A - B_3 + \frac{1}{2}\left(3C_3^A C_3^B - \vec{C}^A \cdot \vec{C}^B\right) ,$$

$$3A\, \rho_{1,0} = \frac{1}{\sqrt{2}} B_- + \frac{3}{2\sqrt{2}}\left(C_-^A C_3^B + C_3^A C_-^B\right) ,$$

$$3A\, \rho_{1,-1} = \frac{3}{2} C_-^A C_-^B ,$$

$$3A\, \rho_{0,-1} = \frac{1}{\sqrt{2}} B_- - \frac{3}{2\sqrt{2}}\left(C_-^A C_3^B + C_3^A C_-^B\right) , \tag{11}$$

where B_- implies $B_1 - iB_2$. The other components can be obtained from the relation $\rho^+ = \rho$.

For the decay process of a vector meson into pseudoscalar particles, P and pion, $V \to P\pi$, the transition amplitude becomes

$$T = f \ \mathscr{E}_\mu \, q_\mu \quad , \tag{12}$$

$$q = p_V - p_\pi \quad , \tag{13}$$

and the square of the transition amplitude becomes

$$|T|^2 = \frac{f^2}{3}\left[q\cdot q + \frac{p\cdot q \ p\cdot q}{m^2} - \frac{k_1 k_2}{1+|k|} \, \eta_{\mu\nu} q_\mu q_\nu \right] \tag{14}$$

In the V rest frame, $P = (0, 0, 0, \ im)$ and

$$A = q^2 \quad , \qquad B_1 = 0 \quad , \qquad C_{ij} = -q_i q_j \quad , \tag{15}$$

$$\vec{q} = q(\sin\theta\cos\zeta, \ \sin\theta\sin\zeta, \cos\theta) \quad . \tag{16}$$

From eqs. (11, 15, 16), one obtains

$$\rho^{decay}_{(\pi)} = \frac{1}{2}\begin{pmatrix} \sin^2\theta & -\frac{1}{\sqrt{2}}\sin 2\theta \, e^{-i\zeta} & -\sin^2\theta \, e^{-2i\zeta} \\ -\frac{1}{\sqrt{2}}\sin 2\theta \, e^{i\zeta} & 2\cos^2\theta & \frac{1}{\sqrt{2}}\sin 2\theta \, e^{-i\zeta} \\ -\sin^2\theta \, e^{2i\zeta} & \frac{1}{\sqrt{2}}\sin 2\theta \, e^{i\zeta} & \sin^2\theta \end{pmatrix} \tag{17}$$

Since one can obtain the density matrix ρ from the production process, the decay distribution for $ab \to VX$ and $V \to P\pi$ becomes

$$I^{\pi}(\theta,\zeta) \sim \mathrm{Tr}\left(\rho\ \rho^{\mathrm{decay}}\right) = \frac{3}{8\pi}\Big[\sin^2\theta + \rho_{0,0}\left(3\cos^2\theta - 1\right) - \sqrt{2}\sin 2\theta\ \mathrm{Re}\left\{e^{-i\zeta}\left(\rho_{1,0} - \rho_{0,-1}\right)\right\} - 2\sin^2\theta\ \mathrm{Re}\left\{e^{-2i\zeta}\left({}_{1,-1}\right)\right\}\Big] \quad . \tag{18}$$

For $V \to P\gamma$, the transition amplitude becomes

$$T' = f'\ \varepsilon_{\mu\nu\lambda\tau} q_\mu p_\nu(v)\ \mathscr{E}_\lambda(v)\ \mathscr{E}_\tau(\gamma) \quad . \tag{19}$$

The previous method applies here to obtain the angular distribution

$$I^{\gamma}(\theta,\zeta) \sim \mathrm{Tr}\left(\rho\ \rho^{\mathrm{decay}}_{(\gamma)}\right) = \frac{3}{8\pi}\Big[1 + \cos^2\theta - \rho_{0,0}\left(3\cos^2\theta - 1\right) + \sqrt{2}\sin 2\theta\ \mathrm{Re}\left\{e^{-i\zeta}\left(\rho_{1.0} - \rho_{0,1}\right)\right\} + 2\sin^2\theta\ \mathrm{Re}\left\{e^{-2i\zeta}\left(\rho_{1,-1}\right)\right\}\Big] \quad . \tag{20}$$

One can obtain the decay distribution of $W \to \mu\nu$ in the same way.

As an example to obtain the density matrix ρ of a vector meson production, consider the production process (4) $e^- e^+ \to D^* \bar{D}$ and the subsequent decay of the charmed meson D^*, $D^* \to D\pi$ or $D^* \to D\gamma$.

For $e^- e^+ \to D^* \bar{D}$

$$T = \frac{e^2}{q^2}\ \bar{v}\ \gamma_\nu\ u \langle pp' |\ J_\mu\ | 0 \rangle \quad , \tag{21}$$

$$\langle pp' |\ J_\mu\ | 0 \rangle = f\ \ \varepsilon_{\mu\nu\lambda\tau} q_\nu p_\lambda \mathscr{E}_\tau \quad . \tag{22}$$

Using the previous tricks, one can obtain ρ and $I(\theta,\zeta)$ explicitly. In particular, in the c.m. frame of e^-e^+ and in the coordinate system where $-z$ is chosen along $\vec{q}$ and the production plane is in the x-z plane, the density matrix elements become

$$\rho_{0,0} = \frac{\sin^2\Theta}{1+\cos^2\Theta} , \tag{23a}$$

$$\rho_{1,0} - \rho_{0,-1} = \frac{\sin 2\Theta}{\sqrt{2}\left(1+\cos^2\Theta\right)} , \tag{23b}$$

$$\rho_{1,-1} = 0 . \tag{23c}$$

where Θ is the polar angle of D^*. Equations (23a through 23c) give the angular distribution near the threshold energy for the process $D^* \to D\pi$ as

$$I^{\pi}(\theta,\zeta) = \frac{3}{8\pi}\left[\sin^2\theta + \frac{1-\cos^2\Theta}{1+\cos^2\Theta}\left(3\cos^2\theta - 1\right) - \sin 2\theta \cos\zeta \; \frac{\sin 2\Theta}{1+\cos^2\Theta}\right]. \tag{24}$$

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