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FULL AVERAGE RADIATION OF ELECTRONS AND POSITRONS CHANNELED BETWEEN THE PLANES OF A CRYSTAL*

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ABSTRACT

The full average radiation intensity of electrons and positrons channeled between crystal planes is calculated. Comparison of the results of these calculations with calculations using the parabolic approximation for the potential well of a channel is presented. The difference can in extreme cases reach two orders of magnitude. Estimates for the positions of the maxima of the frequency spectra are given. The number of quanta averaged over the angular distribution of the particles in a beam are found.

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1. INTRODUCTION

Spontaneous electromagnetic radiation of ultra-relativistic electrons and positrons brought about by passage through properly oriented crystals has aroused a great deal of interest. There are a variety of theoretical papers concerning this phenomenon.[1-8] Recently, two experiments were performed with the aim of detecting the channeling radiation and measuring its characteristics.[9,10]

All the calculations of the radiation intensity performed until now were done in the parabolic well approximation. However, such an approximation is good enough only for positrons and then only for small amplitudes of their oscillations around the crystal plane. The potential of a channel for electrons nowhere resembles the parabolic well. Hence for electrons, the correct results can not be obtained in the parabolic approximation. Also, as we show further, the main part of the channeling radiation for positrons is produced by those positrons with large oscillation amplitudes. Therefore, even for positrons, it is important to calculate the radiation intensity using the real channel potential, rather than approximating it with a parabolic well.

The aim of this work is to find the full average intensity of channeling radiation, which can be done for any given potential. Our approach does not give the spectrum of emitted radiation (which can be found only for the equivalent harmonic oscillator), but it allows one to find the position of the spectral maximum. The shape of the emitted line can be found in the work by Pantell and Alguard,[11] where such a shape was calculated for Si in the parabolic approximation for potential corrected for anharmonicity. Since all the characteristics of the radiation are of interest, the two approaches supplement each other and shed more light on the phenomena.

For an ultra-relativistic particle, the classical treatment is a very good approximation since the corresponding quantum numbers are very large.[12]

Sections 2 and 3 are devoted to potentials of channels for electrons and positrons respectively. Comparison of these potentials with a parabolic well is also presented. In Section 4 we calculate the radiation intensity for the full potential averaged over time for both electrons and positrons. The expected positions of the spectral maxima are estimated in Section 5. In Sections 6 and 7 we discuss numerical examples of the experimental situation for electrons and positrons respectively. In Section 8 we estimate the number of channeling quanta for a beam of particles, assuming their angular distribution to be Gaussian. Some conclusions are drawn in Section 9.

2. THE CONTINUUM POTENTIAL FOR ELECTRONS

The interaction of an electron with the fields of the crystal's atoms can be described by an average continuum potential. For the case of an electron moving in the vicinity of a crystal plane, the following form was suggested by J. Lindhard[13]:

$$U(y) = A\left[\sqrt{y^2 + C^2 a^2} - |y|\right] . \tag{2.1}$$

Here, $A = -2\pi Z e^2 N d_p$, N is the number of atoms with atomic number Z per unit volume, d_p is the distance between crystal planes, y is the distance of the particle from the plane, $C = \sqrt{3}$, and a is the screening length of the electron-atom interaction for the Thomas-Fermi atom model. The negative sign of the constant A makes the potential U(y) attractive for electrons.

The continuum potential (2.1) is a fairly good approximation only for distances y bigger than the characteristic length Ca even though the energy ε is very large :

$$y/Ca \geq 1 \quad . \tag{2.2}$$

For smaller y the deflection angle in the collision with separate atoms could be bigger than the deflection angle due to the collective potential (2.1) . Strictly speaking, for $y < Ca$, either potential (2.1) or the Thomas-Fermi potential for an isolated atom should be used in the equation of motion depending on where the trajectory of the electron happens to go. However, due to the finite value of U(y) for $y \to 0$ and the relatively slow variation of U(y) with y, we may assume that the small impact parameters play a relatively small role in the overall radiation intensity. We shall apply the continuum potential (2.1) for all y.

On the other-hand, y should not be too large compared to the distance d_p between planes, since for $|y/d_p| > 1$ or $z > d_p/Ca$ ($z = y/Ca$), the electron moves through many different planes rather than being channeled in the vicinity of only one plane. For such a motion, instead of the simple form for the potential (2.1), one should consider the periodicity of the crystal planes with y.

The condition for the electron to be channeled between two planes can be formulated in the following way:

$$\dot{y}/c = \alpha < \alpha_p = \left[\left(2U(z_p) - \varepsilon_c\right)/\varepsilon\right]^{\frac{1}{2}} , \qquad (2.3)$$

where $\varepsilon_c = 2U(0)$ is the barrier energy for the potential (2.1), ε is the energy of the particle, and $z_p = d_p/Ca$.

Figure 1 represents the potential $-U/ACa$ as a function of z and also shows one possible energy level in the well.

3. THE CONTINUUM POTENTIAL FOR POSITRONS

To find the potential for positrons we use Lindhard's potential (2.1) for electrons and the fact that it is decreasing quite rapidly with increasing distance from a plane of ions. Therefore, the average potential for positrons in the vicinity of the middle plane is simply the sum of the potentials produced by the two nearest ion planes.

$$U(y) = A\left[\sqrt{\left(\frac{d_p}{2}+y\right)^2 + C^2a^2} - \sqrt{\left(\frac{d_p}{2}+y\right)^2} + \sqrt{\left(\frac{d_p}{2}-y\right)^2 + C^2a^2} - \sqrt{\left(\frac{d_p}{2}-y\right)^2}\right]. \qquad (3.1)$$

Here y is the distance from the middle plane: $-d_p/2 < y < d_p/2$. All other parameters are the same as in (2.1) except that A is now positive to provide the repulsive force on the positron.

Let us now introduce a new variable

$$x = 2y/d_p , \qquad |x| < 1 \qquad (3.2)$$

and a parameter $b = 2Ca/d_p < 1$.

The continuum potential, and consequently all other functions for electrons, depends on only one parameter, a, while the continuum potential for positrons depends on two unrelated parameters, a and d_p. The potential (3.1) can now be rewritten $U(y) = \left(A\, d_p/2\right) U_b(x)$, where

$$U_b(x) = \sqrt{(1+x)^2 + b^2} + \sqrt{(1-x)^2 + b^2} - 2 \quad . \tag{3.3}$$

Figure 2 represents an example of $U_b(x)$ for $b^2 = 0.0552$ (solid curve). For small x (x<<1) $U_b(x)$ can be expanded into the series:

$$U_b(x) \simeq 2\sqrt{1+b^2} - 2 + \frac{b^2 x^2}{\left(1+b^2\right)^{3/2}} \equiv U_0(x) \tag{3.4}$$

For comparison, the function $U_0(x)$ for the same value of b^2 is also plotted in Fig. 2, (dashed curve). The value $1 - [U_0(1) / U_b(1)]$ might serve as a measure of anharmonicity. For $b^2 = 0.0552$ this value equals 37%.

4. INTENSITY OF RADIATION

The expression for the instantaneous radiation intensity of an ultra-relativistic particle in a transverse electric Field E is[14] :

$$I_{in}(t) = \frac{2}{3} \frac{e^4 E^2 \gamma^2}{m^2 c^3} \tag{4.1}$$

where $\gamma = \varepsilon/mc^2$. The field E should be taken on the (classical) trajectory of the particle and hence, in general, we need the solution of the equation of motion in the y direction

$$\frac{dP_y}{dt} = e\,E(y)$$
$$P_y = \frac{\varepsilon}{c^2}\dot{y}\ , \qquad e\,E(y) = \partial U(y)/\partial y \tag{4.2}$$

However, we are interested in the intensity averaged over the period T of the particle oscillations

$$I = \frac{1}{T} \int_0^T I_{in}\, dt \qquad (4.3)$$

First, for electrons we use potential (2.1) in equation (4.2). Let us now change integration over time to integration over y. Then we get:

$$I = \frac{8e^2\ \gamma^2\ A^2}{3m^2\ c^2\ T} \int_0^{y_m} \left[1 - \frac{y}{\sqrt{y^2 + C^2a^2}}\right]^2 \frac{dy}{\dot{y}} \qquad (4.4)$$

We shall further neglect the energy change due to the transverse electric field. Then

$$\dot{y} = \sqrt{\frac{2c^2}{\varepsilon}\ [U(y) - U(y_m)]} \qquad (4.5)$$

$$T = 4 \int_0^{y_m} \frac{dy}{\sqrt{\frac{2c^2}{\varepsilon}\ \left|U(y) - U(y_m)\right|}} \qquad (4.6)$$

The value y_m in the above expressions is the maximum excursion of the particle from the equilibrium point. Its magnitude depends upon the initial value of y_0 and y_0' of the particle. Combining Eqs. (4.4) and (4.6), we get:

$$I = \frac{2}{3}\ \frac{e^2\ A^2\ \gamma^2}{m^2\ c^3}\ F(z_m)\ , \qquad (4.7)$$

where $z_m = y_m / Ca$,

and

$$F(z_m) = \frac{\int_0^{z_m} \left[1 - \frac{x}{\sqrt{x^2+1}}\right]^2 \left[\sqrt{x^2+1} - \sqrt{z_m{}^2+1} - x + z_m\right]^{-\frac{1}{2}} dx}{\int_0^{z_m} \left[\sqrt{x^2+1} - \sqrt{z_m{}^2+1} - x + z_m\right]^{-\frac{1}{2}} dx} \quad . \tag{4.8}$$

For $z_m < 1$ $\quad F(z_m) \simeq 1 - 4\,z_m / 3$

For $z_m > 1$ $\quad F(z_m) \simeq 0.3 / z_m{}^{3/2}$

As was pointed out above, the region $z_m < 1$ has little physical impact and all considerations should start from the value $z_m \simeq 1$.

The period T can also be expressed in the form of:

$$T(z_m) = 4\sqrt{\frac{\varepsilon\ Ca}{2c^2|A|}} \int_0^{z_m} \left[\sqrt{x^2+1} - \sqrt{z_m{}^2+1} - x + z_m\right]^{-\frac{1}{2}} dx \quad . \tag{4.9}$$

Expression (4.7) depends on z_m in quite a different way than the corresponding expression for the parabolic well. The reason for this can be clearly seen from Fig. 1. The shape of the potential well nowhere resembles the parabolic well. Figs. 3 and 4 present functions F(z) and T(z), respectively.

To calculate the radiation intensity of positrons, we use potential (3.1). For the average intensity, we now get:

$$I = I_0 F_b(x_m) \quad , \tag{4.10}$$

where $I_0 = 2\ e^2\gamma^2A^2\ /\ 3\ m^2c^2$, and

$$F_b(x_m) = \frac{\displaystyle\int_0^{x_m}\left[\frac{1+x}{\sqrt{(1+x)^2+b^2}} - \frac{1-x}{\sqrt{(1-x)^2+b^2}}\right]^2 P_b(x,x_m)\,dx}{\displaystyle\int_0^{x_m} P_b(x,x_m)\,dx} \tag{4.11}$$

Here x_m is the maximum excursion of the trapped positron from the middle plane in units of the half distance between planes $d_p/2$, (3.2). The function $P_b(x,x_m)$ is defined by:

$$P_b(x,x_m) = \left[\sqrt{(1+x_m)^2+b^2} + \sqrt{(1-x_m)^2+b^2} - \sqrt{(1+x)^2+b^2} - \sqrt{(1-x)^2+b^2}\right]^{-\frac{1}{2}} \tag{4.12}$$

For small x_m we get:

$$P_b(x,x_m) = \frac{(1+b^2)^{3/4}}{b\sqrt{x_m^2-x^2}} \tag{4.13}$$

The denominator in formula (4.11) is proportional to the period T of the positron oscillations:

$$T(x_m) = 2\sqrt{\frac{m\gamma d_p}{A}}\,T_b(x_m) \tag{4.14}$$

where $T_b(x_m) = \int_0^{x_m} P_b(x,x_m)\,dx$. For small x_m, T is independent of x_m:

$$T = \pi\sqrt{\frac{m\gamma d_p}{A}}\;\frac{(1+b^2)^{3/4}}{b} \tag{4.15}$$

The functions $F_b(x_m)$ and $T_b(x_m)$ are presented in Figs. 5 and 6, respectively, for different values of the parameter b^2.

For small x_m, $F_b(x_m)$ equals:

$$F_b^0(x_m) = \frac{2 b^4 x_m^{\ 2}}{(1+b^2)^3}, \quad x_m \ll 1 \quad . \tag{4.16}$$

This value gives the estimate for the case of the parabolic well approximation.

5. DISCUSSION OF THE SPECTRAL CHARACTERISTICS OF THE RADIATION

Since radiation is emitted in a narrow cone with vertex half-angle $\Delta\theta \sim \gamma^{-1}$ along the instantaneous particle velocity, the frequency range $\Delta\omega$ in which the maximum number of quanta is emitted depends, of course, on the ratio of the angle α, between the trajectory and the crystal plane, to $\Delta\theta$. This ratio can be found from (4.5). Since $\alpha = \dot{y}_{max}/c$, we get

$$\alpha/\Delta\theta \simeq \left[(2\gamma/mc^2)\left|U(y_m) - U(0)\right|\right]^{\frac{1}{2}} \tag{5.1}$$

a) Let us consider first the case where

$$\alpha/\Delta\theta >> 1 \quad . \tag{5.2}$$

Then the radiation in the given direction comes from a very small part of the trajectory parallel to this direction. We may assume that the field on this part of the trajectory is almost constant and apply all the expressions for synchrotron radiation. In particular, the main part of the radiation will be emitted at the frequency

$$\hat{\omega}_1 \simeq \frac{|\partial U/\partial y|_{max}}{mc}\gamma^2 \tag{5.3}$$

For the case of electrons $\partial u/\partial y$ as a function of y for small values of y, is a rather slowly varying function so it is of little importance at which point the derivative is evaluated. If we assume that $(\partial u/\partial y)$ should be taken at $y \simeq Ca$, then $\hat{\omega}_1 \simeq 0.3\ |A|\ \gamma^2/mc$.

For positrons we can use for the value $|\partial u/\partial y|_{max}$ one taken at $y \simeq d_p/2$. Now $\hat{\omega}_1 \simeq A\gamma^2 \Big/ mc\sqrt{1+b^2/4}$.

b) For the opposite limit

$$\alpha/\Delta\theta \ll 1 \tag{5.4}$$

the radiation is gathered from the entire trajectory of the particle. In this case, the spectral maximum occurs at the frequency:

$$\hat{\omega}_2 \sim 2\gamma^2 \Big/ T(z_m) \quad , \tag{5.5}$$

For electrons this expression diverges for $z_m \to 0$. (See Fig. 4). The physical reason for this divergence is connected with the limited validity of potential (2.1) for very small y. It is quite reasonable to assume that the maximum frequency should be taken at $z \simeq 1$.

$$\hat{\omega}_{2max} = 2\gamma^2 \big/ T(1) = \frac{1}{2}\left(\frac{2|A|}{Cam}\right)^{\frac{1}{2}} \frac{\gamma^{3/2}}{\kappa} \quad , \tag{5.6}$$

where

$$\kappa = \int_0^1 \frac{dx}{\left[\sqrt{x^2+1} - x - \sqrt{2} + 1\right]^{\frac{1}{2}}} = 3.7$$

For small values of y/Ca as was discussed above, particle deflections will occur not in the continuum lattice field but rather on separate atoms. In this case, the considered radiation will go smoothly over into the bremsstrahlung type of radiation.[15]

Expression (5.5) for positrons has quite a different feature. As can be seen from Fig. 6, the function $T_b(x_m)$ has a rather slow dependence on x_m, at least for not very small values of parameter b^2. So the value for T(0) from (4.15), which is the value for the corresponding harmonic oscillator, is a good approximation:

$$\hat{\omega}_0 = \sqrt{\frac{A}{m\,d_p}}\;\frac{4\,b\,\gamma^{3/2}}{(1+b^2)^{3/4}} \quad . \tag{5.7}$$

For very small b^2 and for large x_m, the values of $T_b(x_m)$ should be taken from Fig. 6.

6. DISCUSSION OF THE EXPERIMENTAL SITUATION AND A NUMERICAL EXAMPLE FOR ELECTRONS

Let us consider more closely the meaning of our derived expressions from the point of view of the experimental possibility of detecting the channeling radiation. First of all, this type of radiation will occur with a background of bremsstrahlung. The conditions of the experiment should allow for resolving one from the other by using, say, the difference of their spectra, for example. This means that the value for z_m for the electron should not be smaller than 1 or, in other words, an angle α of the trajectory should not be too small.

The situation will be much clearer if we consider a numerical example. For our example let us take the diamond crystal as the device for producing the plane channeling radiation.

Let us assume the following numbers: $Z = 6$, $N = 1.1 \times 10^{23} cm^{-3}$, $d_p = 3.57 \times 10^{-8} cm$, $Ca = \sqrt{3}\, a_0 \times 0.885 Z^{-1/3} = 0.42 \times 10^{-8} cm$ and, consequently $|A| = 2.13 \times 10^4$ MeV/cm. The barrier energy ε_c equals 1.8×10^{-4} MeV.

Let us look first at the maximum large α consistent with (2.3); then z_m will be equal to 8.5 independent of γ. The value $F(z_m)$ from (4.8) is then 1.2×10^{-2}. Table I represents the characteristic quantities for different values of γ for this case. The intensity of radiation can be increased significantly by keeping α smaller. Table I also gives values for the case in which α is chosen in such a way that for all γ, $z_m = 1$ $(F = 0.24)$.

As we see, the wide beam has the disadvantage of producing relatively small amounts of radiation. Besides that, the maximum of the photon spectrum is shifted toward very high frequencies. This will make it difficult to resolve the channeling radiation from the bremsstrahlung type of radiation, especially for very high values of γ. If we try to reduce α more, for example to make it equal to γ^{-1}, then z_m becomes very small. For $\gamma = 10^{-4}$, $z_m = 0.33$; for $\gamma = 4 \times 10^{-4}$, $z_m = 0.075$. It would be very difficult to get convincing results in such a case.

7. DISCUSSION AND NUMERICAL EXAMPLE FOR POSITRONS

The dependence of F_b on x_m suggests an interesting conclusion: the intensity of the channeling radiation for positrons grows very rapidly with x_m; this means that the main part of the radiation is produced by positrons with large angles in the plane of the channel. One must remember, of course, that this angle should not exceed the limit at which the energy of oscillations will be greater than the barrier energy of the potential; or, in other words, the maximum excursion x_m of the particle from the middle plane should be less than 1. This conclusion is the exact opposite of the one for electrons where the main part of the radiation comes from electrons with small amplitudes.

Such features of the radiation once again stress the fact that the calculations of the intensity of radiation in the parabolic approximations are too rough and can underestimate the effect.

Let us again take the example of a crystal with parameters: $A = 2.13 \times 10^4$ MeV/cm, $d_p = 3.57 \times 10^{-8}$cm, $Ca = 0.42 \times 10^{-8}$cm; then $b = 0.235$ and $I_0 = 1.5 \times 10^{13}\ \gamma^2$ MeV/sec.

For any given γ and the trajectory angle α with the crystal axis in the plane of oscillation, one can find first the value of x_m from the equation:

$$\alpha = \sqrt{\frac{Ad_p}{mc^2\gamma}\left(U_b(x_m) - U_b(0)\right)} \quad . \tag{7.1}$$

Then it is easy to find the corresponding values of $F_b(x_m)$ and $T_b(x_m)$. Table II gives the function $x_m = f_1(\gamma,\alpha)$ for different values of γ and α and for $b^2 = 0.0552$. Table III contains values of F_b for the same values of γ and α. Dashes for big values of γ and α mean that for these parameters there is no channeling.

Table IV presents the comparison of radiation intensities and the spectral maximum for different γ and α calculated by means of Eqs. (4.11) and (4.14) ("exact"), and (4.16) and (4.15) ("harmonic oscillator"), respectively.

One can clearly see that the bigger x_m is, the bigger is the difference between the results which one gets from the exact solution and the parabolic well approximation. This is especially true for the whole intensity of radiation where the discrepancy can be two orders of magnitude. On the other hand, the spectral maximum frequency, if corrected for anharmonicity, is determined by the approximate solution quite well.

8. ESTIMATE OF THE RADIATION INTENSITY AND NUMBER OF QUANTA FOR THE PARTICLE BEAM

Until now we have considered the intensity of radiation produced by a single particle. We can now calculate the radiation intensity of the particle beam. To do that we need, of course, to know the distribution of the particles in the angles α in the plane perpendicular to the crystal planes. As an example, we assume the normal distribution with the dispersion σ and beam axis parallel to the crystal plane. Let the normalized probability for the particle to have an angle α be:

$$P(\alpha)\,d\alpha = \frac{1}{(2\pi)^{\frac{1}{2}}\sigma}\; e^{-\alpha^2/2\sigma^2}\,d\alpha \tag{8.1}$$

Now the intensity of radiation of the beam is the sum of the intensities $\bar{I}$ of individual particles with angles α weighted by the probabilities of having these angles:

$$I_b = \int_0^{\alpha_m} I\left(x_m(\alpha)\right)\; P(\alpha)\,d\,\alpha \tag{8.2}$$

Here α_m is the maximum angle for which a particle is still channeled between the crystal planes. We carry out our calculation further for the case of positrons; the case of electrons can be treated in the same manner. It is more convenient to change the integration variable from α to x_m. Then, using (8.1), one gets:

$$I_b = \frac{1}{2\sqrt{2\pi}\,\sigma}\sqrt{\frac{Ad_p}{mc^2\gamma}}\int_0^X I(x_m)\exp\left(-\frac{Ad_p[U_b(x_m)-U_b(0)]}{2\sigma^2\ mc^2\ \gamma}\right)\frac{dU_b/dx_m}{\sqrt{U_b(x_m)-U_b(0)}}\,dx_m \tag{8.3}$$

The upper limit of integration X should be taken equal to $1-b$, since the expression for the effective potential is valid only for $x_m < X$.

Let us now introduce a new dimensionless parameter, which is proportional to the ratio of the square of the characteristic trajectory angle to σ^2:

$$\xi = \frac{A\,d_p}{2\,mc^2\,\gamma\,\sigma^2} \tag{8.4}$$

Now, using the functions $F_b(x_m)$ and $P_b(x, x_m)$, we get:

$$I_b = I_0\sqrt{\frac{\xi}{4\pi}}\ \phi_b(\xi) \tag{8.5}$$

where

$$\phi_b(\xi) = \int_0^{1-b} dx_m F_b(x_m)\,e^{-\frac{\xi}{[P_b(0,\,x_m)]^2}}\,P_b(0,\,x_m)\left[\frac{1+x_m}{\sqrt{(1+x_m)^2+b^2}} - \frac{1-x_m}{\sqrt{(1-x_m)^2+b^2}}\right] \tag{8.6}$$

Figure 7 shows the dependence of $\phi_b(\xi)$ on ξ for several values of b^2. As a function of b, ϕ_b has a maximum around the value $b^2 \simeq 0.18$. It falls rather rapidly both toward bigger and smaller values of b^2.

Let us at last estimate the number dN_f of quanta emitted from the length $d\ell$ of the crystal. For simplicity, we assume that the spectrum of channeled radiation averaged over the incident particle angles does not depend on the frequency of the emitted photon ω in the interval $0 < \omega_f < \omega_{max}$. Then

$$dNf = I_{beam} d\ell/c \quad \omega_{max} \, d\omega_f \, / \, \omega_f \tag{8.7}$$

Using expression (5.7) for ω_{max}, we get

$$dN_f = \frac{\sqrt{\xi}\,\phi_b(\xi)}{137\cdot 3\cdot 4\sqrt{\pi}} \; \frac{\gamma^{\frac{1}{2}} A^{3/2} \sqrt{d_p}\, d\ell}{(mc^2)^{3/2}} \; \frac{(1+b^2)^{3/4}}{b} \; \frac{d\omega_f}{\omega_f} \tag{8.8}$$

where we used $e^2/\ c \simeq 1/137$, the fine structure constant. Since $\xi \sim \gamma^{-1}$ (cf. 8.4) the number of quanta is practically independent of the primary positron energy.

For the numerical example discussed above, and for the value $\xi = 31.3\times 10^{-4}/\gamma$ $(\sigma = 0.5\times 10^{-4})$, formula (8.8) gives $dN_f/d\ell = 2.5(\gamma\xi)^{\frac{1}{2}}\phi_b(\xi)\cdot \left[d\omega_f/\omega_f\right]\cdot$ or $\simeq 0.2$ quanta/cm in the 10% frequency band near ω_{max}.
The photon yield can be increased if the axis of the beam makes a small angle with the crystal plane.

9. CONCLUSION

Our analysis allows us to find the average intensity of channeling radiation as well as the total number of quanta for any given potential of a channel. We did this for Lindhard's potential. However, essentially the same results could be obtained for other descriptions of the force acting on a channeled particle.

The significance of this analysis is that it gives results for the intensity more accurately than all the previous papers do. Maybe more importantly, it helps in finding the optimal conditions for an experiment since the yield and characteristics of the radiation strongly depend on the parameters of both the crystal and particle beam available for the experiment.

The choice between electrons and positrons from the point of view of producing and detecting the channeling radiation depends on the particular conditions of the experiment; however, some general remarks can be made.

The advantage of using electrons is that available electron beams have much smaller transverse phase space than that of positrons. In addition, the intensity of radiation of electrons is somewhat larger than that of positrons; this is simply due to the fact that an electron is captured by an attractive ion potential of the channel and, therefore, moves in a relatively stronger electric field; a positron, on the other hand, is captured by a repulsive potential formed by the two next ion planes and, therefore, moves in a rather weaker field oscillating around the middle plane. At the same time, the remoteness from ions increases the ratio of the channeling radiation intensity to the bremsstrahlung one, making the experiment with positrons easier to interpret.

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TABLE I. The position of frequency maxima and intensity of radiation of electrons for different γ and α.

			$z_m = 8.5$, $F = 1.2 \times 10^{-2}$			$z_m = 1$, $F = 0.24$		
γ	$\hat{\omega}_1$	$\hat{\omega}_2$	α_p	I	$\alpha_p/\Delta\theta$	α_1	I	$\alpha_1/\Delta\theta$
10^2	3.75×10^{18}	3.61×10^{19}	1.81×10^{-3}	6×10^8	.181	1.46×10^{-3}	1.2×10^{10}	.146
10^3	3.75×10^{20}	1.14×10^{21}	5.74×10^{-4}	6×10^{10}	.574	4.62×10^{-4}	1.2×10^{12}	.462
10^4	3.75×10^{22}	3.61×10^{22}	1.81×10^{-4}	6×10^{12}	1.81	1.46×10^{-4}	1.2×10^{14}	1.46
4×10^4	6.00×10^{23}	2.89×10^{23}	0.92×10^{-4}	9.6×10^{13}	3.68	0.73×10^{-4}	1.9×10^{15}	2.92
	sec^{-1}	sec^{-1}	rad	MeV/sec		rad	MeV/sec	

TABLE II

z_m for Different γ and α (b^2 = 0.5552)

γ \ α	10^{-3}	0.3×10^{-3}	10^{-4}	0.25×10^{-4}
10^2	.79	.33	.10	.0
10^3	—	.77	.33	.10
10^4	—	—	.79	.28
2×10^4	—	—	.93	.38
4×10^4	—	—	—	.51

TABLE III

F_b for Different γ and α (b^2 = 0.0552)

γ \ α	10^{-3}	0.3×10^{-3}	10^{-4}	0.25×10^{-4}
10^2	0.028	0.7×10^{-3}	0.5×10^{-4}	0.0
10^3	—	0.027	0.7×10^{-3}	0.5×10^{-4}
10^4	—	—	0.028	0.5×10^{-3}
2×10^4	—	—	0.11	0.10
4×10^4	—	—	—	0.25

TABLE IV. The comparison between "exact" and "approximate" solutions for radiation intensity (F_b) and the spectra maximum frequencies for different γ and α.

γ	γ^{-1}	$\alpha = 10^{-3}$				$\alpha = 0.3 \times 10^{-3}$			
		F_b	F_b^0	$\hat{\omega}$	$\hat{\omega}_0$	F_b^0	F_b	$\hat{\omega}$	$\hat{\omega}_0$
10^2	10^{-2}	28×10^{-3}	3.2×10^{-3}	4.84×10^{19}	2.80×10^{19}	0.7×10^{-3}	0.6×10^{-3}	3.04×10^{19}	2.80×10^{19}
10^3	10^{-3}	—	—	—	—	27×10^{-3}	3.1×10^{-3}	1.46×10^{21}	8.86×10^{20}
10^4	10^{-4}	—	—	—	—	—	—	—	—
2×10^4	0.5×10^{-4}	—	—	—	—	—	—	—	—
4×10^4	0.25×10^{-4}	—	—	—	—	—	—	—	—
		$\alpha = 10^{-4}$				$\alpha = 0.25 \times 10^{-4}$			
γ	γ^{-1}	F_b	F_b^0	$\hat{\omega}$	$\hat{\omega}_0$	F_b^0	F_b	$\hat{\omega}$	$\hat{\omega}_0$
10^2	10^{-2}	0.5×10^{-4}	0.5×10^{-4}	2.80×10^{19}	2.80×10^{19}	0.	0.	2.8×10^{19}	2.80×10^{19}
10^3	10^{-3}	0.7×10^{-3}	0.6×10^{-3}	9.64×10^{20}	8.86×10^{20}	0.5×10^{-4}	0.5×10^{-4}	8.94×10^{20}	8.86×10^{20}
10^4	10^{-4}	28×10^{-3}	3.1×10^{-3}	4.64×10^{22}	2.80×10^{22}	0.5×10^{-3}	0.4×10^{-3}	2.96×10^{22}	2.80×10^{22}
2×10^4	0.5×10^{-4}	—	—	—	—	1×10^{-1}	0.75×10^{-3}	8.68×10^{22}	7.92×10^{22}
4×10^4	0.25×10^{-4}	—	—	—	—	2.5×10^{-1}	1.4×10^{-3}	2.66×10^{23}	2.24×10^{23}
Formulae		(5),(6)	(11)	(12)$\sec^{-1}$	(13)$\sec^{-1}$	(5),(6)	(11)	(12)$\sec^{-1}$	(13)$\sec^{-1}$

FIGURE CAPTIONS

1. The potential $-U/ACa$ as a function of z and one possible energy level in the well.

2. The potential $U(x)$ for a channeled positron (solid curve). The dashed curve represents the corresponding parabolic well.

3. $F(z)$ (See text.)

4. $T(z)$ (See text.)

5. The intensity of the radiation of a channeled positron in units of $2e^2\gamma^2A^2/3m^2c^3$ for different values of the parameter $b = 2Ca/d_p$.

6. The period of oscillations of a channeled positron in units $2\sqrt{m\gamma d_p/A}$ for different values of the parameter $b = 2Ca/d_p$.

7. $\phi_b(\xi)$ for several values of b^2 (as a function of b, ϕ_b has a maximum around the value $b^2 \cong 0.18$).

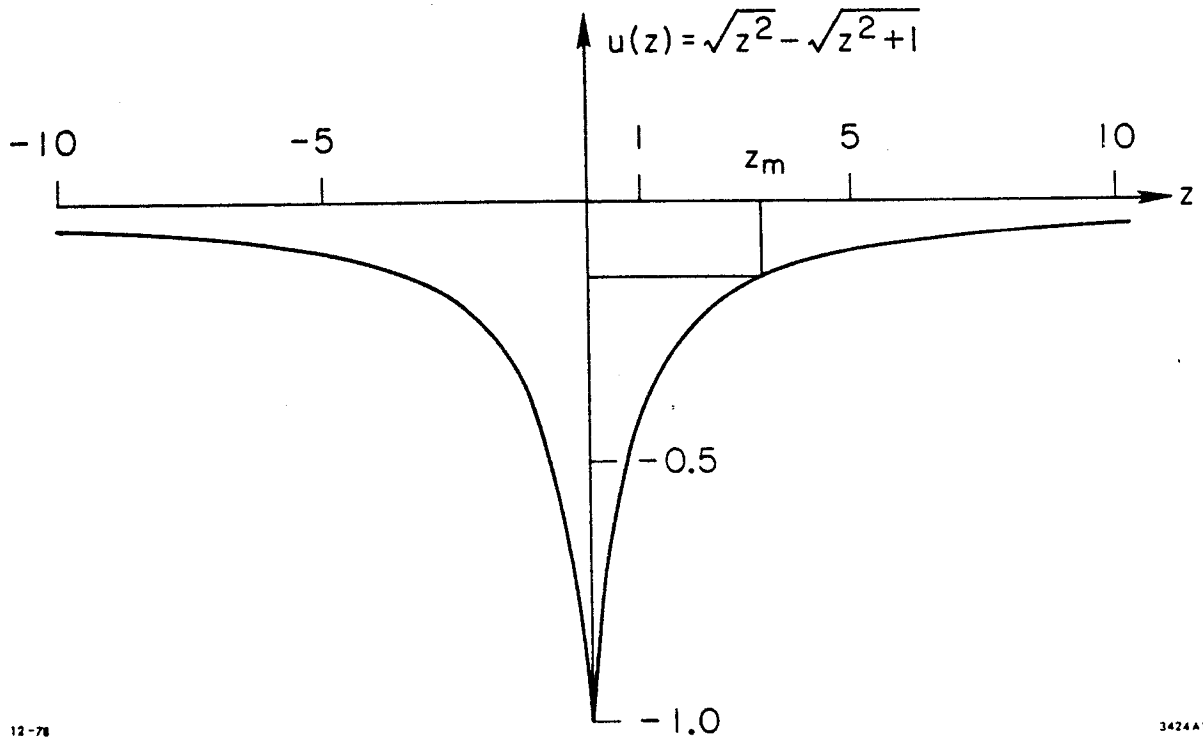


Fig. 1

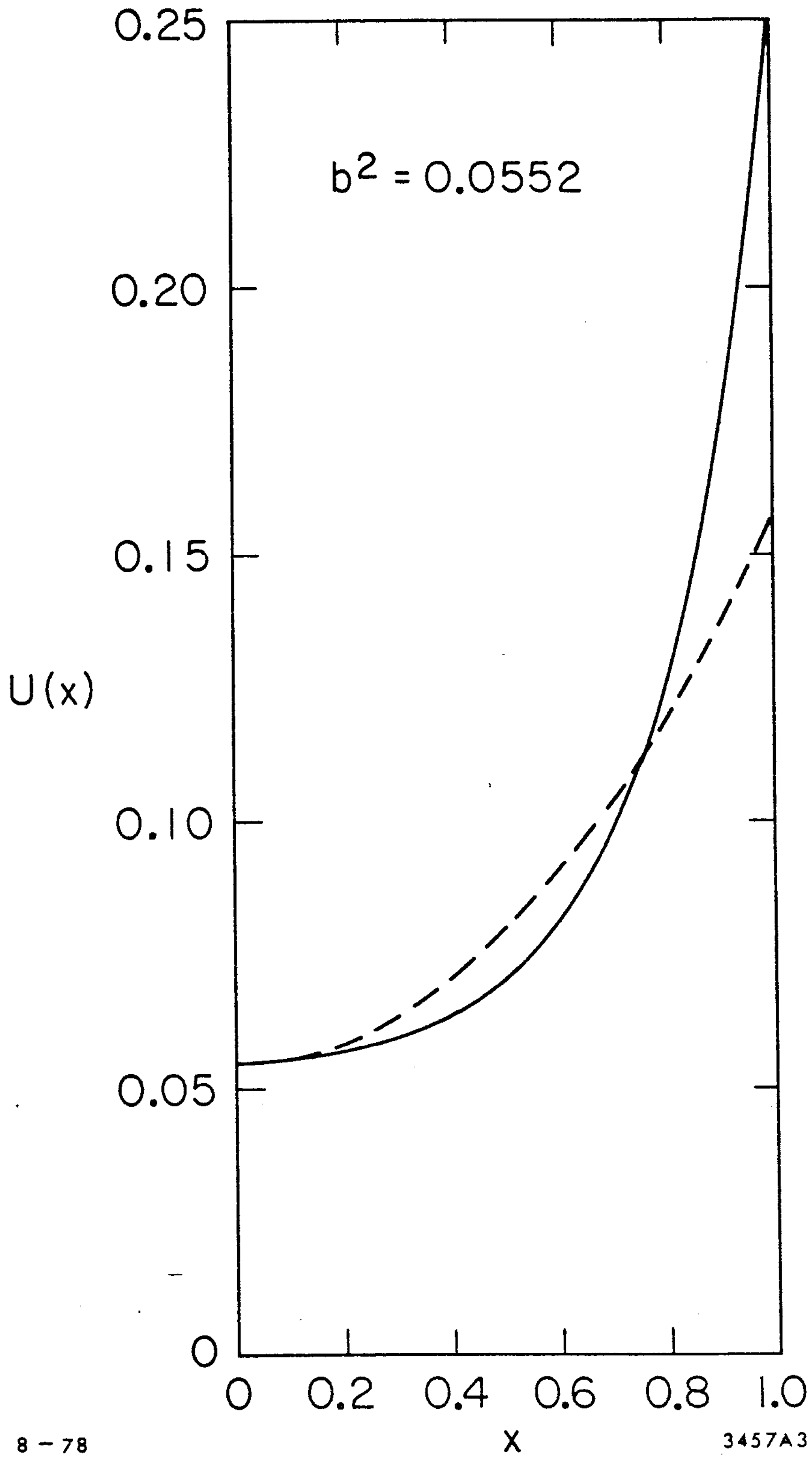


Fig. 2

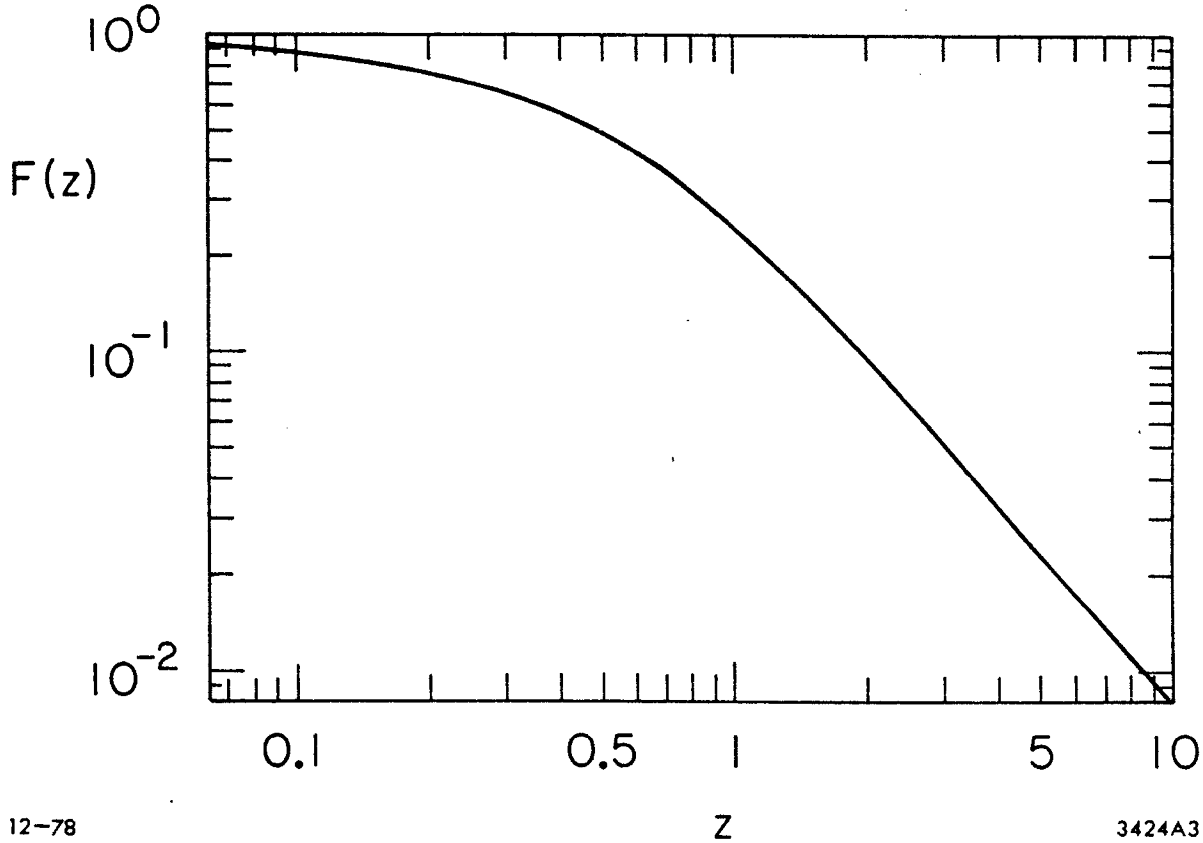


Fig. 3

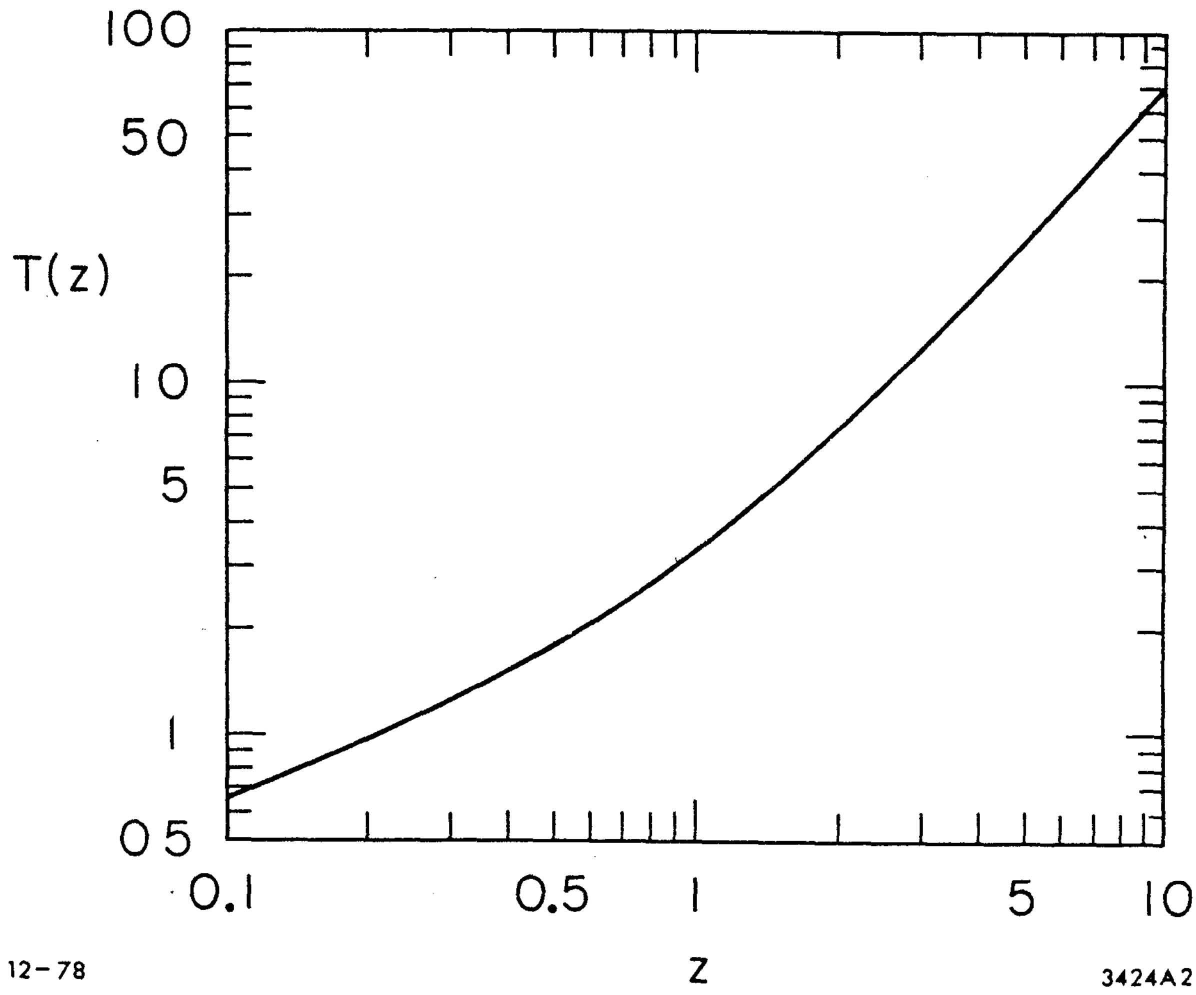
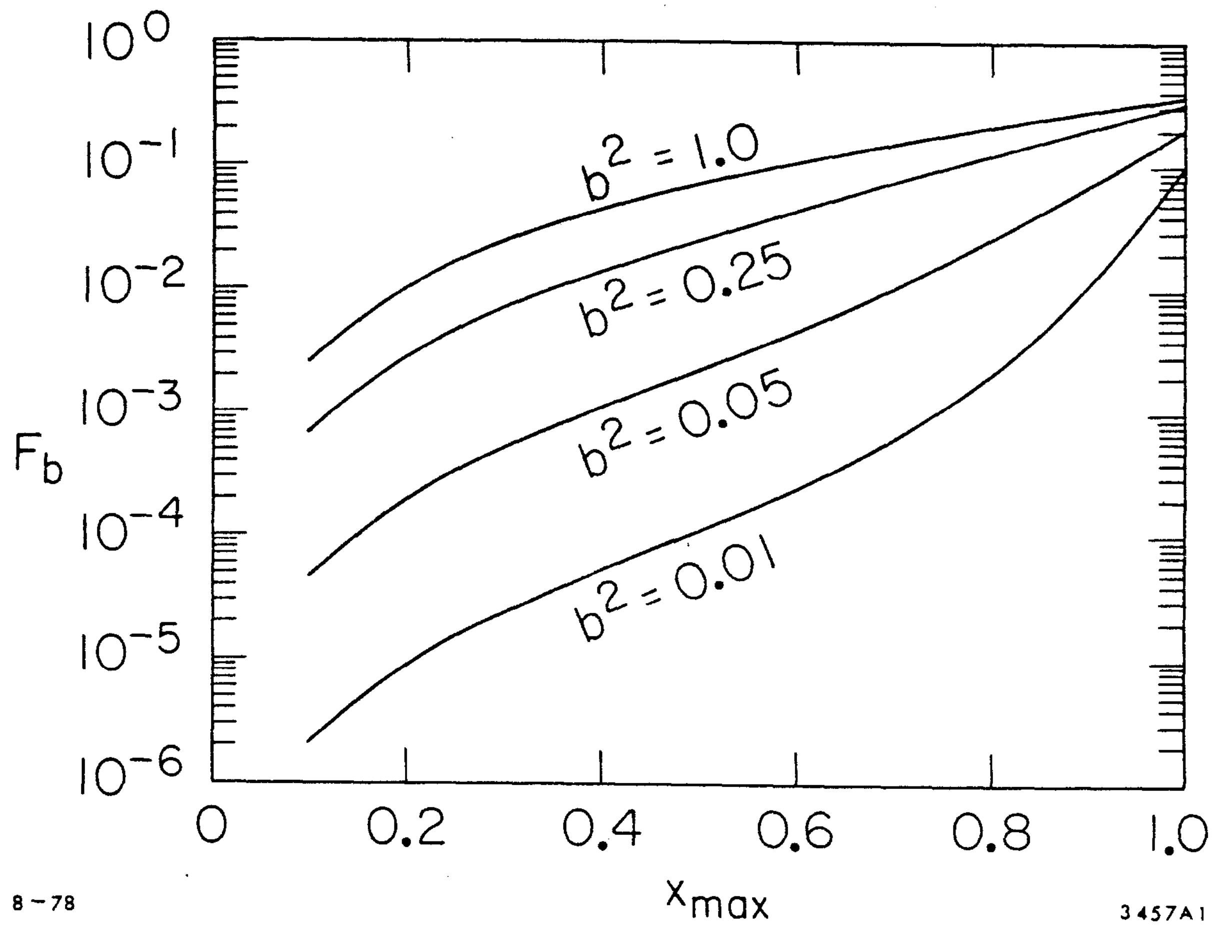


Fig. 4



10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
F_b
$b^2 = 1.0$
$b^2 = 0.25$
$b^2 = 0.05$
$b^2 = 0.01$
0
0.2
0.4
0.6
0.8
1.0
x_{max}
8 – 78
3457A1

Fig. 5

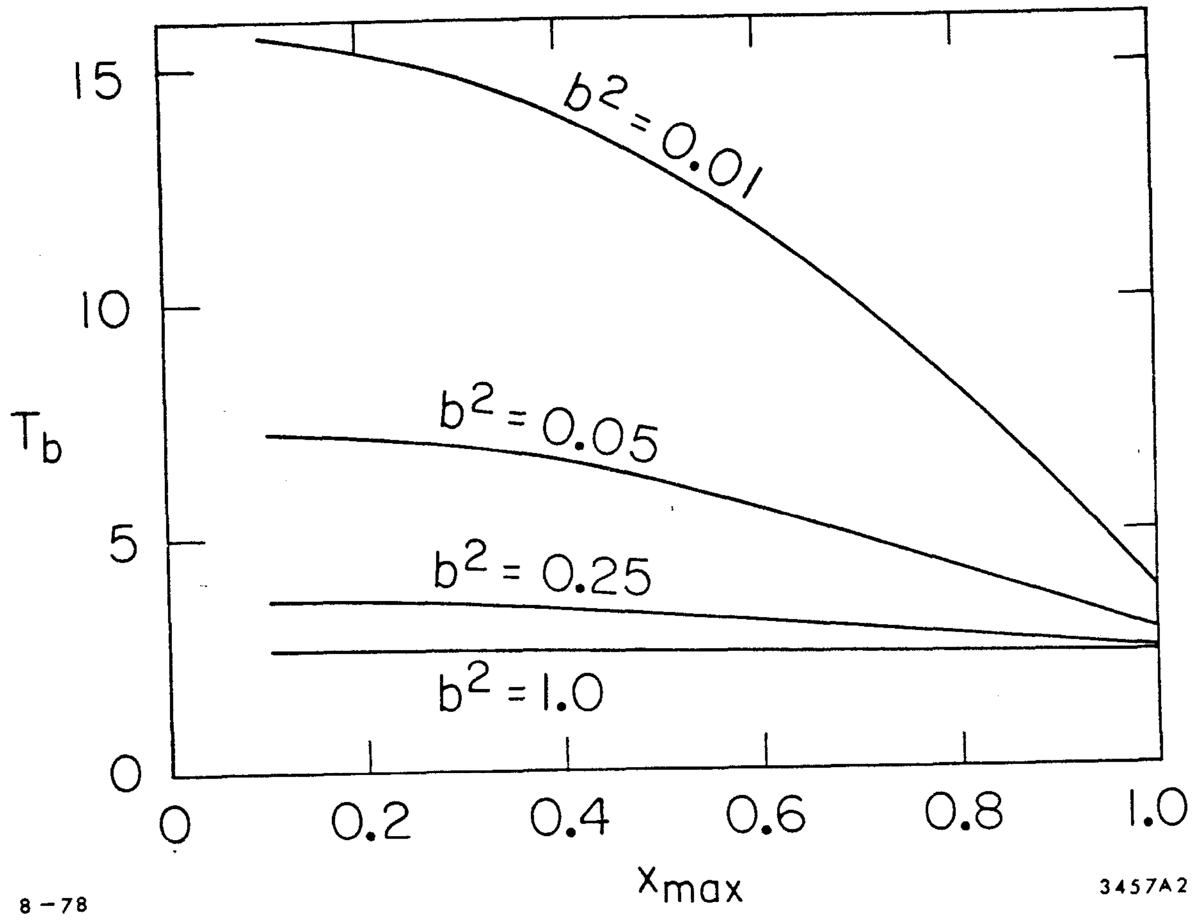
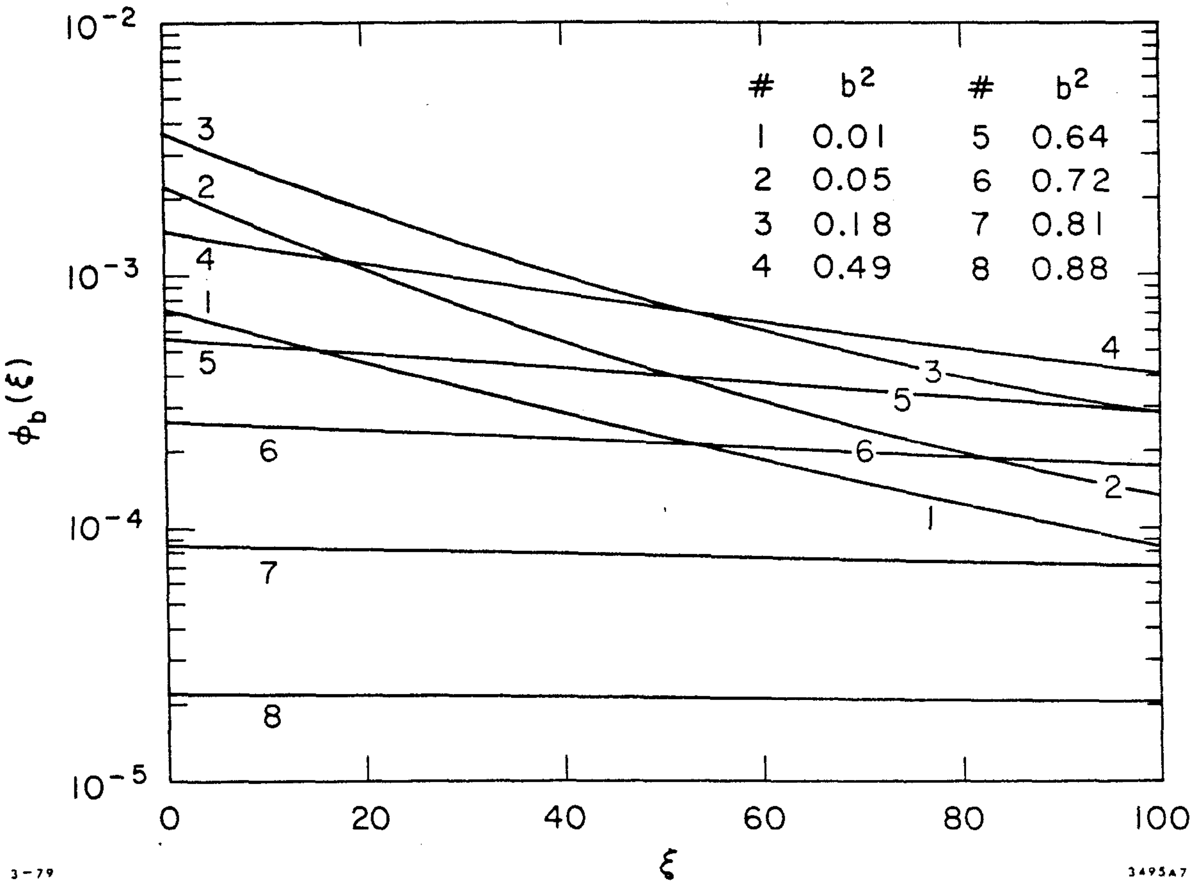

15
10
5
0
T_b
$b^2 = 0.01$
$b^2 = 0.05$
$b^2 = 0.25$
$b^2 = 1.0$
0
0.2
0.4
0.6
0.8
1.0
x_{max}
8 −78
3457A2

Fig. 6



b^2 # b^2
1 0.01 5 0.64
2 0.05 6 0.72
3 0.18 7 0.81
4 0.49 8 0.88
10^{-2}
10^{-3}
10^{-4}
10^{-5}
$\phi_b(\xi)$
0
20
40
60
80
100
ξ
3-79
3495A7

Fig. 7