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MAJORANA LEPTON MEDIATED μ^- TO e^+ CONVERSION IN NUCLEI*

A. N. Kamal†
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

and

J. N. Ng
TRIUMF, and Physics Department
University of British Columbia, Vancouver
British Columbia, Canada V6T 1W5

ABSTRACT

We estimate the branching ratio of the anomalous conversion of μ^- to e^+ in nuclei via the exchange of a virtual Majorana lepton to ordinary muon capture to be $<10^{-13}$ for a 0.5 GeV/c^2 lepton and $<10^{-14}$ for a 1.0 GeV/c^2 lepton. A sequential Weinberg-Salam gauge model with four lepton doublets and neutrino mixing is used. The change in the anomalous capture rate with respect to the mass of the Majorana lepton is also discussed.

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† Permanent Address: Theoretical Physics Institute and Department of Physics, University of Alberta, Edmonton, Alberta, Canada.

1. Introduction

Recently, the question of lepton number conservation has received much attention, both theoretically and experimentally. With the meson factories now in full operation the limits for the conversion of muonic matter into electronic matter in $\mu^+ \to e^+\gamma$ decay[1] and μ^- + nucleons[2] have been pushed to unprecedented levels. The question of total lepton number[3] conservation has been examined in three different kinds of experiments. The first one studies the rare decay of the kaon into two same-charge electrons plus a pion. The second kind searches for no-neutrino double-beta decay of nuclei[4] such as $Ge^{130} \to Se^{130}$; and the last involves the capture of a negative muon by a nucleus and the detection of a positive electron in the final state.[5] Explicitly, the latter is represented by

$$\mu^- + (A,Z) \rightarrow e^+ + (A,Z-2) \qquad \text{(I)}$$

where (A,Z) denotes the target nucleus. The characteristics of all three kinds of experiments involving <u>no</u> neutrinos hinge crucially on the kinematics of the reactions.

Theoretical estimates of nuclear no-neutrino and two-neutrino double-beta decay rates[6] have provided a stringent limit on the level of possible electron number violation. On the other hand, reaction (I) is more interesting since its observation signals a violation of the conservation of total lepton number as well as a break-down of separate electron and muon number conservation. This in turn will mean a positive signature at some level for reactions such as $\mu^+ \to e^+\gamma$, $\mu^+ \to e^+e^-e^+$ and $\mu^- \to e^-$ conversion in nuclei. However, the reverse is not true.

In this paper we study the reaction (I) in a simple extension of the standard Weinberg-Salam (WS) SU(2) × U(1) gauge theory which has enjoyed much phenomenological success in places where it has been tested.[7] However, our result is mostly free of the details of the particular gauge model we construct.

2. Model and Calculations

Consider the following SU(2) × U(1) model of unified weak and electromagnetic interactions where the left-handed leptons are arranged in sequential doublets as follows,

$$\begin{pmatrix} \nu_1 \\ e^- \end{pmatrix}_L , \quad \begin{pmatrix} \nu_2 \\ \mu^- \end{pmatrix}_L , \quad \begin{pmatrix} \nu_3 \\ \tau^- \end{pmatrix}_L , \quad \begin{pmatrix} \nu_4 \\ N^- \end{pmatrix}_L \tag{1}$$

where $L \equiv (1-\gamma_5)/2$. We have extended the usual W-S model with six-quarks and six-leptons by one lepton doublet denoted by the last entry in (1). The neutrinos need not necessarily be massless. The neutrino states in (1), denoted by ν_i, are eigenstates of the weak interactions. We assume that the mass eigenstates represented by n_i are related to the states ν_i by a unitary transformation,[8]

$$(\nu_i)_L = u_{ij} \, (n_j)_L \tag{2}$$

We need not go into the detailed discussion of the Higgs system and how the neutrino may acquire a mass. This is discussed in Ref. 9. The mass matrix generated by the Higgs mechanism is non-diagonal both in the lepton and the quark sector.[9] In the quark sector with six quarks it generates the Cabibbo-like rotations and one CP violating phase.[10]

It was pointed out by Cabibbo[8] that a similar phenomenon can occur for the leptons in that the diagonalization of the neutral lepton mass matrix induces mixing among the corresponding weak eigenstates. In our treatment of the mixing in the lepton sector we shall ignore the CP violating phases. For 2n lepton flavors the mixing is generated by an orthogonal n × n matrix. For the case of four lepton doublets six real parameters determine the neutrino of the last doublet in (1) and ν_1 and ν_2 which we will treat as ν_e and ν_μ. The latter two neutrinos are taken to be very light with masses no larger than 2 eV. Thus we can now rewite our lepton weak doublets as

$$\begin{pmatrix} \nu_e + \beta N^o + \ldots \\ e^- \end{pmatrix}_L , \begin{pmatrix} \nu_\mu + \gamma N^o + \ldots \\ \mu^- \end{pmatrix}_L , \begin{pmatrix} N^o + \beta' \nu_e + \gamma' \nu_\mu + \ldots \\ N^- \end{pmatrix}_L , \ldots \qquad (3)$$

where N^o is now a mass eigenstate of a fourth neutrino and the dots denote other neutrino states which are of no interest to us. In particular the μ_τ mixing with ν_e and ν_μ will be assumed small[11] and its connection with N^o will give rise to phenomena beyond the scope of this paper. The mixing between N^o and ν_e and ν_μ is given by the parameters β and γ respectively both of which are less than unity as we assume universal gauge couplings for weak interactions.

Next we assumed the N^o to be a Majorana particle,[12] i.e., $N^o = N^{oC} \equiv C\tilde{\bar{N}}^o$ where C is the charge conjugation matrix. The general mass term for this lepton $M_\sigma \bar{N}^o N^{oC}$ will serve as a source or a sink for the Majorana particle. This term obviously violates total lepton number by two units.[13]

The model discussed above can induce $\mu^- \rightarrow e^+$ conversion in nuclei via a second order weak process.[14] The interaction Lagrangian density involving the Majorana particle is

$$\mathscr{L} = f\left(\beta \bar{e}_L \gamma_\mu N_L + \gamma \bar{\mu}_L \gamma_\mu N_L\right) W^\mu \tag{4}$$

where f is the gauge coupling. The W-boson field is denoted by W^μ. The terms involving unphysical Higgs boson exchanges would be smaller. The right-handed field N_R will have no effect on the charged weak current processes since it has weak hypercharge $Y = 0$ and weak isospin $I_3 = 0$.

In the μ^- capture reaction (I) the muon is captured in the 1S orbit of a nucleus, usually chosen to be heavy to ensure a high capture rate, and the final state e^+ should in principle be described by a Coulomb wave function appropriately distorted by the nucleus. However, we shall ignore such complexities of the final state and estimate the conversion rate by assuming plane waves for the e^+. The generic Feynman diagram as well as the kinematics are depicted in Fig. 1(a). The μ^- is absorbed by the nucleus via one W-boson exchange and converts into a virtual N^o. The mass insertion represented by the cross in Fig. 1(a) turns the virtual N^o into a $\bar{N}^o$ which scatters from the intermediate nucleus (A,Z-1) and emerges as an e^+. Figure 1(b) shows in detail the coupling of W^- locally to <u>one</u> proton at a time (single nucleon approximation).

For the initial muon we will use the 1S state wave function given by

$$\begin{aligned}\psi_\mu(x) &= \phi_\mu(\vec{x})\, e^{-i E_\mu x^o} \\ &= \frac{Z^{3/2}}{(\pi a_0^3)^{\frac{1}{2}}}\, e^{-\frac{Z}{a_0}|\vec{x}| - i E_\mu x^o}\, u_\mu \end{aligned} \tag{5a}$$

and

$$a_0 = \frac{4\pi}{m_\mu e^2} \qquad (5b)$$

where Z is the atomic number of the initial nuclear state $|i\rangle$ and u_μ the muon spinor. In Eq. (5b) we should strictly speaking use the reduced mass of the muon-nucleus system instead of m_μ. However such corrections are minor and since we are interested in an estimate of the $\mu^- e^+$ conversion rate we shall ignore this kinematic correction. Furthermore we will only treat the case of coherent capture where the final nucleus recoils as a unit, i.e., the nucleus does not break up but can be excited. This implies that the e^+ is emitted with energy of ~100 MeV. The energy difference, ΔE, between the initial and final nuclear states is usually less than 10 MeV. It has been argued that[15] the coherent effect is the dominant one occuring about six times more often than the incoherent effect. Then the e^+ - spectrum will have a peak at $m_\mu - \Delta E$. Selecting the energy of e^+ to be large also serves to cut down the background from the Dalitz pairs from ordinary radiative capture.

With the model and assumptions stated above the matrix element for reaction (2) is given by second order perturbation theory

$$\mathcal{M} = \frac{f^4 \beta\gamma}{16} \int d^4x \int d^4y \int \frac{d^4k_n}{(2\pi)^4} \frac{1}{(k_\mu - k_n)^2 - M_W^2} \frac{1}{(k_n - k_e)^2 - M_W^2} \frac{1}{(k_n^2 - M_\sigma^2)^2}$$
$$\times \left[\tilde{u}(k_e)\, \mathscr{C}\, \gamma_\mu (1-\gamma_5)(\not{k}_n + M_\sigma)\, M_\sigma (1-\gamma_5)(\not{k}_n + M_\sigma)\, \gamma_\nu (1-\gamma_5)\, u_\mu\right]$$
$$\sum_X \Big\{ e^{ik_e x}\, e^{-iE_\mu y^o}\, e^{-ik_n(x-y)}\, e^{-i(E_i - E_x)y^o}\, e^{-i(E_x - E_f)x^o}$$
$$\phi_\mu(\vec{y})\ \langle f|J_\mu(\vec{x})|X\rangle\langle X|J_\nu(\vec{y})|i\rangle \Big\} \qquad (6)$$

where $|f\rangle$ is the final nuclear state with energy E_f, and $|X\rangle$ is a complete set of intermediate states of energy E_X. The hadronic charged weak-current is denoted by $J_\mu(x)$.

The integration in k_n^o can be performed using the usual techniques of contour integration. We shall make the simplifying assumption that all external momenta are small compared to M_W^2. This is a good approximation for μ^- capture since all external momenta involved are of the order of 100 MeV whereas M_W is in the range of 50 to 100 GeV. After the k_n^o integration is done we have

$$\mathcal{M} = \frac{M_\sigma^2 f^4 \beta\gamma}{4(2\pi)^4}\left(-\frac{\pi i}{2}\right)\int d^4x \int d^4y \int d^3\vec{k}_n \, e^{ik_e x} \, e^{-iE_\mu y^o} \, e^{-i\vec{k}_n\cdot(\vec{x}-\vec{y})} \, \phi_\mu(\vec{y})$$

$$e^{-i(E_i - E_X)y^o} \, e^{-i(E_X - E_f)x^o} \, \frac{1}{(M_\sigma^2 - M_W^2)^2} \, \tilde{u}(k_e) \, \mathscr{C} \, \gamma_\mu \Bigg\{ \theta(x^o - y^o)$$

$$\Bigg[\frac{e^{-i\omega_\sigma(x^o-y^o)}}{\omega_\sigma^2} \Bigg\{ \frac{\vec{\gamma}\cdot\vec{k}_n}{\omega_\sigma} - i(x^o-y^o)(\gamma^o\omega_\sigma - \vec{\gamma}\cdot\vec{k}_n)$$

$$- \frac{4\omega_\sigma}{M_\sigma^2 - M_W^2}(\gamma^o\omega_\sigma - \vec{\gamma}\cdot\vec{k}_n)\Bigg\} + (M_\sigma \rightleftarrows M_W)\Bigg] + \theta(y^o - x^o)\Bigg[\frac{e^{+i\omega_\sigma(x^o-y^o)}}{\omega_\sigma^2}$$

$$\Bigg\{ -\frac{\vec{\gamma}\cdot\vec{k}_n}{\omega_\sigma} + i(x^o-y^o)(\gamma^o\omega_\sigma - \vec{\gamma}\cdot\vec{k}_n) - \frac{4\omega_\sigma}{M_\sigma^2 - M_W^2}(\gamma^o\omega_\sigma + \vec{\gamma}\cdot\vec{k}_n)\Bigg\}$$

$$+ \; (M_\sigma \rightleftarrows M_W)\Bigg]\Bigg\} \cdot \gamma^\nu(1-\gamma_5)\, u_\mu \langle f|J_\mu(\vec{x})|X\rangle\langle X|J_\nu(\vec{y})|i\rangle \qquad (7)$$

and

$$\omega_\sigma^2 = \vec{k}_n^2 + M_\sigma^2 \qquad (8a)$$

Similarly with $(M_\sigma \mp M_W)$,

$$\omega_W^2 = \vec{k}_n^2 + M_W^2 \tag{8b}$$

We have kept the M_W term which will enable us to investigate the range of M_σ from smaller to larger than M_W.

Next we invoke the closure approximation[16] where we can approximate the energy of the intermediate E_X by some average value $\langle E_X\rangle$ which is of the same order as $\langle E_i\rangle$. Studies of ordinary muon capture indicate that the energy difference $E_i - E_X$ is usually no greater than 10 MeV and probably much smaller.[17] Hence, replacing E_X by $\langle E_X\rangle$ should not introduce a gross error. The closure approximation then allows us to use completeness on $|X\rangle$ and obtain

$$\sum_X e^{-i(E_i - E_X)y^o} e^{-i(E_X - E_f)x^o} \langle f|J_\mu(\vec{x})|X\rangle\langle X|J_\nu(\vec{y})|i\rangle$$

$$\simeq e^{-i(E_i - \langle E_X\rangle)y^o} e^{-i(\langle E_X\rangle - E_f)x^o} \langle f|J_\mu(\vec{x})\ J_\nu(\vec{y})|i\rangle \tag{9}$$

The x^o, y^o and $\vec{k}_n$ integrations can then be done by contour integrations. Performing them in succession leaves us with

$$\mathcal{M} = -\frac{if^4\,\gamma\beta\,M_\sigma^2}{16\pi^3}\int d^3\vec{x}\ d^3\vec{y}\ \delta(E_\mu + E_i - E_e - E_f)\ e^{-i\vec{k}_e\cdot\vec{x}}\ \phi_\mu(\vec{y})$$

$$\frac{1}{(M_W^2 - M_\sigma^2)^2}\ \tilde{u}(k_e)\,\mathscr{C}\gamma_\mu\ L\,\gamma_\nu(1-\gamma_5)\ u_\mu\langle f|J_\mu(\vec{x})\ J_\nu(\vec{y})|i\rangle \tag{10}$$

where

$$L = \left(\sum_{i=1}^{5} J_i + (M_\sigma \mp M_W)\right) \tag{11a}$$

and

$$J_1 = 16\pi\ E\ \gamma^o \int_0^\infty dk \frac{k^2\ j_0(kr)}{(E^2 - \omega_\sigma^2)^2} \tag{11b}$$

$$J_2 = 8\pi i\ \vec{\gamma} \cdot \hat{r} \int_0^\infty dk \frac{k^3\ j_1(kr)}{\omega_\sigma^2 (E^2 - \omega_\sigma^2 - i\varepsilon)} \tag{11c}$$

$$J_3 = -8\pi i\ \vec{\gamma} \cdot \hat{r} \int_0^\infty dk \frac{E^2 + \omega_\sigma^2}{\omega_\sigma^2} \frac{k^3\ j_1(kr)}{(E^2 - \omega_\sigma^2)^2} \tag{11d}$$

$$J_4 = \frac{32\pi\ E\ \gamma^o}{(M_W^2 - M_\sigma^2)} \int_0^\infty dk \frac{k^2\ j_0(kr)}{(E^2 - \omega_\sigma^2 - i\varepsilon)} \tag{11e}$$

$$J_5 = - \frac{32\pi i\ \vec{\gamma} \cdot \hat{r}}{(M_W^2 - M_\sigma^2)} \int_0^\infty dk \frac{k^3\ j_1(kr)}{(E^2 - \omega_\sigma^2 - i\varepsilon)} \tag{11f}$$

and

$$E \equiv E_f + E_e - \langle E_X \rangle \tag{11g}$$

with $r = |\vec{r}| = |\vec{x}-\vec{y}|$. We can now divide the discussion into four cases:

(i) the mass of the Majorana lepton is in the intermediate region $E < M_\sigma \le M_W$; e.g., from 0.5 to 10 GeV/c^2;

(ii) super heavy lepton range of $M_\sigma >> M_W$, say $M_\sigma \approx \frac{1}{2}$ TeV/c^2;

(iii) $M_\sigma \simeq M_W$ about 80 GeV/c^2; and

(iv) a very light M_σ less than 1 MeV/c^2.

The case of immediate experimental interest is the first one. We also note that the actual value of M_W is of less importance although in our numerical results for cases (ii) and (iii) we will use $M_W \simeq 84$ GeV/c^2 which is the value in the W-S model with the weak mixing angle, θ_W, given by $\sin^2\theta_W = 0.25$.

Consider first both M_σ and M_W large (cases (i) to (iii)). Then we have

$$J_1 \simeq \frac{2\pi^2 E \gamma^o}{M_\sigma} e^{-M_\sigma r} \tag{12a}$$

$$J_2 \approx J_3 \simeq -2\pi^2 i \vec{\gamma} \cdot \hat{r} e^{-M_\sigma r} \tag{12b}$$

$$J_4 = \frac{16\pi^2 E \gamma^o}{(M_\sigma^2 - M_W^2)} \frac{e^{-M_\sigma r}}{r} \tag{12c}$$

and

$$J_5 = \frac{16\,{}^2 i \vec{\gamma} \cdot \hat{r}}{(M_W^2 - M_\sigma^2)} \frac{e^{-M_\sigma r}}{r} \left(M_\sigma + \frac{1}{r}\right) \tag{12d}$$

Thus keeping only J_2, J_3 and J_5 one gets

$$L = -4\pi^2 i \vec{\gamma} \cdot \hat{r} e^{-M_\sigma r} \left\{ 1 + \frac{4}{M_\sigma^2 - M_W^2} \left(\frac{M_\sigma}{r} + \frac{1}{r}\right) + (M_\sigma \rightleftarrows M_W) \right\} \tag{13}$$

In the limit $M_\sigma << M_W$, J_5 drops out and $(M_\sigma \rightleftarrows M_W)$ term is small giving

$$L \approx -4\pi^2 i \vec{\gamma} \cdot \hat{r} e^{-M_\sigma r} \tag{13a}$$

In the limit $E << M_W << M_\sigma$ we obtain simply

$$L \simeq -4\pi^2 i \vec{\gamma} \cdot \hat{r} e^{-M_W r} \tag{13b}$$

For $M_\sigma \simeq M_W$ we can expand (13) in $M_\sigma^2 - M_W^2$ and set $M_\sigma = M_W$. This gives

$$L \simeq -\frac{8\pi i}{3}\vec{\gamma}\cdot\hat{r}\; e^{-M_W r} \tag{13c}$$

Next we consider in detail the evaluation of the matrix element for case (i). For super-heavy lepton cases of (ii) and (iii) the treatment is identical and only the results will be presented. Hence, putting (13a) into (10) we have, explicitly

$$\mathcal{M} = -\frac{i\beta\gamma f^4 M_\sigma^2}{16\pi^3 M_W^4}\int d^3x \int d^3y\; \delta(E_\mu + E_i - E_e - E_f)\; e^{-i\vec{k}_e\cdot\vec{x}}\; \frac{e^{-M_\sigma r}}{r}$$

$$\phi_\mu(\vec{y})\; \tilde{u}(k_e)\mathscr{C}\gamma_\mu\vec{\gamma}\cdot\hat{r}\;\gamma_\nu\;(1-\gamma_5)\;u_\mu\;\langle f|J_\mu(\vec{x})\;J_\nu(\vec{y})|i\rangle \tag{14}$$

We observe in Eq. (14) that the exchange of a heavy lepton gives rise to an effective Yukawa interaction in accordance with Ref. 18. This is independent of the treatment of the nuclear physics. To proceed we need to know the two-current correlation function $\rho^f_{\mu\nu} \equiv \langle f|J_\mu(\vec{x})\;J_\nu(\vec{y})|i\rangle$. The superscript f labels the particular final state to which transition is made. Next we assume that each of the weak current $J_\mu(\vec{x})$ couples locally to one nucleon at a time in the nucleus. Then in reaction (I) we have two protons changing into two neutrons and we can write in the usual V-A current form

$$J_\mu^{(-)}(\vec{x}) = \frac{1}{2}\bar{\psi}_n(\vec{x})\;\gamma_\mu(1-\gamma_5)\;\psi_p(\vec{x}) \tag{15}$$

where ψ_p and ψ_n are proton and neutron wavefunctions. We note that due to the non-relativistic nature of the problem, the dominant term in $\rho^f_{\mu\nu}$ is given by ρ^f_{00}. The terms involving $\vec{J}(\vec{x})$ will be proportional to the spin of the nucleon which samples the small components of the nucleon wavefunctions. Hence in neglecting ρ_{0i} and ρ_{ij} $(i,j = 1,2,3)$ we will not be making a gross error. The hadronic current is given by Eq. (15). The nucleons are treated non-relativistically. The matrix γ_5 which mixes large and small components of the Dirac spinor in the hadron current can be ignored. Then with $\mu=0$, $\nu=0$ we write

$$\langle f|J_0(x)\ J_0(y)|i\rangle \simeq \frac{1}{4}\langle f|\sum_{k,\ell} \tau_k^- \tau_\ell^- \,\delta(\vec{x}-\vec{x}_k)\ \delta(\vec{y}-\vec{y}_\ell)|i\rangle$$

$$= \frac{1}{4}\langle f|\sum_{k,\ell} \tau_k^- \tau_\ell^-|i\rangle \frac{\langle f|\sum_{k,\ell} \tau_k^- \tau_\ell^- \,\delta(\vec{x}-\vec{x}_k)\ \delta(\vec{y}-\vec{y}_\ell)|i\rangle}{\langle f|\sum_{k,\ell} \tau_k^- \tau_\ell^-|i\rangle}$$

$$\approx \frac{Z(Z-1)}{8}\langle f|\rho^2(\vec{y})|i\rangle\ f(r) \tag{16}$$

where f(r) is a two-nucleon correlation function which we assume to be spherically symmetrical in $\vec{r} = \vec{x}-\vec{y}$. We have also set $\vec{x} \approx \vec{y}$ in the nuclear density since the matrix element is multiplied by a rapidly damping exponential in $\vec{r}$. As the nucleon-nucleon potential has a hard core of radius r_c we take the correlation function to be

$$\begin{aligned} f(r) &= 0 \quad , \quad \text{if} \quad r < r_c \text{ and } r < 2R \\ &= 1 \quad , \quad \text{if} \quad r_c < r < 2R \end{aligned} \tag{17}$$

where the nuclear radius R is taken to be related to A through $R = b_0 A^{1/3}$ with $b_0 \approx 1.2\times 10^{-13}$ cm. The nuclear density $\rho(y)$ is assumed to be constant within the nucleus. Using Eqs. (16) and (17) in (14) one gets

$$\mathcal{M} = \frac{9Z(Z-1)}{16 b_0^6 A^2}\,\frac{f^4(\beta\gamma)}{M_W^4\, M_\sigma^2}\,\frac{Z^{5/2}}{(\pi a_0^5)^{\frac{1}{2}}}\,\frac{\left\{2\tilde{u}(k_e)\ \mathscr{C}\,\vec{\gamma}\cdot\vec{k}_e(1-\gamma_5)\ u_\mu\right\}}{\left(\vec{k}_e^2 + \frac{Z^2}{a_0^2}\right)^2}\ F\ G \tag{18}$$

where

$$F = \left[\left(1 - \cos k_e R\, e^{-ZR/a_0}\right) - \frac{Z}{a_0 k}\, e^{-ZR/a_0} \sin k_e R + \frac{\left(\vec{k}_e^{\,2} + \frac{Z^2}{a_0^2}\right)}{2Z/a_0}\, e^{-ZR/a_0} \left\{\left(1 - \frac{ZR}{a_0}\right) \sin k_e R - k_e R \cos k_e R\right\}\right] \tag{19}$$

and

$$G = e^{-M_\sigma r_c}\left(1 + r_c M_\sigma + \frac{1}{2} r_c^2 M_\sigma^2 + \frac{1}{6} r_c^3 M_\sigma^3\right) - (r_c \to 2R) \tag{20}$$

F represents the finite nucleus effect and G the correlation effect. In the "large nucleus" limit F and G are,

$$F \xrightarrow{\frac{ZR}{a_0} \gg 1} 1 \tag{21}$$

$$G \xrightarrow{2M_\sigma R \gg 1} e^{-M_\sigma r_c}\left(1 + r_c M_\sigma + \frac{1}{2} r_c^2 M_\sigma^2 + \frac{1}{6} r_c^3 M_\sigma^3\right) \tag{22}$$

The "large nucleus" limit is easily met for $M_\sigma > 0.5$ GeV; however, for medium heavy nuclei $A = 64$, $Z = 30$, $\frac{ZR}{a_0} \simeq 0.57$ and one cannot use the limit of Eq. (21).

The capture rate into a particular final state $\langle f|$ is then given by

$$R_f(\mu^- \to e^+) = \pi \int \frac{d^3\vec{k}_e}{(2\pi)^3} \frac{1}{2E_e} \delta(E_\mu + E_i - E_e - E_f) \sum_{\text{spins}} \left|\mathcal{M}_{fi}\right|^2 \tag{23}$$

However, since the final state nucleus is not detected we have to sum over all the final states to obtain the total anomalous capture rate

$$R_{tot}(\mu^- \to e^+) = \sum_f R_f(\mu^- \to e^+) \tag{24}$$

In doing this final sum we invoke closure once more[19] by using an average $\langle E_f \rangle \approx E_f$ which allows us to use $\sum_f |f\rangle\langle f| = 1$. Finally, we obtain

$$R_{tot}(\mu^- \to e^+) = \frac{1}{4}\left(\frac{9Z(Z-1)}{b_0^6\ A^2}\right)^2 \frac{(8G_F^2)^2}{\pi^2} \frac{(\beta\gamma)^2(Z\alpha)^5}{M_\sigma^2} m_\mu F^2 G^2 \tag{25}$$

where the gauge coupling f has been replaced by the Fermi coupling, G_F via $f^2/M_W^2 = 4\sqrt{2}\ G_F$ and k_e has been approximated by m_μ.

The capture rate decreases as M_σ^{-4} in this mass range ($E_1 < M_\sigma << M_W$). The normal capture rate for $\mu^- + (A,Z) \to \nu_\mu + (A,Z-1)$ is given by

$$R(\mu^- \to \nu_\mu) = \frac{Z_{eff}^4}{2\pi^2} \alpha^3\ m_\mu^5\ G_F^2(C_V^2 + 3C_A^2) \tag{26}$$

where $C_V \simeq 1$ and $C_A \simeq 1.2$. In writing these rates we have ignored the Pauli blocking factor. The last two equations give

$$\frac{R(\mu^- \to e^+)}{R(\mu^- \to \nu_\mu)} \simeq \frac{1}{2}\left(\frac{9Z(Z-1)}{b_0^6\ A^2}\right)^2 \frac{(8G_F^2)^2(\beta\gamma)^2\alpha^2 Z}{M_\sigma^4\ m_\mu^4(C_V^2 + 3C_A^2)} \left(\frac{Z}{Z_{eff}}\right)^2 F^2 G^2 \tag{27}$$

Pauli blocking will in general lower the normal capture rate by forbidding transitions to filled neutron levels. In the anomalous capture one may argue that Pauli blocking will come into play by forbidding transitions to some of the states $|X\rangle$ introduced in Eq. (6) and finally forbidding transitions to some other states of $|f\rangle$. Thus the Pauli blocking may enter twice and will lower the ratio given in Eq. (27) by a single Pauli blocking factor. Thus we can calculate an upper bound for anomalous capture from Eq. (27).

The parameters yet to be determined are β and γ. The model of Eq. (3) permits neutrinoless double-beta-decay of heavy nuclei into two electrons as well as neutrinoless double-beta-decay of kaon into two muons. The latter will occur at level of $\gamma^2 G_F^2$ for rare decay of kaons. Recent analysis[20] shows that γ can be as large as unity for $M_\sigma > 0.5$ GeV/c^2 to tens of GeV/c^2. On the other hand, β can be determined by analysis of various no neutrino double-beta-decay of naturally occuring nuclei as is done in Ref. 18. Taking the results of their analysis we have

$$\beta^2 \lesssim 2.7 \times 10^{-3} \quad \text{for} \quad M_\sigma = 1 \text{ GeV/c}^2$$

and

$$\beta^2 \lesssim 1.0 \times 10^{-3} \quad \text{for} \quad M_\sigma = 0.5 \text{ GeV/c}^2 \qquad (28)$$

Hence the product $(\beta\gamma)^2$ has the limit

$$\begin{aligned} (\beta\gamma)^2 &\lesssim 2.7 \times 10^{-3} \quad \text{for} \quad M_\sigma = 1 \text{ GeV/c}^2 \\ &\lesssim 1 \times 10^{-3} \quad \text{for} \quad M_\sigma = 0.5 \text{ GeV/c}^2 \end{aligned} \qquad (29)$$

Alternatively, we can use experimental information from neutrino hadron reactions[21]

$$\frac{\nu_\mu + N \to e^- + N}{\nu_\mu + N \to \mu^- + N} \lesssim 2 \times 10^{-3} \qquad (30)$$

Assuming this limit for violation of μ-e universality gives[22]

$$(\beta\gamma)^2 \leq 2 \times 10^{-3} \qquad (31)$$

From Eqs. (25)-(27) and (31), the branching ratio for anomalous capture for medium heavy nuclei with $Z = 30$ and Z_{eff} taken from Ref. 23 is

$$\text{B.R} = \frac{R_{tot}(\mu^- \to e^+)}{R(\mu^- \to \nu_\mu)} \leq 0.7 \times 10^{-14} \text{ for } M_\sigma = 1 \text{ GeV/c}^2$$

$$\leq 2 \times 10^{-13} \quad \text{for } M_\sigma = 0.5 \text{ GeV/c}^2 \quad (32)$$

Using the value on $(\beta\gamma)^2$ from Eq. (29) we have instead

$$\begin{aligned} \text{B.R} &\leq 0.95 \times 10^{-14} \quad \text{for} \quad M_\sigma = 1 \text{ GeV/c}^2 \\ &\leq 7.5 \times 10^{-14} \quad \text{for} \quad M_\sigma = 0.5 \text{ GeV/c}^2 \end{aligned} \quad (33)$$

These are to be compared with the current experimental value of[5]

$$\text{B.R} \leq 1 \times 10^{-9} \qquad \text{(experiment)} \qquad (34)$$

The ratio given in Eq. (32) is within reach of the next generation of experiments on this reaction that are now in progress at the meson factories.[5] We emphasize here that a bare minimum of nuclear physics is put into our estimate. However, we expect our results to be good to within an order of magnitude.

So far we have discussed the two nucleon mechanism as a mode of inducing the conversion process (I). As pointed out in Ref. 6 there may exist a non-negligible probability, P_Δ, of finding a virtual $P_{\Delta^{++}} \simeq 1\%$. The $\mu^- \to e^+$ conversion can proceed via a $\Delta^{++} \to n$ in the nucleus. This mechanism is depicted in Fig. 2. Following the treatment of the Δ in N-N potential,[24] the $W^- \Delta^{++} n$ vertex is expected to have the form

$$\frac{f^2}{M_W^2} \langle N | (\vec{\sigma} \times \vec{\nabla}) \cdot \vec{W} | \Delta^{++} \rangle \qquad (35)$$

In the non-relativistic reduction of the wavefunctions Eq. (35) is proportional to $\frac{(\Delta p)^2}{M_W^2} \sim 10^{-2}$ where Δp is the difference in momentum between the Δ^{++} in the nucleus and the intermediate nucleon. Hence we estimate this mechanism to give a rate smaller than the two nucleon mechanism;[25] therefore we can neglect this as a source for $\mu^- \to e^+$ conversion at least in the region where the mass scale is set by M_σ or M_W.

Next we discuss the effects of a superheavy Majorana lepton, $M_\sigma >> M_W$. From perturbative treatment of gauge theories[26] one expects that $M_\sigma < 0.5$ TeV/c^2. Now we will use the value of $(\beta\gamma)^2$ in Eq. (31) as a guide and the calculation follows as in the previous case with L given by (13b) and (13c). Observe that now the effective Yukawa potential has a range that is set by M_W. The M_σ^{-4} behavior is still obtained in the rate since other factors in the amplitude are symmetrical under the interchange $M_\sigma \rightleftarrows M_W$. There is, however, an additional suppression factor $e^{-2M_\sigma r_c}$ in the rate, which strongly suppresses the effect of a superheavy Majorana lepton. With $r_c = 2.5$ GeV^{-1} the branching ratio is many orders of magnitude below present experimental capabilities. Even when the core softens, one obtains a branching ratio of the order 10^{-22} for $r_c \to 0$. However, we caution that this is a very conservative estimate based on extrapolation of current knowledge of quarks and leptons and is very model dependent.

For completeness we examine the case of a light Majorana lepton. The mass, M_σ, cannot be in the range between the mass of the kaon and the electron. Otherwise the kaon would decay into $\mu^- N^o$ which will be detected. On the other hand for $M_\sigma < 50$ KeV/c^2 the N^o will be stable and one will not be able to distinguish it from the usual ν_μ or ν_e by just studying the kaon or pion decays. Current data do not exclude this possibility. To discuss the effects of such a light Majorana lepton we return to Eqs. (10)-(11g). In evaluating J_i one has to keep E^2 in the denominators. One now has complex poles in k-plane resulting in J_i having oscillations of frequency E, dampened slowly by M_σ. Note that E being ~100 MeV and a typical nuclear radius being of the order of a few fermis $\left(\sim \frac{1}{200}\ (\mathrm{MeV})^{-1}\right)$, Er becomes unity and the oscillations are slow. The scale dimension of J_i will be set by E, i.e., these in integrals will not depend on inverse powers of M_σ. Instead the matrix element will behave as M_σ^2 and since $M_\sigma \ll E$ this case will give an undetectable rate for (I).

3. Speculations and Discussion

More exotic reactions can also take place at the hadronic vertex. If one assumes that exotic quarks of charge +5/3 or anti-quark of charge 4/3 exist in the sea quark ($q\bar{q}$) components of the nucleons, then reaction (I) can proceed via the Feynman diagram of Fig. 2 with the Δ^{++} being replaced by an exotic quark line and the nucleon lines by ordinary u or d quark lines. The spectator partner of this quark will then most likely decay non-leptonically or interact with other quarks and lose its identity. Energetically the semi-leptonic decay into ($e\bar{\nu}$) of this latter quark is allowed but only at the few percent level. Moreover, this will appear as the weak decay of the recoiling nucleus. Experimentally the existence of these quarks will be difficult to establish. The rate of $\mu^- \rightarrow e^+$ conversion proceeding with this kind of exotic quark mechanism is proportional to the probability of finding them in the nucleon. Current experimental information from high energy inclusive ν_μ and $\bar{\nu}_\mu$ reactions together with dimuon and trimuon events indicate[27] that the total sea quark content is about 10%. The major part of this consists of u,d and s quarks with the $\bar{c}c$ + other $\bar{q}q$ of the

1 to 2% level. The exotic quarks will certainly be massive as indicated by e^+e^- annihilation.[28] We do not expect it to occupy a large portion of the sea content of a nucleon and no more than 1% would be a fair estimate. Thus we see that such a mechanism will not contribute to a larger conversion rate than we have calculated.

Besides all the rare decays of the muon and neutrinoless double-beta in nuclei that have been looked for, the existence of $(N^o,N^-)_L$ doublet will result in other spectacular signatures owing to the Majorana nature of N^o. Firstly, there will be the neutrino sharing phenomenon.[29] If N^o is light (case (iv)) the following scenario can take place

$$\mu^+ \rightarrow e^+ \nu_e \bar{N}^o \tag{36a}$$

$$\mu^+ \rightarrow e^+ \bar{\nu}_e N^o \tag{36b}$$

together with

$$N^o(\bar{N}^o) + p \rightarrow e^+ + n \tag{37}$$

simulating the multiplicative scheme. Reaction (36a) will proceed with rate proportional to γ^2 and (36b) goes as β^2 so does reaction (37). Similar considerations also apply to the pion. For heavier mass, N^o, the reactions (36) are forbidden but non-orthogonality can still occur via direct e and μ mixing which was considered by many authors.[29] We note in passing that all the classical tests[3] for other known lepton numbers schemes can be induced as a second order weak interaction by a Majorana lepton that mixes with both ν_e and ν_μ. This includes the anti-muonium to muonium conversion.

Since in our model we do not have flavor changing neutral current, N^o cannot be produced in a first order weak process in ν_μ scattering.

Production of N^- and N^o via the Bethe-Heitler mechanism is possible but the rate will be very small.[30] The more promising possibilities are:

(i) pair production of N^+N^- in e^+e^- annihilations[1,2] and subsequent decays into N^o and/or $\bar{N}^o$ followed by the $e^\mp$ $(\mu^\mp)$ $\pi^\pm$ decay mode of N^o depicted by, for example,

$$e^+e^- \rightarrow N^+N^- \tag{38a}$$

$$\hookrightarrow e^+ \nu_e \bar{N}^o \tag{38b}$$

$$\hookrightarrow e^\mp \pi^\pm \tag{38c}$$

This takes place if the charged $N^\pm$ is heavier than the Majorana lepton N^o. Otherwise single production via the sequence

$$e^+e^- \rightarrow N^o \nu_e \, (\bar{\nu}_e) \tag{39a}$$

$$\hookrightarrow \mu^\mp \pi^\pm \; (e^\mp \pi^\pm) \tag{39b}$$

is possible and an exotic resonance $e^\mp$ $\pi^\pm$ can be searched for.[33]

(ii) If the mass of N^o is light enough it can also be produced in deep inelastic charged lepton scattering on nucleons. Here single production of N^o can take place via

$$e^-(\mu^-) + \text{Nucleon} \rightarrow N^o + \text{hadrons}, \quad N^o \hookrightarrow \ell^\pm \pi^\mp \tag{40}$$

where again an electron-pion or muon-pion resonance will be a signature. These can certainly be looked for at SLAC and the Fermi laboratory for M_σ in the GeV/c^2 range.

We have seen that $\mu^- \rightarrow e^+$ conversion in a nucleus can be induced in a generalized sequential W-S model if neutrinos are not a priori massless. If one of the massive neutrinos, N^o, is a Majorana particle the conversion proceeds as a second order weak effect and at a branching ratio of about 10^{-14} compared to ordinary muon capture for a 1 GeV/c^2 lepton and a branching ratio of 10^{-13} for a 0.5 GeV/c^2 lepton. Both a light N^o and ultraheavy N^o give too low a branching ratio for the conversion to be achievable by current machines.

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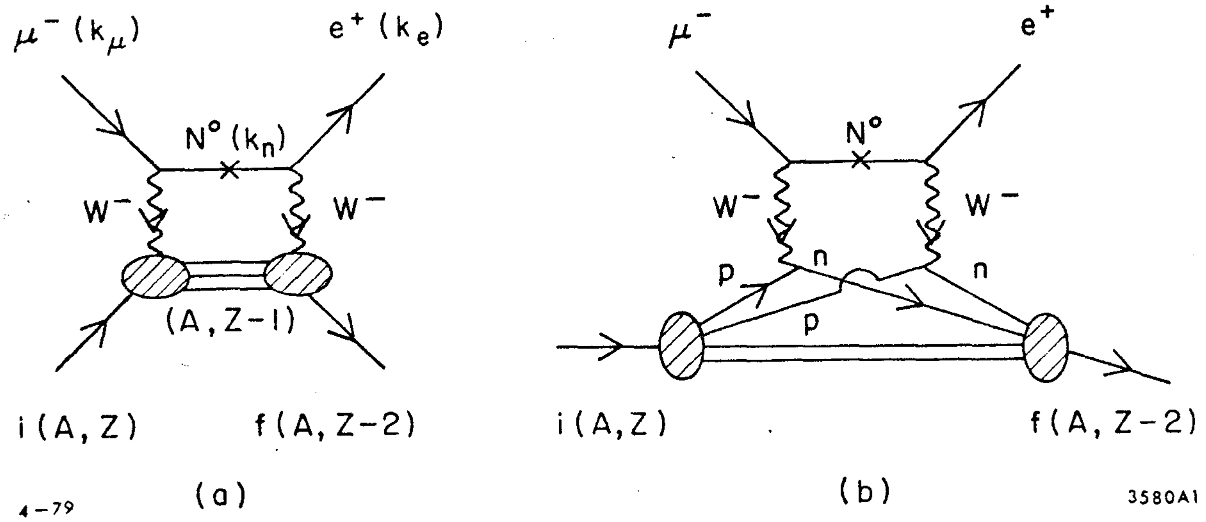
Figure Captions

Fig. 1. (a) Generic Feynman diagram for μ^- to e^+ conversion in a nucleus via the exchange of a Majorana lepton N^o. The cross denotes a mass insertion (see text).

(b) The one nucleon mechanism for the coupling of the W^- boson to the nucleus is depicted.

Fig. 2. Feynman diagram for $\mu^- \to e^+$ via interaction of the charged weak current with a virtual Δ^{++} in the nucleus.



μ^- (k_μ)
e^+ (k_e)
N° (k_n)
W^-
W^-
(A, Z−1)
i (A, Z)
f (A, Z−2)
(a)
4−79
μ^-
e^+
N°
W^-
W^-
p
n
n
p
i (A, Z)
f (A, Z−2)
(b)
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Fig. 1

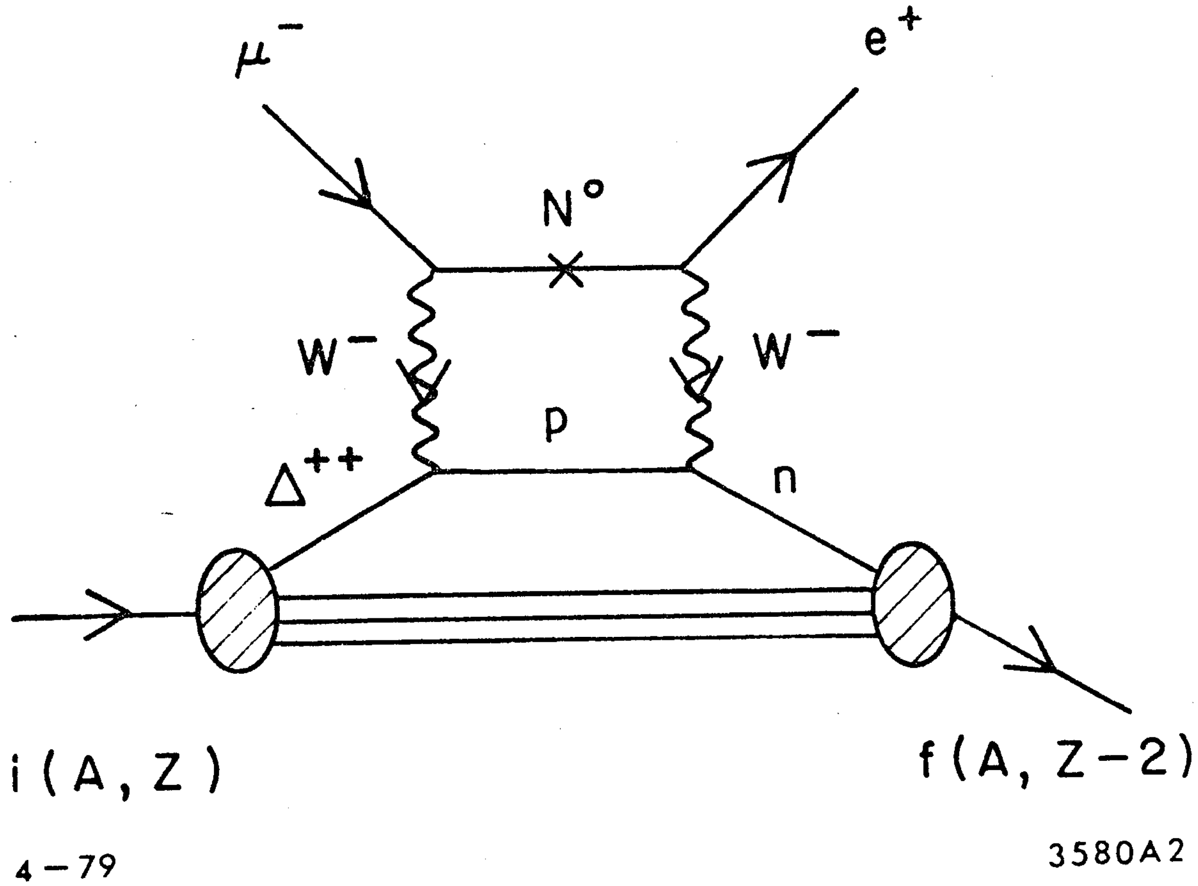

Fig. 2