SLAC-PUB-2268
March 1979
(A)

A DEVICE TO MEASURE QUADRUPOLE GRADIENT-LENGTH PRODUCT*

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Summary

This instrument measures the "focusing strength," $\int g dz$, of a quadrupole by moving a bundle of wires across its bore and recording the integrals of the two voltage loops generated by each stroke, together with the length of the stroke. A discussion of errors is given that indicates that it is possible to do .01% measurements upon electromagnets of moderately good quality with the device if amplitudes and phases of non-quadrupole field components are known from other measurements.

Introduction

Quadrupole focusing strengths, or gradient-length products, are commonly measured by recording the e.m.f. produced by long moving or rotating coils which are threaded through the quadrupoles and extended well beyond the ends of the electromagnets. To establish the calibration constant of such a "long-coil," it is necessary to measure its average width and stroke or average radius of rotation. For example, to do .01% measurements with a 5 cm wide rotating coil, it would be necessary to measure the average radius of rotation of its moving conductors to 2.5 μm or better. Since it would be difficult and expensive to build and measure a long-coil to such accuracy, the device that has been developed as a standard for PEP quadrupole measurements depends upon a single moving bundle of wires to generate its e.m.f. With this device, it is necessary to make only two "stroke" measurements, one at each end of the moving bundle of wires. The instrument (see Fig. 1)

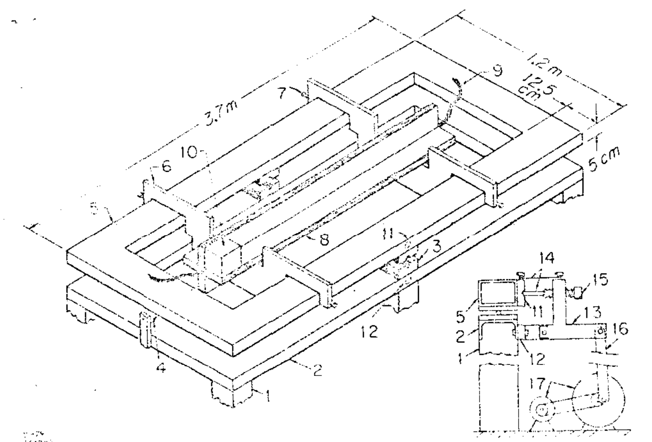
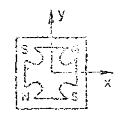


Fig. 1. Base, bearings, and moving framework. 1) legs; 2) *fixed* frame; 3) Vee-block-and-ball bearings; 4) end ball bearing races; 5) moving frame; 6) epoxy and glass fiber "crux" supports; 7) tensioning screws and gauge points; 8) epoxy and glass fiber stiffening "crux"; 9) wire-bundle; 10) Pb counterweight (one of eight shown); 11) drive point; 12) bell crank mounting point; 13) bell crank; 14) push-rod and elastomer retainer; 15) centering screw; 16) connecting rod; 17) motor drive and flywheel.

consists of a fixed frame with bearings for the moving parts, and a moving frame supporting the bundle of wire, which is held in a cruciform stiffener.

Rationale

If a long wire is stretched along the z axis of a quadrupole and moved in the x direction (see sketch), it will generate a voltage which is



$$V = \ell \langle B_y \rangle v_x = k\ell x\dot{x} \quad , \tag{1}$$

where ℓ is the effective length of the quadrupole, $\langle B_y \rangle$ is the average vertical component of the magnetic induction, $v_x = \dot{x}$ is the speed of motion of the wire, and $k\ell$ is the gradient-length product of the magnet. In an ideal quadrupole, Eq. (1) would be exact. In actual magnets, deviations will occur, which will be discussed below. If V is integrated over time for an ideal quadrupole,

$$\int V dt = (k\ell/2)\left(x_2^2 - x_1^2\right) \quad , \tag{2}$$

where x_2 and x_1 are the endpoints of the motion in x of the wire, assuming the wire remains parallel to the quadrupole axis and extends beyond the fringing fields. In the device to be described, many loops of wire are used, with the return members all bundled together and held fixed within the bore of the magnet.

Electrical measurements can provide information which makes it possible to center the stroke and eliminate the necessity for measuring mid-stroke position. As the wire-bundle moves from one end of its travel to the other, it generates two loops of voltage having opposite polarity. The area of each is integrated by a digital voltmeter and recorded as a (signed) number by a computer. The two numbers can be written as

$$n_1 = -mk\ell\left[\eta w x_1 + \frac{x_1^2}{2} + \frac{\gamma x_1^3}{3w} + \frac{\eta^2 w^2}{2} + \mathcal{O}(\eta^3\gamma w^2)\right] \tag{3}$$

and

$$n_2 = mk\ell\left[\eta w x_2 + \frac{x_2^2}{2} + \frac{\gamma x_2^3}{3w} + \frac{\eta^2 w^2}{2} + \mathcal{O}(\eta^3\gamma w^2)\right] \quad . \tag{4}$$

where m is the number of loops of wire, x_1 and x_2 represent the extremes of the motion, measured from the magnetic axis, the full stroke is $S = 2w = |x_2 - x_1|$, and η and γ are the average magnitudes of the vertical dipole and in-phase sextupole magnetic field components expressed as fractions of the quadrupole field when all three are measured at x=w, y=0. The number η is proportional to Earth's magnetic field, and is of order 10^{-4}. The sextupole coefficient, γ, must be measured by some other device such as rotating long-coil,[1] and will be 10^{-3} or smaller in a good quadrupole. Sextupole effects will be discussed further below. Now let $x_1 = -(1-\varepsilon)w$, and $x_2 = (1+\varepsilon)w$, and assume $\varepsilon \ll 1$. Substituting and combining Eqs. (3) and (4) gives

$$n_1 + n_2 \simeq 2mk\ell w^2\left[\varepsilon + \eta + \frac{\gamma}{3} + \gamma\varepsilon^2 + \mathcal{O}(\gamma\eta^3)\right] \quad , \tag{5}$$

and

$$n_1 - n_2 \simeq -2m\ell w^2\left[1 + (\varepsilon+\eta)^2 + 2\gamma\varepsilon + \frac{2\gamma\varepsilon^3}{3} + \mathcal{O}(\gamma\eta^3)\right] \quad . \tag{6}$$

Solving Eqs. (5) and (6) by successive approximations,

$$\varepsilon \simeq -\eta - \gamma/3 - \delta/2 \quad , \tag{7}$$

and

$$k\ell \simeq 8\langle n\rangle\left(1 - \frac{\delta^2}{4} + \frac{2\gamma\delta}{3} + \frac{5\gamma^2}{9} + 2\gamma\eta\right)/(mS^2) \quad , \tag{8}$$

*Work supported by the Department of Energy under contract no. EY-76-C-03-0515.
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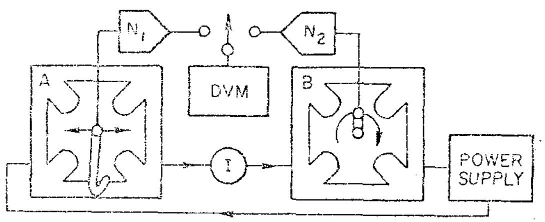
(Presented at the 1979 Particle Accelerator Conference, San Francisco, California, March 12-14, 1979.)

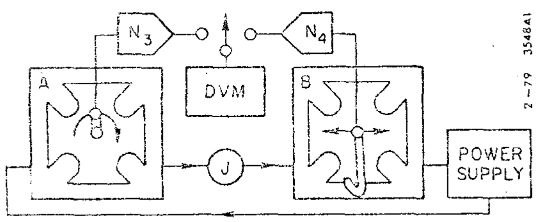
where $\langle n\rangle = (n_2-n_1)/2$ is the average integral of the voltage loops in volt-seconds, and $\delta = (n_1+n_2)/(n_1-n_2)$. An adjusting screw is provided (see Fig. 1) to adjust the centering of the stroke of the wire-bundle with respect to the quadrupole so that (n_1+n_2) can be made very nearly equal to zero.

Rotating Coil vs. Reciprocating Wire-Bundle Calibration

The calibration of a rotating long-coil, using two similar quadrupoles, is illustrated in Fig. 2. To



First comparison: Moving wires in magnet "A" give N_1 counts per integration, rotating coil in magnet "B" gives N_2 counts per integration at current I.



Second comparison: Rotating coil in magnet "A" gives N_3 counts per integration, moving wires in magnet "B" give N_4 counts per integration at current J.

Fig. 2. Calibrating a rotating long-coil.

reduce errors due to DVM drift and non-linearity, the rotating coil and the reciprocating wire-bundle device are designed so that their signal frequencies and peak signal voltages will be closely matched. Note that the reciprocating wire and rotating coil instruments are exchanged between quadrupoles for the second half of the procedure. Let C be the calibration constant of the rotating coil in m^2, and N_1, N_2, N_3, and N_4 be the induced voltage waveform integrals as indicated. Then

$$C \cong \frac{mS_1S_4}{8}\left(\frac{N_2N_3}{N_1N_4}\right)^{1/2}\left[1 + \mathcal{O}(\delta^2,\gamma\delta,\gamma^2,\gamma\eta)\right] \quad , \tag{9}$$

where S_1 and S_4 are the strokes corresponding to N_1 and N_4, and the correction terms can be worked out using Eq. (8).

Some Sources of Error

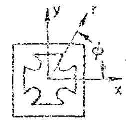
If the path of the moving wire-bundle is not parallel with the magnetic x-axis of the quad, the apparent sensitivity of the device will be low. A one degree error in this parameter will cause a .06% sensitivity error. This error goes as the square of the angle. If the strokes measured at each end of the wire-bundle differ in length, the proper "average S^2" is

$$\langle S^2\rangle \cong \left(S_1^2 + S_1S_2 + S_2^2\right)/3 \tag{10}$$

where S_1 and S_2 are measured at the ends of the magnet. Tests must be made to verify that any yaw at the ends of the stroke introduces no error. In a perfect quadrupole, if the wire-bundle has an angular error in pitch, the error due to increased wire-length in the field is cancelled by the cosine dependence of the induced e.m.f. This is also true of sag.

Some Wire-Bundle Size and Position Effects

In an ideal quadrupole where $B_\phi = k_{21}r\cos 2\phi$ and $B_y = k_{21}x$ (see sketch), the size or shape of the wire-bundle will not affect the signal. To show this, let the bundle consist of m wires, each having a position $x_i(t)$, y_i, with y_i constant, let $x_i = x(t) + u_i$, and choose $x(t)$ so that $\Sigma u_i = 0$. The integrated signal produced by one half-stroke will be



$$U_{21} = \sum_i k_{21}\ell\int_{x=0}^{x=w} x_i\dot{x}_i\,dt = k_{21}\ell\sum_i\left[(w+u_i)^2-u_i^2\right]/2$$

$$= mk_{21}\ell w^2/2 \quad . \tag{11}$$

Note that U_{21} is independent of y.

In an "in-phase" pure sextupole where $B_\phi = k_{31}r^2\cos 3\phi$ and $B_y = k_{31}(x^2-y^2)$, the signal would be

$$U_{31} = \sum_i k_{31}\ell\int\left(x_i^2-y_i^2\right)\dot{x}_i\,dt$$

$$= mk_{31}\frac{\ell}{3}w^3\left(1 + 3\langle u_i^2\rangle/w^2 - 3\langle y_i^2\rangle/w^2\right) \quad . \tag{12}$$

This result indicates that an m-wire-bundle that was flattened horizontally would be somewhat more than m times as sensitive to an in-phase sextupole component than a single small wire would be, and that a perfectly circular bundle moving with its center on the x-axis would have exactly m times the in-phase sextupole sensitivity as a single wire. In any event, in this device, $\langle u_i^2\rangle/w^2 \cong \langle y_i^2\rangle/w^2 < 3\times 10^{-3}$, and for practical purposes, no correction is required in the value of γ which appears in Eq. (8).

An "out-of-phase" sextupole component, where $B_\phi = k_{32}r^2\sin 3\phi$ and $B_y = 2k_{32}xy$, may produce a signal which is similar to that of a pure quadrupole:

$$U_{32} = mk_{32}\ell w^2\left(\langle y_i\rangle + 2\langle u_iy_i\rangle/w\right) \quad . \tag{13}$$

Note that U_{32} has a first order term in $\langle y_i\rangle$. To reduce the error from this source, it is necessary to make $\langle y_i\rangle$ small by adjusting the average elevation of the wire-bundle to agree with that of the magnetic axis of the quadrupole. The $\langle u_iy_i\rangle$ term will be zero if the wire-bundle is symmetrical about a vertical plane.

An "in-phase" octupole component will have $B_\phi = k_{41}r^3\cos 4\phi$ and $B_y = k_{41}(x^3-3xy^2)$. The integrated signal is

$$U_{41} = mk_{41}\ell\left(\frac{w^4}{4} + \frac{3w^2}{2}\langle u_i^2\rangle - \frac{3w^2}{2}\langle y_i^2\rangle + w\langle u_i^3\rangle - 3w\langle u_iy_i^2\rangle\right) . \tag{14}$$

Here again, as with an in-phase sextupole component, bundle size effects make small changes in octupole signals which, for most quadrupoles, are already small.

An "out-of-phase" octupole component will have $B_\phi = k_{42}r^3\sin 4\phi$ and $B_y = k_{42}(3x^2y-y^3)$. It will produce a signal of magnitude

$$U_{42} = mk_{42}\ell\left(w^3\langle y_i\rangle + 3w^2\langle u_iy_i\rangle + 3w\langle u_i^2y_i\rangle - w\langle y_i^3\rangle\right) \quad . \tag{15}$$

Again, the largest term in this signal can be eliminated by installing the wire-bundle at the proper elevation. The other terms will then be zero for circular symmetry.

Mechanical Arrangement

Except for the linear motion transducers, the moving parts of the device are illustrated in Fig. 1. Most of the ~100 kg weight of the moving frame (items 5-11) is supported upon two hardened steel balls which are captive in Vee grooves in steel bearing blocks (item 3). The ball bearing assemblies at each end (item 4) prevent tipping. The moving frame is welded from .32 cm rectangular Al tubing. The moving assembly is braced by tightening the tensioning screws (item 7), so that the epoxy-glass fiber crux supports (item 6) are maintained in tension. The epoxy-glass cruciform stiffening structure, or "crux" (item 8), in which the wire-bundle is clamped, is counterweighted and supported in such a way that its average inertial deflection is zero, when the average is taken over the span between its supports. Its peak horizontal deflection is ~.15 μm at the center.

The reciprocating drive mechanism is shown in the inset of Fig. 1. The push-rod (item 14) has spherical ends and is held in conical sockets by elastomer bands. The centering adjustment screw is item 15. The design speed is 1 revolution in 5 seconds. The flywheel was found to be necessary for smooth motion, storing about twice as much as the peak kinetic energy of the moving framework.

The displacement of the moving wire-bundle is measured at each end by a Sony Magnascale.[2] These instruments operate on the principle of a magnetic tape player, measure distances to 1 μm, and produce digital output. The measuring probes are held in aluminum fixtures which are bolted to the frame of the quadrupole. Elastic deformations of the crux, the crux supports, the gauging-point screws, the transducer mounting fixtures, and the quadrupole frame can all contribute to error in the measurements. It is estimated that in the configuration shown the aggregate of these deformations is less than 20% of the 1 μm resolution of the transducers.

A future report will describe later modifications made to the device to adapt it for routine direct measurements of PEP interaction region quadrupoles.

Acknowledgments

J. K. Cobb was responsible for developing and programming the data acquisition system. Z. Vassilian improved the drive mechanism to make the device run smoothly.

References

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2. Trademark, Sony Magnascale, Inc.