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MULTIFLAVOR QCD IN TWO DIMENSIONS: BREAKDOWN OF LOCAL COLOR SYMMETRY AND LIBERATION OF MASSIVE QUARKS*

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ABSTRACT

It is shown that in two-dimensional, multiflavor QCD with one massless flavor, local color symmetry is broken spontaneously and the gluons rendered massive by means of the Schwinger mechanism. The massive quarks are thereby liberated.

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In a study of two-dimensional SU(N) Yang-Mills theory [to be designated here as QCD2(N)] with massless fermions [1,2], motivated by a conjecture [3] regarding the weak-coupling nature of the 't Hooft model [4], we found that (a) local color symmetry is broken spontaneously, the chiral current possesses an anomaly and the gauge field is rendered massive by means of the Schwinger mechanism, and (b) the (massless) fermion is confined, massive color-singlet states do not exist, and the only massive states are the "colored" ones. The latter are exemplified by the massive gauge particles which may also be looked upon as the fermion-antifermion bound states of the confined field. The appearance of these massive bosons in place of the confined fermion is a Higgs phenomenon, as in the (massless) Schwinger model [5].

In the present note, we consider the addition of massive flavors to the single-flavor QCD2(N) just described to arrive at a two-dimensional version of standard QCD wherein light (here massless) as well as heavy (here massive) quark flavors are included (the dimension N of the gauge group being arbitrary and finite). The essential structure of this theory is determined by the above-mentioned breakdown of local color symmetry and the dynamical replacement of massless quarks with massive gluons. The massive quarks are thereby left to interact through massive gluons. This is precisely the picture that has been contemplated as a possibility for imperfect confinement and eventual liberation in four dimensional theories [6]. Aside from the massive gluons (or equivalently the quark-antiquark bound states of the massless flavor) and their composites, the spectrum includes the massive quark states and their composites, the relative masses of which will depend on the relative magnitude of the quark masses and the coupling constant.

Just as in the exposition of local color symmetry breaking in ref. [2], it is useful to first illustrate the liberation mechanism within the context of an Abelian analog. The analog model is two-dimensional QED with one massless fermion, $\psi^{(0)}$, and M massive ones, $\psi^{(a)}$, $a=1,\ldots,M$, interacting through the electromagnetic field $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The basic point is the simple observation that, as in the Schwinger model (i.e., in the absence of the massive fermions), local charge conservation is spontaneously broken and the massless fermion is effectively replaced by a massive "photon," thus leaving the massive fermions to interact by means of a massive vector field. In symbols, the original equations of motion for the massless fermion and the gauge field (written in an obvious notation),

$$(i\partial\!\!\!/ - eA\!\!\!/)\,\psi^{(0)} = 0 \quad , \tag{1}$$

$$\partial^\mu F_{\mu\nu} = j_\nu^{(0)} + \sum_{a=1}^{M} j_\nu^{(a)} \quad , \tag{2}$$

are replaced, respectively, with the well-known solution (here written in the transverse gauge)

$$\psi^{(0)}(x) = \exp[-ie\gamma^5\phi(x)]\,\psi_0^{(0)}(x) \quad , \tag{1'}$$

and the modified field equations

$$\partial^\mu F_{\mu\nu} + \left(\frac{e}{\sqrt{\pi}}\right)^2 A_\nu = \sum_{a=1}^{M} j_\nu^{(a)} \quad , \tag{2'}$$

while the massive fermions obey the usual equations of motion

$$(i\partial\!\!\!/ - m^{(a)} - eA\!\!\!/)\,\psi^{(a)} = 0 \quad . \tag{3}$$

In eq. (1'), $\psi_0^{(0)}$ is a massless fermion field and ϕ is defined by $A_\mu = \varepsilon_{\mu\nu}\partial^\nu\phi$, where ε is the antisymmetric symbol. Note that the mass term appearing in (2') is none other than the current contributed by the confined fermion. Equations (2') and (3) clearly exhibit the theory as one of massive fermions interacting with a massive vector field, as was to be illustrated [7].

Having illustrated the basic mechanism in an Abelian model, we now turn to multiflavor QCD2(N). The Lagrangian density for one massless flavor and M massive ones may be written (using the notation of ref. [2] when appropriate)

$$\mathscr{L} = -\frac{1}{4} G_i^{\mu\nu} G_{i\mu\nu} + \sum_{A=0}^{M} \bar{q}^{(A)} (i\not{\partial} - m^{(A)}) q^{(A)} + B_i^\mu \sum_{A=0}^{M} j_{i\mu}^{(A)} \quad , \tag{4}$$

where

$$G_{i\mu\nu} = \partial_\mu B_{i\nu} - \partial_\nu B_{i\mu} + g f_{ijk} B_{j\mu} B_{k\nu} \quad ,$$
$$j_{i\mu}^{(A)} = g\bar{q}^{(A)} \lambda_i \gamma_\mu q^{(A)} \quad , \quad A = 0,\ldots,M \quad , \tag{5}$$

and $m^{(0)} = 0$, $m^{(A)} > 0$, $A \neq 0$.

We now proceed to bosonize the massless flavor. To begin, we adopt the light-cone gauge

$$B_{i-} = B_{i0} - B_{i1} = 0 \quad , \tag{6}$$

which leads to the field (more properly, constraint) equation

$$\frac{1}{2}\partial_-^2 B_{i+} = \sum_{A=0}^{M} j_{i-}^{(A)} \quad , \tag{7}$$

and also to

$$\left(i\not{\partial} - m^{(A)} + \frac{1}{2} g\lambda_i B_{i+} \gamma_- \right) q^{(A)} = 0, \quad A = 0,\ldots,M \quad . \tag{8}$$

Using the results of ref. [2] [cf., eq. (28) therein], we can construct the solution of (8) for the massless flavor as follows:

$$q^{(0)}(x) = \left[\mathscr{T}_+(x) \frac{1+\gamma^5}{2} + \frac{1-\gamma^5}{2} \right] q_0^{(0)}(x) \quad , \tag{9}$$

where $\mathscr{T}_+(x) = T_+(x)\tau_+(x_+)$, with

$$T_+(x_+, x_-) = T\left\{ \exp\left[\frac{1}{2} ig\lambda_i \int_{-\infty}^{x_-} dx_-' \; B_{i+}(x_+, x_-') \right] \right\} , \tag{10}$$

and τ_+ determined by

$$\tau_+(z) = 1 - \frac{i}{2\pi} \int \frac{dz'}{z - z' - i\epsilon} \left[T_+(z',+\infty) - T_+(z',-\infty) \right] \tau_+(z') \ . \tag{11}$$

Note that the right-hand side of eq. (10) is an ordered exponential. Note further that, as in the Abelian case, a free massless fermion field $q_0^{(0)}$ has appeared.

Next, we turn to the calculation of $j_i^{(0)}$. Using the necessary limiting procedure to insure gauge invariance, we obtain as in ref. [2] [cf., eqs. (34) and (71) therein]

$$j_{i-}^{(0)}(x) = \frac{ig}{\pi} \, \mathrm{tr}\left[\lambda_i \frac{\partial \mathscr{T}_+(x)}{\partial x_+} \mathscr{T}_+^{-1}(x) \right] \quad , \tag{12}$$

and the associated property

$$\partial_+ j_{i-}^{(0)}(x) = -\frac{g^2}{4\pi} \partial_- B_{i+} - gf_{ijk} B_{j+} j_{k-}^{(0)} \quad . \tag{13}$$

Substituting $j^{(0)}$ in (7), we obtain the modified gauge field equation

$$\frac{1}{2}\,\partial_-^2 B_{i+}(x) - \frac{ig}{\pi}\,\mathrm{tr}\left[\lambda_i \frac{\partial \mathscr{T}_+(x)}{\partial x_+}\,\mathscr{T}_+^{-1}(x)\right] = \sum_{A=1}^{M} j_{i-}^{(A)} \quad . \qquad (14)$$

Equations (8) together with (14) redefine the theory in terms of M massive quark flavors coupled to the field B_{i+}, which, according to the results of refs. [1] and [2], is also massive with a bare mass of $\mu_0 = g/\sqrt{2\pi}$.

The argument showing the last-mentioned property is most simply stated in terms of the gauge invariant field strength $G_i = \frac{1}{2}\,\partial_- B_{i+}$. Thus eq. (14) without the source terms $j^{(A)}$ may be put in the form

$$\left(\partial^2 + \frac{g^2}{2\pi}\right) G_i^{(b)}(x) = -g f_{ijk}\, B_{j+}^{(b)}(x)\; \partial_-\, G_k^{(b)}(x) \quad , \qquad (15)$$

where the superscript (b) has been used to indicate the turning off of the source terms in (14). As argued in refs. [1] and [2], for large $|x|$, the strong-coupling (i.e., the converse of the asymptotic freedom) regime prevails, in which case the right-hand side of (15), which is linear in g, is dominated by the mass term, thus revealing the bare mass given above.

The actual value of the mass acquired by the gauge field will of course differ from μ_0 owing to the effect of the sources in eq. (14). However, since the theory is finite, the magnitude of the resulting change is directly governed by the ratio of the quark masses $m^{(A)}$ to μ_0. Clearly, if $m^{(A)}/\mu_0 >> 1$, corresponding to weak coupling for the massive flavors, the modification to μ_0 will be small (and calculable perutrbatively). The resulting spectrum will contain a light gluon, massive quarks, and their loosely bound composites (i.e., mesons, baryons, and glue balls).

The converse regime, characterized by $m^{(A)}/\mu_0 << 1$, presumably gives rise to a significant modification of μ_0 but one which is bounded (and calculable in mass perturbation theory). Indeed, recalling the origin of μ_0 as the contribution of the massless flavor to the source currents, one can see that in this regime the remaining flavors will contribute magnitudes comparable to μ_0 since they may be considered nearly massless. Evidently, the resulting spectrum will include a heavy gluon (and its composites), quarks of smaller mass, and deeply bound composites of the latter with still smaller masses. An Abelian case, somewhat similar to this possibility, has been treated in the last work cited in ref. [7] as an example of imperfect confinement. It must be pointed out, however, that the intimate connection between the gluon mass the gauge coupling constant in the present theory is not shared by models which start with massive gluons ab initio or rely on fundamental scalars for symmetry breaking.

Notwithstanding the details of the resulting spectrum, the Schwinger mechanism of spontaneous symmetry breaking and acquisition of mass by the gauge field is seen to be a rather simple as well as univeral phenomenon within two-dimensional gauge theories. A necessary element, one which has played a crucial role in our treatment of both the Abelian and non-Abelian cases, is the massless fermion. Is the massless fermion actually necessary? To the extent that the gauge field mass generation proceeds through the anomaly in the fermion chiral current and this requires a massless fermion [8], the answer is yes. It is therefore probably (but not certainly) the case that QCD2(N) involving only massive flavors leaves the gauge field massless and confines color, as found in the 't Hooft model. In this respect, it may be stated that it is the very strong coupling of the massless flavor which breaks the local color symmetry and liberates the massive flavors in the present model.

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