LOW MASS LEPTON PAIRS FROM HADRON AND NEUTRINO BEAMS[*]

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ABSTRACT

A simple model based on meson annihilation is proposed to explain low mass production of lepton pairs for both hadron and neutrino beams. Resonances, such as the rho and omega, are naturally taken into account and are shown to give rise to a large but unexpected contribution at very low pair masses. Numerical results are given together with comparisons to data. For the most part, the normalizations are taken from other experimental data.

I. INTRODUCTION

The simplicity and appeal of the Drell - Yan (D - Y) model[1] for lepton pair production in hadronic scattering has fostered its application well outside the kinematic regimes for which it was originally justified. Indeed, as Drell and Yan stressed, the approximations made in the parton model (in the impulse approximation or the hard scattering expansion[2] (HSE) are not valid at large pair transverse momenta nor at low pair masses.

The HSE is illustrated in Fig. la. In the sum over states labeled by a, b, and d, one must insure that the individual terms are incoherent as well as gauge invariant. This requires that partons a and b have a small transverse momentum k_T. The D - Y model analyzes only the subprocess $q\bar{q} \rightarrow \ell^+ \ell^-$, and since there is no recoil d parton, the pair must also have a small p_T.

Recently, a more general analysis of this expansion (but without the small effects due to QCD radiative corrections, for example) has been carried out.[3] By summing over parton types and independent, incoherent subprocesses (including, for example, meson-quark, Fig. 1b, and diquark - antiquark, Fig. 1c, significant additional physical information is included; it is simple to achieve a gauge invariant extension of the D - Y model to large p_T pairs. The experimental results are quite well reproduced.

In this paper we wish to extend the discussion to low mass pairs at low p_T and explicitly include resonance effects. We argue that at low mass and low p_T, the most important intermediate states are

those which have small mass but are strongly interacting, that is, those states that determine the large distance structure of the hadron. Since the most important of these are expected to be light mesons, we shall study here the contribution of the subprocess $M + M' \to "\gamma" \to \ell^+\ell^-$.[4]

As the pair mass and/or transverse momentum increases, one expects in this picture to probe smaller and smaller distances, and the most important subprocess should change smoothly from meson – meson to meson – quark and diquark – antiquark, and finally to quark – gluon and quark – antiquark processes.

The general HSE formula for the fully differential pair production cross section is written as

$$Q^4 \frac{d\sigma}{d^4Q}\left(AB \to \ell^+\ell^- X\right) = \sum_{a,b,d} \int dx\, d^2k_T\, dy\, d^2\ell_T\, P_{a/A}\left(x, k_T\right) P_{b/B}(y, \ell_T)$$

$$\times\, Q^4 \frac{d\sigma}{d^4Q}\left(ab \to \ell^+\ell^- d; s', t', u'\right)\quad, \tag{1}$$

where, for sufficiently small k_T and ℓ_T,

$$s' = xys$$

$$t' = xt + (1-x)Q^2$$

$$u' = yu + (1-y)Q^2 \tag{2}$$

Some trivial kinematic factors which tend to unity as a and b approach their mass shells have been omitted in the above. The function $P_{a/A}$ is the probability function of finding constituent a in hadron A with (infinite) momentum fraction x and transverse momentum k_T.

In Section II, the above model is applied to hadron beams and

and numerical results are given for pion-nucleon scattering using a $P_{\pi/N}(x)$ distribution function consistent with inclusive production of large p_T mesons. In Section III, the model is applied to neutrino scattering by computing the probability of finding a meson in a neutrino, $P_{M/\nu}(x)$. We do this in two stages. Using the (presumed) exact lepton current-quark current coupling, we first calculate $P_{q/\nu}(x)$ [in a manner analogous to the equivalent particle calculations of Chen and Zerwas][5]. We then convolute this with $P_{M/q}(x)$ [inferred from experimental results in e^+e^- annihilation into hadronic jets][6] to obtain $P_{M/\nu}(x)$. For the case $M = \pi$, we obtain a (common) lower bound for the ratio of tri-muon to single muon cross sections in charged current (CC) neutrino-hadron scattering and of dimuon to muonless cross sections in neutral current (NC) scattering. We find that this lower bound is consistent with the experimentally observed ratio in the measured CC case indicating that there may be no other significant contributions beyond π-π annihilation in that case. Finally, we indicate some tests of our model and summarize our conclusions in Section IV.

II. HADRON BEAMS

The primary goal of this section is to give a theoretical treatment of low mass pair production based on the physical picture described in the Introduction. Since there is no requirement of any large transverse momentum in the process, the expansion in Eq. (1) is properly defined (there are no subtle coherence problems), and the structure functions are properly restricted, i.e., the intermediate particles are only

slightly off shell and enforcing gauge invariance is no problem.

The contribution of meson – meson, M – M', annihilation is given by

$$Q^4 \frac{d\sigma}{dQ^2 d\xi} \approx \frac{4\pi\alpha^2}{3} \sum_{M \neq M'} \left| F_{MM'}(Q^2) \right|^2 dxdy\delta(\xi - x + y)xy\delta(xy - Q^2/s)$$

$$[P_{M/A}(x)P_{M'/B}(y) + P_{M'/A}(x)P_{M/B}(y)] \quad , \tag{3}$$

where off – shell effects in the form factor have been neglected.

These subprocesses automatically included multiple soft scattering corrections to the Drell – Yan process in which soft particles accompany the quarks in the form of correlated pairs (the mesons). What we have picked out here is a computable and physically sensible subset of all possible processes which, as we shall see, seem to be dominant at low p_T and for small pair masses.

The main contribution to the above sum is anticipated to be charged pion annihilation. For this process there are direct measurements of the relevant form factor in electron – positron annihilation.[7] There are other interesting contributions such as pion –"rho" annihilation which contains the omega resonance, et cetera. [It should be pointed out that these resonance effects will also play some role in the Drell – Yan process, quark – antiquark annihilation, but this is a small fraction of the cross section at these low masses in our model.] From uncertainty principle arguments, such as given in the Introduction, these contributions are expected to be unimportant at low masses and small momentum transfers and numerically they are small.

The probability functions $P_{M/A}(x)$ are not well studied, but an analysis[8] of large transverse momentum scattering yielded the

approximate form for a proton target which is consistent with the spectator counting rules [9]

$$xP_{M/P}(x) = P_0(1-x)^2 F_2(x)$$

where $F_2(x)$ is the proton structure function. P_0 is a normalization constant chosen so that this probability behaves as

$$xP_{M/P}(x) \sim C_{M/P}(1-x)^5 \quad , \tag{4}$$

where $C_{M/P}$ is a constant fitted to the data ($C_{M/P} \sim 0.2$). The above form and value of $C_{M/P}$ are consistent with the large momentum transfer scattering results.[10] For pion beams, we also need

$$xP_{M/\pi}(x) = x\delta(1-x) + C_{M/\pi}(1-x)^3 \quad ,$$

where $C_{M/\pi} \sim 0.2$. One expects that the probability function for pions, rhos and omegas will be of the same order of magnitude and that all are described by the above approximate forms (omitting the δ-function when $M \neq \pi$, of course).

The formula for the cross section, Eq. (3), will be evaluated and compared with data below. Before doing this, however, it is amusing to note several properties of the model. If Eq. (3) is evaluated at $\xi \sim 0$, and for $x = (Q^2/s)^{\frac{1}{2}}$, the result can be weitten in terms of the Drell-Yan amplitude for point quarks

$$Q^4 \frac{d\sigma}{d^2Qd\xi}\bigg|_{\xi=0} \simeq Q^4 \frac{d\sigma}{d^2Qd\xi}\bigg|_{\xi=0}^{D-Y} \sum_{M,M'} \left| F_{MM'}(Q^2) \right|^2 C_{MM'}$$

where $C_{MM'}$ contains the relative normalization factors. The D-Y process itself, can be included in the sum by setting $F_{q\bar{q}} = 1$ and $C_{q\bar{q}} = 1/3$

(for color) It will be shown to be amsll for pair masses below 1 GeV.
The crossover between the $q - \bar{q}$ and $M - M'$ annihilation contributions occurs
for masses between 1.0 and 1.5 GeV.

The rho resonance will be a dominant feature of $\pi^+\pi^-$ annihilation
(as well as $K\bar{K}$, etc.) and data exists for the relevant form factor.[7]
The omega resonance will likewise be a dominant feature of π - rho
annihilation but the relevant experimental form factor is not directly
available. One can, however, proceed by direct analogy with the $\pi - \pi$
case by using the omega width in the relevant form factor and normalizing
to the (narrow) omega peak. This neglects the small effects of rho - omega
mixing but is adequate for our purposes.

For numerical calculation, we use a fit to the pion form factor as
determined at Orsay[11]

$$\left| F_{\pi\pi}(Q^2) \right|^2 = F_0^2 M_\rho^2 \Gamma_\rho^2 \left[\left(M_\rho^2 - Q^2 \right)^2 + M_\rho^2 \Gamma_\rho^2 \left(p/p_0 \right)^6 \left(M_\rho/Q \right)^2 \right]^{-1} \quad , \qquad (6)$$

where p is the pion momentum and M_ρ = 0.775 GeV, Γ_ρ = 0.15 GeV,
F_0 = 5.83 and p_0 = 0.36 GeV/c.

The numerical results for $\pi + p \rightarrow \mu^+\mu^-$ are given in Fig. 2 for
an energy of E = 16 GeV and compared to the data of K. Bunnel, et al.[12]
The sharp rise near the π-pair threshold can be explained from the Dalitz
decay of ω and η. The normalization is achieved by fitting with the
value[10] $C_{\pi/p}$ = 0.2 and assuming only π-π annihilation in the region
of the rho. Due to the very fine resolution of this streamer chamber
experiment, an additional ω peak, due to π - ρ annihilation is also

clearly visible. We have not explicitly included these contributions in our calculation since they involve a term with independent renormalization and form factor, do not check our model in any way, and do not contribute significantly to the total cross section.

Using the same value of $C_{\pi/p}$ as above, we find that the calculated cross section is consistent with the high energy $\pi - p$ data, e.g., K. J. Anderson, et al.,[13] after taking scaling violations into account. In such higher energy experiments, the pair-mass resulution is not suffucient to resolve the ω-contribution.

These results lend credence to the idea that the meson – meson mechanism dominates low-mass μ-production, and fixes the hadronic distribution functions that we need in the next section.

III. NEUTRINO BEAMS

We now turn to the discussion of low mass μ-pair production in neutrino scattering. For charged current (CC) events this means a three – muon final state with its attendant difficulty of separating the identification of the pair and the "leading" muon. For neutral current (NC) events, the situation is entirely analogous to hadron – hadron scattering μ-pair production as discussed above.

In order to parallel the preceeding calculations and discussions, we need to compute the probability of finding a meson in a neutrino as a function of momentum fraction x. This is analogous to the equivalent photon and electron calculations of Chen and Zerwas.[5] The calculation here is somewhat more involved, since the fundamental coupling (in the approximation appropriate to present energies) generates a one-to-three

body transition $(\nu \to \mu q \bar{q})$ rather than a one-to-one body transition $(\gamma \to e \bar{e}$ or $e \to \gamma e)$. We will make things easier by working in the Finite Momentum Frame (FMF) and combining two of the three bodies into a single effective object.

In analogy with the equivalent photon calculations, we analyze $\nu - T$ scattering into $\mu - q - \bar{q}$ (see Fig. 3) where an intermediate, off-shell $\bar{q}'$ (q') strikes the massless scalar target T to make the final state on-shell $\bar{q}$ (q). The target T is chosen for calculational convenience; since we will lose all spin information when we convert the derived quark distrubution into a pion distribution, we calculate only a scalar structure function, rather than the full tnesor object. We do not wish to calculate the whole of the Feynman diagrams of Fig. 3, but rather, in the sense of the Hard Scattering Expansion (HSE),[2] only the part where the $\bar{q}'$ (q') are "alomst" on-shell so that we may write, approximately,

$$\sigma_{\nu T \to \mu q \bar{q}} = \int_0^1 dx \left[P_{q/\nu}(x) \sigma(q'T \to q) + P_{\bar{q}/\nu}(x) \sigma(\bar{q}T \to \bar{q}) \right] \qquad (7)$$

and identify the function $P_{q/\nu}$ and $P_{\bar{q}/\nu}$ for further use.

We use the standard V-A charged-current quark-lepton coupling and parameterize the momenta in the FMF as (for Fig. 3b)

$$\nu = \left(P, \ 0_T, \ P \right)$$

$$q' = \left(xP - \frac{L^2 + L_T^2}{4(1-x)P}, \ -\vec{L}_T, \ xP + \frac{L^2 + L_T^2}{4(1-x)P} \right)$$

$$L = \left((1-x)P + \frac{L^2 + L_T^2}{4(1-x)P}, \quad \vec{L}_T, \quad xP - \frac{L^2 + L_T^2}{4(1-x)P} \right)$$

$$T = \left(P, \quad 0_T, \quad -P \right) \quad , \tag{8}$$

where $L = \mu + \bar{q}$. Straightforward calculation then yields

$$\sigma_{\nu T \to \mu \bar{q} q} \simeq 3 \frac{2 G_F^2}{\pi^6} \int d^4\mu \, d^4\bar{q} \, \delta\left(\bar{q}^2 - m_q^2\right) \delta\left(\mu^2 - m_\mu^2\right) \frac{q' \cdot T \nu \cdot \bar{q} [\mu \cdot (2T + q')]}{\nu \cdot T [q'^2 - m_q^2]^2}$$

$$\left\{ \frac{\pi}{2q' \cdot T} \int \frac{d_q^3}{2E_q} \delta^4 (T + q' - q) \, g^2 m_q^2 \right\} \quad , \tag{9}$$

where the factor of 3 counts quark colors, the last factor in braces is $\sigma(q' + T \to q)$, and for simplicity we have used the symbol for each particle to also represent the corresponding four-momentum. Note that $q' = \nu - L$. Defining $\ell = (\mu - \bar{q})/2$, we replace the $d^4\mu d^4\bar{q}$-integration by $d^4L d^4\ell$-integrations, and after completing the $d^4\ell$-integral, identify from Eq. (9)

$$\int dx P_{q/\nu}(x) \simeq 3 \frac{s G_F^2}{16\pi^4} \quad dx \, dL^2 \, dL_T^2 \quad \frac{x(1-x) \quad [2L_T^2 + L^2]}{[L_T^2 + xL^2 + m_q^2(1-x)]^2} \quad ,$$

where we have also used $\int d^4 L = (\pi/2) \int dL^2 dL_T^2 \, dx/(1-x)$, and $q'^2 = -(L_T^2 + xL^2) / (1-x)$.

In equivalent photon calculations, such as those of Chen and Zerwas,[5] the corresponding formula would not have L^2, L_T^2 dependence in the numerator function of the integrand, and the denominator function would force the dominant contribution to be from low q'^2 automatically. Here we must recognize that only the small q'^2 part of the above expression

is sensible in the HSE picture that we are using; the factorization of

Eq. (9) into the form of a term of Eq. (7) and treatment of $\sigma(T + q' \rightarrow q)$

as a constant is justified only for q'^2 less than or on the order of a

natural hadronic (mass)^2 scale. We take this scale to be $\langle m^2 \rangle \sim 1 \, \text{GeV}^2$.

Thus, although the integrand above does not force this restriction, the

physics of the HSE tells us that only in the L^2, $L_T^2 \lesssim 1 \, \text{GeV}^2$ region do

we have a consistent picture. Note that this restriction is just the

standard one limiting the validity of the equivalent particle approxi-

mation to low mass objects (but here, composed of a pair). We, therefore,

cut off the integration at that value and keep only that part of the

calculated cross-section; this gives us lower bounds on $P_{q/\nu}(x)$. The

omitted region is present in a higher order term in the HSE where it is

consistently treated.

Completing the integration, we find

$$P_{q/\nu}(x) \simeq s \, \frac{G_F^2 \langle m^2 \rangle}{16\pi^4} \, (1 - x) \quad . \tag{10}$$

Equation (10) is not the actual calculated form which is quite

complicated but is a simple interpolating function, accurate to $\pm 30\%$

over the range of x. This form was chosen in accord with spectator

counting rules.[9]

Note that the hard vertex has given rise to the typical weak

interaction sG_F^2 dependence and that there is no x^{-1} factor. We have

assumed that s is sufficiently small so that the effects of the

intermediate-vector=boson propagator do not need to be included. If s

were much larger, the calculation would then resemble that for finding

say a positron in an electron,[5] by convoluting the probability of

finding a quark (e+) in a W-boson [eventually with negligible mass]

(γ), with that of finding a W-boson (γ) in a neutrino (e⁻).

If we imagine the T in Fig. 3 to be a hadron, it becomes apparent

that Eq. (7), with $P_{q/\nu}$ as given by Eq. (10), describes a portion of the

total CC ν-hadron cross-section. This allows us a consistency check

on the normalization of Eq. (10) as we must have

$$2 \int dx \, P_{q/\nu} \, \sigma(q + hadron) \ll \sigma(\nu + hadron) \qquad (11)$$

since we have not included the entire contribution of all of the

relevant Feynman diagrams. The factor of 2 accounts for the contribution

of quarks and antiquarks (Figs. 3a and 3b) since $P_{\bar{q}/\nu} \simeq P_{q/\nu}$. This last

point argues that the inequality in (11) should be well satisfied:

a lack of distinction between q and $\bar{q}$ is characteristic of "wee" quark

effects which are known to contribute $\lesssim 15\%$ of the total cross-section.

In fact, estimating the left side of (11), we find

$$\frac{sG_F^2 \langle m^2 \rangle}{8\pi^4} \left(\frac{1}{2} \sigma(\pi - hadron) \right) \simeq \left(.006 \text{ mb-GeV}^2 \right) sG_F^2$$

$$\leq \left(.075 \text{ mb-GeV}^2 \right) sG_F^2 \quad , \qquad (12)$$

where the r.h.s. describes the experimental value.

The calculation for $P_{\bar{q}/\nu}$ is very similar to that above. The result

for <u>neutral</u> current neutrino scattering is similar in form, although

reduced in scale by the relative strength of the weak neutral current.

That is, the <u>detailed</u> distribution function is <u>process-dependent</u>.[14]

However, since our procedure is approximate, rather than an exact

Feynman diagram calculation, we shall below uniformly use a common,

approximate form

$$P_{q/\nu}(x) \simeq P_{\bar{q}/\nu}(x) \simeq \frac{sG_F^2 \langle m^2 \rangle}{16\,\pi^4}\,(1-x) \qquad (13)$$

for CC scatterings, and $\sigma(NC)/\sigma(CC)$ times this for neutral current

reactions. Recall that this already includes a factor of 3 for color.

Equation (13) is sufficient for us to be able to calculate <u>massive</u>

lepton pair production via $\bar{q} - q$ annihilation in neutrino – hadron

scattering. However, the quark annihilation process is certainly

inadequate for <u>low mass</u> pairs even though it may also have resonance

effects present. Its normalization is small at small masses. As we

have done in hadron – hadron scattering, this can be handled by using

a meson – meson annihilation picture for the low pair mass regime.

Fortunately, it is quite simple to compute $P_{M/\nu}$ from Eq. (10) since we

know that[15]

$$P_{M/q}(y) \simeq P_{M/\bar{q}}(y) \simeq 0.5\,(1-y)/y \qquad (14)$$

for any given meson M; we need only convolute the two distributions

$$P_{M/\nu}(x) \simeq P_{M/\bar{\nu}}(x) = 2 \int_0^1 dy\,dw\,\delta(x-wy)\,P_{M/q}(w)\,P_{q/\nu}(y) \quad , \qquad (15)$$

where the factor of 2 accounts of M coming from a q or a $\bar{q}$.

As in Eq. (10), we again rewrite the result in Eq. (15) as a simple

interpolating function good to 30% accuracy, where the form is as

suggested by the spectator counting rules[9]

$$P_{M/\nu}(x) \simeq \frac{s G_F^2 \langle m^2 \rangle}{32 \pi^4} \frac{(1-x)^{3.5}}{x} \tag{16}$$

We will use this result in our calculations below. Note the appearance of the $(1/x)$ factor.

The contribution of meson annihilation to the differential cross-section for low mass muon pairs in neutrino scattering on a nucleon N is therefore

$$\frac{d^2\sigma}{dQ^2 d\xi} = \frac{4\pi\alpha^2}{3} \sum_{MM'} \int dx\, dy\, \delta(\xi - x + y)\, \delta(xy - Q^2/s) \left(\frac{Q^2}{s}\right)$$

$$\left(\frac{1}{Q^4}\right) \left| F_{MM'}(Q^2) \right|^2 P_{M/\nu}(x)\, P_{M'/N}(y) \quad , \tag{17}$$

where Q is the pair mass, ξ is the longitudinal momentum fraction of the pair (along the ν-beam) in the center-of-momentum frame and $F_{MM'}(Q^2)$ is the transition form factor for $M - M'$ annihilation into an off-shell photon of mass Q^2 [F = 0 if $M + M'$ is a state of nonzero charge, etc.

A reliable lower bound in (17) may be found by restricting MM' to the states $\pi^{\pm} \pi^{\mp}$ so that $F = F_{\pi}$, the pion form factor with time-like argument. Using $P_{\pm\pi/N}$ from Eq. (4) and $\left| F_{\pi} \right|^2$ as parametrized in Eq. (6), we have calculated this lower bound by numerical integration. We display the results as singly differential cross-section $d\sigma/dQ^2$ in Fig. 4 for E_{ν} = 50 and 300 GeV with $\left| F_{\pi} \right|^2$ set equal to unity in the dotted curves; the $\bar{q}q$ contribution at high Q^2 is shown by the dashed curves. Note that there is a crossover between these two contributions in the region of

the 1 - 1.5 GeV. This is the same behavior found in Sec. II for

π-induced pairs. In Fig. 5, we show $d\sigma/d\xi$ with the same labelling

conventions as in Fig. 4. Note the enhanced forward "throw" effect of

the relatively hard $P_{q/\nu}$ distribution in the $\bar{q}q$ case.

When we integrate to form a total muon - pair cross-section, we

find the high mass contribution negligible and a total value for the

cross-section which corresponds to a trimuon-to-charged-current cross-

section ratio of

$$\frac{\sigma(\nu N \to \mu^{-}\mu^{+}\mu^{-}x)}{\sigma(\nu N \to \mu^{-}x)} \geq 10^{-4} \tag{18}$$

at very high energies. This is about a factor of five larger than would

be the case with $|F_{\pi}|^2 = 1$ (no rho form factor effect). This result

should be reduced in any comparison with uncorrected experimental values

since our calculations have not included the effects of experimental

cuts on the individual muon momenta. Our detailed results as a function

of s are shown in Fig. 6, along with some recent data.[16]

For comparison with the result (18), we recall that Smith et al.,[17]

and Barger et al.,[18] find a cross-section ratio of $5 - 7 \times 10^{-5}$

(see Fig. 6) using a quark and muon bremsstrahlung calculation which

also has a steeper mass dependence than our result at very low pair

masses. (The muon contribution is clearly present in the data at about

the right level.) Those calculations include the entire contribution

of (quark) Feynman diagrams and so give larger results than Drell - Yan

type quark calculations. Nonetheless, they do not include the mesonic

contributions that we have calculated, and since Fig. 4 showed the

quark and pionic annihilation contributions occur in complementary regions, the two curves may be added to give the total trimuon ratio. Associated charm production followed by semi-muonic decay of <u>both</u> charmed particles yields a ratio of $\sim 10^{-6}$ (see Ref. (18)); this mechanism is suppressed both in production and by the small semi-muonic charm decay branching ratios.

Since (18) is a lower bound, we conclude that the mechanism of meson annihilation explains a large fraction of the observed tri-muon event rate in neutrino - nucleon scattering. This mechanism is best tested by comparing the data with our predictions for ξ and Q^2 dependence of the differential cross-sections. Finally, we note that in a ratio such as (18) the absolute cross-section normalization cancels out. Thus we predict that the di-muon to no-muon ratio in neutral current scattering is

$$\frac{\sigma(\nu N \to \nu \mu^+ \mu^- x)}{\sigma(\nu N \to \nu x)} \gtrsim 10^{-4} \qquad (19)$$

from the assumed mechanism. Unfortunately, charged-current charm production, occuring at a level of several percent, will completely swamp this source. The approximately 1% of observed $\mu^+ \mu^-$ events which are due to the mechanism in Eq (19) are, however, characterized by a low μ-pair mass ($\lesssim M_\rho$).

On the other hand, since $P_{q/\nu}$ given in Eq. (13) is not sensitive to whether we start with ν or $\bar{\nu}$, the smaller absolute $\bar{\nu}$ CC cross-section implies that the $(3\mu/1\mu)$ ratio in $\bar{\nu}$ CC events will be ~ 2 times larger than for ν CC - scattering.

IV. CONCLUSION

We have shown that the $\pi^+\pi^-$ parton annihilation mechanism provides the largest single contribution to the production cross-section in hadronic and neutrino – hadron scattering for low mass, low p_T lepton pairs, and is consistent with the experimental data. In particular, the neutrino induced charged current trimuon-single muon ration ratio of $\sim 10^{-4}$ which was once of such great concern is well explained by our mechanism. It also predicts approximately the same for the neutral current dimuon-muonless ratio and a factor of $\sigma^\nu(CC)/\sigma^{\bar\nu}(CC)$ larger anti-neutrino induced trimuon ratio.

The validity of the application of this mechanism may be further tested by comparing the ξ and Q^2-distributions predicted with other experiments. In particular, we note the following salient features: (1) The μ-pair mass distribution is very similar to that in $\pi - p$ scattering. More than half of the events should have pair masses below 1 GeV and $\sim 25\%$ should be in the rho mass region; (2) In the $W^\pm$ N center-or-mementum frame, the mean $Q_\perp$ (mementum transverse to the W-direction) of the μ-pair should rise to a limiting value of ~ 1.2 GeV/c (with a slight energy dependence) as the μ-pair masses increase above ~ 3 GeV, since this has been shown to occur for the corresponding case in pion-induced pair production based on the same meson annihilation mechanism; (3) The μ-pairs from meson – messon annihilation tend to follow the hadronic shower direction. In addition, of course, there are bremsstrahlung μ-pairs associated with the leading muon direction.

Also, it should be noted that, except at resonances, the $\pi^+\pi^-$ annihilation mechanism predicts a $(1-\cos^2\theta)$ decay angular distribution for the virtual photon, which should be contrasted with the $(1+\cos^2\theta)$ distributions for $q\bar{q}$ annihilation.[19] Except on resonances, we expect $A(Q^2)$, where $(1+A\cos^2\theta)$ is fit to the decay angular distribution, to fall from $\simeq +1$ in the high mass continuum to a negative value (although possibly small in absolute value) for low (≤ 1 GeV) pair masses.

Finally we note that if charm production occurs at the $\sim 10\%$ level in ν-interactions, and since charmed mesons have $\sim 10\%$ semileptonic branching ratios, our mechanism for tri-leptons also predicts tetra-leptons at the $\sim 10^{-6}$ level.[20]

ACKNOWLEDGMENTS

FIGURE CAPTIONS

Fig. 1(a)   A general contribution to the Hard Scattering Expansion.

Fig. 1(b)   The meson - quark basic subprocess.

Fig. 1(c)   The diquark - antiquark basic subprocess.

Fig. 2   Numerical results of the model, Eq. (3) compared with
            the data of Ref. 12.   The solid and dashed curves are
            limits from the Dalitz decay of the omega and eta.

Fig. 3   Dominant diagrams for $q - \bar{q}$ production by neutrino
            beams on the target T.

Fig. 4   Muon pairs produced by the $\pi\pi$ and $q\bar{q}$ (Drell - Yan)
            processes from neutrino beams.   The dotted curve
            assumes a point pion ($F_\pi = 1$), and Q = pair mass.
            (a)   50 GeV incident neutrino beam.
            (b)   300 GeV incident neutrino beam.

Fig. 5   The ξ distribution for muon pairs at selected pair masses Q.
            (a)   50 GeV incident neutrino beam.
            (b)   300 GeV incident neutrino beam.

Fig. 6   Comparison of experimental data (Ref. 16) with
            theoretical calculations for $(3\mu/1\mu)$ ratio of
            total cross-sections.

FOOTNOTES AND REFERENCES

1. S. D. Drell and T. - M. Yan, Phys. Rev. Lett. $\underline{25}$, 316 (1970);
    Ann. Phys. (N.Y.) $\underline{66}$, 578 (1971).

2. W. E. Caswell, R. R. Horgan and S. J. Brodsky, Phys. Rev. D$\underline{18}$,
    2415 (1978) and references therein.

3. C. T. Sachrajda and R. Blankenbecler, Phys. Rev. D$\underline{12}$, 3624 (1975).

4. For early work on a similar approach, see H. Thacker, Phys. Rev.
    D$\underline{9}$, 2567 (1973); G. Chu and J. Koplik, Phys. Rev. D$\underline{11}$, 3134
    (1975). See also, N. S. Craigie and D. Schildknecht,
    Nucl. Phys. $\underline{B118}$, 311 (1977).

5. M. - S. Chen and P. Zerwas, Phys. Rev. D$\underline{12}$, 187 (1975).

6. A. Seiden, Phys. Lett. $\underline{68B}$, 157 (1977); M. Duong-van, in
    Proceedings of the XVIII International Conference on High Energy
    Physics, Tblisi, 1976, ed. by N. N. Bogolubov et al. (JINR, Dubna,
    U.S.S.R., 1977), Vol. II, p. B61.

7. D. Benaksas et al., Phys. Lett. $\underline{39B}$, 289 (1972); G. Cosme et al.,
    ibid. $\underline{40B}$, 685 (1972) and $\underline{63B}$, 349 (1976); V. Sidorov, in
    Proceedings of the XVIII International Conference on High Energy
    Physics, Tblisi, 1976, ed. by N. N. Bogolubov et al. (JINR, Dubna,
    U.S.S.R., 1977), Vol. II, p. B13; V. Alles - Borelli et al., Phys.
    Lett. $\underline{40B}$, 433 (1972); G. Barbiellini et al., Lett. Nuovo Cimento
    $\underline{6}$, 557 (1973); M. Bernardini et al., Phys. Lett. $\underline{44B}$, 393 (1973)
    and $\underline{46B}$, 261 (1973); D. Bollini et al., Lett. Nuovo Cimento $\underline{14}$,
    418 (1975).

8. M. Duong-van, K. V. Vasavada and R. Blankenbecler, Phys. Rev. D$\underline{16}$, 1389 (1977); M. Duong-van and R. Blankenbecler, Phys. Rev. D$\underline{17}$, 1826 (1978).

9. R. Blankenbecler and S. J. Brodsky, Phys. Rev. D$\underline{16}$, 2973 (1974).

10. The $(1-x)^5$ form is best tested in high p_T hadron scattering at high x. Since the effective power is reduced at low x for reasonably shaped structure functions, the fits to hadron data give $C_{M/p} \sim 1.0$; this is consistent with the value given in the text [for a different shape $P(x)$]. See, e. g., J. F. Gunion and D. Jones, SLAC-PUB-2157 (July 1978).

11. See first paper in Ref. 7.

12. K. Bunnel et al., Phys. Rev. Lett. $\underline{40}$, 136 (1978).

13. K. J. Anderson et al., Phys. Rev. Lett. $\underline{37}$, 799 (1976).

14. Process dependence of parton distribution functions has been noted previously: J. Babcock and D. Sivers, Phys. Rev. D$\underline{18}$, 2301 (1978).

15. See second paper in Ref. 8.

16. T. Hansl et al., Phys Lett., $\underline{77B}$, 393 (1978); A. K. Mann in Proceedings of the XIX International Conference on High Energy Physics, Tokyo, 1978, to be published; A. Benvenuti et al., Phys. Rev. Lett., $\underline{40}$, 488 (1978); $\underline{38}$, 1110 (1977); B. Barish et al., ibid. $\underline{38}$, 577; 1037(E) (1978).

17. J. Smith and J. A. M. Vermaseren, Phys. Rev. D17, 2288 (1978).

18. V. Barger, T. Gottschalk and R. J. N. Phillips, Phys. Rev. D17, 2284 (1978).

19. However, the fact that scaling violations affect this has been pointed out by E. Berger and S. Brodsky, SLAC-PUB-2247 (Jan 1979).

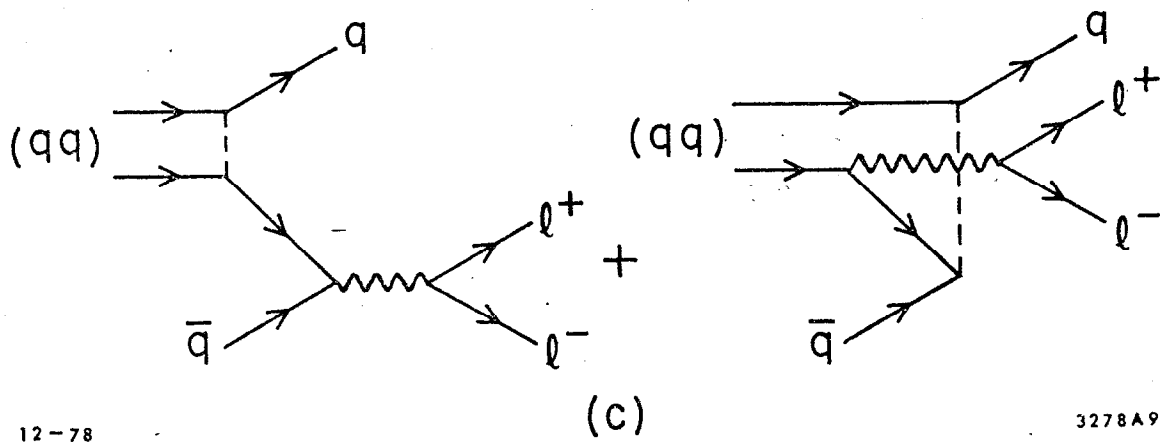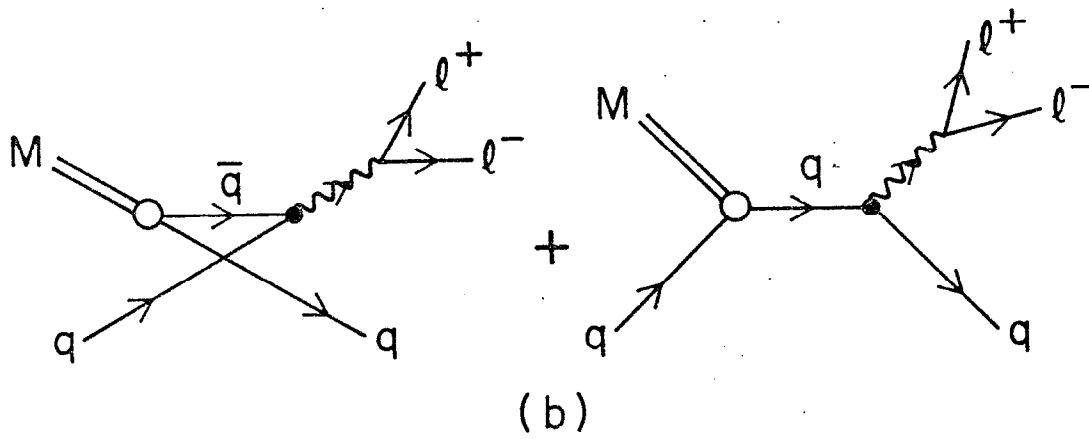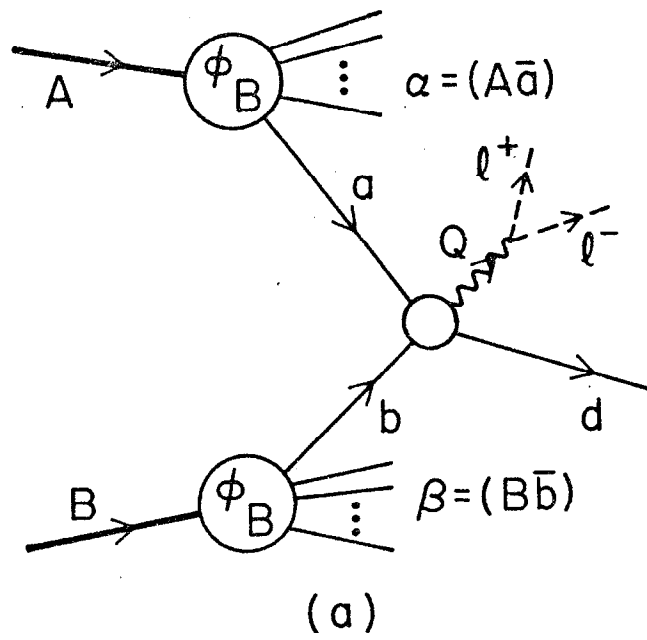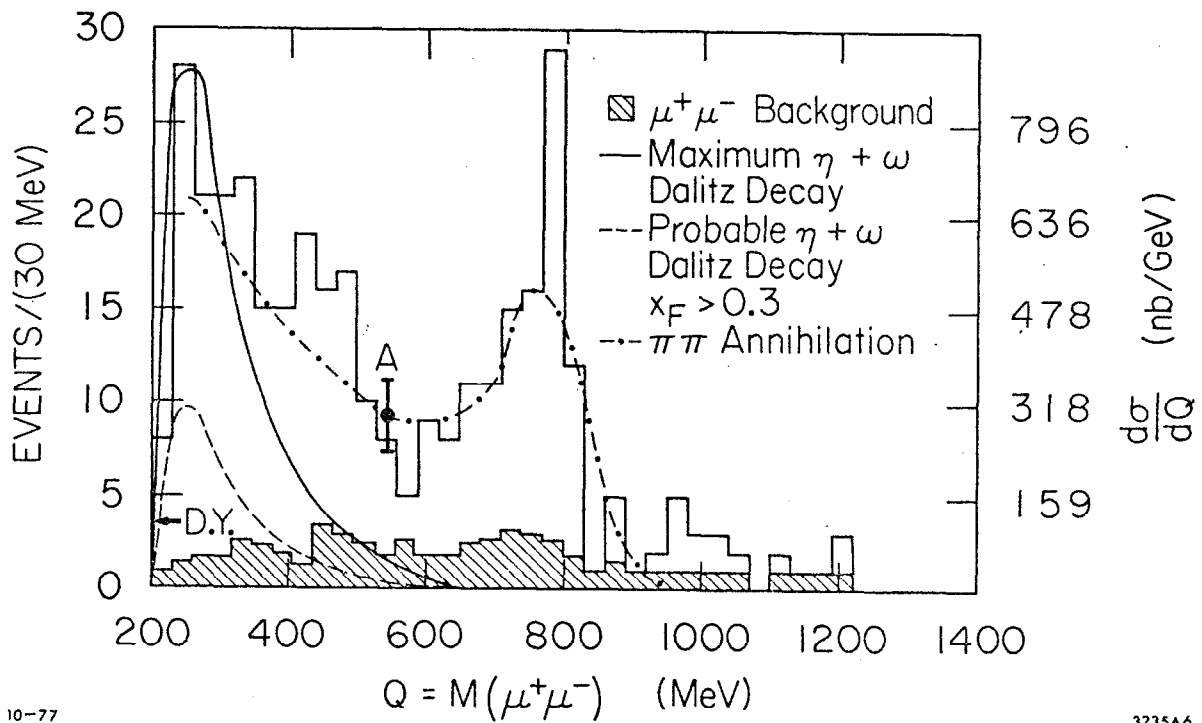20. M. Holder et al., Phys. Lett. 73B, 105 (1978).

$\alpha = (A\bar{a})$

$\beta = (B\bar{b})$

(a)

(b)

(c)

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Fig. 1

Fig. 2

The figure shows a plot with axes labeled:
- Left vertical axis: EVENTS/(30 MeV), ranging from 0 to 30
- Right vertical axis: $\frac{d\sigma}{dQ}$ (nb/GeV), with marked values 159, 318, 478, 636, 796
- Horizontal axis: $Q = M(\mu^+\mu^-)$ (MeV), ranging from 200 to 1400

Legend:
- ⊠ $\mu^+\mu^-$ Background
- —— Maximum $\eta + \omega$ Dalitz Decay
- --- Probable $\eta + \omega$ Dalitz Decay, $x_F > 0.3$
- –·– $\pi\pi$ Annihilation

Labels on plot: A, D.Y.

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μ⁻
ν ——●——→ q
         q̄
         T
(a)

μ⁻
ν ——●——→ q̄
         ↓
         q
         T
(b)

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Fig. 3

(a) $E_\nu = 50$ GeV

—— $\pi\pi$
····· $F_\pi \equiv 1$
—— $q\bar{q}$

(b) $E_\nu = 300$ GeV

$\frac{d\sigma}{dQ}$ (cm²/GeV)

10^{-39}
10^{-40}
10^{-41}
10^{-42}
10^{-43}

0   1   2

0   1   2   3

Q   (GeV)

Fig. 4

Fig. 5

Fig. 6