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HIGHER ORDER QCD CORRECTIONS IN e^+e^- ANNIHILATION *

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ABSTRACT

Nonleading QCD corrections to e^+e^- annihilation into hadrons are computed. Comparison with experiment is briefly discussed.

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In quantum chromodynamics (QCD), processes which probe the structure of hadrons at short distances may be investigated using perturbation theory and the renormalization group.[1] The photon vacuum polarization tensor,

$$\Pi_{\mu\nu}(q) = i(q^2 g_{\mu\nu} - q_\mu q_\nu)\ \Pi(-q^2)$$

for q^2 large and spacelike, is one such short distance probe. $\Pi(-q^2)$ and $R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$ can be related through dispersion relations or smearing methods.[2] In regions between new quark thresholds, where the cross section is reasonably smooth, one may hope to obtain R directly from the discontinuity of $\Pi(-q^2)$.

The leading QCD corrections to $\Pi(-q^2)$ are well known,[3] and arise from the renormalization group improvement of the graphs of Fig. 1a. In this paper we report a calculation of $\Pi(-q^2)$ through order g^4, arising from the graphs of Fig. 1b. This calculation is necessary in order to determine if higher order corrections are small, and in order that one may compare the strong coupling constant determined from measurement of R with that measured in other processes, such as deep inelastic scattering. To address the first issue we will employ two renormalization schemes, the minimal (MS) scheme of 't Hooft and a modified scheme ($\overline{\text{MS}}$) due to Bardeen et al.[4] This latter scheme has been shown to lead to a more satisfactory perturbation series than MS in deep-inelastic and photon-photon scattering, and the same will be seen to be true here.

The problem of determining the strong coupling constant can be understood in terms of the mass, Λ, which is frequently used to parametrize the running coupling. The running coupling constant, $\alpha_s(-q^2)$, may be written,[4] graphs of Fig. 1b. Such a calculation is necessary for two reasons: first, to assure that higher order corrections are reasonably small, and second, to be able to make a meaningful comparison of the strong coupling constant determined from measurements of R to that measured in other processes, such as deep inelastic scattering. The first point can only be addressed once a renormalization procedure has been chosen, for there is a good deal of freedom in the definition of the coupling constant. We will present our results in the two renormalization schemes which have been used for the analysis of photon-photon and deep inelastic scattering, the minimal (MS) scheme of 't Hooft, and a modified scheme ($\overline{\text{MS}}$) due to Bardeen _et. al._[4] In these processes it was found that the $\overline{\text{MS}}$ scheme tended to give much smaller results for higher order corrections than the MS scheme, and we will see that the same is true here.

The importance of the second question can be understood in terms of the mass, Λ, which is frequently used to parameterize the running coupling constant of QCD. Neglecting quark masses, we may write for $\alpha_s(-q^2) = \bar{g}^2(-q^2)/4\pi$ in the one loop approximation

$$\alpha_s^o(-q^2) = \frac{g^2/4\pi}{1 + \beta_0\, g^2/16\pi^2\, \ell n(-q^2/\mu^2)}$$

$$\equiv \frac{4\pi}{\beta_0\, \ell n(-q^2/\Lambda^2)} \tag{1}$$

where β_0 is defined in Eq. (14.a). However, in terms of $\Lambda' = e^a\Lambda$ we have

$$\alpha_s^o(-q^2) = \frac{4\pi}{\beta_0\, \ell n(-q^2/\Lambda'^2)} - \frac{4\pi a}{\beta_0\, \ell n^2(-q^2/\Lambda'^2)} + \mathcal{O}\left(\frac{1}{\ell n^3(-q^2/\Lambda'^2)}\right) \tag{2}$$

We work in Feynman gauge and set all quark masses to zero. Integrals involving self-energy insertions were performed using spectral representations. The remaining integrals were performed by introducing Feynman parameters and performing the momentum integrals. Subdivergences in the resulting parameter integrals were treated by adding and subtracting from the integrand simpler functions with the same singularity structure, along the lines of Reference 8. This procedure yielded a finite integral which was evaluated numerically,[9] along with divergent integrals which were performed analytically.

For each diagram we obtained only the coefficient of $q_\mu q_\nu$, which can be identified with $q_\mu q_\nu - q^2 g_{\mu\nu}$ in gauge invariant sets of diagrams. The results are presented in Table I, where the coefficients of $1/\varepsilon^3$, $1/\varepsilon^2$ and $1/\varepsilon$ are given. The numerical errors in each diagram are less than 0.4%, though, due to large cancellations, the error in the sum is 2%.

Calling the sum of the graphs of Fig. 1b $\Pi_{un}^{(6)}$ we find

$$\Pi_{un}^{(6)} = \frac{\alpha}{\pi}\left(\frac{g^2}{4\pi^2}\right)^2 C_F\left[-\frac{\beta_0}{12\varepsilon^2}+\frac{B}{\varepsilon}+\mathcal{O}(1)\right]\Gamma\left(1+\frac{3\varepsilon}{2}\right)\left(4\pi\right)^{3\varepsilon/2}\left(-\frac{q^2}{\mu^2}\right)^{-3\varepsilon/2} \tag{4}$$

where

$$B = .0212\ C_F - .0506\ C_A + .00579\ N_f \tag{5}$$

In this expression, C_A and C_F are the quadratic Casimir operators for the adjoint and the fermion representation, respectively, and N_f is the number of quark flavors $\left(\text{for } SU(N),\ C_A = N,\ C_F = \frac{N^2-1}{2N}\right)$.

The mass, μ, is arbitrary, and is introduced to give the coupling constant correct dimensions in $4-\varepsilon$ dimensions. To take account of the QCD counterterms, we add to $\Pi_{un}^{(6)}$

$$\Pi_{CT}^{(6)} = \frac{\alpha}{\pi}\left(\frac{g^2}{4\pi^2}\right) Z\, C_F\left[-\frac{1}{4\varepsilon}+D+\mathcal{O}(\varepsilon)\right]\Gamma(1+\varepsilon)(4\pi)^{\varepsilon}\left(-\frac{q^2}{\mu^2}\right)^{-\varepsilon} \tag{6}$$

where

$$D = .0564 \tag{7a}$$

$$Z = -\frac{\beta_0}{2\varepsilon}\left(\frac{g^2}{4\pi^2}\right)(1+M\varepsilon) \tag{7b}$$

The term in brackets in Eq. (6) is the unsubtracted two-loop contribution. Z is the sum of all one-loop counterterms (the two-loop counterterms cancel). The prescription dependence of the calculation enters through the finite part of Z, which is arbitrary. In the minimal scheme, counterterms are introduced in each order so as to cancel only the pole parts of the divergent quantities. The $\overline{MS}$ scheme is defined by absorbing all factors of $\ln 4\pi - \gamma$, where γ is Euler's constant, into the renormalized coupling constant. For MS, M = 0, while for $\overline{\text{MS}}$, $M = (\ln 4\pi - \gamma)/2$. Before subtraction, we have, expanding $\Pi_R = \Pi^{(6)}_{un} + \Pi^{(6)}_{CT}$ in powers of ε,

$$\Pi_R = \left(\frac{\alpha}{\pi}\right)\left(\frac{g^2}{4\pi^2}\right)^2 C_F\left[\frac{\beta_0}{24\varepsilon^2} + \frac{8B+\beta_0(M-4D)}{8\varepsilon} - \frac{\beta_0}{32}\ln^2\left(-\frac{q^2}{\mu^2}\right)\right.$$
$$\left. + \ln\left(-\frac{q^2}{\mu^2}\right)\left(\frac{\beta_0}{16}(\ln 4\pi - \gamma - 2M + 8D) - \frac{3B}{2}\right) + \mathcal{O}(\varepsilon)\right] \tag{8}$$

The $\ln^2(-q^2/\mu^2)$ represents the explicit begining of the renormalization group improvement of Fig. 1a. For $C_A = 0$, $C_F = 1$, $N_f = 0$, this expression reproduces a result due to Rosner for QED.[9]

In both the MS and the $\overline{\text{MS}}$ schemes, the μ-dependence, and hence the scaling properties of the renormalized $\Pi(-q^2)$ are completely determined by $C_1(g^2)$, the residue of the simple pole in Equation (3). In either scheme,

$$\left[\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g}\right]\Pi(-q^2) = \frac{\partial}{\partial g^2}\left(g^2 C_1(g^2)\right) \quad . \tag{9}$$

Writing

$$C_1(g^2) = C_{10} + C_{11}g^2 + C_{12}g^4 + \ldots , \tag{10}$$

from our calculation and well-known QED results,[10] we have

$$C_{10} = -2/3 \tag{11a}$$

$$C_{11} = -1/4\, C_F\left(\frac{1}{4\pi^2}\right) \tag{11b}$$

$$C_{12} = C_F\left[B + \frac{\beta_0}{8}(M - 4D)\right]\left(\frac{1}{4\pi^2}\right)^2 \tag{11c}$$

If we assume that we may obtain R by taking the discontinuity of Π,[12] we obtain, specializing to SU(3),

$$R = \sum Q_i^2 \left(1 + \frac{\alpha_s(s)}{\pi} + \begin{cases} (7.35 - .442\, N_f) \left(\frac{\alpha_s(s)}{\pi} \right)^2 & \text{MS} \\ (1.98 - .115\, N_f) \left(\frac{\alpha_s(s)}{\pi} \right)^2 & \overline{\text{MS}} \end{cases} \right) \tag{11}$$

Thus just as in deep inelastic and photon-photon scattering the perturbation theory appears more satisfactory in the $\overline{\text{MS}}$ than in the MS scheme.[13]

In order to confront theory with the experimental value of R, several effects must be taken into account. To illustrate their relative importance, we work in the $\overline{\text{MS}}$ scheme, taking $\sqrt{s} = 6$ GeV, $\Lambda = .5$ GeV. This choice of Λ is motivated by recent analyses, including higher order QCD corrections, of deep inelastic scattering data.[4] Then, in order of decreasing importance, one must consider

(1) The lowest order result. With the presently accepted four quarks the contribution is $R_1 = 10/3$.

(2) The first QCD correction. Using α_s^o one gets a contribution to R, $R_2 = .32$.

(3) QED radiative corrections.[14] In the experimental analysis of R, account is taken only of radiation from the initial electrons and the electron loop contribution to vacuum polarization. Therefore, to the

theoretical prediction of R we must add vacuum polarization contributions from muons, taus, and hadrons, along with radiation from the quark lines. The last effect is negligible, but the vacuum polarization terms give $R_3 = .13$.

(4) Mass corrections. The first QCD correction is computed in the zero mass limit, and while this approximation is satisfactory for u, d, s, quarks, it is not valid for the charmed quark. Following the treatment of Poggio, Quinn, and Weinberg,[2] the effect of taking the charmed quark mass to be 1.5 GeV is $R_4 = .088$.

(5) Higher order QCD corrections. In $\overline{\text{MS}}$, although the correction due to the graphs of Fig. 1b is .047, inclusion of the effect of the 2 loop β function cancels the effect so that $R_5 = -.029$.

The smallness of R_5 relative to the other contributions to R is the most important feature of our calculation; higher order QCD corrections are small and make no qualitative change in the results obtained from the first order analysis.[15] Adding the above corrections gives R = 3.84 at $s = 36\ \text{GeV}^2$, to be compared with the experimental value[16] $R = 4.17 \pm .09 \pm .42$, where the first error is statistical and the second systematic. Clearly the data does not rule out the existence of an additional charge 1/3 quark or spinless boson, but the large systematic error prohibits a definite conclusion. The use of smearing techniques and dispersion relations is currently under study, but we do not expect that our conclusions will be qualitatively modified.

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	WEIGHT	$\frac{1}{\varepsilon^3}$	$\frac{1}{\varepsilon^2}$	$\frac{1}{\varepsilon}$
A	$2C_F^2$	-1/18	-4/27	-.1839
B	C_F^2	-1/18	-4/27	-.1978
C	$2C_F^2$	-1/36	-35/432	-.1226
D	$2C_F(C_F - C_A/2)$	1/18	29/216	.1642
E	$4C_F^2$	1/12	47/144	.3016
F	$C_F(C_F - C_A/2)$	0	1/9	-.0625
G	C_F^2	-1/9	-115/216	-.5757
H	$2C_F(C_F - C_A/2)$	-1/18	-59/216	-.0322
I	$2C_FC_A$	-1/12	-61/144	-.4240
J	$2C_FC_A$	1/12	43/144	.4714
K + L	C_FC_A	0	$\frac{2N_f - 5C_A}{36\,C_A}$	$.00679\ N_f/C_A - .0447$

Table 1. Pole terms from the graphs of Fig. 1b, where a common factor $\left[\frac{\alpha}{\pi}\left(\frac{g^2}{4\pi^2}\right)^2 \Gamma\left(1+\frac{3\varepsilon}{2}\right)(4\pi)^{3\varepsilon/2}\left(-\frac{q^2}{\mu^2}\right)^{-3\varepsilon/2}\right]$ has been taken out.

FIGURE CAPTIONS

Fig. 1a. Graphs whose discontinuity gives R to order g^2.

Fig. 1b. Graphs whose discontinuity gives R to order g^4.

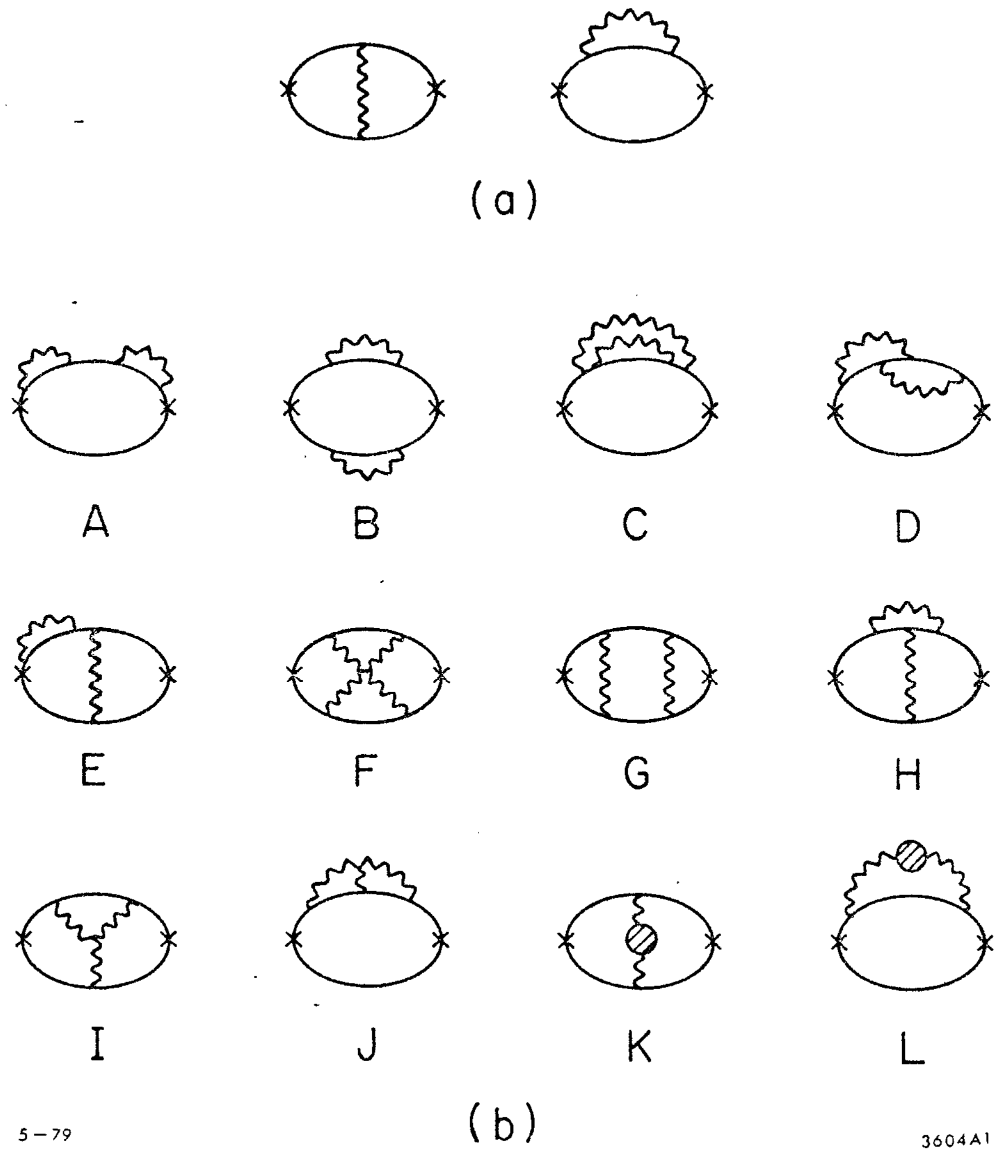

Fig. 1