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DIQUARK CONTRIBUTIONS TO SCALING VIOLATIONS AND TO σ_L/σ_T*

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ABSTRACT

Arguments for significant dynamical diquark substructure in baryons are reviewed. If present, integer-spin diquarks will absorb longitudinally polarized currents resulting in a relatively large value of $R = \sigma_L/\sigma_T$ in certain kinematic regions of deep-inelastic reactions. We provide simple parametrizations for this higher-twist contribution to structure functions and to R. We present fits to the x and Q^2 dependences of SLAC-MIT data on electroproduction. Further tests are suggested, and implications are discussed for the interpretation of R in perturbative QCD.

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I. INTRODUCTION

Our purpose is to discuss a simple model for $R = \sigma_L/\sigma_T$ in deep-inelastic lepton scattering processes. The model yields a relation between R and scaling violations which is different from that in standard perturbative quantum chromodynamics (QCD). If currents are absorbed by elementary spin one-half constituents of hadrons, and if transverse momenta are neglected, then σ_L is expected to be essentially zero. Contributions to R from kinematical and constituent transverse momentum effects, both from confinement and from perturbative QCD, have been studied in detail.[1] Definite predictions have been made,[2] and found wanting[3] especially in some kinematic regimes.

On the other hand, if there is an important diquark (qq) substructure in baryons, then the integer-spin diquark system will absorb longitudinally polarized currents, and the resulting R may be large in certain kinematic domains. Diquark scattering must be present at some level in any discussion of deep-inelastic scattering. Diquarks appear as a twist 6 term in the operator-product expansion. Their contribution to structure functions is proportional to $\left(\mu_d^2/Q^2\right)^2$, where μ_d is a mass parameter determined by properties of the nucleon. Simple physical arguments suggest the x dependence of these terms as well. If no dynamical diquark substructure exists one might expect $\mu_d \lesssim 1$ GeV. However, a "bound" diquark system of nonperturbative dynamical origin could lead to larger values of μ_d. To account for the large R values[4] measured at SLAC we will need a significant but not unreasonable diquark contribution. To set an upper bound on the size of this higher-twist term we fit SLAC-MIT data[5] on $F_2(x,Q^2)$ assuming that all of the scaling violation for $Q^2 \geq 3\,\text{GeV}^2$ and $x \geq 0.4$ comes from a higher twist-term of order $\left(\mu_d^2/Q^2\right)^2$.

Clearly, this sets an upper limit since other sources of scaling violation (such as the logarithmic scaling violation of QCD) are likely to be present. In this way we find that a value of $\mu_d \simeq 2$ GeV is consistent with F_2 data and leads to a reasonable fit to R in the large x region. Contributions to R in the low x region are likely to be dominated by QCD effects which are not considered here.

Theoretical reasons for a significant "bound" diquark substructure of a nonperturbative origin arise from several different points of view. First, in color SU(3), a diquark can be in an antisymmetric $\bar{3}_c$ or a 6_c. Nambu[6] and Lipkin[7] have argued that the $\bar{3}_c$ is more strongly bound than the 6_c. In a flavor-spin SU(6) theory, the diquark will be in a $(21)_{f-s}$ or a $(15)_{f-s}$. Fermi statistics then chooses the former. The symmetric $(21)_{f-s}$ decomposes into a flavor triplet, spin singlet and a flavor sextet, spin triplet. Since the higher mass decuplet is pure spin triplet, we will assume that the diquark sector of the baryon wave function is dominated by a $\bar{3}$ of color, a 3 of flavor, and is spin singlet, although spin triplet components may also be present. Such ideas have been extended to detailed questions of hadron spectroscopy.[8,9,10] The relevant literature can be traced from these papers. Incidentally, the quark-diquark baryon model has been used to give a natural explanation for the universality of the Regge trajectory slopes for baryons and mesons.[11] From the more abstract point of view of lattice gauge theory, Drell, Quinn and Weinstein[12] have argued recently for a dynamical diquark structure in a baryon. Similar conclusions have been reached on the basis of instanton arguments.[13] All of these arguments are essentially nonperturbative. The diquark seems to be more than just a useful bookkeeping device; it appears also to have dynamical relevance.

II. ANALYSIS

The standard definition of R is $R = \sigma_L/\sigma_T$, where σ_T and σ_L are total cross-sections for the scattering of transversely polarized and longitudinally polarized photons of mass Q^2. R may also be expressed in terms of the usual structure functions F_1 and F_2 which are functions of x and Q^2. A simple parton model calculation with elementary spin one-half constituents yields $F_2 = xF_1$ for large Q^2. In such a model R is nonzero only because of kinematic effects, and $R \propto m^2/Q^2$. Here m is the nucleon mass. Target mass effects[14] and final-state mass effects can substantially affect the coefficient of m^2/Q^2. Such a term is normally small, and it will be neglected in our analysis. This approximation should be kept in mind.

In a parton model calculation, the structure function F_2 is usually expressed approximately as $(1-x)^3$, with the power 3 provided by constituent counting rules.[15] In Ref. 16, the diquark contributions to the structure functions of the nucleon were discussed. In the simple model used, the diquark provides an additional non-scaling contribution to F_2 of the form

$$F_2^d(x,Q^2) = D(x)\ (d^2 + Q^2)^{-2} \quad , \tag{1}$$

where the Q^2 dependence arises from the diquark form factor. In a spinless model, $D(x) \propto (1-x)$ for x near 1, and thus at fixed Q^2 this term becomes dominant over the conventional quark scaling term near $x = 1$. D(x) was found to peak near $x \simeq 2/3$, so that the diquark most likely carries 2/3 of the total momentum of the nucleon.[17] If other sources of nonscaling are ignored, Eq. (1) (plus the scaling term) was found[16] to fit the electroproduction data for $x > 0.2$ and $Q^2 > 1-3\ (\mathrm{GeV})^2$ with $D(x) = 10x^2\,(1-x)$ and $d^2 = 2-1\ \mathrm{GeV}^2$. Sum rules involving F_2 were checked by Schmidt.[16] Motivated by this interpretation of scaling violations and by the importance[18] of higher-twist effects in $\pi N \to (\mu\bar{\mu})X$, we proceed here to analyze $R(x,Q^2)$.

In general there are both spin-zero and spin-one diquarks in the nucleon. A complete analysis must include the absorption of longitudinally polarized currents by both of these components, as well as transitions between them. These latter possibilities will give rise to a transverse cross section as well. In order to make definite predictions, the simplest possible models will be assumed, as described in the previous section. We write

$$\sigma_T = A(x)(1-x)^3 + D_T(x)[F(Q^2)]^t \quad , \tag{2}$$

$$\sigma_L = D_L(x)[F(Q^2)]^\ell \quad , \tag{3}$$

$$F(Q^2) = (d^2 + Q^2)^{-1} \quad , \tag{4}$$

and express the structure function $F_2(x,Q^2)$ as

$$F_2(x,Q^2) = \sigma_T + \sigma_L \quad .$$

The function $A(x)$ is slowly varying; the forms of $D_T(x)$ and $D_R(x)$, as well as the powers t and ℓ are specified below. We shall present two models which provide an adequate fit to the SLAC-MIT electroproduction data. If gluonic radiation terms associated with perturbative QCD were included, an additional term would appear in the equation for σ_L, and various terms, such as $A(x)$ and $D(x)$, would develop explicit dependence on log Q^2. We ignore QCD effects in this paper. Specific calculations[2] indicate that they contribute to R principally at small x, whereas our diquark contribution dominates at larger x.

In the two models discussed below, the power behaviors of $D_T(x)$ and $D_L(x)$ are well specified as $x \to 1$. However, the behaviors as $x \to 0$ are not determined. We choose to write $D(x) \propto x^r$ for small x. In our fits to the data, best values of r were found near the integer values which we list. In the form factor $F(Q^2)$ in Eq. (4), we set[19] $d^2 = 2\ \mathrm{GeV}^2$, but acceptable fits may also be obtained with $d^2 = 1\ \mathrm{GeV}^2$. We limit our attention to values of $Q^2 \geq 3\ \mathrm{GeV}^2$, and $x \geq 0.4$.

Model 1 - Massless Spin 1/2 Quarks

An explicit calculation analogous to that in Ref. 18 may be carried out to obtain the structure functions for a spin-zero diquark in a spin-1/2 baryon. Single gluon exchange between the diquark system and the free quark in the nucleon is used to describe the far off-shell behavior of the diquark system for large x. Expressing the results of this calculation in terms of Eqs. (2) and (3), we obtain $D_L(x) = d_L x^2 (1-x)^2$ and $D_T(x) = d_T x^2$, with $\ell = 2$ and $t = 3$. In the model, d_T and d_L are related by the expression $(d_T/d_L) = 1/2\ \langle k_T^2 \rangle$, where $\langle k_T^2 \rangle$ is the mean squared transverse momentum of the diquark system in the nucleon. The large x behavior of $D_L(x)$ and $D_T(x)$ is specified by the model, whereas the factor x^2 is introduced by hand, guided by the fit to the data. A good fit to $F_2(x,Q^2)$ and $R(x,Q^2)$ is obtained with $d_L \simeq 15\ \mathrm{GeV}^4$, $d_T \simeq 5\ \mathrm{GeV}^6$, and $A(x) \simeq 2(1+x)/3$. These parameters imply that $\langle k_T^2 \rangle \approx 0.7\ \mathrm{GeV}^2$. This value is consistent with our expectations and is similar to that deduced from fits to the pion structure function.[18]

Since the errors on R are relatively large, the parameters of the fit are determined largely by the data on $F_2(x,Q^2)$. We obtain values of chi-squared comparable to the number of degrees of freedom. For a few selected values of Q^2, our fit to $F_2(x,Q^2)$ is shown in Fig. 1. The data on R and our fit are shown in Fig. 2.

Model 2 - Simplified Counting Rules

In the model developed in Ref. 16, one predicts $\ell = t = 2$, $D_L(x) = d_L x(1-x)$ and $D_T(x) = d_T x(1-x)$. Our fit to the electroproduction data for $x \geq 0.4$ and $Q^2 \geq 3$ GeV^2 yields $d_L \simeq d_T \simeq 4$ GeV^4. The function $A(x)$ is parameterized conveniently as $[1+5\ (x-0.6)^2]^{-1}$. The fit is again determined largely by the data of $F_2(x,Q^2)$. The results of this model are also shown in Fig. 2.

Differences are apparent in Fig. 2 between the two models, especially at small Q^2 and at very large x. The data appear to be more constant in x and Q^2 than our expectations. However, what we judge to be important is that the two models reproduce the magnitude of R reasonably well at large x and at low Q^2. This success is associated in part with the fact that the quark and diquark structure functions have different dependences on $(1-x)$ as $x \to 1$. The magnitude of R in our models is related to the size of the non-scaling contribution to F_2.

Using our parameters, we can estimate from Eq. (2) the relative probabilities for striking a quark or a diquark. At $Q^2 = 2$ GeV^2 and $x = 0.5$, we find that these probabilities are about equal, for both models. As Q^2 grows, the chance of striking a diquark falls rapidly, governed by the form factor, Eq. (4). The relatively large size of our diquark term suggests that there is substantial diquark substructure in the nucleon.

In general, nonscaling contributions to F_1 and F_2 due to QCD gluonic radiation and intrinsic transverse momentum effects should also be included in our expressions, especially at small x. To isolate such fundamental effects at larger x, it is necessary first to subtract any diquark contribution. In any case, the (qq) contribution is of considerable physical interest by itself. As outlined in the first section, it is important for our understanding of hadrons to know if such a nonperturbative contribution is present with the expected size and dependence on x and Q^2.

III. COMMENTS

A few brief remarks are perhaps relevant.

(1) In general, R is a function of x and Q^2. The deep-inelastic cross section for leptoproduction depends on x, Q^2 and y. One can only be confident of R by extracting it from the y dependence at fixed x and Q^2. This requires combining data from different energies with known relative normalization. Otherwise, some care is required to avoid misleading conclusions. In particular, we note that

(a) If the average x grows with Q^2, R may also increase with Q^2 if a diquark contribution is present.

(b) Integrations over x are a different test from examining the distributions at fixed x. For example, at x = 3/4 and $Q^2 = 20\ \mathrm{GeV}^2$, $R \simeq 0.10$, whereas if one integrates over x, large values are not obtained for the ratio

$$\bar{R} = \frac{\int dx\, \sigma_L}{\int dx\, \sigma_T}$$

Although our models do not apply at small x, for purposes of illustration we integrate over the full interval $0 \le x \le 1$ and obtain $\overline{R} = 0.005$ at $Q^2 = 20\ \mathrm{GeV}^2$ (for Model 1). Thus binning of data may allow one to observe or to exclude significant effects.

(c) The diquark contributions to R fall with Q^2 at essentially all x. Confusion with a QCD effect is possible for $x > 1/3$.

(2) In ν and $\bar{\nu}$ reactions, F_3 measures the difference of absorption of left- and right-handed currents. A significant contribution from spin-one diquarks may show up in F_3 with the x and Q^2 dependence given in Eq. (2). Recently it has been shown[20] that although data on F_3 are consistent with perturbative QCD calculations, they are also consistent with the presence of significant higher-twist contributions such as those from a diquark system.

For charged-lepton scattering the absorption of photons is proportional to the charge squared. Spin-zero diquarks (ud) have charge (1/3) while spin-one diquarks (uu) have charge (4/3). Consequently, the effect of the spin-one component is enhanced in this case. Scaling violations due to diquark dynamics will be different in e,μ and in $\nu,\bar{\nu}$ data (especially at the larger x values).

(3) An independent test of any diquark contribution is the identification of fast baryons in the photon fragmentation region[16] with the expected dependence on x and Q^2. In the symmetric quark model, the Λ contains spin-zero diquarks only and the Σ only spin-one. The relative yield of these provides information on the diquark spin. Ordinary quark processes and QCD will also yield final-state baryons from the decay of the recoil quark, but the baryon's momentum (z) and Q^2 dependence will be very different from that described here. Properties of diquark jets in hadron reactions are studied in Ref. 21.

(4) The ratio of the average diquark $(\text{charge})^2$ for the neutron to that of the proton is 1/3, whereas the corresponding ratio of the quark $(\text{charge})^2$ is 2/3. A careful analysis of the nonscaling behavior of the neutron vs. the proton structure function can provide further tests of the model. If diquarks are important, the ratio F_2^n / F_2^p should drop from 2/3 towards 1/3 as $x \to 1$, as appears to be true in the data.[5] A relativistic model of the deuteron was developed in Ref. 16, and should be used in the extraction of the neutron distributions.

(5) In neutrino processes, for example, the final state hadron (say, π) distribution will receive contributions both from quark scattering and decay, and from diquark scattering and decay. We may express the cross-section as a sum of these two terms:

$$\frac{d^2\sigma}{dxdz} \propto P_{q/h}(x)\, D_{\pi/q}(z) \;+\; P_{d/h}(x,Q^2)\, D_{\pi/d}(z) \quad . \tag{6}$$

We have assumed here that the scattering (P) and decay (D) processes factorize. Nevertheless, the sum in Eq. (6) does not factorize. Our analysis of F_2 and R suggests that the two terms in Eq. (6) are of comparable strength at modest Q^2. Their effects in Eq. (6) may be separated by a detailed study of the x and Q^2 dependences of $d^2\sigma/dxdz$. An attempt should be made to subtract the diquark contribution from the data before conclusions are reached on the possible non-factorizing nature of the quark term in Eq. (6).

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REFERENCES

1. For a review, see O. Nachtmann, in Proceedings of the 1977 International Conference on Lepton and Photon Physics at High Energies, Hamburg, ed. F. Gotbrod; R. E. Taylor in Proceedings of the 19th International Conference on High Energy Physics, Tokyo, 1978, ed. S. Homma, M. Kawaguchi and H. Miyazawa; and E. Gabathuler, Tokyo, 1978, ibid.
2. For computations of R in perturbative QCD see, e.g., G. C. Fox, Nucl. Phys. B131, 107 (1977), and B134, 269 (1978); K. J. Kim, Phys. Lett. 76B, 95 (1978); and A. DeRujula, H. Georgi and H. D. Politzer, Ann. of Phys. (N.Y.) 103, 315 (1977).
3. This comparison was made in spite of the counsel that since the prediction for R is "one of the cleaner predictions of the theory," it is certainly more accurate than the data and should be used "when processing unseparated data," A. DeRujula, et al., Ref. 2.
4. M. D. Mestayer, SLAC-Report No. 214 (Aug. 1978)(Ph.D Dissertation).
5. J. S. Poucher et al., Phys. Rev. Letters 32, 118 (1974), and SLAC-PUB-1309 (unpublished); E. M. Riordan, et al., Phys. Rev. Letters 33, 561 (1974); E. M. Riordan, MIT LNS Report No. COO-3069-176, MIT Ph.D Thesis (unpublished); W. B. Atwood et al., Phys. Letters 64B, 497 (1976); W. B. Atwood, SLAC Report No. SLAC-185, Stanford Ph.D. thesis, 1975 (unpublished); A. Bodek, et al., SLAC-PUB-2248 (1979), submitted to Phys. Rev. D; A. Bodek, MIT LNS Report No. COO-3069-116, MIT Ph.D. Thesis (unpublished). We have used a compilation of these data made by W. B. Atwood.

6. Y. Nambu, Preludes in Theoretical Physics, ed. A. deShalit, H. Feshbach and L. Van Hove, 1966.

7. H. Lipkin, Phys. Rev. Lett. 45B, 267 (1973).

8. J. D. Bjorken, unpublished manuscript, November 1974.

9. R. E. Cutkosky and R. E. Hendrick, Phys. Rev. D16, 2902 (1977).

10. A. Hendry and I. Hinchliffe, Phys. Rev. D18, 3453 (1978).

11. G. F. Chew and C. Rosenzweig, Phys. Rept. 41C, 263 (1978), and references therein.

12. S. D. Drell, H. Quinn and M. Weinstein, private communication and SLAC report in preparation.

13. R. Carlitz and D. Creamer, University of Pittsburgh preprint (PITT-208).

14. H. Georgi and H. D. Politzer, Phys. Rev. D14, 1829 (1976).

15. S. Brodsky and R. Blankenbecler, Phys. Rev. D10, 2973 (1974).

16. I. Schmidt and R. Blankenbecler, Phys. Rev. D16, 1318 (1977); I. Schmidt, SLAC Report No. SLAC-203, (1977) Ph.D. Thesis submitted to Stanford University; A. Fernandez-Pacheco, J. A. Grifols and I. A. Schmidt, Lett. Nuovo Cimento 22, 339 (1978).

17. Some of the aspects of diquarks in deep inelastic scattering have been studied from a very different point of view by S. Ono, Phys. Rev. D10, 3124 (1974) and by M. I. Pavkovic, Phys. Rev. D13, 2128 (1976).

18. E. L. Berger and S. J. Brodsky, Phys. Rev. Lett. 42, 940 (1979). The prediction of the higher-twist $\sin^2\theta$ behavior in $\pi N \to (\mu\mu) X$ at large x_F has been verified experimentally with the size expected. Private communication from J. Pilcher, Chicago—Illinois—Princeton collaboration.

19. One might expect intuitively that $d^2 \sim 1$ GeV^2 for a hadronic form factor. However, transitions from spin-zero to spin-one diquarks will also occur with form factors that behave as $Q^2(1+Q^2/d^2)^{-2}$; thus when the total Q^2 dependence is fit by a monopole form, a larger effective value of d is required to fit the smaller Q^2 values. In Ref. 16, a value of $d^2 = 2$ GeV^2 was found to fit down to $\bar{Q}^2 \sim 1$, where as a value of $d^2 = 1$ GeV^2 fits only down to $Q^2 \sim 3$. We adopt the larger value.

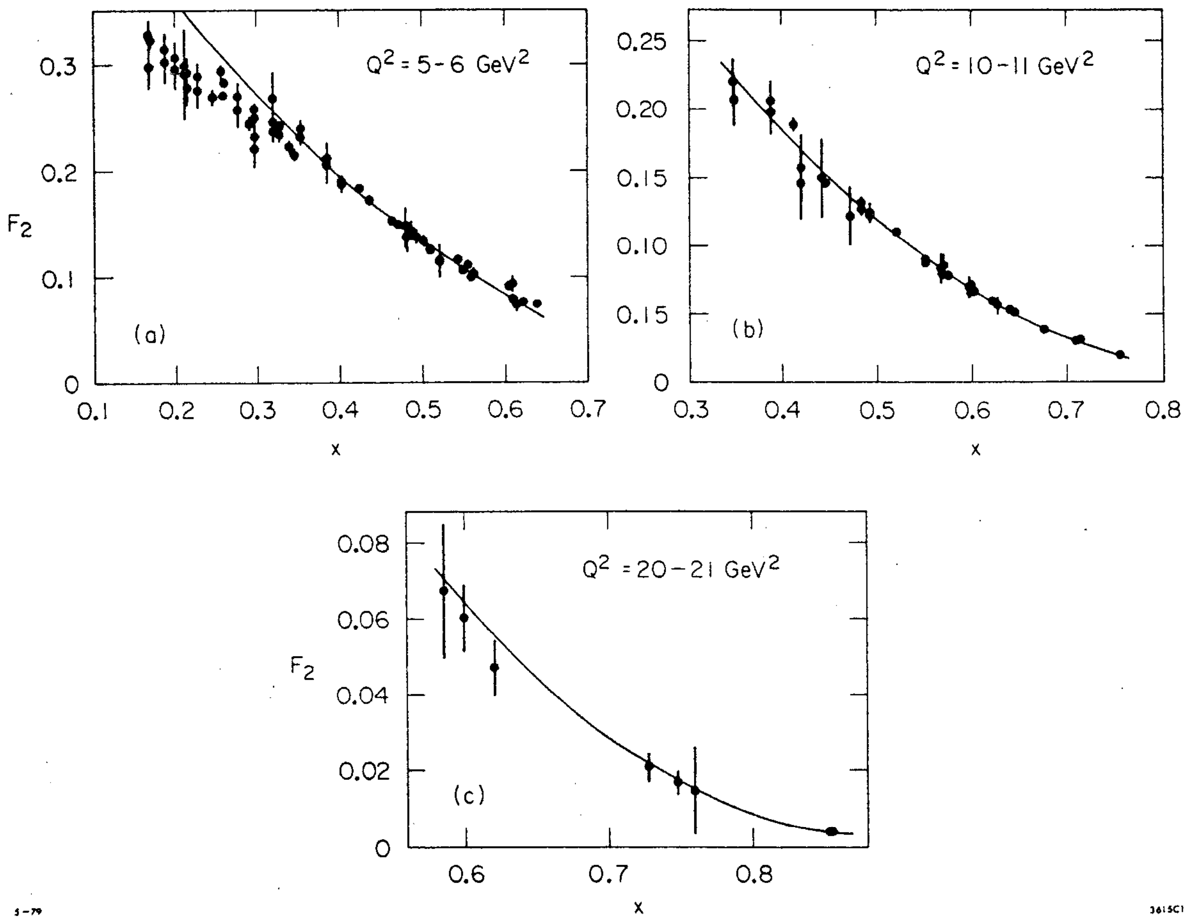
20. L. F. Abbott and R. M. Barnett, SLAC-PUB-2325, submitted to Ann. of Phys. (1979).

21. J. Gunion, Phys. Rev. D10, 3684 (1974); M. Fontannaz, B. Pire and D. Schiff, Phys. Lett. 77B, 315 (1978); S. J. Brodsky, in Color Symmetry and Quark Confinement, Moriond meeting, 1977, ed., Tran Thanh Van.

FIGURE CAPTIONS

1. Shown are SLAC-MIT data (Refs. 4 and 5) on $F_2(x,Q^2)$ for three intervals in Q^2. The solid line is our fit to these data with Model 1, described in the text. The fit is done for $x > 0.4$ and $Q^2 > 3$ GeV^2, but our solid line is extended below $x = 0.4$, nevertheless. No significance should be attached to any discrepancy below $x = 0.4$.

2. Data (Ref. 4) on $R(x,Q^2)$ are shown as a function of x for six values of Q^2. The solid line is obtained from Model 1, and the dashed curve from Model 2.



$Q^2 = 5-6$ GeV^2
(a)
F_2
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
x
$Q^2 = 10-11$ GeV^2
(b)
0.25
0.20
0.15
0.10
0.15
0
0.3
0.4
0.5
0.6
0.7
0.8
x
$Q^2 = 20-21$ GeV^2
(c)
F_2
0.08
0.06
0.04
0.02
0
0.6
0.7
0.8
x
5-79
3615C1

Fig. 1

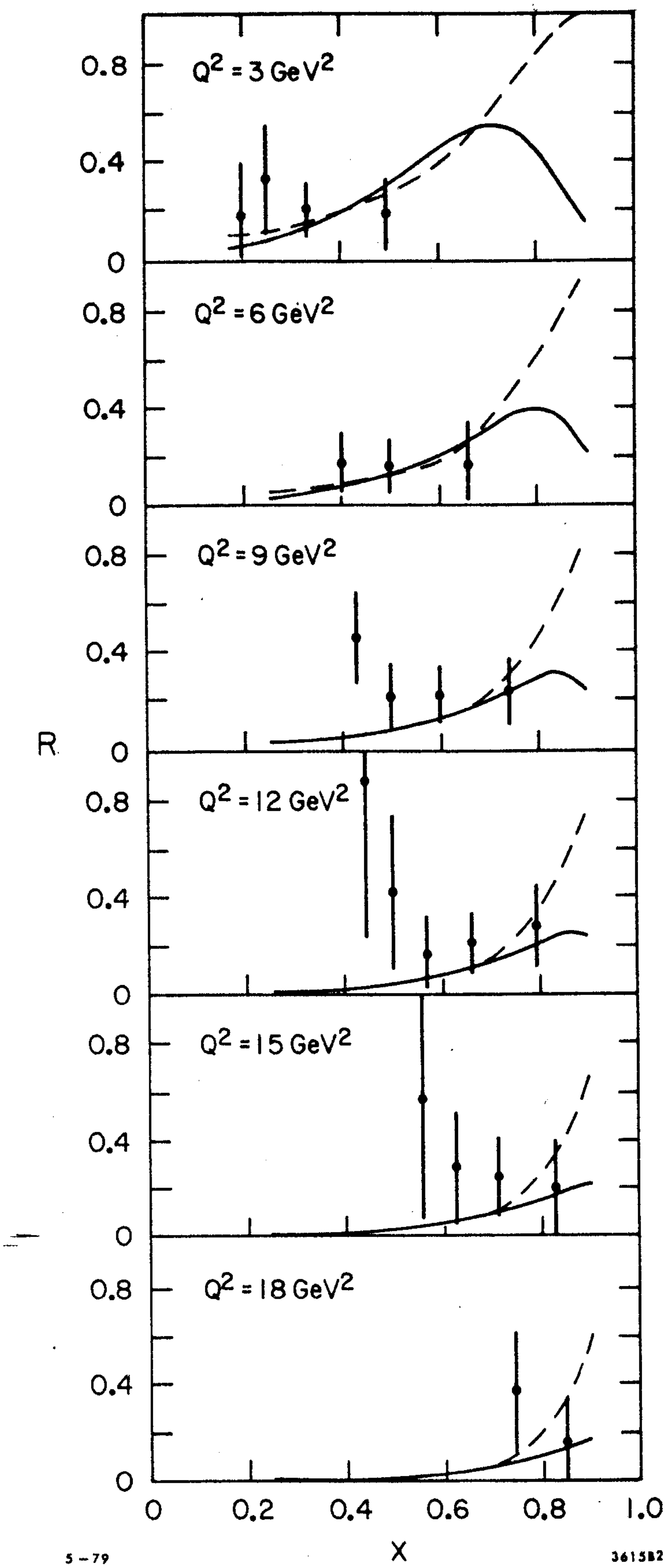

Q² = 3 GeV²
Q² = 6 GeV²
Q² = 9 GeV²
Q² = 12 GeV²
Q² = 15 GeV²
Q² = 18 GeV²
R
0.8
0.4
0
0
0.2
0.4
0.6
0.8
1.0
X
5-79
3615B2

Fig. 2