

MESON RADIATIVE DECAYS*

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ABSTRACT

The status of decays of the kind $V \rightarrow P\gamma$ and $P \rightarrow V\gamma$ is reviewed with special emphasis on the work done by the authors in this field. The low experimental value of $\Gamma(\rho \rightarrow \pi\gamma)$ remains the outstanding problem. The latest preliminary numbers from a Fermi Laboratory experiment go in the right direction but not far enough.

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1. Introduction

In this paper only the decays of the kind $V \rightarrow P\gamma$ and $P \rightarrow V\gamma$ will be considered. Among the older generation of mesons (made up of u, d, and s quarks) there are eleven measurable rates. These are: $\Gamma(\omega \rightarrow \pi\gamma)$, $\Gamma(\omega \rightarrow \eta\gamma)$, $\Gamma(\eta' \rightarrow \omega\gamma)$, $\Gamma(\eta' \rightarrow \rho\gamma)$, $\Gamma(\phi \rightarrow \pi\gamma)$, $\Gamma(\phi \rightarrow \eta\gamma)$, $\Gamma(\phi \rightarrow \eta'\gamma)$, $\Gamma(\rho \rightarrow \pi\gamma)$, $\Gamma(\rho \rightarrow \eta\gamma)$, $\Gamma(K^{*\pm} \rightarrow K^\pm\gamma)$ and $\Gamma(K^{*0}(\bar{K}^{*0}) \rightarrow K^0(\bar{K}^0)\gamma)$. Of these eleven rates unambiguous measurements exist on the following: $\Gamma(\omega \rightarrow \pi\gamma)$ ¹, $\Gamma(\rho \rightarrow \pi\gamma)$ ², $\Gamma(\phi \rightarrow \pi\gamma)$ ¹, $\Gamma(\phi \rightarrow \eta\gamma)$ ¹ and $\Gamma(K^{*0} \rightarrow K^0\gamma)$ ³. See the last column of Table I. There are measurements⁴ on $\Gamma(\omega \rightarrow \eta\gamma)$ and $\Gamma(\rho \rightarrow \eta\gamma)$ up to an ambiguity of a phase. This arises from the coherent photo-production of ω and ρ . The smaller solution (see the last column of Table I) comes from the assumption of constructive interference and the larger solution from the assumption of destructive interference. The measurements of $\Gamma(\eta' \rightarrow \rho\gamma)/\Gamma(\eta' \rightarrow \omega\gamma)$ by Zanfino et al.⁵ (see the last column of Table I) proves to be an important constraint. Only upper bounds¹ exist on $\Gamma(K^{*+} \rightarrow K^+\gamma)$, $\Gamma(\eta' \rightarrow \omega\gamma)$, and $\Gamma(\eta' \rightarrow \rho\gamma)$. There is no information on $\Gamma(\phi \rightarrow \eta'\gamma)$.

In the charm sector the following measurements exist:⁶ $\Gamma(\psi \rightarrow \pi\gamma) = 5 \pm 3.2$ eV, $\Gamma(\psi \rightarrow \eta\gamma) = 55 \pm 12$ eV, and $\Gamma(\psi \rightarrow \eta'\gamma) = 152 \pm 117$ eV. There is an upper bound on $\Gamma(\psi \rightarrow \eta_c\gamma) < 3.5$ eV.⁷ There are no measurements of $\Gamma(D^{*\pm} \rightarrow D^\pm\gamma)$, $\Gamma(D^{*0} \rightarrow D^0\gamma)$ or $\Gamma(F^{*\pm} \rightarrow F^\pm\gamma)$.

Throughout this paper the following mixing convention will be used.

$$|\omega\rangle = \sin\theta_v |8\rangle + \cos\theta_v |0\rangle$$

$$|\phi\rangle = \cos\theta_v |8\rangle - \sin\theta_v |0\rangle$$

$$\text{For ideal mixing } \tan\theta_v = \frac{1}{\sqrt{2}}$$

$$|\eta'\rangle = \sin\theta_p |8\rangle + \cos\theta_p |0\rangle$$

$$|\eta\rangle = \cos\theta_p |8\rangle - \sin\theta_p |0\rangle \quad (1.1)$$

We use $\theta_p = -10^\circ$.

The organization of this paper is as follows. In Section 2 we discuss the nonet symmetry scheme and its implications. In Section 3 we discuss the symmetry breaking schemes. In Section 4 we extend the scheme to the charm sector and end with conclusions in Section 5.

2. Nonet Symmetry

Consider a decay $V \rightarrow P\gamma$ where both the vector and the pseudoscalar mesons may be mixed states of singlets and octets. In general, we can maintain $SU(3)$ symmetry by assuming a single coupling constant where a vector octet ($V^{(8)}$) couples to a pseudoscalar octet ($P^{(8)}$) and a photon. We are still at liberty to introduce two other coupling constants; one where a vector singlet ($V^{(0)}$) couples a pseudoscalar octet and a photon and the other where a vector octet couples to a pseudoscalar singlet ($P^{(0)}$) and a photon. Thus maintaining $SU(3)$ symmetry one has an effective lagrangian for $V \rightarrow P\gamma$,

$$\begin{aligned} \mathcal{L}_{VP\gamma} = & \epsilon^{\mu\nu\rho\sigma} \left[g_0 \text{Tr} \left(\left\{ \partial_\mu V_\nu^{(8)}, \partial_\rho A_\sigma \right\} P^{(8)} \right) \right. \\ & + g_1 \text{Tr} \left(\left\{ \partial_\mu V_\nu^{(0)}, \partial_\rho A_\sigma \right\} P^{(8)} \right) \\ & \left. + g_2 \text{Tr} \left(\left\{ \partial_\mu V_\nu^{(8)}, \partial_\rho A_\sigma \right\} P^{(0)} \right) \right] \end{aligned} \quad (2.1)$$

The anti-commutator results from invoking charge conjugation invariance and

$$\begin{aligned} V^{(8)} &= \frac{1}{\sqrt{2}} \sum_{i=1}^8 \lambda_i V_\mu^i \\ V^{(0)} &= \frac{1}{\sqrt{2}} \lambda_0 V_\mu^0 \end{aligned} \quad (2.2)$$

Similarly for $P^{(8)}$ and $P^{(0)}$. A_μ is the electromagnetic field. Nonet symmetry implies $g_0 = g_1 = g_2$. If nonet symmetry is invoked the above lagrangian takes a simple form⁸

$$\mathcal{L}_{VP\gamma} = \epsilon^{\mu\nu\rho\sigma} g_0 \text{Tr} \left(\left\{ \partial_\mu V_\nu, \partial_\rho A_\sigma \right\} P \right) \quad (2.3)$$

where singlets are now included by defining

$$V_\mu = \frac{1}{\sqrt{2}} \sum_{i=0}^8 \lambda_i V_\mu^i \quad (2.4)$$

and similarly for P . The above lagrangian has a piece $V^0 P^0 \gamma$ which was absent in Eq. (2.1). This piece makes no contribution if the photon does not have a singlet piece. With the lagrangian of Eq. (2.3) one gets,

$$\begin{aligned} \Gamma(V^m \rightarrow P^i \gamma) &= \frac{(g_0 d_{\min})^2}{96\pi} \left(\frac{M_m^2 - M_i^2}{M_m} \right)^3 \\ \Gamma(P^i \rightarrow V^m \gamma) &= \frac{(g_0 d_{\min})^2}{32\pi} \left(\frac{M_i^2 - M_m^2}{M_i} \right)^3 \end{aligned} \quad (2.5)$$

where m and i are the internal symmetry labels of V and P respectively and $n = (3) + \frac{1}{\sqrt{3}}(8)$ is the internal symmetry label of the photon.

d_{\min} is the usual symmetric $SU(3)$ structure function.

If nonet symmetry and ideal vector mixing are assumed then the following predictions result.

$$\begin{aligned} \Gamma(\omega \rightarrow \pi\gamma) / \Gamma(\rho \rightarrow \pi\gamma) &\approx 9 \\ \Gamma(\phi \rightarrow \pi\gamma) &= 0 \quad (0Z1 - \text{rule}) \\ \Gamma(K^{*0} \rightarrow K^0\gamma) / \Gamma(\omega \rightarrow \pi\gamma) &\approx 0.24 \\ \Gamma(K^{*0} \rightarrow K^0\gamma) / \Gamma(K^{*+} \rightarrow K^+\gamma) &\approx 4 \end{aligned} \quad (2.6)$$

These are also the predictions of a quark model calculation in the limit $m_u = m_d = m_s$. For the nonet symmetry and ideal θ_v predictions see column 2 of Table I.

The main points to notice are that the measurements of $\Gamma(\rho \rightarrow \pi\gamma)$, $\Gamma(K^{*0} \rightarrow K^0\gamma)$, and $\Gamma(\phi \rightarrow \eta\gamma)$ are too low (roughly by a factor $2\frac{1}{2}$) compared to the nonet symmetry predictions. Nonet symmetry does well in predicting the Zanfino et al.⁵ result. We shall see later that $\Gamma(K^{*0} \rightarrow K^0\gamma)$ and $\Gamma(\phi \rightarrow \eta\gamma)$ pose no particular theoretical problems but it is hard to understand $\Gamma(\rho \rightarrow \pi\gamma)$ and the Zanfino et al.⁵ experiment simultaneously.

3. Symmetry Breaking Schemes

One of the ways to break nonet symmetry is to use the effective lagrangian of Eq. (2.1).⁹ The constant g_0 governs the two rates $\Gamma(\rho \rightarrow \pi\gamma)$ and $\Gamma(K^{*0} \rightarrow K^0\gamma)$. Either rate could be used to determine g_0 . Also since both these rates appear to be on the lower side of the nonet prediction by about the same factor it makes little difference which rate is used. $\Gamma(\omega \rightarrow \pi\gamma)$ and $\Gamma(\phi \rightarrow \pi\gamma)$ involving mixing in the vector meson are governed by g_0 and g_1 . One could therefore use $\Gamma(\omega \rightarrow \pi\gamma)$ to determine g_1 and have a prediction on $\Gamma(\phi \rightarrow \pi\gamma)$. This last prediction comes out too large.⁹ One could then use $\Gamma(\phi \rightarrow \eta\gamma)$ to pin down the coupling constant g_2 . The bad prediction for $\Gamma(\phi \rightarrow \pi\gamma)$ can be overcome by varying the mixing angle θ_v . In the third column of Table I we show the best fit we obtained in this model. The value of θ_v was 24° . Note that the Zanfino et al.⁵ ratio is predicted to be too high and $\Gamma(\rho \rightarrow \eta\gamma)$ too low. The model favors the higher solution for $\Gamma(\omega \rightarrow \eta\gamma)$.

More involved symmetry breaking schemes have been proposed by us in a series of papers.¹⁰ A scheme that allows us to break SU(3) symmetry is to use an I=0, Y=0 scalar spurion, U_8 , which transforms like λ_8 , i.e., consider $V \rightarrow P + \gamma + U_8$. If one then writes the most general effective lagrangian which has (i) charge conjugation invariance and (ii) nonet symmetry (i.e., terms of kind $\text{Tr}(p^0)$ or $\text{Tr}(\partial_\mu V_\nu^0)$ are disallowed) one gets

$$\begin{aligned}
 \mathcal{L}_{VP\gamma} = & \epsilon^{\mu\nu\rho\sigma} \left[f_0 \text{Tr} \left(\left\{ \partial_\mu V_\nu, \partial_\rho A_\sigma \right\} P \right) \right. \\
 & + f_1 \left\{ \text{Tr} \left(\partial_\mu V_\nu \partial_\rho A_\sigma P \lambda_8 \right) + \text{Tr} \left(\partial_\mu V_\nu \lambda_8 P \partial_\rho A_\sigma \right) \right\} \\
 & + f_2 \left\{ \text{Tr} \left(\partial_\mu V_\nu P \partial_\rho A_\sigma \lambda_8 \right) + \text{Tr} \left(\partial_\mu V_\nu \lambda_8 \partial_\rho A_\sigma P \right) \right\} \\
 & + f_3 \left\{ \text{Tr} \left(\partial_\mu V_\nu \partial_\rho A_\sigma \lambda_8 P \right) + \text{Tr} \left(\partial_\mu V_\nu P \lambda_8 \partial_\rho A_\sigma \right) \right\} \\
 & + f_4 \text{Tr} \left(\partial_\mu V_\nu \partial_\rho A_\sigma \right) \text{Tr} \left(P \lambda_8 \right) \\
 & + f_5 \text{Tr} \left(\partial_\mu V_\nu P \right) \text{Tr} \left(\partial_\rho A_\sigma \lambda_8 \right) \\
 & \left. + f_6 \text{Tr} \left(\partial_\mu V_\nu \lambda_8 \right) \text{Tr} \left(\partial_\rho A_\sigma P \right) \right] \quad (3.1)
 \end{aligned}$$

If one invokes boson symmetry, i.e., photon interacting like hadrons in vector meson dominance then $f_1 = f_3$ and $f_5 = f_6$. One, therefore, gets down to five parameters. The matrix element obtained from $\mathcal{L}_{VP\gamma}$ of Eq. (3.1) has the internal symmetry structure (with boson symmetry) for $V^m \rightarrow P^i_\gamma$,

$$\begin{aligned}
 g_{\text{min}} = & f_0 d_{\text{min}} \\
 & + (2f_1 + f_2) (d_{8nk} d_{kim} + d_{8mk} d_{kin} - d_{8ik} d_{kmn}) \\
 & - 2f_1 (d_{8nk} d_{kim} + d_{8mk} d_{kin} - 2d_{8ik} d_{kmn}) \\
 & + f_4 \delta_{8i} \delta_{mn} + f_5 (\delta_{8n} \delta_{im} + \delta_{8m} \delta_{in}) \quad (3.2)
 \end{aligned}$$

where $k = 0, \dots, 8$.

It turns out that the coefficient of f_1 in Eq. (3.2) is zero for all rates except $K^{*+} \rightarrow K^+\gamma$. Thus if we do not fit this rate (it has not yet been measured) we need only four parameters if θ_v is fixed to be ideal.

In Table II column 2 shows the best fit with our 4 parameter model. Notice that $\Gamma(\rho \rightarrow \pi\gamma)$ stays high. Zanfino et al.⁵ ratio is predicted fairly well. $\Gamma(\rho \rightarrow \eta\gamma)$ and $\Gamma(\omega \rightarrow \eta\gamma)$ favor the lower solution but are too low.

The reason for our inability to fit $\Gamma(\rho \rightarrow \pi\gamma)$ in the four parameter model is the following constraint which is particular to this model,

$$g_{\rho\pi\gamma} = \frac{1}{\sqrt{3}} \left(\sin\theta_v g_{\omega\pi\gamma} + \cos\theta_v g_{\phi\pi\gamma} \right) \quad (3.3)$$

As $g_{\phi\pi\gamma}$ is small ($\Gamma(\phi \rightarrow \pi\gamma) \approx 6$ KeV) it is very difficult to get away from the relation (for θ_v ideal),

$$g_{\rho\pi\gamma} \approx \frac{1}{3} g_{\omega\pi\gamma} \quad (3.4)$$

Also in the simplest model with only one parameter g_0 (the first term of Eq. (3.1)) the Zanfino et al.⁵ ratio is predicted to be = 11 independent of θ_p provided θ_v is ideal. Thus the Zanfino et al.⁵ ratio prefers no symmetry breaking and a θ_v close to the ideal value.

One way to relax the model is to give up boson symmetry. This can be implemented in the extended vector meson dominance by allowing both ρ and ρ' to couple to the photon. Relaxing boson symmetry releases two parameters but an identity, $d_{8ik} d_{kmn} = d_{8mk} d_{kin}$ ($k = 0, \dots, 8$), reduces the number of new parameters to 1. In the third column of Table II,

we show the best fit in the 5 parameter model. Note that $\Gamma(\rho \rightarrow \pi\gamma)$ can now be fitted but the Zanfino et al.⁵ ratio is very poorly predicted. $\Gamma(K^{*0} \rightarrow K^0\gamma)$ has also risen to 150 KeV.

One of us¹¹ tried a scheme in the quark model in which quarks were allowed to have anomalous magnetic moments. This is equivalent to allowing a divergence of an anti-symmetric tensor in the electromagnetic current. Again it was found that though one had the freedom to fit both $\Gamma(\rho \rightarrow \pi\gamma)$ and $\Gamma(K^{*0} \rightarrow K^0\gamma)$ the Zanfino et al.⁵ ratio came out poorly.¹¹

It is worth pointing out that the model of Ref. 11 can be summarized by saying that the internal symmetry structure of $V^m \rightarrow P^i\gamma$ is

$$g_{\min} = g_0 \left[(\mu_1 - \mu_3) d_{\min} + \sqrt{3} (\mu_3 - \mu_2) d_{8nk} d_{kim} + \frac{2}{\sqrt{3}} (\mu_1 + 2\mu_3) \delta_{8n} \delta_{im} \right] \quad (3.5)$$

where $k=0, \dots, 8$ and μ_i are the magnetic moments of the three quarks.

(1) If μ_i are in the ratio of quark charges, 2 : -1 : -1, then

$$g_{\min} \propto d_{\min} \quad (3.6)$$

and the model reduces to the nonet symmetry scheme.

(2) If only μ_1 and μ_2 are in the ratio 2 : -1 (degeneracy of m_u and m_d) then the last term in (3.5) is absent. One has two parameters and one can fit $\Gamma(\omega \rightarrow \pi\gamma)$ and $\Gamma(K^{*0} \rightarrow K^0\gamma)$. One still predicts $\Gamma(\omega \rightarrow \pi\gamma)/\Gamma(\rho \rightarrow \pi\gamma) \simeq 9$ and the Zanfino et al.⁵ ratio $\simeq 11$ independent of θ_p , provided θ_v is ideal.

(3) If μ_i are not in the ratio 2 : -1 : -1, then one has three parameters.

It is now possible to fit $\Gamma(\omega \rightarrow \pi\gamma)$, $\Gamma(K^{*0} \rightarrow K^0\gamma)$ and $\Gamma(\rho \rightarrow \pi\gamma)$

but the Zanfino et al.⁵ is poorly predicted.¹¹ The last term of (3.5) contributes to both $\Gamma(\rho \rightarrow \pi\gamma)$ and the Zanfino et al.⁵ ratio.

4. Generalization to SU(4)

A straightforward SU(4) generalization¹² of our model would be to introduce a scalar spurion U_{15} in addition to the spurion U_8 . If we invoke boson symmetry then the internal symmetry structure of the decay matrix element for $V^m \rightarrow P^i \gamma$ is

$$\begin{aligned}
 g_{min} &= g_0 d_{min} + g_1 d_{8ik} d_{kmn} \\
 &+ g_2 (d_{8mk} d_{kin} + d_{8nk} d_{kim}) \\
 &+ g_3 \delta_{8i} \delta_{mn} + g_4 (\delta_{8m} \delta_{in} + \delta_{8n} \delta_{im}) \\
 &+ g_5 d_{15ik} d_{kmn} + g_6 (d_{15mk} d_{kin} + d_{15nk} d_{kim}) \\
 &+ g_7 \delta_{15i} \delta_{mn} + g_8 (\delta_{15m} \delta_{in} + \delta_{15n} \delta_{im}) \quad (4.1)
 \end{aligned}$$

The photon index is

$$n = (3) + \frac{1}{\sqrt{3}} (8) - \sqrt{\frac{2}{3}} (15) + \frac{\sqrt{2}}{3} (0) .$$

The mixing angles are such that ϕ is a pure $s\bar{s}$ state, ω is a pure $(u\bar{d} + \bar{u}d)$ state, ψ and η_c are pure $c\bar{c}$ states.

All SU(3) rates except $K^{*+} \rightarrow K^+ \gamma$ use the following combination of parameters: $g_0 + \frac{1}{\sqrt{6}} (g_5 + 2g_6)$, $(g_1 + 2g_2)$, g_3 , g_4 , g_7 and g_8 . In the charm sector $\psi \rightarrow \pi\gamma$ uses g_8 alone and can be made to give a value consistent with 5 ± 3.2 eV, $\psi \rightarrow \eta\gamma$ and $\psi \rightarrow \eta' \gamma$ use g_3 and g_7 only and can be made to be consistent with 55 ± 12 eV and 152 ± 117 eV respectively. $\psi \rightarrow \eta_c(2.83)\gamma$ uses $\frac{1}{3} g_0 - \frac{1}{\sqrt{6}} (g_5 + 2g_6)$, g_4 , g_7 and g_8 and can be

controlled (< 3.5 KeV). If $\Gamma(\psi \rightarrow \eta_c \gamma)$ is made to vanish then we predict $\Gamma(D^{*0} \rightarrow D^0 \gamma) = 18$ KeV. Using vector meson dominance we also predict $\Gamma(\eta_c \rightarrow \gamma \gamma) = 280$ eV. Independent predictions on $\Gamma(D^{*+} \rightarrow D^+ \gamma)$ and $\Gamma(F^{*+} \rightarrow F^+ \gamma)$ cannot be made because these rates depend on g_6 as does $\Gamma(K^{*+} \rightarrow K^+ \gamma)$ on g_2 . If both g_2 and g_6 are set equal to zero then we predict $\Gamma(K^{*+} \rightarrow K^+ \gamma) = 21$ KeV, $\Gamma(D^{*+} \rightarrow D^+ \gamma) = 0.76$ KeV and $\Gamma(F^{*+} \rightarrow F^+ \gamma) = 0.09$ KeV.

5. Conclusions

Unless the $\rho \rightarrow \pi \gamma$ rate goes up to about 70 KeV we find it difficult to understand this rate consistently with the Zanfino et al.⁵ ratio. The preliminary number for a new measurement¹³ of $\Gamma(\rho \rightarrow \pi \gamma)$ at Fermi Laboratory is 50 ± 10 KeV which goes in the right direction but not enough. The strong interaction background at the Fermi Laboratory energies is much lower than that in the Gobbi et al.² experiment done at Brookhaven. There is a need for further measurements of this rate.

A measurement of $\Gamma(K^{*+} \rightarrow K^+ \gamma)$ is very desirable as it will provide a check (along with $\Gamma(K^{*0} \rightarrow K^0 \gamma)$) on the SU(3) structure of the decay matrix elements.

We also note that the Zanfino et al.⁵ ratio has proved to be rather a stringent limit on the models.

In the charm sector η_c remains somewhat of a phantom particle. It has not been seen at SPEAR.

A measurement of $\Gamma(D^{*0} \rightarrow D^0 \gamma)$ would be useful. Again a measurement of $\Gamma(D^{*+} \rightarrow D^+ \gamma)$ together with $\Gamma(D^{*0} \rightarrow D^0 \gamma)$ would be very useful in understanding the symmetry structure.

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TABLE I

The numbers in brackets indicate the rates that were used in the fit.

Mode	Nonet Symmetry and θ_v Ideal	Nonet Symmetry Breaking, $\theta_v = 24^\circ$	Rate in KeV
$\omega \rightarrow \pi\gamma$	(880)	(870)	880 ± 60
$\rho \rightarrow \pi\gamma$	92	(35)	35 ± 10
$K^{*0} \rightarrow K^0\gamma$	210	(78)	75 ± 35
$\phi \rightarrow \pi\gamma$	0	(6.5)	5.9 ± 2.1
$\phi \rightarrow \eta\gamma$	170	(81)	64 ± 10
$\omega \rightarrow \eta\gamma$	7.2	24	3.0 ± 2.5 ± 1.8 or 29 ± 7
$\rho \rightarrow \eta\gamma$	55	26	50 ± 13 or 76 ± 15
$\frac{\Gamma(\eta' \rightarrow \rho\gamma)}{\Gamma(\eta' \rightarrow \omega\gamma)}$	11	50	9.9 ± 2.0
$K^{*+} \rightarrow K^+\gamma$	51	20	< 80
$\eta' \rightarrow \omega\gamma$	10	2.6	< 50
$\eta' \rightarrow \rho\gamma$	120	130	< 300

TABLE II

The numbers in brackets indicate the rates that were used in the fit.

Mode	4 Parameter Model	5 Parameter Model	Rate in KeV
$\omega \rightarrow \pi\gamma$	(810)	(860)	880 ± 60
$\rho \rightarrow \pi\gamma$	(70)	(41)	35 ± 10
$K^{*0} \rightarrow K^0\gamma$	(75)	(150)	75 ± 35
$\phi \rightarrow \pi\gamma$	(6.7)	(5.6)	5.9 ± 2.1
$\phi \rightarrow \eta\gamma$	(64)	(61)	64 ± 10
$\omega \rightarrow \eta\gamma$	0.3	(1.7)	3.0 ± 2.5 ± 1.8 or 29 ± 7
$\rho \rightarrow \eta\gamma$	10	(49)	50 ± 13 or 76 ± 15
$\frac{\Gamma(\eta' \rightarrow \rho\gamma)}{\Gamma(\eta' \rightarrow \omega\gamma)}$	13	24	9.9 ± 2.0
$K^{*+} \rightarrow K^+\gamma$			< 80
$\eta' \rightarrow \omega\gamma$	10	5.2	< 50
$\eta' \rightarrow \rho\gamma$	140	130	< 300