SLAC-PUB-2259
February 1979
(T/E)

# ON WEAK EXCHANGE DEGENERACY IN THE REACTIONS $\pi^+ p \to K^+\Sigma^+$ AND $K^- p \to \pi^-\Sigma^+$ AT 4, 7, 11.5 AND 70 GeV/c†

B. F. L. Ward

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

## ABSTRACT

The existing data on the line-reversed reactions $\pi^+ p \to K^+\Sigma^+$ and $K^- p \to \pi^-\Sigma^+$ near 4, 7, and 11.5 GeV/c are discussed from the point of view of weak exchange degeneracy (WEXD). It is noted that a smooth and simple generalization of the model of Navelet and Stevens is able to describe adequately, in the Regge region, the data at 7 and 11.5 GeV/c. (Recall that the Navelet and Stevens model appears to fit the pronounced violation of WEXD for leading $K^*$ and $K^{**}$ exchanges in the data near 4 GeV/c whereas the data at 7 and 11.5 GeV/c are in agreement with this particular WEXD.) A prediction for 70 GeV/c (namely, essentially that WEXD for leading $K^*$ and $K^{**}$ exchanges should hold true) is given. Further, an effort is made to indicate how this particular generalization of the model of Navelet and Stevens may arise in a dual multiperipheral bootstrap model of the general H. Lee-Veneziano-Chan-Paton variety as formulated by Balázs.

(Submitted to Phys. Rev. D)

†Work supported by Department of Energy under contract number EY-76-C-03-0515.

# I. INTRODUCTION

Sometime ago, it was pointed out by Gilman[1] that, if K* and K** Regge exchanges were indeed weak exchange degenerate, then the line-reversed reactions

$$\pi^+ p \to K^+ \Sigma^+ \qquad (1)$$

$$K^- p \to \pi^- \Sigma^+ \qquad , \qquad (2)$$

when dominated by these two exchanges, should have equal differential cross sections $d\sigma/dt$ and mirror symmetric polarizations, where $t$ is the momentum transfer squared. (The K* and K** exchanges are said to be weak exchange degenerate if their trajectories are equal.) According to the lore, the kinematic region in which the two exchanges should dominate would be the region of high $s$ and relatively low $|t|$, where $s$ is the squared center of momentum energy. Thus, it has come to pass that a number of experiments[2,3,4,5] have been done which probe the region

$$s \gtrsim 6\ (\text{GeV}/c)^2 \quad , \qquad |t| \lesssim 1\ (\text{GeV}/c)^2 \quad .$$

We should mention that the ideas of weak exchange generacy also make similar predictions about other line-reversed pairs of reactions and that we, by focusing on $\pi^+ p \to K^+ \Sigma^+$ and $K^- p \to \pi^- \Sigma^+$, in no way mean to imply that the other reactions are less significant. Rather, we take the view that all of the predictions of the (weak) exchange degeneracy idea are important and that, in order to have a complete picture of the data in relation to the idea, one should evidently consider all of the relevant predictions in relation to observation. For a more complete picture of the various predictions, we refer the reader to Ref. 1. Our work must be viewed in this broader context.

The single most interesting aspect of the data on the two line-reversed reactions (1) and (2) is that the lower energy data, near a laboratory momentum of 4 GeV/c, exhibit marked violations of weak exchange degeneracy (WEXD) for K* and K** leading trajectories whereas the higher energy data of Baker, et al.,[4] at 7 and 11.5 GeV/c laboratory momenta are in general agreement with WEXD.[††] The agreement appears to be better at the higher laboratory momentum. Further, we should also mention that the data of Berglund, et al.,[3] taken at 7.0 and 10.1 GeV/c laboratory momenta, are in general agreement with the data of Baker, et al.,[4] and, hence, are also in general agreement with the expectations of WEXD. Thus, for the sake of simplicity we shall work with the data of Baker, et al.,[4] with the understanding that, at the level of our discussions, everything we say about the data of reference 4 will apply in general also to the data of reference 3. A more complete discussion of both sets of data may be appropriate at a later time. In other words, here we simply take the data of Baker, et al.,[4] as representative of the level of agreement of WEXD with observation at the respective type of laboratory momenta, i.e., 7.0 and 11.5 GeV/c.

Now, Navelet and Stevens,[6] for example, have shown that the data at laboratory momenta $\sim 4$ GeV/c, although in disagreement with the expectations of WEXD, nonetheless, can be fitted with a flip amplitude, $H_F$, which is essentially weak exchange degenerate and a non-flip amplitude, $H_{NF}$, which is the sum of an amplitude (a) that is esentially what one expects from weak exchange degenerate K* and K** exchange and an amplitude (b) which represents the effect of Regge-cuts. The latter amplitude (b) is taken as a sum of effective Regge poles, giving two distinct trajectories for each signature. Now, if we ignore the issue of summing the diagrams responsible for generating the cut to all orders, it is to be expected that a cut can be represented as a pole only in a limited kinematic region. For, there are logarithms which accompany such a cut. Thus, it is not surprising that the fit of Navelet and Stevens may fail at some higher energy.

Of course, it can happen that the logarithms which are well-known to characterize the true effect of a Regge-cut at the one loop level, when all loops are summed, become a Regge pole with, perhaps, a trajectory which is quite different from the one-loop effective cut trajectory! Depending on the manner in which the two differ, a given phenomenologist may arrive at an effective representation of the data in one kinematic region which does not appear to bear a simple dynamical relationship to the representation of the data in another kinematic region, although all data are in the Regge region. It is from this particular point of view that we shall discuss, in this paper, the data on reactions (1) and (2) in the range $6 \lesssim s \lesssim 23$ $(\mathrm{GeV}/c)^2$.

It will happen that we shall be able, strictly from the phenomenological point of view, to predict $d\sigma/dt$ and the polarization P for reactions (1) and (2) at even higher energy, in particular, at a laboratory momentum of 70 GeV/c where an upcoming result from Fermilab[5,7] will be a direct check. And, indeed, a comparison or our generalized version of the Navelet and Stevens model with the preliminary results for reaction (1) alone at 70 GeV/c from the Fermilab experiment will be presented already in this paper.

What we shall do is the following. First, we shall, by looking at the data at 4 GeV/c, 7 GeV/c and 11.5 GeV/c laboratory momenta arrive at a simple phenomenological description. This description will be a simple generalization of the work of Navelet and Stevens. Then, we shall attempt to illustrate how such a generalization might arise theoretically by examining a simple toy model, the dual multiperipheral bootstrap advocated by H. Lee,[8] G. Veneziano,[9] and Chan Hung-Mo and J. Paton,[10] as formulated by L. A. P. Balázs,[11] to be specific. We would like to emphasize that we take this type of model no more seriously than do its authors. It will be necessary to extend the model, in a certain way, beyond the region investigated by Balázs. This extension is such that we feel the conclusions which we arrive at may have something to do with nature! For, as one can see from the work of Refs. 8-11, the dual multiperipheral model does have some experimental support.

Our work proceeds as follows. In Section II, we give our phenomenological generalization of the work of Navelet and Stevens and discuss our predictions for a laboratory momentum of 70 GeV/c. In Section III, we analyze the dual multi-peripheral bootstrap to see if our phenomenological results can be a reasonable prediction of such a theory. Finally, Section IV contains some concluding remarks.

## II. THE PHENOMENOLOGICAL MODEL AND THE DATA

Two experimentally accessible quantities for reactions (1) and (2) which are immediately relevant to the weak exchange degeneracy idea are $d\sigma/dt$ and the polarization, P, of the $\Sigma^+$. Specifically, one may write, following Ref. 6, for each reaction,

$$\frac{d\sigma}{dt} = |H_{NF}|^2 + |H_F|^2 \tag{3}$$

$$P\,\frac{d\sigma}{dt} = -2\,\mathrm{Im}\,(H_{NF}H_F^*) \quad , \tag{4}$$

where $H_{NF}$ is the s-channel non-flip amplitude, $H_F$ is the s-channel flip amplitude, * denotes complex conjugation, and Im denotes the imaginary part. As we indicated in Section I, t is the 4-momentum transfer squared and s is the squared center of momentum energy. Our metric is that of Bjorken and Drell. Thus, in order to distinguish the two amplitudes in each reaction, for $\pi^+ p \to K^+ \Sigma^+$, reaction (1), we shall use the obvious superscripts on $H_{NF}$ and $H_F$: $H_{NF}^{(1)}$ and $H_F^{(1)}$. Similarly, for $K^- p \to \pi^- \Sigma^+$, reaction (2), we shall use the obvious notation $H_{NF}^{(2)}$ and $H_F^{(2)}$ for the non-flip and flip amplitudes, respectively.

As one can see from Ref. 6, Navelet and Stevens are able to achieve a reasonable description of the data near 4 GeV/c laboratory momentum for $d\sigma/dt$ and P by taking $H_{NF}^{(\ell)}$ and $H_F^{(\ell)}$ in Regge form as follows:

$$
\begin{aligned}
H_{NF}^{(\ell)} = \kappa \Big\{ & (-1)^{\ell+1} \gamma_{K^*,NF} (-1 + \exp(-i\pi\alpha_{K^*}(t)))\, s^{\alpha_{K^*}(t)} \exp(a_{K^*,NF} t) \\
& + \gamma_{K^{**},NF} (1 + \exp(-i\pi\alpha_{K^{**}}(t)))\, s^{\alpha_{K^{**}}(t)} \exp(a_{K^{**},NF} t) \\
& + (-1)^{\ell+1} \gamma_{K^*,NF} \gamma^c_{K^*}\, i \exp(-i\pi\alpha^c_{K^*}(t)/2)\, s^{\alpha^c_{K^*}(t)} \exp(a^c_{K^*} t) \\
& - \gamma_{K^{**},NF} \gamma^c_{K^{**}} \exp(-i\pi\alpha^c_{K^{**}}(t)/2)\, s^{\alpha^c_{K^{**}}(t)} \exp(a^c_{K^{**}} t) \Big\}
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
H_F^{(\ell)} = \kappa \sqrt{-t'} \Big\{ & (-1)^{\ell+1} \gamma_{K^*,F} (-1 + \exp(-i\pi\alpha_{K^*}(t)))\, s^{\alpha_{K^*}(t)} \exp(a_{K^*,F} t) \\
& + \gamma_{K^{**},F} (1 + \exp(-i\pi\alpha_{K^{**}}(t)))\, s^{\alpha_{K^{**}}(t)} \exp(a_{K^{**},F} t) \Big\},
\end{aligned}
\tag{6}
$$

$\ell = 1, 2.$

Here,

$$\alpha_{K*}(t) = .375 + .678\,t \quad , \qquad \alpha_{K**}(t) = .322 + .678\,t \quad ,$$

$$\gamma_{K*,NF} = 10.34 \quad , \qquad \gamma_{K**,NF} = 11.15 \quad ,$$

$$\gamma_{K*,F} = -9.862 \quad , \qquad \gamma_{K**,F} = -7.114 \quad ,$$

$$a_{K*,NF} = 2.414 \quad , \qquad a_{K**,NF} = 1.331 \quad ,$$

$$a_{K*,F} = 1.895 \quad , \qquad a_{K**,F} = 1.895 \quad ,$$

$$\alpha^c_{K*}(t) = .099 + .532\,t \quad , \qquad \alpha^c_{K**}(t) = .127 + .532\,t \quad ,$$

$$\gamma^c_{K*} = 1.01 \quad , \qquad \gamma^c_{K**} = 1.44 \quad ,$$

$$a^c_{K*} = 1.134 \quad , \quad \text{and} \quad a^c_{K**} = .091 \quad . \tag{7}$$

also,

$$t' = t - t_{min} \tag{8}$$

and

$$\kappa = \frac{.38935\,M_p^2}{16\pi p_i^2 s}\ \mathrm{mb/(GeV/c)^2} \quad , \tag{9}$$

where $p_i$ is the incident momentum in the center of momentum frame and $M_p$ is the proton rest mass. Thus, all invariants and momenta are to be expressed in GeV/c units. At this point, let us emphasize that we take the fit described by (5), (6) and (7) no more seriously than do its authors. For our purposes, it is a convenient representation of the lower energy data which is not unreasonable from the point of view of the standard Regge lore.

Indeed, the form of the trajectory functions $\alpha_{K*}(t)$ and $\alpha_{K**}(t)$ in (5) is reasonably consistent with the expectations of weak exchange degeneracy for the K* and K** vector and tensor exchanges. Thus, interpreting the odd and even signatured terms as due to K* and K** exchange, respectively, as our notation indicates, we see that the flip amplitudes are essentially weak exchange degenerate. However, the non-flip amplitudes, containing, as they do, the contributions of the effective poles $\alpha^c_{K*}$ and $\alpha^c_{K**}$ in addition to approximately weak exchange degenerate K* and K** exchange contributions, exhibit pronounced violations from what one expects from the weak exchange degenerate view of K* and K** leading exchanges. We remind the reader that, by weak exchange degeneracy, we should have $\alpha_{K*}(t) = \alpha_{K**}(t)$.

If one compares the prediction of (5) and (6) for the polarization P, for example, with the data of Baker, et al.,[4] at 11.5 GeV/c, one sees that the parameterization of Navelet and Stevens appears to fail at such laboratory momenta. See Fig. 1. Thus, we are invited by this observation to modify the model described by (5) and (6).

We should like to do this entirely phenomenologically, at first. For, in this way, we shall probably create the most interesting theoretical challenge. Specifically, the main characteristic of the data of Baker, et al.,[4] from our point of view, is the agreement with the expectations from weak exchange degeneracy for the K* and K** exchanges, assumming they are the leading Regge exchanges in reactions (1) and (2). Recall this agreement appears to be better at $p_{Lab}$ = 11.5 GeV/c than at $p_{Lab}$ = 7 GeV/c. Here, $p_{Lab}$ is the laboratory incoming momentum. Hence, in the simplest situation, which is the one we shall discuss, we need non-flip amplitudes $H^{(i)}_{NF}$ which extrapolate smoothly from equation (5) at $p_{Lab} \cong 4$ GeV/c to essentially weak exchange degenerate K* and K** exchange at $p_{Lab}$ = 7 GeV/c. The flip amplitudes $H^{(i)}_F$ in equations (6) are already essentially weak exchange degenerate. So, we shall leave them as they are given in (6). Further, we wish to stay close to the Regge form (5).

Thus, we replace, for simplicity, the effective poles $\alpha^c_{K*}(t)$, $\alpha^c_{K**}(t)$ with $\alpha^c_{K*}(t) - n$, $\alpha^c_{K**}(t) - n$, respectively, for some integer n. If we require that the phase of the effective pole amplitudes be unchanged at $p_{Lab} = 4$ GeV/c, we shall assure that the asymmetric polarization predictions of (5) and (6) are unchanged at this momentum, provided we correct $\gamma^c_{K*}$ and $\gamma^c_{K**}$ in (7) by an appropriate power of $s_1 = m_1^2 + M_p^2 + 2M_p\sqrt{(4\ \text{GeV/c})^2 + m_1^2}$ in $H^{(1)}_{NF}$, $1 = 1, 2$, where $m_1$ is the $\pi^+$ rest mass and $m_2$ is the $K^+$ rest mass; clearly, the correct power is $s_1^n$ in $H^{(1)}_{NF}$. Thus, we require

$$-i\pi(-n)/2 = 2m\pi i \tag{10}$$

or

$$n = 4m \quad , \tag{11}$$

for some integer m. Notice that, since $\gamma^c \sim 1$, in order for the effective poles to be at most a few percent of the remaining approximately weak exchange degenerate parts of $H^{(i)}_{NF}$ at $p_{Lab} = 7$ GeV/c, we must have

$$(s_i/s)^n \lesssim \text{a few percent} \tag{12}$$

at $p_{Lab} = 7$ GeV/c. In other words, we need

$$(4/7)^n \lesssim \text{a few percent} \tag{13}$$

For the sake of discussion we take m = 1 in (11), giving n = 4:

$$(4/7)^4 \sim 10\% \quad .$$

The data of reference 4 are expected to be accurate to about this level.

Now, clearly, if we had extremely accurate data, one might wish to try replacing the effective poles $\alpha^c_{K*}$, $\alpha^c_{K**}$ with the sum of a series of poles $\left\{\alpha^c_{K*} - n \mid n = 0, 1, 2, \ldots\right\}$ , $\left\{\alpha^c_{K**} - n \mid n = 0, 1, 2, \ldots\right\}$ , each with $\gamma^c_{K*}(n) = a_n s^n_\ell \gamma^c_{K*}$ , $\gamma^c_{K**}(n) = b_n s^n_\ell \gamma^c_{K**}$ in $H^{(\ell)}_{NF}$

and require

$$\sum_{n=0}^{\infty} a_n e^{-i\pi n/2} = 1 , \qquad \sum_{n=0}^{\infty} b_n e^{-i\pi n/2} = 1 . \tag{14}$$

In other words, for example, one might wish to carry out a fit to the detailed t- and s-dependence of the data by replacing, in (5),

$$(-1)^{\ell+1}\gamma_{K*,NF}\gamma^c_{K*}\ i\ \exp(-i\pi\alpha^c_{K*}(t)/2) s^{\alpha^c_{K*}(t)} \exp(a^c_{K*} t) \tag{15}$$

with

$$(-1)^{\ell+1}\gamma_{K*,NF}\gamma^c_{K*}\sum_{n=0}^{\infty} a_n s^n_\ell\ i \exp(-i\pi(\alpha^c_{K*}(t)-n)/2) s^{(\alpha^c_{K*}(t)-n)} \exp(a^c_{K*} t) \tag{16}$$

and by replacing

$$-\gamma_{K**,NF}\gamma^c_{K**}\exp(-i\pi\alpha^c_{K**}(t)/2)\ s^{\alpha^c_{K**}(t)} \exp(a^c_{K**} t) \tag{17}$$

with

$$-\gamma_{K**,NF}\gamma^c_{K**}\sum_{n=0}^{\infty} b_n s^n_\ell \exp(-i\pi(\alpha^c_{K**}(t)-n)/2) s^{(\alpha^c_{K**}(t)-n)} \exp(a^c_{K**} t) \tag{18}$$

subject to (14).

In the notation of (17) and (18), the choice $n=4$ in (11) corresponds to

$$a_4 = 1,\ a_n = 0 \text{ for } n \neq 4,\ b_4 = 1,\ b_n = 0 \text{ for } n \neq 4. \tag{19}$$

We now have

$$H^{(m)}_{NF} = \kappa \Big\{ (-1)^{m+1}\gamma_{K*,NF}(-1+\exp(-i\pi\alpha_{K*}(t))) s^{\alpha_{K*}(t)} \exp(a_{K*,NF} t)$$

$$+ \gamma_{K**,NF}(1+\exp(-i\pi\alpha_{K**}(t)))\ s^{\alpha_{K**}(t)} \exp(a_{K**,NF} t)$$

$$+ (-1)^{m+1}\gamma_{K*,NF}\gamma^c_{K*}\ i \exp(-i\pi\alpha^c_{K*}(t)/2) s^{\alpha^c_{K*}(t)} (s_i/s)^4 \exp(a^c_{K*} t)$$

$$-\gamma_{K^{**},NF}\gamma^{c}_{K^{**}}\exp(-i\pi\alpha^{c}_{K^{**}}(t)/2)s^{\alpha^{c}_{K^{**}}(t)}(s_i/s)^4\exp(a^{c}_{K^{**}}t)\Big\}\ , \qquad (20)$$

where all parameters are given by (7) and (9). The comparisons of (20) and (6) with experiment at $p_{Lab}$ = 7 and 11.5 GeV/c are shown in Fig. 2. Figs. 3 and 4 show the predictions of (20) and (6) for $p_{Lab}$ = 70 GeV/c, where only the Fermilab data on $\pi^+p \to K^+\Sigma^+$ are available. The data on $K^-p \to \pi^-\Sigma^+$ should be available soon. One sees that the agreement between the amplitudes (20), (6) and the data is rather reasonable in the strict Regge region.[‡‡]

Having achieved a phenomenological description of the data at 7 and 11.5 GeV/c we turn, in the next section to the obvious theoretical question, "From where could such an amplitude as (20) come?" For, at this stage, one must consider the forms (16) and (18) completely ad hoc so that the amplitude (20) is a special case of an ad hoc, empirical analysis.

## III. A TOY MODEL

The amplitude (20) and the more general forms (16) and (18) have the structure that might be associated with the daughter trajectories of the effective pole trajectories $\alpha^c_{K^*}(t)$, $\alpha^c_{K^{**}}(t)$. But, the latter poles are actually supposed to be approximations to Regge cuts. Thus, we are looking for a model which might have "daughters" of the cut trajectories. For simplicity, we will ignore the difference between $\alpha^c_{K^*}(t)$ and $\alpha^c_{K^{**}}(t)$ and consider ourselves to have the cut trajectory

$$\alpha^c(t) \doteq .1 + .5\,t \qquad , \qquad (21)$$

which is taken to be weak exchange degenerate, i.e., which gives contributions of both odd and even signature in the same way as a weak exchange degenerate leading trajectories would. A theoretical framework which is set-up to consider the effects of cuts to all orders in Regge exchange is the dual multiperipheral bootstrap of H. Lee,[8] Veneziano,[9] and Chan and Paton[10] as represented by Balázs.[11] We will work in the planar approximation, for we are not interested in the Pomeron. Balázs' representation is particularly appropriate to our needs because it pays special attention to certain threshold effects.

Specifically, in this scheme the amplitude for the production of clusters is given by the standard multiperipheral diagrams illustrated in Fig. 5. In our case, we will be interested in generating Reggeons actually, so that the vertical lines are clusters and the horizontal lines are linear combinations of exchange degenerate sets of Regge exchanges with Regge propagators

$$R = e^{-i\pi\alpha_e(t)} s^{\alpha_e(t)} \quad . \tag{22}$$

We are, of course, interested in the case of exchange degenerate K* and K** exchange. The diagrams illustrated in Fig. 5 give the two-body absorptive part, via multiparticle unitarity, as the sum of ladders as illustrated in Fig. 6. Since we are only interested in generating Reggeons, only the (planar) quark-duality diagrams shown in Fig. 7 are relevant.

The model has the following additional constraint. The vertical lines in Fig. 6 are dual to Regge behavior in the sense of a finite energy sum rule (FESR), as illustrated in Fig. 8. Specifically, we have (We will use the notation of Ref. 11 wherever possible.)

$$\Gamma(t_1, t_2, t_1', t_2', t) = F(t)\, g_1(t_1, t_1', t)\, g_2(t_2, t_2', t) \ , \tag{23}$$

where $\Gamma$ represents the Reggeon-Reggeon-cluster-Reggeon-Reggeon coupling in Fig. 8, $g_1$ and $g_2$ are triple-Regge couplings, and F is a kinematic factor. In the model, it can be argued that F is independent of $t_i$ and $t_i'$, i = 1, 2, so that Γ factorizes; this permits a simple solution of the model.

Specifically, we assume the absorptive part A(t,s) of the process $1\ 2 \to 1'\ 2'$ is given by Fig. 9, wherein only one narrow-resonance cluster of type a is produced, with the exception of the end clusters R and Q. A more realistic assumption might be to sum Fig. 9 over several such clusters a. We take the squared mass of a, $s_a$, to be $s_a \doteq .5\ \text{GeV}^2$, corresponding to ignoring the difference between the squared masses of the ρ and K*. A better choice might be $s_a \doteq .65 = (m_\rho^2 + m_{K^*}^2)/2$, where $m_a$ = mass of a, a = ρ,K*. Clearly in each loop in Fig. 9, one of the exchanged Reggeons carries strangeness, the other does not. However, since the empirical result in (7)

$$\alpha_{K^*}(t) = .375 + .678\,t \quad , \tag{24}$$

is not too far from the canonical form

$$\alpha_V(t) = .5 + t \tag{25}$$

for the leading vector trajectory, we shall, for simplicity, consider that we have a single exchange trajectory function $\alpha_e(t) = \alpha_e^0 + \alpha_e' t$. We then require that the cut trajectory $\alpha^c(t)$ in (21) be associated with $\alpha_e$ in the usual way

$$\begin{aligned} .1 + .5\,t = \alpha^c(t) &= 2\alpha_e(\tfrac{1}{4}t) - 1 \\ &= 2\alpha_e^0 + \tfrac{1}{2}\alpha_e' t - 1 \quad . \end{aligned} \tag{26}$$

This gives

$$\alpha_e = .55 + t \tag{27}$$

which is not too different from (24) and (25). Thus, in our toy calculation, we will simply use $\alpha_e = \alpha_V$.

We shall take R = a and Q will play the role of an appropriate strange or non-strange baryon. In other words, if particle 1' is $K^+$, particle 2' is $\Sigma^+$ so, if the last Reggeon coupled to 2' is strange, we could take Q to be the nucleon. If this Reggeon were non-strange, we must take Q to be a baryon with $S \neq 0$. In our toy calculation, we simply choose Q to have mass squared $s_Q \doteq 1\ \mathrm{GeV}^2$.

Let us now describe the actual computation of A(t,s). One first notes that in the narrow resonance approximation the graph in Fig. 8a, out of which A(t,s) is constructed, gives

$$V(t,s) = \Gamma_a \delta(s-s_a) \quad . \tag{28}$$

Therefore, from (23) the Mellin transform of V is

$$V(t,j) = g(t_1,t_2,t)\, g(t_1',t_2',t) F_a(t)\, s_a^{-j-1} \quad , \tag{29}$$

where

$$V(t,j) = \int_0^\infty ds\, s^{-j-1} A(t,s) \tag{30}$$

and g is the $\alpha_e - \alpha_e - K^{**}$ triple Regge coupling. Hence, if one takes the Mellin transform of A(t,s)

$$A(t,j) = \int_0^\infty ds\, s^{-j-1} A(t,s) \quad , \tag{31}$$

then, with the standard high energy approximations one obtains the result of Balázs:[11]

$$A(t,j) = \gamma_{11'K^{**}}(t)\, s_a^{-j-1}\, F_a(t)\, B(t,j)\, F_Q(t)\, s_Q^{-j-1}\, \gamma_{22'K^{**}}(t) \quad . \tag{32}$$

Here, $\gamma_{11'K^{**}}$, $\gamma_{22'K^{**}}$ are the 11'K**, 22'K** Regge couplings respectively and B(t,j) is given by

$$B(t,j) = K(t,j) \,/\, D(t,j) \tag{33}$$

where K(t,j) has the general structure

$$K(t,j) = \int gRRg \tag{34}$$

and

$$D(t,j) = 1 - s_a^{-j-1} F_a(t) K(t,j) \quad . \tag{35}$$

Thus, B corresponds to summing the series of graphs shown in Fig. 10; K is evidently just the one loop graph in this series. In other words, one can write B as

$$B(t,j) = K(t,j) + K(t,j)F_a(t)\, s_a^{-j-1} K(t,j) + \ldots \quad . \tag{36}$$

The function K(t,j), whose treatment by Balázs represents the central point of his form of the dual multiperipheral planar bootstrap, is the function which we wish to concentrate on in this toy model of ours for the hypercharge exchange reactions () and (2). Specifically, for the case of $\pi$-$\pi$ scattering, Balázs solved for the output f-trajectory $\alpha = .5 + t$ with $s_a$ taken to be the same as in our toy model. Working to first order in t, from Balázs' work we know that if one takes

$$K(t,j) = \frac{k_0(1+\frac{1}{2}\pi\tau t)}{j - \alpha^c}\,(x_0 + x_1 t)^{\alpha^c - j} \tag{37}$$

and require $D(t, \alpha_V(t)) = 0$ so that A(t,j) has a pole at $j = \alpha_V$, then a conventional first moment finite energy sum rule will give, approximately,

$$k_0 = .36, \quad \tau = 1.39, \quad x_0 = 1.34, \quad x_1 = 1.76\, x_0 \quad , \tag{38}$$

completely solving the model. (See Ref. 11 for more details on the derivation of the results (38). We have used the FESR

$$F_a(t) \cong (s_a + \tfrac{1}{2}t)^{-1}\,(2s_a + \tfrac{1}{2}t)^{\alpha_V+2}(\alpha_V + 2)^{-1}$$

for simplicity.) The choice (37) is motivated by two things:

(1) K(t,j) is known to have a logarithmic singularity at $j = \alpha^c$,

which is approximated by the pole in (37). (It is expected that this approximation fails for $j < \alpha^c$.); and (2), $K(t,s)$ should have a threshold factor at $s = x(t) \doteq x_0 + x_1 t$. Thus, we are invited to consider a more realistic choice for $K(t,j)$.

Specifically, since we know that in (37) $1/(j-\alpha^c)$ should be $\sim \ln\left((1/(j-\alpha^c)\right)$, we could simply replace $1/(j-\alpha^c)$ by the logarithm. This we do as follows: we take

$$K(t,j) = k_0'(1+\tfrac{1}{2}\pi\tau t)(x_0+x_1 t)^{\alpha^c-j}\ln\left|\frac{j-\alpha_m}{j-\alpha^c}\right| \tag{39}$$

where $k_0'$ and $\alpha_m$ are constrained by the requirement that

$$k_0' \ln\left|\frac{\alpha_V-\alpha_m}{\alpha_V-\alpha^c}\right| \doteq \frac{k_0}{\alpha_V-\alpha^c} \quad . \tag{40}$$

For simplicity, we work with (40) to zeroth order in $t$ only and find at $t = 0$

$$k_0'\ln\left|1-2\alpha_m\right| = 2k_0 \quad . \tag{41}$$

The condition (40) insures that $D(t,j)$ will still have a pole at $j = \alpha_V$ for the values of $k_0$, $\tau$, $x_0$ and $x_1$ in (38).

In case the reader is wondering about the origin of $\alpha_m$, let us remark that it is simply an approximation to the lower support limit $\zeta_m(t)$ of the Amati-Stanghellini-Fubini-Bertocchi-Tonin[13] Regge weight function $\rho(t,s)$ in their multiperipheral theory of K: namely, following their notation, the first iteration of a Regge pole

$$T(s,t) = C(t)\, s^{\alpha(t)}\left(-\cot\left(\frac{\pi\,\alpha(t)}{2}\right) + i\right) \tag{42}$$

gives the asymptotic result

$$\rightarrow T\ (s,t) \quad = \int_{\zeta_m(t)}^{\zeta_M(t)} s^{\zeta}\rho(t,\zeta)\left[-\cot\left(\frac{\pi\zeta}{2}\right) + i\right] d\zeta \quad , \tag{43}$$

where

$$\rho(t,\zeta) \quad = \frac{2}{8\pi^2}\int_{-\infty}^{0} dt' \int_{-\infty}^{0} dt''\,\delta\Big(\zeta + 1 - \alpha(t') - \alpha(t'')\Big)\ \Delta(t,t',t'')$$

$$C(t')\ C(t'')\left[\cot\left(\frac{\pi\alpha(t')}{2}\right)\ \cot\left(\frac{\pi\alpha(t'')}{2}\right) + 1\right] \tag{44}$$

with

$$\Delta(t,t',t'') \quad = \frac{\theta(-t^2 - t'^2 - t''^2 + 2tt' + 2tt'' + 2t't'')}{\sqrt{-t^2 - t'^2 - t''^2 + 2tt' + 2tt'' + 2t't''}} \quad . \tag{45}$$

The functions $\zeta_M(t)$ and $\zeta_m(t)$ are the upper and lower support limits of $\rho(t,\zeta)$ and to make contact with our work, we should point out that

$$\zeta_M(t) \quad = \quad \alpha^c(t) \tag{46}$$

$$\zeta_m(t) \quad = \quad \alpha_m(t) \quad . \tag{47}$$

An advantage of (39) is that, unlike (37), it may be valid for $j < \alpha^c$. In other words, the toy model for A(t,j) does not obviously fail for $j < \alpha^c$ if we use (39) whereas the model does appear to fail for $j < \alpha^c$ if we use (37). Indeed, using (39) we have, again at $t = 0$, that, for example, there is a pole in A(t,j) also at $j \doteq \alpha^c - 4$ if

$$1 \quad = \quad (s_a x_0)^{.5+4}\,\ell n\left|\frac{-4-\alpha_m}{-4}\right| \Big/ \ell n\left|\frac{.5-\alpha_m}{.5}\right| \tag{48}$$

or

$$\alpha_m(0) \quad = \quad 0 \quad . \tag{49}$$

This value of $\alpha_m(0)$ is not inconsistent with $\alpha^c(0)$, although we would naively expect that $\alpha_m(0) < 0$. That $\alpha_m(0) = 0$ is probably just an artifact of our approximations; in a more complete treatment, more of the smooth dependence of $\rho(t,\zeta)$ in $\zeta$ would have to be taken into account. It is clear, however, that when properly extended to $j < \alpha^c$, models like the dual multipheripheral planar bootstrap may indeed contain further Regge poles at places like $j = \alpha^c-4$, i.e., at places like the empirical extra trajectories that appear to be required by the data on the hypercharge exchange reactions (1) and (2).

## IV. DISCUSSION

This paper has attempted to do two things. First, it has attempted to present an empirical fit to the apparently weak exchange degenerate data on the hypercharge exchange reactions $\pi^+ p \to K^+ \Sigma^+$ and $K^- p \to \pi^- \Sigma^+$ at laboratory momenta like 7, 11.5 GeV/c which, at the same time, incorporates apparent violations of weak exchange degeneracy at laboratory momenta of order 4 GeV/c. The type of fit arrived at requires a trajectory at $\alpha^c-4$, where $\alpha^c$ may be identified with the cut trajectory of the leading K*, K** trajectory exchanges. Thus, the second part of the paper was devoted to a toy model calculation, which by the way is nothing but a simple version of the Chew-Goldberger-Low[14] model applied to cluster production. The result of the model calculation is that if the j-plane structure of the two-Regge exchange graph is properly extended to the region $j < \alpha^c$, one may indeed find trajectories like $\alpha^c-4$! Thus, the emperical fit in Sect. II may not be without theoretical support.

There is one more thing we should like to emphasize. This is that, from our empirical analysis, the trajectory $\alpha^c-4$, if it were the only trajectory persent in the reactions $\pi^+ p \to K^+ \Sigma^+$ and $K^- p \to \pi^- \Sigma^+$, would give essentially weak exchange degenerate predictions. Thus, in some sense, the notion that each trajectory function contributes both odd and even signatured contributions in the weak exchange degeneracy sense is still true. What differs from the naive weak exchange degenerate idea is that at lower laboratory momenta, two different trajectories, each by itself respecting weak exchange degeneracy, nonetheless are both important and interfere to give apparent violations of the predictions one would make if one only had to consider a single weak exchange degenerate trajectory.

Finally, there is an interesting coincidence to which we wish to call attention. This is that, if one follows the work of Balázs[15] on threshold effects, one finds that the D* trajectory is, approximately,

$$\alpha_{D*}(t) \doteq -.75 + .433\,t$$

$$\simeq \alpha^{c} - 1 \qquad (50)$$

where $\alpha^{c}$ is given by (21) for example. At this time, we will simply leave this as a coincidence!

## ACKNOWLEDGMENTS

This paper is the result of an investigation carried out at the suggestion of and with the encouragement of Group BC at SLAC — especially of its director, Professor J. Ballam. The author is grateful to Professor Ballam and his Group. The hospitality of Professor S. D. Drell of the SLAC Theory Group is also appreciated. The author further acknowledges a helpful conversation with Professor F. J. Gilman.

REFERENCES


1. F. J. Gilman, Phys. Lett. 29B, 673 (1969).

2. S. M. Pruss et al., Phys. Rev. Lett. 23, 189 (1969); P. Kalbaci et al., ibid. 27, 74 (1971); A. Bashian et al., Phys. Rev. D4, 2667 (1971); K. S. Han et al., Phys. Rev. Lett. 24, 1353 (1970); M. Aguilar-Benitez et al., Phys. Rev. D6, 29 (1972); L. Moscoso et al., Nucl. Phys. B36, 332 (1972); R. J. Miller et al., Nuovo Cimento Lett. 6, 49 (1973); R. Blokzijl et al., Nucl. Phys. B51 535 (1973); D. Birnbaum et al., Phys. Lett. 31B, 484 (1970).

3. A. Berglund et al., Phys. Lett. 73B, 369 (1978).

4. A. Baker et al., "Experimental Test of Exchange Degeneracy in Hypercharge Exchange Reactions at 7 and 11.5 GeV/c", SLAC-PUB 2169, July 1978.

5. M. W. Arenton et al., "Measurement of Exclusive Hypercharge Exchange Reactions at 35 - 140 GeV/c", ANL-HEP-RP-78-24, July 1978.

6. H. Navelet and P. R. Stevens, Nucl. Phys. B104, 171 (1976).

7. See, for example, K. C. Moffeit, "Experimental Test of Exchange Degeneracy in Hyperchange Exchange Reactions," SLAC-PUB 2208, October 1978.

8. H. Lee, Phys. Rev. Lett. 30, 719 (1973).

9. G. Veneziano, Phys. Lett. 43B, 413 (1973).

10. Chan Hong-Mo and J. E. Paton, Phys. Lett. 46B, 228 (1973).

11. L. A. P. Balázs, Phys. Rev. D15, 309 (1977).

12. J. D. Bjorken and S. D. Drell, Relativistic Quantum Fields, (McGraw-Hill, Inc., New York, 1965).

13. D. Amati, A. Stanghellini and S. Fubini, Nuovo Cimento 26, 896 (1962); L. Bertocchi, S. Fubini and M. Tonin, ibid. 25, 626 (1962).

14. G. F. Chew, M. L. Goldberger and F. E. Low, Phys. Rev. Lett. 22, 208 (1969).

15. L. A. P. Balázs, Nuovo Cimento Lett. 23, 56 (1978).

†† By agreement with WEXD we shall henceforth simply mean agreement with the expectations of weak exchange degeneracy for leading K* and K** trajectories.

‡‡ Assumming only $I_t = \frac{1}{2}$ exchange is relevant, the agreement of the theory with $d\sigma/dt$ at low $|t|$ must be considered to be modulo to a 50% normalization correction in comparing with the lower energy data at 4 GeV/c, because the parameters in (7) refer to $\pi^- p \to K^0 \Sigma^0$.

## FIGURE CAPTIONS

1. Comparison of the Navelet and Stevens model with the polarization data of Ref. 4 for $\pi^+ p \to K^+ \Sigma^+$. This figure is taken from Ref. 7.

2. Comparison of the amplitudes (20) and (6) with the data of Ref. 4.

3. Comparison of the amplitudes (20) and (6) with the data of Ref. 5 on $d\sigma/dt$ for $\pi^+ p \to K^+ \Sigma^+$. The prediction for $K^- p \to \pi^- \Sigma^+$ is also shown.

4. Comparison of the amplitudes (20) and (6) with the data of Ref. 5 on the $\Sigma^+$ polarization for $\pi^+ p \to K^+ \Sigma^+$. The prediction for $K^- p \to \pi^- \Sigma^+$ is also shown.

5. Typical type of diagram in the multiperipheral model for cluster production.

6. Absorptive part generated by the diagrams illustrated in Fig. 4.

7. Planar quark-duality diagram relevant to the generation of Reggeons.

8. Cluster (a) and Reggeon (b) average duality.

9. Absorptive part A corresponding to the production of a-clusters.

10. Series of graphs corresponding to B(t,j) in (32).

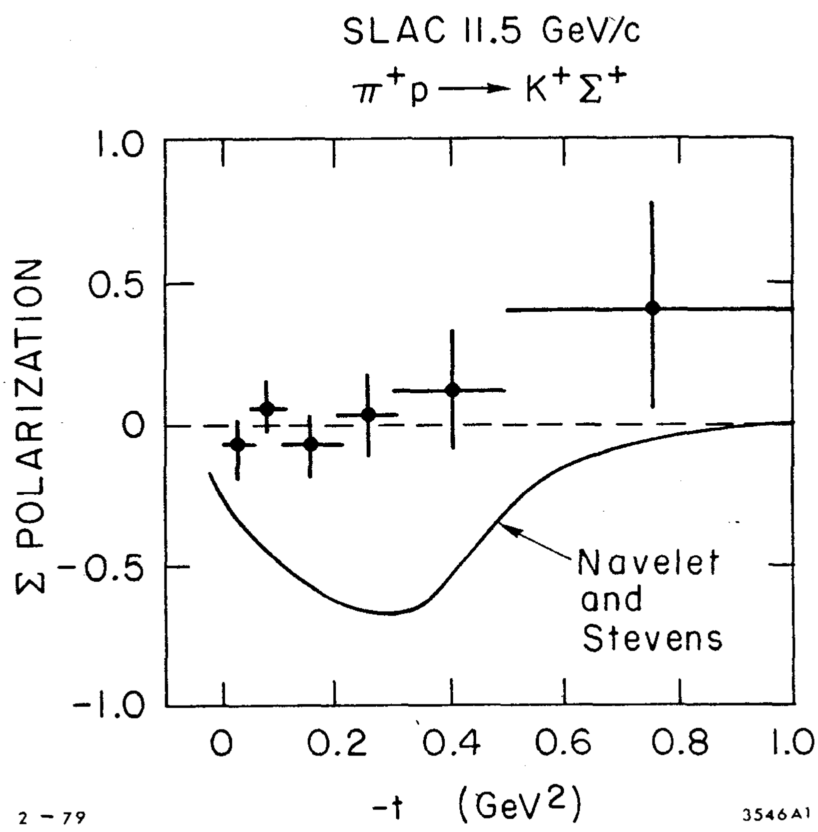
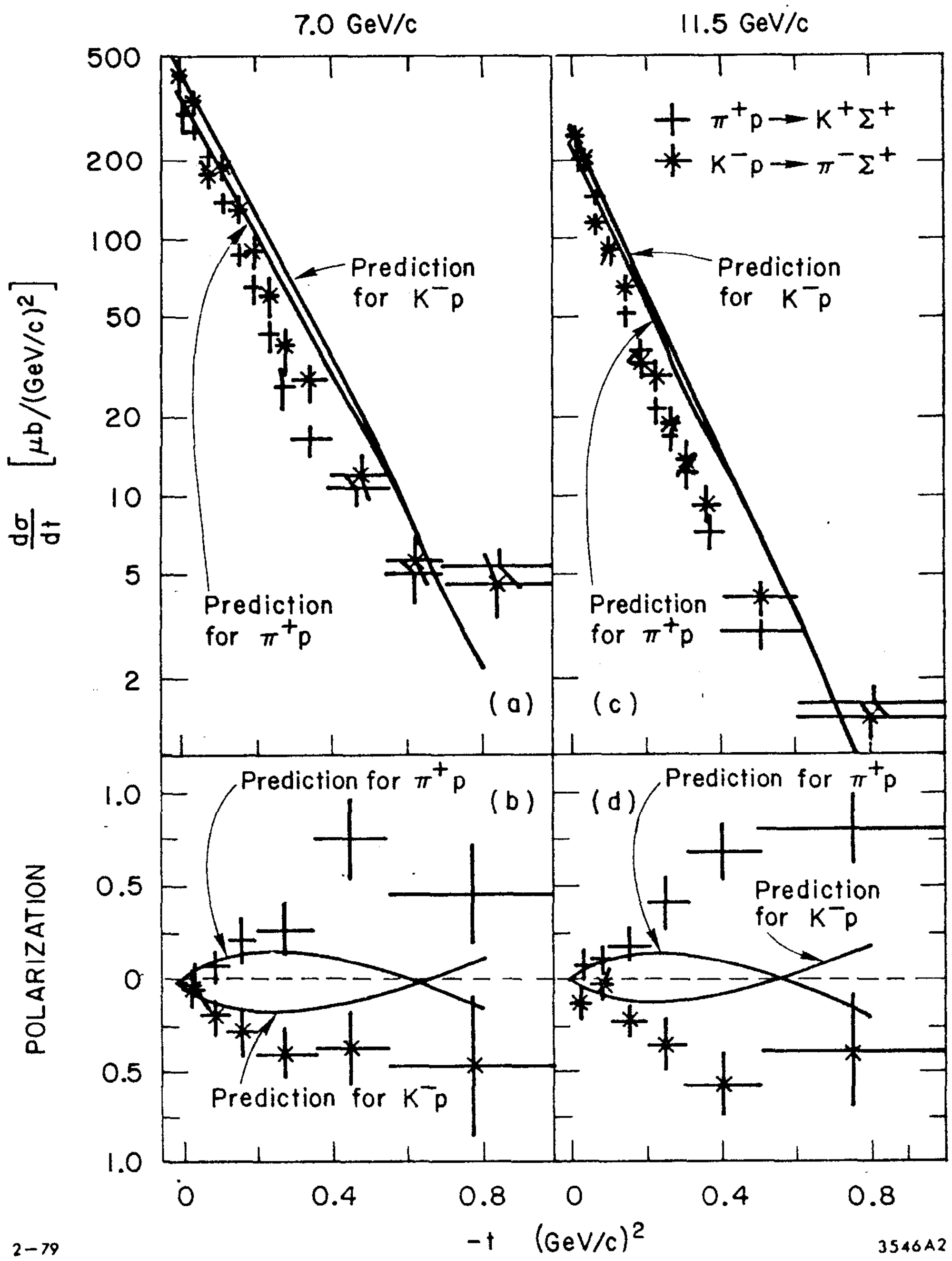
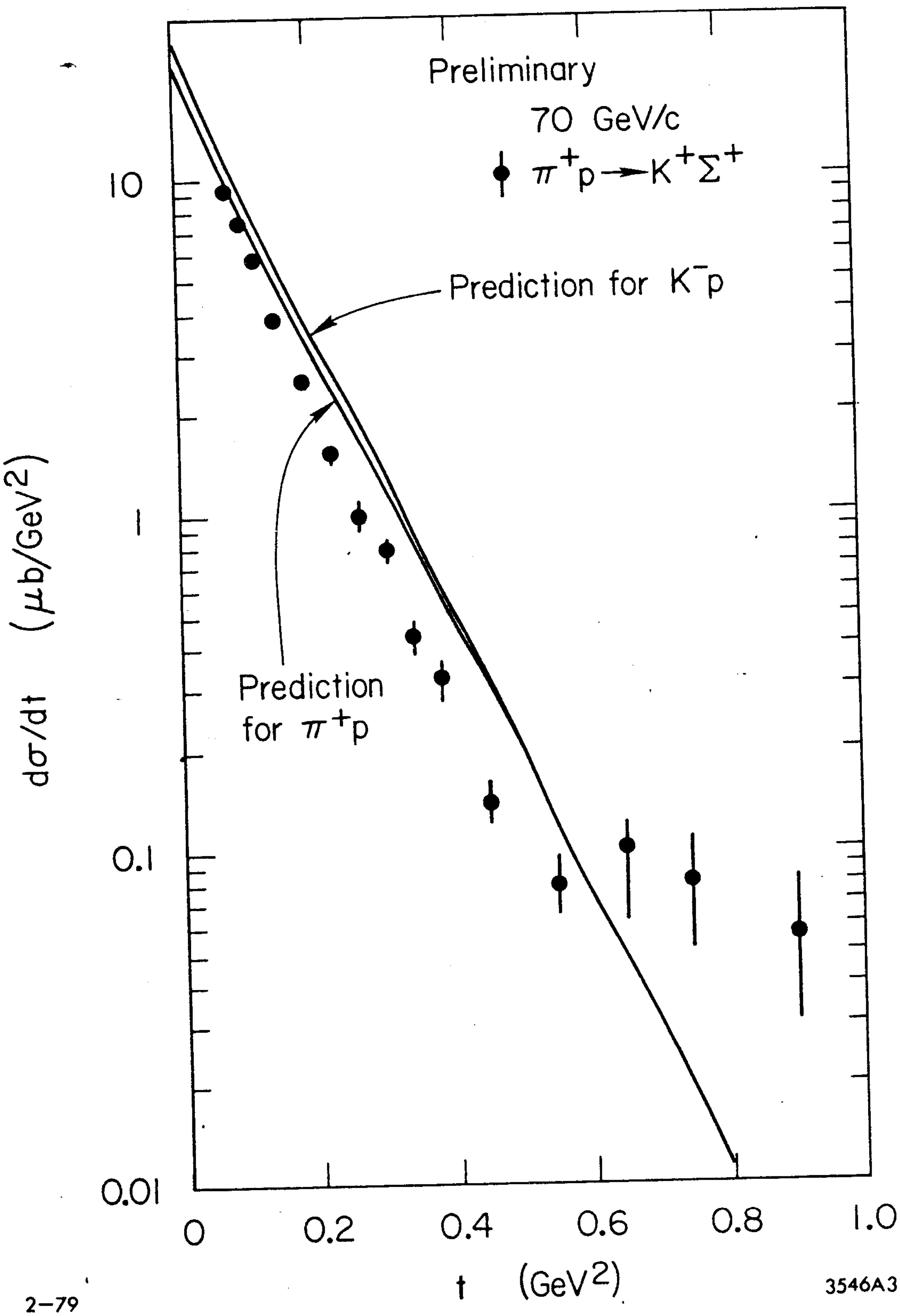

SLAC 11.5 GeV/c
π⁺p ⟶ K⁺Σ⁺
Σ POLARIZATION
1.0
0.5
0
−0.5
−1.0
0
0.2
0.4
0.6
0.8
1.0
−t (GeV²)
Navelet and Stevens
2 − 79
3546A1

Fig. 1



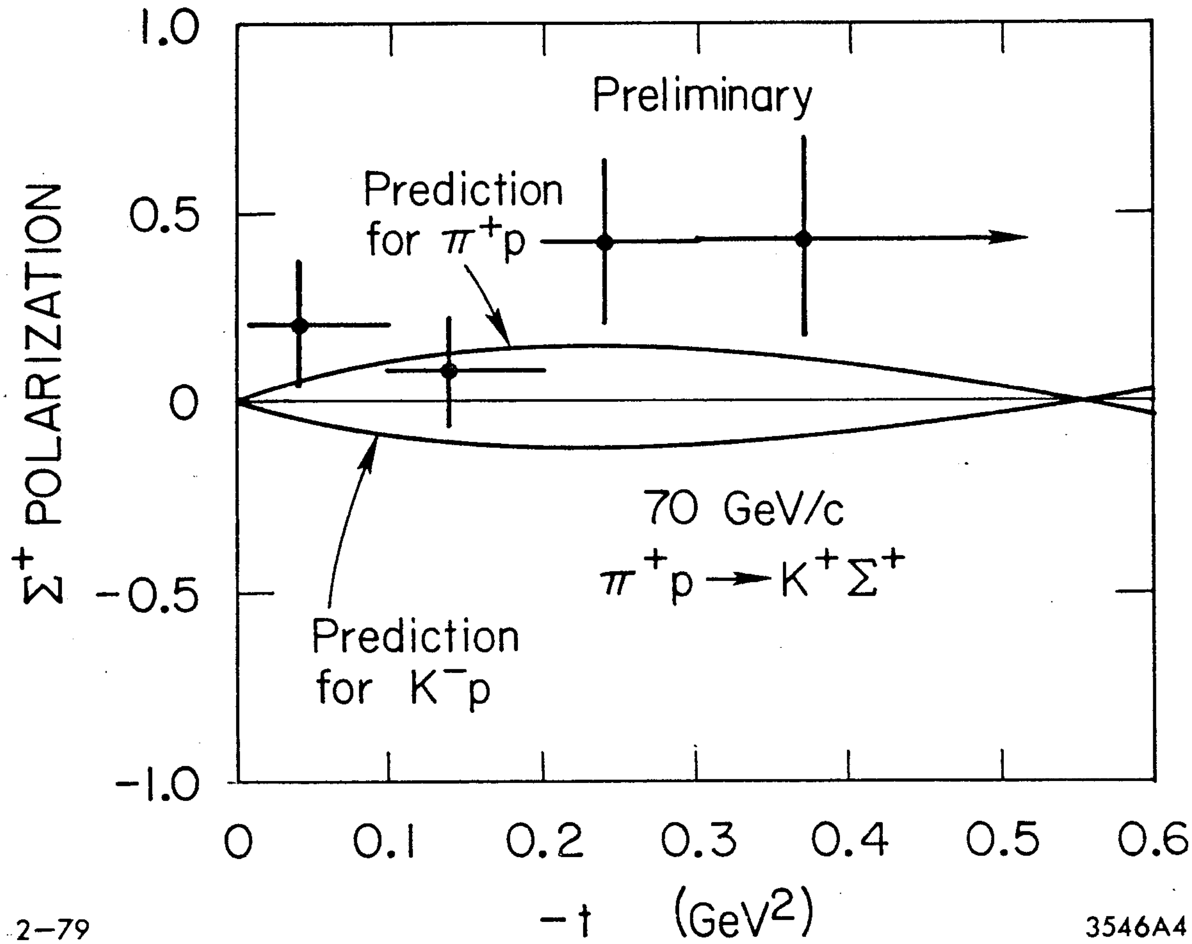
7.0 GeV/c
11.5 GeV/c
500
200
100
50
20
10
5
2
$\frac{d\sigma}{dt}$ [$\mu$b/(GeV/c)$^2$]
$\pi^+ p \to K^+ \Sigma^+$
$K^- p \to \pi^- \Sigma^+$
Prediction for $K^- p$
Prediction for $\pi^+ p$
(a)
(c)
(b)
(d)
1.0
0.5
0
0.5
1.0
POLARIZATION
Prediction for $\pi^+ p$
Prediction for $K^- p$
0
0.4
0.8
$-t$ (GeV/c)$^2$
2-79
3546A2


Fig. 2



Preliminary
70 GeV/c
$\pi^+ p \to K^+ \Sigma^+$
Prediction for $K^- p$
Prediction for $\pi^+ p$
10
1
0.1
0.01
$d\sigma/dt$ ($\mu b/GeV^2$)
0
0.2
0.4
0.6
0.8
1.0
t ($GeV^2$)
2-79
3546A3


Fig. 3



Preliminary
Prediction
for $\pi^+p$
Prediction
for $K^-p$
70 GeV/c
$\pi^+p \rightarrow K^+\Sigma^+$
$\Sigma^+$ POLARIZATION
1.0
0.5
0
-0.5
-1.0
0
0.1
0.2
0.3
0.4
0.5
0.6
$-t$ (GeV$^2$)
2-79
3546A4


Fig. 4

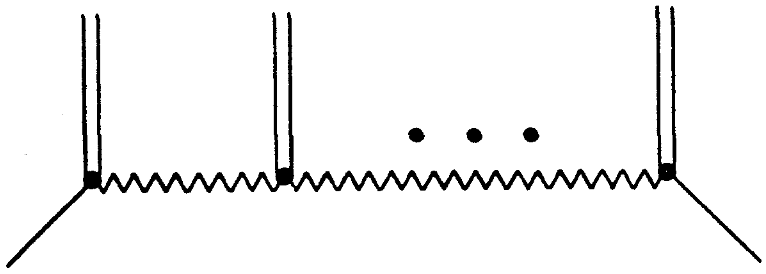



Fig. 5

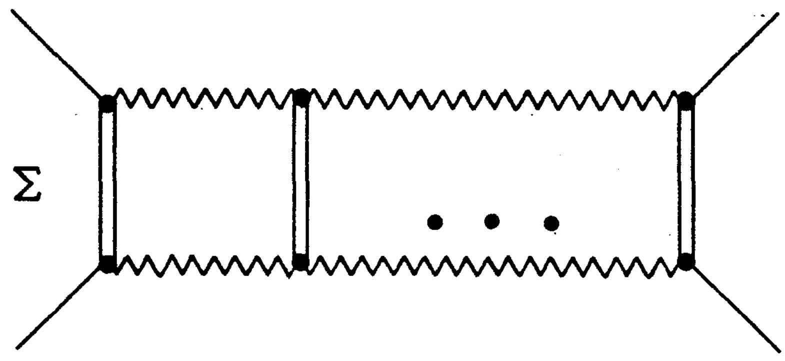
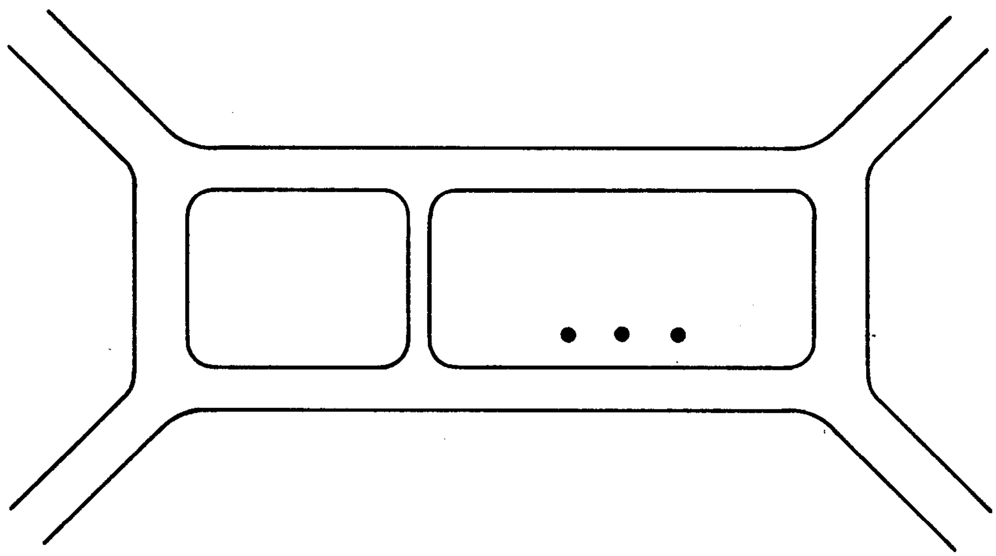



Fig. 6



2 – 79 Fig. 7 3546A7

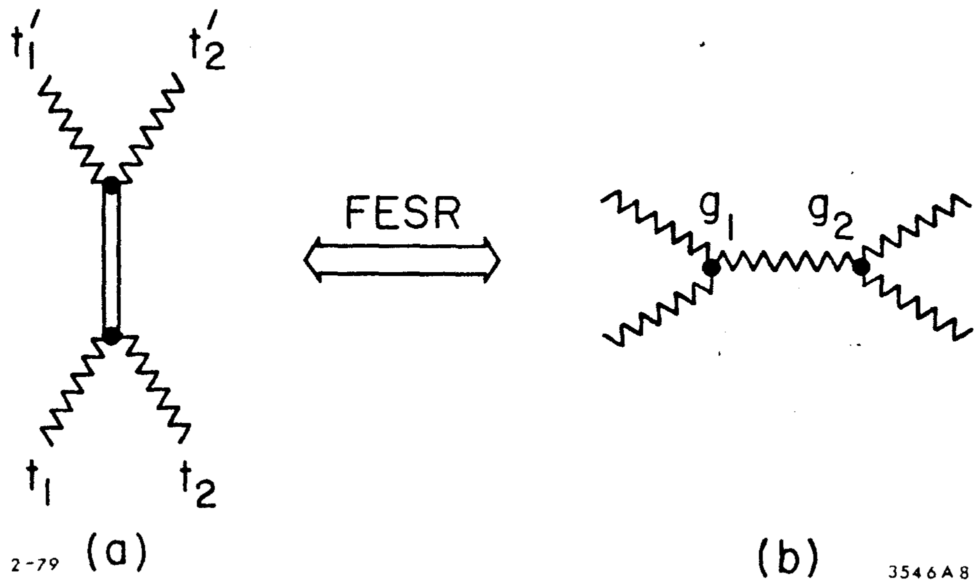



Fig. 8

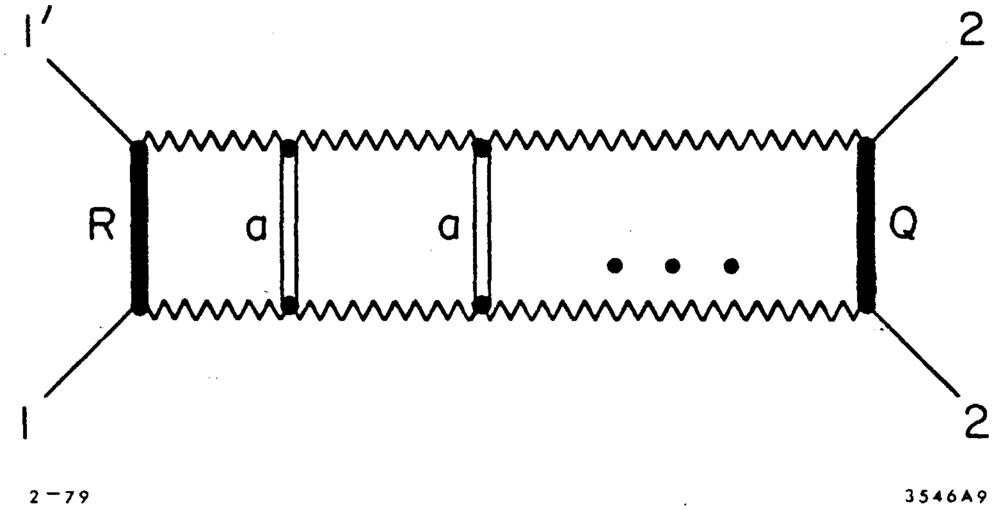



Fig. 9

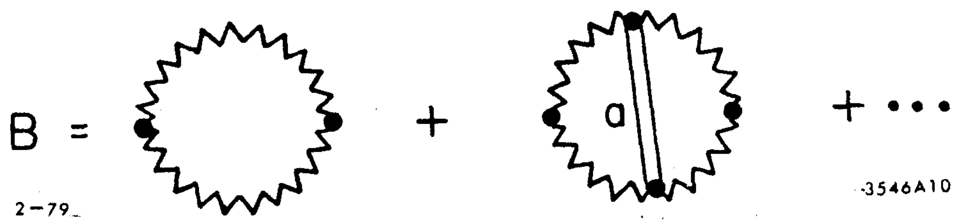



Fig. 10