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SINGLE FEEDBACK SYSTEMS FOR SIMULTANEOUS DAMPING OF HORIZONTAL AND LONGITUDINAL COHERENT OSCILLATIONS*

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Introduction

In a storage ring, the center of charge of a particle bunch may oscillate in the transverse-betatron, or the longitudinal-synchrotron degrees of freedom. In case of a coherent instability, the amplitude of these oscillations may grow indefinitely in time, leading to the loss of the particle bunch. In many machines, feedback systems have been successfully used to damp unstable coherent bunch oscillations. The basic principle is quite simple: one first measures the deviation of the bunch's position in some coordinate from its ideal trajectory and then tries to perturb the bunch in such a way that the deviation becomes smaller after the perturbation.[1] In practice, however, the situation may be slightly more complicated. The complication comes mainly from the fact that the position deviations measured or the pertubation applied to the bunch are often not pure coordinates of the degree of freedom which one wants to damp. For example, an easily measurable quantity is the horizontal displacement x of the bunch, but the value of x contains both horizontal-betatron and synchrotron contributions. Similarly, an easily applicable perturbation is to kick the bunch horizontally by an angle $\Delta x'$, but this kick in general excites both horizontal-betatron and synchrotron motions. It is clear that the horizontal-betatron and the synchrotron motions are intrinsically coupled and a consistent analysis of a feedback system for these degrees of freedom must take both dimensions simultaneously into consideration.[2] The same difficulty does not appear in feedback damping of vertical-betatron oscillation in most rings because the vertical dimension is coupled to the other two degrees of freedom only by electromagnetic field errors existing in the machine.

To describe the horizontal motion of the bunch, we need four coordinates, which can be written as a vector $\{x,x',z,\delta\}$. Where x and z are the horizontal and longitudinal displacements of the bunch center relative to the ideal trajectory; x' is the angle between the bunch's direction of motion and the ideal trajectory; and $\delta=\Delta E/E$ is relative energy error of the bunch. Among the four variables, x and z are easy to measure by position monitors, while x' and δ are easy to change by electromagnetic devices. In combination, this suggests four possible types of feedback systems:

- Type (x,δ) : measuring x and changing δ
- Type (x,x'): measuring x and changing x'
- Type (z,δ) : measuring z and changing δ
- Type (z,x'): measuring z and changing x'

In the following, we will present a complete analysis of the Type (x,δ) feedback system, using a matrix method. The analyses of other types are similar to that of Type (x,δ) and only the results are included. We then include some comparisons of these types of feedback schemes in terms of power consumptions and the effectiveness in damping the horizontal-betatron and synchrotron oscillations. We will also discuss some effects of position measuring errors on the performance of the feedback systems.

Matrix Method

Consider a Type (x,δ) feedback system, which consists of a beam position monitor and an rf cavity driven by a generator which is phase modulated by the position signal. The horizontal displacement of the beam is measured at the monitor and the measurement is sent to the rf cavity. The phase of the rf cavity is then adjusted so that an electron changes its energy by the additional amount $\Delta\delta=\zeta x_{monitor}$. We assume that there is only one active rf cavity in the storage ring which supplies the longitudinal restoring force as well as the feedback action; and that the measurement signal from the monitor reaches the cavity before the beam completes one turn.

The transfer matrix of the vector $\{x,x',z,\delta\}$ for one complete revolution starting from the rf cavity is given by

$$T_{tot} = T_{cav}\, T_o, \tag{1}$$

where T_{cav} is the transfer matrix across the cavity and T_o is the transfer matrix for the rest of the storage ring.

The energy kick the electron receives at the cavity is

$$\Delta\delta = \frac{2\pi\nu_s^2}{\alpha R} z + \zeta x_{monitor}, \tag{2}$$

where ν_s is the unperturbed synchrotron tune, α is the momentum compaction factor and $2\pi R$ is the circumference of the machine. The first term in Eq. (2) is the usual rf focusing term and the second term is the feedback contribution. The transfer matrix T_{cav} can therefore be written as

$$T_{cav} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{2\pi\nu_s^2}{\alpha R} & 1 \end{bmatrix} + \zeta \begin{bmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 1&0&0&0 \end{bmatrix} T_{CM}^{-1} \tag{3}$$

where T_{CM} the transfer matrix from the monitor to the cavity.

The definitions of the elements for the matricies T_o, T_{CM} and T_{tot} are given in Ref. 3. To find the Type (x,δ) feedback damping rates, we need to find the eigenvalues of the matrix T_{tot}. The eigenvalues are given by the solution of the secular equation

$$\det\,(T_{tot} - \lambda) = 0, \tag{4}$$

Solving Eq. (4) for $\lambda = \exp\{-\alpha_k \pm 2\pi(\nu_k + \Delta\nu_k)\}$ with k=(x or s), yields an exact solution for the damping constants α_k and the coherent tune shifts $\Delta\nu_k$. The 4 eigenvalues obey the property $\lambda_1\lambda_2\lambda_3\lambda_4 = \det(T_{tot})$, which for Type (x,δ) becomes

$$e^{-2\alpha_x}\, e^{-2\alpha_s} = 1 + \zeta\left[\eta_m + Q\right], \tag{5}$$

with

$$Q = \sqrt{\beta_c\beta_m}\,\eta_c' \sin\Delta\psi - \sqrt{\frac{\beta_m}{\beta_c}}\,\eta_c(\cos\Delta\psi - \alpha_c \sin\Delta\psi),$$

and $\Delta\psi = 2\pi\nu_x - \psi_m + \psi_c$ the betatron phase advance from the monitor to the feedback cavity. For weak damping, $|\alpha_{x,s}| \ll 1$, we have the following sum rule

$$\alpha_x + \alpha_s = -\frac{\zeta}{2}(\eta_m + Q), \tag{6}$$

If the cavity and the monitor are located at the same location $(\Delta\psi=0, \eta_c=\eta_m, \beta_c=\beta_m)$ we find $\alpha_x + \alpha_s = 0$, which means that damping for one mode necessarily causes anti damping for the other mode.

It turns out that more practical approximate expressions for the damping rates can be obtained from Eq. (4) provided that

(i) $2\pi\nu_s \ll 1$.

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(ii) the synchrotron and betatron tunes are very different so that the synchrotron-betatron coupling effect can be ignored (See Ref. 4).

(iii) the feedback system is reasonably weak so that that damping rates $|\alpha_{x,s}| \ll 1$.

The results of this approximation are, for Type (x,δ) feedback system, Type (x,δ) $(\Delta\delta_c = \zeta x_m)$:

$$\alpha_x \approx -\frac{\zeta Q}{2}$$
$$\alpha_s \approx -\frac{\zeta \eta_m}{2} \qquad (7)$$

These results are consistent with the sum rule, Eq.(6).

The matrix method used above to analyze the Type (x,δ) feedback system can be applied to other types as well. We make the same approximations as before to obtain the damping rates for the other three types of feedback systems (See Ref. 3 for detail results). In general the feedback systems damp or anti-damp both the horizontal-betatron and the synchrotron oscillations. As a result, installation of such feedback systems requires careful arrangements. Putting feedback components at favorable positions damps both modes, while unfavorable arrangements may damp one mode but anti-damp the other mode.

Comparison of Feedback Schemes

In this section, we will compare the four types of feedback schemes in terms of their effectiveness in damping the horizontal-betatron and the synchrotron modes and their required power consumption.

We demand that the system damps a mode oscillation with a "one-sigma" amplitude by a factor of e in N_d turns. We will make order-of-magnitude estimates, letting β-functions $\approx R/\nu_x$, η-functions $\approx R/\nu_x^2$, momentum compaction factor $\alpha \approx 1/\nu_x^2$ and $\Delta\psi$ arbitrary. We will also assume that $\nu_x \gg 1$ and $\nu_s \ll 1$. Under these conditions, the maximum deviations in x, x', z and δ for a one-sigma horizontal-betatron oscillation are $\sigma_x, \sigma_x\nu_x/R, \sigma_x/\nu_x$ and $\sigma_x/f\nu_x$ where σ_x is the rms betatron beam size and $f=R/\nu_x^2\nu_s$ is the ratio of the rms bunch length to the rms energy spread, σ_δ. Similar values for the synchrotron mode are $R\sigma_\delta/\nu_x^2, \sigma_\delta/\nu_x$, $f\sigma_\delta$ and σ_δ.

The order of magnitude expressions of the damping rates for the horizontal-betatron mode are shown in Table 1. In order to have a damping rate of $\alpha_x=1/N_d$, the required feedback strengths $\Delta x'_{max}$ for Types (x,x') and (z,x') and $\Delta\delta_{max}$ for Types (x,δ) and (z,δ) are also shown in Table 1. To damp a n-sigma oscillation in N_d turns, the required feedback strengths must be increased by a factor of n. Similar results for the synchrotron mode are shown in Table 2, where α_{cm} is the partial momentum compaction from the monitor to the cavity.

The power consumption of a feedback system is directly proportional to the electromagnetic field energy, U, stored in the system. In order to feed back on a turn-by-turn basis, this energy U is dissipated before the next particle bunch arrives. The required feedback power is therefore given by $P=U\,N_b/T_o$, where N_b is the number of particle bunches and T_o is the revolution period. For a feedback system which uses a kicker magnet {Types (x,x') and (z,x')}, the maximum stored field energy is given by, in the cgs units,

$$U_{max} = \frac{1}{8\pi} V_{mag}\, B_{max}^2 \qquad (8)$$

where V_{mag} is the effective volume in the kicker magnet filled with a magnetic field of strength

$$B_{max} = \frac{E}{eL_{mag}} \Delta x'_{max}$$

with L_{mag} the length of the magnet kicker, E the beam energy and e the unit charge. For a feedback system which uses a cavity {Types (x,δ) (z,δ)}, we find

$$U_{max} = \frac{1}{8\pi} V_{cav}\, E_{max}^2 \qquad (9)$$

where V_{cav} is the cavity volume filled with an electric field of strength

$$E_{max} = \frac{E}{e\,L_{cav}} \Delta\delta_{max}$$

with L_{cav} the length of the cavity. For a rough estimate, let us assume $V_{mag} = V_{cav} = V_o$ and $L_{mag} = L_{cav} = L_o$. The ratio of the power required in a magnet system to the power required in a cavity system is

$$\left(\frac{B_{max}}{E_{max}}\right)^2 = \left(\frac{\Delta x'_{max}}{\Delta\delta_{max}}\right)^2 \qquad (10)$$

In Tables 1 and 2. the estimates of the power consumption of the various types of feedback systems are given for the horizontal-betatron mode and synchrotron mode respectively, where the quantity C is given by

$$C \equiv \frac{N_b\, V_o\, E^2}{2\pi\, T_o e^2 L_o^2 N_d^2} \qquad (11)$$

The betatron frequency ν_x is usually much larger that unity so that for damping the horizontal-betatron oscillations the Type (x,x') and (z,x') feedback systems, in which the variable x' is changed, will require the least power. The synchrotron frequency ν_s is generally much less than one so that for damping the synchrotron oscillation the Type (x,δ) feedback system, in which x is measured and δ changed, will require the least power.

Effects Caused By Errors

So far we have assumed that the beam position measurements by the monitors do not contain errors. In reality, the noise in the position measuring signal sent to the feedback device causes a diffusion in the bunch motion. In equilibrium, this diffusion effect is balanced by the feedback damping effect, giving rise to a gaussian distribution in the synchrotron and betatron amplitudes of the bunch motion.

As an example, consider a Type (x,δ) feedback system. Let $\bar{x}$ be the position measuring noise, corresponding to a contribution of $\zeta\bar{x}$ to the energy gain at the feedback cavity. The synchrotron energy spread then has a diffusion rate per turn given by

$$\frac{d}{dn}\langle\delta^2\rangle = \frac{1}{2}\zeta^2\langle\bar{x}^2\rangle$$

The damping rate per turn, on the other hand, due to the feedback is

$$\frac{d}{dn}\langle\delta^2\rangle = -2\alpha_s\langle\delta^2\rangle$$

where α_s is the damping constant given by Eq. (7). In equilibrium, the sum of the above two expressions vanishes, yielding

$$\langle\delta^2\rangle = \frac{\zeta^2\langle\bar{x}^2\rangle}{4\alpha_s} \qquad (12)$$

The position measuring noises also give rise to a spread in the betatron amplitude of the bunch. The diffusion rate is

$$\frac{d}{dn}\langle x_\beta^2\rangle = \frac{1}{2}\eta_c^2\zeta^2\langle\bar{x}^2\rangle$$

where η_c is the dispersion function at the cavity and the damping rate is

$$\frac{d}{dn}\langle x_\beta^2\rangle = -2\alpha_x\langle x_\beta^2\rangle$$

where α_x is given by Eq. (7). In equilibrium, we find

$$\langle x_\beta^2 \rangle = \frac{\eta_c^2 \zeta^2 \langle \bar{x}^2 \rangle}{4 \alpha_x} \qquad (13)$$

These results, Eqs. (12) and (13), together with the results for the other types of feedback systems, are summarized in Table 3. The feedback damping constants for different feedback types are given by Eq. (7) and Ref. 3, except that, for Type (x, x') α_s is dominated by the radiation damping since the feedback damping is not effective. In practice, these noises in bunch motion usually are small compared with the natural incoherent spread within the beam and should not impose serious problems.

Table I. Horizontal-Betatron Mode

Feedback Type	Damping Rate	Required Feedback Strength	Required Feedback Power
Type (x, δ) $\Delta\delta_c = \zeta x_m$	$\left(\frac{\zeta R}{2 \nu_x^2}\right)$	$\Delta\delta_{c,max} = \left(\frac{2\nu_x^2 \sigma_x}{N_d R}\right)$	$C \nu_x^4 \left(\frac{\sigma_x}{R}\right)^2$
Type (x, x') $\Delta x'_k = \zeta x_m$	$\left(\frac{\zeta R}{2 \nu_x^2}\right)\nu_x$	$\Delta x'_{k,max} = \left(\frac{2\nu_x^2 \sigma_x}{N_d R}\right)\frac{1}{\nu_x}$	$C \nu_x^2 \left(\frac{\sigma_x}{R}\right)^2$
Type (z, δ) $\Delta\delta_c = \zeta z_m$	$\left(\frac{\zeta R}{2 \nu_x^2}\right)\frac{1}{\nu_x}$	$\Delta\delta_{c,max} = \left(\frac{2\nu_x^2 \sigma_x}{N_d R}\right)$	$C \nu_x^4 \left(\frac{\sigma_x}{R}\right)^2$
Type (z, x') $\Delta x'_k = \zeta z_m$	$\left(\frac{\zeta R}{2 \nu_x^2}\right)$	$\Delta x'_{k,max} = \left(\frac{2\nu_x^2 \sigma_x}{N_d R}\right)\frac{1}{\nu_x}$	$C \nu_x^2 \left(\frac{\sigma_x}{R}\right)^2$

References

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Table II. Synchrotron Mode

	Damping Rate	Required Feedback Strength	Required Feedback Power
Type (x,δ) $\Delta\delta_c = \zeta x_m$	$\left(\frac{\zeta R}{2\nu_x^2}\right)$	$\Delta\delta_{c,max} = \left(\frac{2}{N_d}\sigma_\delta\right)$	$C \sigma_\delta^2$
Type (x,x') $\Delta x'_k = \zeta x_m$	o	$\Delta x'_{k,max} = \infty$	∞
Type (z,δ) $\Delta\delta_c = \zeta z_m$	$\left(\frac{\zeta R}{2\nu_x^2}\right)\frac{2\pi\alpha_{cm}}{\alpha}$	$\Delta\delta_{c,max} = \left(\frac{2}{N_d}\sigma_\delta\right)\frac{\alpha}{2\pi\alpha_{cm}\nu_s}$	$C\sigma_\delta^2\left(\frac{\alpha}{2\pi\alpha_{cm}}\right)^2\frac{1}{\nu_s^2}$
Type (z,x') $\Delta x'_k = \zeta z_m$	$\left(\frac{\zeta R}{2\nu_x^2}\right)$	$\Delta x'_{k,max} = \left(\frac{2}{N_d}\sigma_\delta\right)\frac{1}{\nu_s}$	$C \sigma_\delta^2 \frac{1}{\nu_s^2}$

Table III

	Synchrotron Mode	Betatron Mode
Type (x,δ) $\Delta\delta_c = \zeta x_m$	$\langle\delta^2\rangle = \frac{\zeta^2\langle\bar{x}^2\rangle}{4\alpha_s}$	$\langle x_\beta^2\rangle = \frac{\eta_c^2\zeta^2\langle\bar{x}^2\rangle}{4\alpha_x}$
Type (x,x') $\Delta x'_k = \zeta x_m$	$\langle z^2\rangle = \frac{\eta_k^2\zeta^2\langle\bar{x}^2\rangle}{4\alpha_s}$	$\langle x'^2_\beta\rangle = \frac{\zeta^2\langle\bar{x}^2\rangle}{4\alpha_x}$
Type (z,δ) $\Delta\delta_c = \zeta z_m$	$\langle\delta^2\rangle = \frac{\zeta^2\langle\bar{z}^2\rangle}{4\alpha_s}$	$\langle x_\beta^2\rangle = \frac{\eta_c^2\zeta^2\langle\bar{z}^2\rangle}{4\alpha_x}$
Type (z,x') $\Delta x'_k = \zeta z_m$	$\langle z^2\rangle = \frac{\eta_k^2\zeta^2\langle\bar{z}^2\rangle}{4\alpha_s}$	$\langle x'^2_\beta\rangle = \frac{\zeta^2\langle\bar{z}^2\rangle}{4\alpha_x}$