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THE LOWEST LEVEL OF THE COMBINATORIAL HIERARCHY AS PARTICLE ANTIPARTICLE QUANTUM BOOTSTRAP*

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Abstract

It is shown that the lowest level of the combinatorial hierarchy can be given an explicit dynamical interpretation, using an on-shell version of the three body relativistic equations developed by Brayshaw, as representing the quantum number flow in which two particles and an antiparticle bind to form a single particle with the mass of one of the three.

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The combinatorial hierarchy[1] contains four levels with $2^2 - 1 = 3$, $2^3 - 1 = 7$, $2^7 - 1 = 127$ and $2^{127} - 1 \approx 10^{38}$ entities in the four communicating levels. The representation of the hierarchy by means of column vectors containing only the existence symbols 0 and 1 has been interpreted[2] as the quantum numbers of (boson) systems. The first level represents charge, the second baryon-antibaryon pairs with associated mesons, the third a baryon-antibaryon pair together with an (externally neutral) lepton-antilepton pair and the associated bosons. The lowest mass exemplars of the bosons are π, ρ, ω, and the electromagnetic or weak boson; the quantum numbers are baryon and antibaryon number, lepton and antilepton number, the z-component of spin and the z-component of isospin, and hence represent the absolutely conserved quantum numbers. Strangeness and other quark quantum numbers are expected to come in at the fourth (unstable) level, but here there are as yet only a few representatives of the $2^{127} - 1$ available entities, and this interpretation is at a very primitive stage. While, in the reference cited,[2] it was only possible to suggest possible dynamical interpretations, these allowed an heuristic interpretation of the calculation by Parker-Rhodes[3] that $m_p/m_e = 137\pi/\left[\frac{3}{14}\left(1+\frac{2}{7}+\left(\frac{2}{7}\right)^2\right)\left(\frac{4}{5}\right)\right] = 1836.1516$. In this paper we take another step toward a rigorous dynamics by connecting the lowest level of the hierarchy to a relativistic three particle bound state calculation.

The lowest level of the hierarchy consists of the basis vectors $\begin{pmatrix}1\\0\end{pmatrix}$, $\begin{pmatrix}0\\1\end{pmatrix}$ and the three discriminately closed subsets $\left\{\begin{pmatrix}1\\0\end{pmatrix}\right\}$, $\left\{\begin{pmatrix}0\\1\end{pmatrix}\right\}$, $\left\{\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}, \begin{pmatrix}1\\1\end{pmatrix}\right\}$. By discrimination we mean the combination of two columns to form a third obtained by adding (modulo 2) the elements row by row. Explicitly, if we represent a column by $(x)_n = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$ where the x_i are 0 or 1, discrimination is defined by

$$D_n(x,y) \quad = \quad (x +_2 y)_n \quad = \quad \begin{pmatrix} x_1 +_2 y_1 \\ x_2 +_2 y_2 \\ \vdots \\ x_n +_2 y_n \end{pmatrix} \qquad .$$

A set of columns is discriminately closed if it is a single column or if the discrimination between any two columns in the set gives another (non-null) member of the set. This is true of the three vectors in the third set given above, as can be readily checked. Alternatively, we could have used as a basis the two linearly independent vectors $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Either could be used for the calculation below.

To go from this abstract scheme to dynamics, we must first interpret the discrimination operation in such a way that the quantum numbers are conserved. For this we assume, following Feynman, that $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ represents either a positively charged particle moving forward in time (—>) or a negative particle moving backward in time (<--). These are, of course antiparticles of each other. Then $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ is an antiparticle moving forward in time (-->) or a particle moving backward in time (<—). In this simple environment particle-antiparticle conjugation (interchange of rows) is equivalent to time reversal is equivalent to charge conjugation. The third vector $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ or (∿∿∿) we interpret as a bound state of a particle antiparticle pair, or quantum. It is, of course, self-conjugate, so the line can be assigned either direction. We next interpret the discrimination operation as corresponding to either of two Feynman vertices in which all particles are either incoming or outgoing (cf. Fig. 1(a),(b)). Interpreted as discrimination diagrams,[2] these express the fact that discrimination between any two columns gives the third, and there is no reference to time. Interpreted as Feynman diagrams, we see that charge is indeed conserved at the vertex, and that the time direction is arbitrary.

To go from this to the particle-antiparticle-particle bootstrap we must first reinterpret these vertices as time-ordered diagrams, as in Fig. 1(c). For the dynamical theory we use the relativistic generalization of the zero range theory for three body scattering,[4] which generates unique, unitary three-particle amplitudes under the assumption that for the two-body scatterings $q \operatorname{ctn} \delta = -1/a = \text{constant}$. The corresponding relativistically invariant two-body amplitude is $A(s) = s^{\frac{1}{2}}/[(m^2 - s/4)^{\frac{1}{2}} - 1/a]$, assuming equal masses m and, in the c.m. system $s = 4(q^2 + m^2)$, q being the momentum of either particle. This is a special case of the separable approximation discussed by Brayshaw in his exact relativistic three-body scattering theory.[5] The S-wave equation we use can be obtained from Ref. 5, Eq. 6.14, which reduces in the non-relativistic limit to the theory just mentioned.[4] In the three-particle c.m. system, $s = (P_0^3 - k)^2 = M_3^2 + m^2 - 2\varepsilon_k M_3 = [(M_3^2 + \tilde{k}^2)^{\frac{1}{2}} - (m^2 + \tilde{k}^2)^{\frac{1}{2}}]^2$ where M_3^2 is the invariant four momentum squared, k is the momentum of the third particle, $\tilde{k}$ that momentum in the two-body c.m. system, and $\varepsilon_k = (m^2 + k^2)^{\frac{1}{2}}$. As $\tilde{k}$ varies from 0 to ∞, we see that s varies from $(M_3 - m)^2$ to 0, so that the maximum value k can have in the three-body c.m. system when it refers to another Faddeev channel is $(M_3^2 - m^2)/2M_3$. Hence in the limit of three equal masses we wish to take, the integrals vanish and we are left with driving terms! To obtain a three-body bound state amplitude we must iterate the equation once to get rid of the singular δ-function driving term before taking this limit, which leads to the Feynman diagram Fig. 1(d). Evaluated on-shell, as we must in our zero range theory, this gives the three-body amplitude

$$T = AG_oA = \frac{\sqrt{s_1}}{\left(\sqrt{m^2 - s_1/4} - 1/a\right)} \frac{1}{\varepsilon_k \varepsilon_a} \frac{1}{(\varepsilon_k + \varepsilon_a - M_3)} \frac{\sqrt{s_2}}{\left(\sqrt{m^2 - s_2/4} - 1/a\right)}$$

$$\varepsilon_a = \sqrt{(\underline{K} - \underline{k})^2 + (2m - s_b^{1/2})^2} \quad ; \quad s_b = 4(m^2 - 1/a^2)$$

In order to impose our bootstrap condition which makes this into a three-body bound state vertex, we first require that the particle-antiparticle state be bound with mass $2m - s_b^{1/2}$, as already implied by our two-body amplitude. This is an old idea. In more familiar language, Fermi and Yang[6] suggested long ago that we consider the pion to be a bound state of a nucleon-antinucleon pair, to which we add the familiar idea that a nucleon, in first approximation, can be considered to be a bound state of a nucleon and a pion. We see immediately that if we require the mass of the particle-antiparticle bound state to be m by taking $1/a^2 = 3m^2/4$, impose overall momentum conservation $\underline{K} = \underline{k}$, and evaluate the scattering amplitudes at $s_1 = m$, our bootstrap condition $M_3 = m$ gives $T \propto 1/(m^2 - s)(k^2 + m^2)$. Hence the particle is also bound to the particle-antiparticle pair with mass m, and the particle-antiparticle-particle system binds with the unique mass m. Thus the mass spectrum of this system in M_3 consists of a three-body bound state at mass m (with the charge of one of the particles), elastic scattering threshold for the scattering of the particle and the quantum at 2m, and breakup threshold at mass 3m. Numerical work to show that the three-body equations with $1/a < \sqrt{3}\,m/2$ do converge to this limit is in progress. This will support what we already believe to be a rigorous conclusion that in this particular kinematic limit the first iteration of the driving term produces the three-body bound state directly without solving an integral equation, a novel result that obviously only occurs in a zero range theory.

It may be objected that we have not here justified the use of the relativistic on-shell theory in terms of fundamental concepts. This has been done in two earlier papers.[7,8] In particular, in the second[8] we have proved that starting from free particle wave functions operationally defined, we obtain the deBroglie relation from counts in detectors with the usual Born statistical interpretation, and further that, defining scattering boundary conditions, we can derive the usual scattering wave function as given, for example, by Goldberger and Watson[9] by requiring translational invariance and the absence of hidden variables, with the important distinction that the T which appears is kinematic, and can describe any process whether unitary or not. The need for a unitary relativistic three-particle dynamics for T has been met by Brayshaw,[5] using - significantly we believe - the Goldberger-Watson propagator, which Brayshaw shows preserves the cluster property in an n-body relativistic scattering theory.

A second objection is that, having stripped down the theory to such a basic level, we have no guarantee that we can build it back up to approach both the quantum theory and the relativistic kinematics used in the equations as a "correspondence limit." We believe that both problems have been solved, at least in principle, by Finkelstein.[10,11] He has shown that given <u>any</u> partial ordering relation between two sets of entities, one can, by a theorem due to Galois, construct the lattice logic, and if that lattice logic is that of bra and ket, by a theorem due to Birkhoff, construct the Hilbert space.[10] We believe it significant that in interpreting the hierarchy so as to conserve quantum numbers <u>and</u> provide an ordered interpretation for dynamics we have been led to the flow of quantum numbers through time-ordered Feynman diagrams. It remains to find out whether the lattice implied by the hierarchy leads directly to quantum mechanics or is more general. The problem of relativistic kinematics has also been solved by Finkelstein[11] by showing that the discrete moves of a dicotomic spinor on a checkerboard with finite step size gives, in the limit as step size goes to zero, the full forward light cone in Minkowski 4-space.

Before demonstrating either possibility, we must articulate the hierarchy much farther. At the second level, we have the quantum numbers needed to describe four baryons (B^+B^o and their antiparticles $B^-\bar{B}^o$) and three mesons $m^+m^om^-$. To see if we can extend the bootstrap we will first need a four-body theory. If we assume, <u>a priori</u>, that there is only one baryon mass and <u>one</u> meson mass at this level, we can try to get the mass ratio along the lines indicated in the three-body situation. A unique result would indicated success, and ambiguity might indicate the need for additional postulates. For comparison with experiment we would have to go to the third level, which includes both strong and electromagnetic effects and spin. The success of the m_p/m_e calculation might then be justified on a firmer basis. For precise results we clearly have to go to the fourth level.

Whether or not the speculative ideas in the last paragraph bear fruit in the uncertain future, we hope that this paper will at least be of interest as illustrating a novel bootstrap mechanism which works only if the particle, antiparticle, bound state of particle and antiparticle (quantum), and bound state of two particles and an antiparticle (or of a quantum and a particle) all have precisely the same mass.

The integration of the combinatorial hierarchy with physical ideas drawn from particle physics would have been completely impossible without continuing and intimate collaboration with Ted Bastin. Less frequent, but often intense, discussion and correspondence with John Amson, Clive Kilmister, and Fredrick Parker-Rhodes was equally vital. The idea of producing an on-shell (or single time) relativistic quantum scattering theory was stimulated by a paper by T. E. Phipps, Jr.,[12] as has been explained in more detail elsewhere;[7,8] continuing correspondence and discussion with him has been most rewarding. Clearly the exact three-body relativistic scattering theory constructed by David Brayshaw[5] is essential for the technical result reported in this paper. I am also indebted to many colleagues in many countries both for encouragement and for much needed criticism.

References

1. T. Bastin, Studia Philosophica Gandensia 4, 77 (1966).

2. T. Bastin and H. Pierre Noyes, "On the Physical Interpretation of the Combinatorial Hierarchy," SLAC-PUB-2225 (1978); presented at the conference on "Quantum Theory and the Structures of Time and Space 3," Tutzing, 1978 and submitted, with an appendix by John Amson, to the International Journal of Theoretical Physics.

3. A. F. Parker-Rhodes, private communication, 1978.

4. E. M. Zeiger and H. Pierre Noyes, in preparation; a preliminary report appears in the Proceedings of the Workshop on "Few Body Nuclear Physics," Trieste, 1978, published by International Atomic Energy Agency, Vienna, and as SLAC-PUB-2114 (1978).

5. D. D. Brayshaw, "Diffractive Production and Rescattering of Three-Particle Systems," SLAC-PUB-2094 (to be published in Phys. Rev. D).

6. E. Fermi and C. N. Yang, Phys. Rev. 76, 1739 (1949).

7. H. Pierre Noyes, Foundations of Physics 5, 37 (1975) [Erratum 6, 125 (1976)].

8. H. Pierre Noyes, Foundations of Physics 6, 83 (1976).

9. M. L. Goldberger and K. M. Watson, Collision Theory, Wiley, New York, 1964.

10. D. Finkelstein, seminar at Stanford, February 1979.

11. D. Finkelstein, Phys. Rev. 184, 1261 (1969).

12. T. E. Phipps, Jr., Dialectica 23, 189 (1969).

Figure Captions

1(a). Discrimination diagram for the first level of the hierarchy interpreted as a Feynman vertex with incoming lines and charge conservation.

1(b). Same as 1(a) with outgoing lines.

1(c). Discrimination diagram as a time-ordered diagram representing a particle-antiparticle bound state vertex.

1(d). Diagramatic representation of the particle-antiparticle-particle bound state vertex (bootstrap condition).

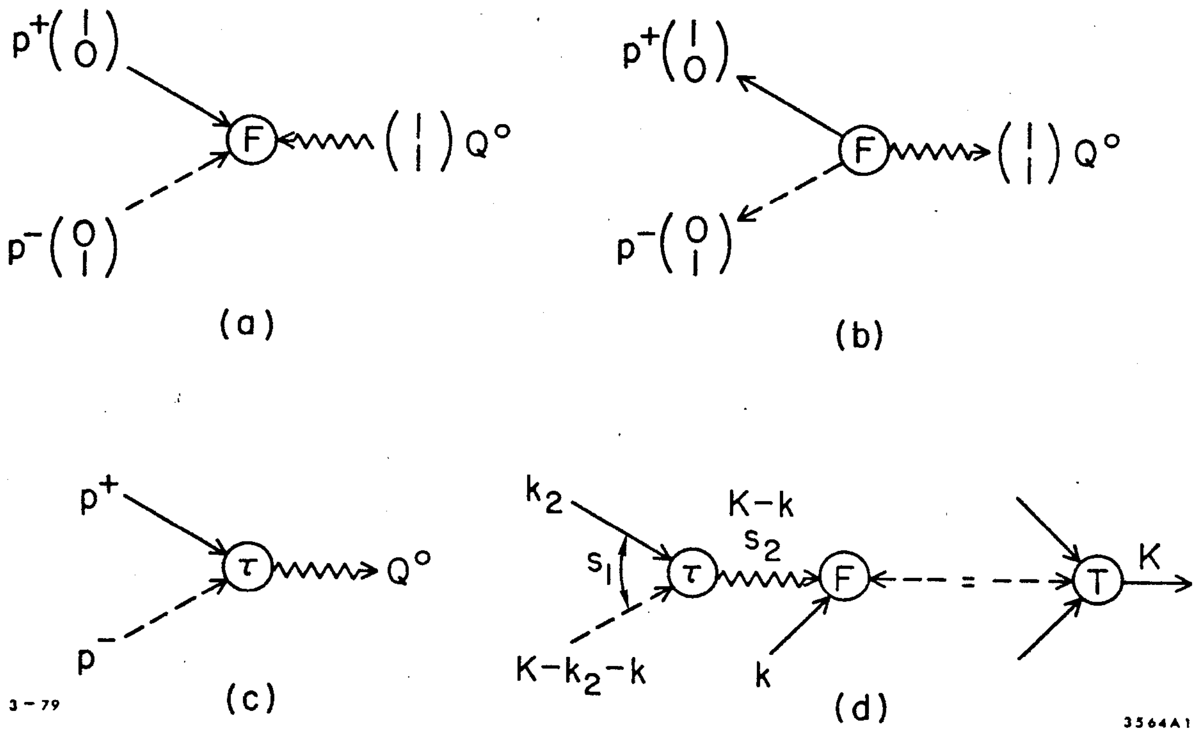

Fig. 1