SLAC-PUB-2257
February 1979
(I/A)

A SECOND-ORDER MAGNETIC OPTICAL ACHROMAT*

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ABSTRACT

A design procedure is given for the elimination of all of the second-order transverse geometric and chromatic aberrations in a particular class of static-magnetic transport systems for charged-particle beams (1).

Submitted to Nuclear Instruments and Methods**

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* Work supported by the Department of Energy under contract number EY-76-C-03-0515.

** A condensed version will appear as paper F-11 in the 1979 Particle Accelerator Conference.

INTRODUCTION

There are numerous applications for magnetic-optical systems that transport beams of charged particles from one location to another such that the transverse phase-space configuration of the beam at the final position is a faithful reproduction of the beam at the point of origin. The precision to which this objective may be achieved depends upon the magnitude of the phase-space volume to be transmitted and upon the optical distortions (aberrations) introduced by the intervening transport system. It is the purpose of this paper to describe a relatively simple method of devising a class of beam-transport systems which approach this ideal objective by eliminating all of the second-order geometric and chromatic aberrations at the end point of the system.

BASIC DESIGN CONCEPTS

In this report we restrict the discussion to systems whose transverse phase-space volume is conserved. We furthermore assume that space-charge effects are negligible. Under these circumstances the following of a charged particle through a system of magnetic lenses may be reduced to a process of matrix multiplication (2,3), such that at any specified position in the system an arbitrary charged-particle trajectory may be represented by a vector (single column matrix) X, whose components are the positions, angles, and momentum of the particle being considered with respect to some reference particle (the central trajectory). In this application, we take the components of the vector X to be the same as those used in the TRANSPORT program (3), i.e.,

$$X = \begin{bmatrix} x \\ x' \\ y \\ y' \\ \ell \\ dp/p \end{bmatrix}$$

where

x= the horizontal displacement of the arbitrary trajectory with respect to the assumed central trajectory.

x'= the angle this trajectory makes in the horizontal plane with respect to the assumed central trajectory.

y= the vertical displacement of the trajectory with respect to the assumed central trajectory.

y'= the vertical angle of the trajectory with respect to the assumed central trajectory.

ℓ= the path length difference between the arbitrary trajectory and the assumed central trajectory.

dp/p= the fractional momentum deviation of the particle from the assumed central trajectory. As we are concerned with only the transverse phase-space variables, the longitudinal component $X_5 = \ell$ will be ignored for the remainder of this report.

In this TRANSPORT formalism, the linear properties of each magnetic lens are represented by a square matrix R, which describes the action of the magnet on the particle coordinates:

$$X_1 = RX_0 \qquad (1)$$

i.e., the ith component of the vector X is

$$X_{i,1} = \sum R_{ij} X_{j,0}$$

where X_0 is the initial coordinate vector and X_1 is the final coordinate vector of the particle under consideration. The same linear transformation matrix R is applicable to all such particles traversing the system (one particle being distinguished from another by its initial coordinate vector X_0).

The traversing of several magnets and interspersing drift spaces is described by the same basic equation, but with R now being the product matrix $R_t = R_n \ldots R_3 R_2 R_1$ of the individual matrices representing each of the system elements.

This linear matrix formalism is conveniently extended to include second-order terms (aberrations) by the addition of a matrix T_{ijk} as follows:

$$X_{i,1} = \sum R_{ij} X_{j,0} + \sum T_{ijk} X_{j,0} X_{k,0} \quad , \qquad (2)$$

where T is a matrix representing the second-order geometric and chromatic aberrations. The vector components of interest are $X_1 = x$, $X_2 = x'$, $X_3 = y$, $X_4 = y'$, and $X_6 = dp/p$. The geometric terms are those for which i,j or k are equal to 1,2,3 or 4; and the chromatic terms are those for which j or k is equal to 6.

We now define a second-order achromat as any system for which all R_{ij} and all T_{ijk} vanish for i = 1,2,3 or 4 and j or k equals 6, i.e., any system for which all of the first- and second-order transverse chromatic terms vanish.

The particular solution we present here is further restricted to the special case where the transformation matrix, from the beginning to the end point of the system, is the unity matrix to second-order for both the x and y transverse planes; that is, to those systems where $R_{ij} = 1$ for i = j, and $R_{ij} = 0$ for i not equal to j, and all $T_{ijk} = 0$ for i = 1,2,3 or 4.

ELIMINATION OF THE SECOND-ORDER GEOMETRIC ABERRATIONS

Now consider a static magnetic-optical beam-transport system composed of a series of N identical unit cells where each unit cell contains dipole and quadrupole magnetic field components. It is then possible to choose the dipole and quadrupole components for each cell such that the linear transfer matrix R, representing the first-order transverse optics of the total system, is equal to the unity matrix: i.e., such that $R_{ij} = 1$ for $i = j$ and $R_{ij} = 0$ for i not equal to j. This corresponds to a 2π betatron phase shift between the beginning and the end of the transport system. It then follows from the general theory of second-order beam-transport optics (2) that the resulting system will have vanishing second-order transverse geometric aberrations provided that the number of unit cells, N, comprising the total system, does not equal one or three. Furthermore, it can be shown that if N = 4 or more, the addition of two sextupole components to each unit cell, one for the x-plane and one for the y-plane, combined with the dispersion introduced by the dipoles is sufficient to eliminate all of the second-order chromatic aberrations and at the same time still have vanishing second-order geometric aberrations.

The proof that all second-order geometric aberrations will vanish under these circumstances is seen by writing the integrals which are used to calculate these terms in a form involving the phase shift ψ and the multipole strengths $K_n(\psi)$. Where $n = 0$ is the dipole term, $n = 1$ is the quadrupole term, and $n = 2$ is the sextupole term. For a system of N repetitive unit cells making up a total phase shift of $\psi = 2\pi$, the second-order geometric terms in T_{ijk} are generated by integrals of the form:

$$\int_0^{2\pi} K_n(\psi) \cos^{\ell}\psi \sin^{m}\psi \, d\psi \qquad (3)$$

where

$$K_n(\psi) = \left(\frac{1}{n!}\right)\left(\frac{1}{B\rho}\right) \frac{\partial^n B_y}{\partial x^n}\Bigg|_{x=y=0}$$

and $(\ell+m) = 3$ for the dipole and sextupole contributions. (See ref. 4 for a derivation of K_n.)

The first important observation to make is that quadrupole components do not contribute to the second-order geometric terms but that the dipole and sextupole components do. (See ref. 2 for a general derivation of these integrals.)

Transforming this integral to the complex plane, it assumes the form

$$\int_0^{2\pi} K_n(\psi)\left[e^{i\psi} + e^{-i\psi}\right]^{\ell} \cdot \left[e^{i\psi} - e^{-i\psi}\right]^{m} d\psi \qquad (4)$$

Expanding and ignoring the numerical coefficients, the final result may be expressed as a sum of terms containing two basic integral forms, i.e.,

$$\int_0^{2\pi} K_n(\psi)\ e^{\pm i\psi}\ d\psi \qquad (5)$$

and

$$\int_0^{2\pi} K_n(\psi)\ e^{\pm 3i\psi}\ d\psi \qquad (6)$$

Evaluating these integrals for a repetitive unit cell structure, it is observed that the dipole or sextupole components may each be viewed as "vector additions in the complex plane", where $K_n(\psi)$ is the amplitude of the vector, ψ is its phase for eq. (5), and 3ψ is its phase for integral (6). Both integrals vanish when N, the number of unit cells comprising a 2π betatron phase shift, does not equal one or three. N = 1 is excluded because there is no possibility for a vector cancellation in either integral, and N = 3 is excluded because all of the vector components in equation (6) add constructively even though vector cancellation does occur for equation (5). But both integrals vanish for any other integer value of N.

ELIMINATION OF THE SECOND-ORDER CHROMATIC ABERRATIONS

All of the second-order chromatic aberrations in a unity transform system composed of four or more identical first-order unit cells may be eliminated, without introducing new second-order geometric aberrations, by the proper distribution of sextupole components through the cell structure. One obvious way of achieving this is as follows:

(1) Identical dipoles are introduced into each cell of the system to provide momentum dispersion and to allow the above integrals to vanish. The dispersion provides coupling between the chromatic terms of the T_{ijk} matrix and the

sextupoles. The strength of this coupling is proportional to the magnitude of the momentum dispersion at the location of each sextupole component.

(2) Two sextupole components are then introduced into each unit cell, one for the x-plane and one for the y-plane. The x-plane sextupoles are positioned where the x-plane monoenergetic beam envelope is large compared to the y-plane beam envelope. Similarly the y-plane sextupole components are positioned at a location where the y-plane beam envelope is large compared to the x-plane envelope. This procedure maximizes the relative coupling coefficients to the chromatic terms in each transverse plane and thereby minimizes the strength of the sextupole components required for the correction process.

These sextupole components may be thought of as providing additional "quadrupole-like" gradient focusing elements for the off-momentum trajectories. The strengths of the two sextupole components are then adjusted to make the chromatic terms vanish in both the x and y planes. This consists of solving two simultaneous linear equations via an appropriate beam optics program such as TRANSPORT (3). The remarkable result is that all of the second-order chromatic terms vanish simultaneously with the introduction of only two variables, the x-plane and y-plane sextupole strengths that are introduced into each unit cell structure.

The above solution is perhaps the easiest to comprehend. However, other solutions are also possible, all of which have the common characteristic that at least four appropriately positioned sextupole components are needed in each transverse plane to correct for the chromatic aberrations. There are a number of ways to see why four or more sextupole components in each transverse plane are needed. One is the observation that the transformation matrix for the total system is the unity matrix to second-order. To achieve this it is necessary to have a sextupole array that allows a unity transform matrix to be achieved in both the x and y planes for the off-momentum trajectories. By analogy with the monoenergetic case, a minimum of eight sextupoles is required, four for the x plane and four for the y plane. Another way of viewing the problem is to note that the solution of the homogeneous beam optics equation for any given momentum has two normal-mode solutions, the so-called sine-like function and the cosine-like function (2), from which all possible monoenergetic trajectories may be derived by a linear combination of these two characteristic trajectories. Therefore, in order to couple to all possible off-momentum trajectories, the sextupole correcting elements must couple to both the sine- and cosine-like trajectories and at the same time not introduce new geometric aberrations. With some thought it is again evident that at least four sextupole correcting elements in each plane are required.

HIGHER ORDER OPTICAL ABERRATIONS

Aberrations of higher than second order should also be considered when formulating a particular solution for an achromat. They arise from two primary sources: (A) those which are inherent in the basic design of the first-order optics solution, and (B) those which arise from the introduction of the sextupole correcting elements. Aberrations of type A are best minimized by gaining design experience, whereas the type B aberrations can be uniquely eliminated in some applications by choosing the pattern in which the sextupoles are introduced into the lattice structure. This is discussed below in greater detail.

In the recipe given in the above paragraphs for formulating a second-order achromat, it is implicitly assumed that the lattice structure remains essentially linear. This is a valid assumption if the strength of the sextupoles needed for the correction process is sufficiently small. If this is not the case, then the sextupole components may introduce non-linear distortions into the system and thereby limit the usefulness of the design. Fortunately there is a solution to this particular problem if there is enough space available and the budget is adequate. This special solution will be discussed in subsequent paragraphs.

In the discussions above it has been assumed that the total length of the achromat corresponds to a 2π phase shift. But it is obvious that the results quoted are equally valid for systems whose length is a multiple of a 2π phase shift. Under these circumstances the sextupole correcting elements may be distributed over a longer distance, measured in units of phase shift. Consider, for example, the interesting case where the number of first-order unit cells N making up each 2π phase shift section is four or more and is an even integer. The sextupole components may then be introduced in pairs, the elements of each pair being identical and separated by a phase shift of π in both transverse planes. The transformation matrix between them is then equal to minus the unity matrix. If under these circumstances the two sextupoles are of equal strength and of the same polarity, then for all monoenergetic trajectories, corresponding to the momentum of the central trajectory, the effect of the first sextupole on the trajectory at the end of the system is uniquely cancelled by the second sextupole. This is valid to all orders in the monoenergetic optics to the extent that the phase shift over the length of the sextupole is negligible. Using this principle , it is then possible to formulate achromats which have no higher order monoenergetic geometric aberrations caused by the introduction of the sextupole correcting elements. This can be achieved in a 6π phase shift lattice by correctly positioning two pairs of non-interlaced sextupoles into each transverse plane, x and y, making a total of eight sextupoles. See example 3 at the end of the report.

SOME EXAMPLES OF SECOND-ORDER ACHROMATS

Before giving specific examples of achromats, it is perhaps useful to review the fundamental purpose of the various multipoles and list the different ways in which these multipole components may be introduced into a lattice structure.

The primary function of the dipole is to bend the optic axis of the beam and to introduce momentum dispersion into the system. Dipoles, however, always have a first-order focusing action in addition to their zero'th order bending properties. For small angles of bend, the first-order focusing action is usually very small compared to the focusing strength of the quadrupoles in the lattice. However for large angles of bend and/or combined-function lattice structures, the first-order focusing of the dipoles can be dominant.

The purpose of quadrupoles is to provide first-order focusing to supplement that provided by the dipoles. A quadrupole component may be defined as any physical element that introduces a first derivative of the magnetic field with respect to the transverse coordinates x and y. This can occur in any one of three ways: by an actual four-pole quadrupole magnet, or by a rotated entrance or exit face of a bending magnet, or finally by a linear field variation in the transverse field expansion of a bending magnet.

Sextupole components affect the second- and higher order optics of the system. Sextupole components may be introduced via a six-pole magnet, or by a second-order curved surface on the entrance or exit face of a dipole, or by introducing a second-order field derivative into the transverse field expansion of a dipole or quadrupole magnet.

The examples given below use either the most convienent and/or the most economical method of introducing the multipole components for the particular case illustrated. A TRANSPORT printout of each example is given at the end of the report.

EXAMPLE 1

One typical example of an achromat is a separated function FODO array of alternating strong-focusing quadrupoles (Q) with interspersed dipoles (B), sextupoles (S), and drift spaces. An acceptable unit cell is the following symmetric array of magnetic elements:

Q(x) S(x) B(x) S(y) Q(y)Q(y) S(y) B(x) S(x) Q(x)

where

Q(x) is a quadrupole focusing in the x plane and defocusing in the y plane.

Q(y) is a quadrupole focusing in the y plane and defocusing in the x plane.

S(x) is a sextupole with strong coupling to the x plane and weak coupling to the y plane.

S(y) is a sextupole with strong coupling to the y plane and weak coupling to the x plane.

B(x) is a dipole whose magnetic midplane lies in the x plane. The optical equivalent of the above FODO array is shown in fig. 1, where the "lenses" represent the quadrupole components, the triangles the dipole components, and the hexagons the sextupole components.

An assembly of four or more such unit cells adjusted to a total phase shift of 2π constitutes a second-order achromat when the sextupole components are adjusted to make the second-order chromatic aberrations vanish.

As an alternative, the sextupoles may be introduced into the unit cell in an asymmetric manner as follows:

Q(x) S(x) B(x) Q(y) S(y) B(x)

Another alternative solution is to combine the quadrupole and sextupole components into the same physical element. All three cases are acceptable achromats and will be essentially equivalent in system performance. The advantage of the last two cases is simply that the number of physical elements needed is less than in the first case.

If in the example given above, the first-order focusing action is achieved predominately via a FODO array of focusing and defocusing quadrupoles of equal focal length f separated by a distance ℓ, then in the thin-lens approximation, and to the extent that the focusing action of the dipoles may be ignored, the phase shift per unit cell, μ, is given by the equation

$$\sin \mu/2 = \ell/2f \ .$$

If now the system is composed of N unit cells such that $N\mu = 2\pi$ radians, the length L of the total system is

$$L = 2N\ell = 4Nf \sin(\pi/N)$$

EXAMPLE 2

A unit cell may also be generated by using a combined function magnet as shown in fig. 2. The strength of the dipole component is equal to the bending angle α. The dipole also provides first-order focusing in the radial plane. A quadrupole component, focusing in the non-bend plane and defocusing in the bend plane, is introduced via the rotated input face of the magnet; and two sextupole components are introduced via the curved surfaces, R_1 and R_2, on the entrance and exit faces of the magnet. The unit cell then consists of the combined-function magnet and a drift space preceeding and following it. The total achromat is composed of at least four such unit cells adjusted to a total phase shift of 2π.

EXAMPLE 3

An example of a 6π phase-shift achromat, having non-interlaced sextupole pairs, is illustrated in fig. 3. The phase shift in each transverse plane is chosen to be the same. The correcting sextupoles are introduced in pairs with the individual members of each pair being identical and separated by a phase shift of π. This corresponds to a minus unity first-order transform matrix between the members of the pair. The respective pairs, labeled Sx_1, Sx_2, Sy_1, and Sy_2 are separated (non-interlaced) and therefore do not introduce higher-order geometric distortions. The distance of separation is chosen such that the strengths of Sx_1 and Sx_2 are the same as Sy_1 and Sy_2. The ratio of the sextupole strengths in the x and y planes is then determined by the magnitude of the coupling coefficients averaged over all four sextupoles in each plane. The coupling coefficient is proportional to the magnitude of the momentum dispersion at each correcting element.

The 6π phase-shift achromat is most applicable to those systems where it is desirable to avoid higher order geometric aberrations caused by the interlacing of the sextupoles. An example of this is a chromatic correction system for large storage rings (6). Another example is in the design of secondary charged particle beams where residual tails in the transverse spatial distribution at the end point is important.

SUMMARY

Several examples of second-order achromats have been studied using the computer programs TRANSPORT (3) and TURTLE (5). Other studies have been made using the achromat principle to make chromaticity corrections for large storage rings (6). In addition secondary beams have been designed based on the achromat principle which have significant improvement in the transmitted phase-space volume(7). From the study of these few examples it is evident that there are many potential applications for the achromat concept.

ACKNOWLEDGEMENTS

It is a pleasure to acknowledge many discussions with various colleagues during the development of the achromat concept. But I particularly wish to thank Roger Servranckx who enthusiastically accepted my suggestion that the

achromat concept be adapted to the problem of making chromatic corrections for large storage rings (6), and also for his suggestion that my original concept of the achromat be analysed in the complex plane.

TRANSPORT PRINTOUTS OF EXAMPLES

(TRANSPORT INPUT FOR EXAMPLE 1)

```
'A 4 CELL ACHROMAT USING SEPARATED FUNCTION MAGNETS 9/26/77'
0
15 1 'MM';
1 0 0 0 0 0 0 10;
17;
9 4;
5  .4  8.02262 100 'QX';
18 .2  1.22642 100 'SX';
3 .3;
2 1; 4   5.82179 2 'BX'; 2 1;
3 .3;
18 .2 -2.38768 100 'SY';
5  .8 -8.01363 100 'QY';
18 .2 -2.38768 100 'SY';
3 .3;
2 1; 4   5.82179 2 'BX'; 2 1;
3 .3;
18 .2  1.22642 100 'SX';
5 .4  8.02262  100 'QX';
9 0;
13 4;
SENTINEL
```

(TRANSPORT OUTPUT FOR EXAMPLE 1)

```
'A 4 CELL ACHROMAT USING SEPARATED FUNCTION MAGNETS 9/26/77'

*BEAM*          1.                  10.00000 GEV
*2ND ORDER*    17.
*QUAD*          5.     "QX  "        0.40000 M    8.02262 KG   100 MM
     0.400 M
*SEXT*         18.     "SX  "        0.20000 M    1.22642 KG   100 MM
     0.600 M
*DRIFT*         3.                   0.30000 M
     0.900 M
*ROTAT*         2.                   1.00000 DEG
     0.900 M
*BEND*          4.      "BX  "       5.82179 M    2.00000 KG
   ( 166.782 M BEND RADIUS   2.000 DEGREE BEND)
     6.722 M
*ROTAT*         2.                   1.00000 DEG
     6.722 M
*DRIFT*         3.                   0.30000 M
     7.022 M
*SEXT*         18.     "SY  "        0.20000 M   -2.38768 KG   100 MM
     7.222 M
*QUAD*          5.     "QY  "        0.80000 M   -8.01363 KG   100 MM
     8.022 M
*SEXT*         18.     "SY  "        0.20000 M   -2.38768 KG   100 MM
     8.222 M
```

```
*DRIFT*            3.                         0.30000 M
     8.522 M
*ROTAT*            2.                         1.00000 DEG
     8.522 M
*BEND*             4.      "BX    "           5.82179 M     2.00000 KG
   ( 166.782 M BEND RADIUS    2.000 DEGREE BEND)
    14.344 M
*ROTAT*            2.                         1.00000 DEG
    14.344 M
*DRIFT*            3.                         0.30000 M
    14.644 M
*SEXT*            18.      "SX    "           0.20000 M     1.22642 KG   100 MM
    14.844 M
*QUAD*             5.      "QX    "           0.40000 M     8.02262 KG   100 MM


  ( THE ABOVE UNIT CELL IS REPEATED 4 TIMES, THE RESULTANT
      TRANSFORM MATRIX IN FIRST- AND SECOND-ORDER IS GIVEN BELOW. )


*TRANSFORM 1*
                0.99999    0.00024    0.0        0.0        0.0        0.00007
               -0.00000    0.99999    0.0        0.0        0.0        0.00000
                0.0        0.0        0.99998   -0.00002    0.0        0.0
                0.0        0.0        0.00000    0.99998    0.0        0.0
               -0.00000   -0.00000    0.0        0.0        1.00000   -0.14072
                0.0        0.0        0.0        0.0        0.0        1.00000
*2ND ORDER TRANSFORM*
  1 11 -4.065E-09
  1 12  1.183E-06    1 22 -5.373E-06
  1 13  0.0          1 23  0.0          1 33  1.593E-08
  1 14  0.0          1 24  0.0          1 34 -4.838E-07    1 44 -1.444E-06
  1 16  7.048E-08    1 26 -2.048E-05    1 36  0.0          1 46  0.0
  1 66 -1.159E-07

  2 11  2.023E-10
  2 12  1.777E-09    2 22 -3.902E-07
  2 13  0.0          2 23  0.0          2 33 -1.576E-09
  2 14  0.0          2 24  0.0          2 34 -2.143E-08    2 44 -7.299E-08
  2 16  2.859E-09    2 26  2.727E-08    2 36  0.0          2 46  0.0
  2 66 -3.817E-08

  3 11  0.0
  3 12  0.0          3 22  0.0
  3 13  5.051E-09    3 23 -1.923E-07    3 33  0.0
  3 14  9.394E-08    3 24  1.259E-06    3 34  0.0          3 44  0.0
  3 16  0.0          3 26  0.0          3 36 -7.719E-08    3 46 -1.005E-05
  3 66  0.0
```

```
 4 11  0.0
 4 12  0.0         4 22  0.0
 4 13 -3.832E-09   4 23  7.034E-08   4 33  0.0
 4 14 -3.596E-08   4 24  1.571E-07   4 34  0.0         4 44  0.0
 4 16  0.0         4 26  0.0         4 36  4.883E-07   4 46  5.245E-08
 4 66  0.0

*LENGTH*         60.97400 M
```

(TRANSPORT INPUT FOR EXAMPLE 2)

```
'A 4 CELL ACHROMAT USING COMBINED FUNCTION MAGNETS 10/5/77'
0
15 1 'MM'; 15 11 'MEV'; 15 8 'CM'; 15 5 'MM';
1 1 1 1 1 0 0 40.51097;
17;
16  5 5;
16 7 .4;
16 12 .02177;
16 13 -.025908;
9 4;
3 21.23306;
2 30.97464; 4 16.55821 8.54609; 2 0;
3 21.23306;
9 0;
13 4;
SENTINEL
```

(TRANSPORT OUTPUT FOR EXAMPLE 2)

```
'A 4 CELL ACHROMAT USING COMBINED FUNCTION MAGNETS 10/5/77'

*BEAM*           1.                 40.51093 MEV
*2ND ORDER*     17.                      GAUSSIAN DISTRIBUTION
* G/2 *         16.                  5.     0.50000E+01
* K1 *          16.                  7.     0.40000E+00
* 1/R1 *        16.                 12.     0.21770E-01
* 1/R2 *        16.                 13.    -0.25908E-01
*DRIFT*          3.                 21.23303 CM
    21.233 CM
*ROTAT*          2.                 30.97461 DEG
    21.233 CM
                                    16.55817 CM        8.54609 KG
```

```
*BEND*          4.
   ( 15.812 CM BENDING RADIUS,  60 DEGREE BEND ANGLE )
   37.791 CM
*ROTAT*         2.                  0.0      DEG
   37.791 CM
*DRIFT*         3.                 21.23303 CM
   59.024 CM
*DRIFT*         3.                 21.23303 CM
   80.257 CM
*ROTAT*         2.                 30.97461 DEG
   80.257 CM
*BEND*          4.                 16.55817 CM          8.54609 KG
   ( 15.812 CM BENDING RADIUS,  60 DEGREE BEND ANGLE )
   96.815 CM
*ROTAT*         2.                  0.0      DEG
   96.815 CM
*DRIFT*         3.                 21.23303 CM
  118.048 CM
*DRIFT*         3.                 21.23303 CM
  139.281 CM
*ROTAT*         2.                 30.97461 DEG
  139.281 CM
*BEND*          4.                 16.55817 CM          8.54609 KG
   ( 15.812 CM BENDING RADIUS,  60 DEGREE BEND ANGLE )
  155.840 CM
*ROTAT*         2.                  0.0      DEG
  155.840 CM
*DRIFT*         3.                 21.23303 CM
  177.073 CM
*DRIFT*         3.                 21.23303 CM
  198.306 CM
*ROTAT*         2.                 30.97461 DEG
  198.306 CM
*BEND*          4.                 16.55817 CM          8.54609 KG
   ( 15.812 CM BENDING RADIUS,  60 DEGREE BEND ANGLE )
  214.864 CM
*ROTAT*         2.                  0.0      DEG
  214.864 CM
*DRIFT*         3.                 21.23303 CM
  236.097 CM

*TRANSFORM 1*
             0.99999  -0.00000   0.0       0.0       0.0        0.00001
             0.00004   1.00000   0.0       0.0       0.0       -0.00012
             0.0       0.0       1.00000  -0.00001   0.0        0.0
             0.0       0.0       0.00006   0.99999   0.0        0.0
             0.00001   0.0       0.0       0.0       1.00000  -12.93663
             0.0       0.0       0.0       0.0       0.0        1.00000
```

```
*2ND ORDER TRANSFORM*
 1 11 -6.206E-08
 1 12 -4.526E-09   1 22  2.721E-09
 1 13  0.0         1 23  0.0         1 33 -5.415E-08
 1 14  0.0         1 24  0.0         1 34  6.396E-08   1 44 -3.417E-10
 1 16 -8.695E-08   1 26 -3.772E-07   1 36  0.0         1 46  0.0
 1 66  7.507E-08

 2 11  2.779E-07
 2 12  1.213E-07   2 22  7.570E-09
 2 13  0.0         2 23  0.0         2 33  7.801E-07
 2 14  0.0         2 24  0.0         2 34 -1.827E-07   2 44 -2.378E-08
 2 16  2.830E-06   2 26  1.022E-07   2 36  0.0         2 46  0.0
 2 66 -1.051E-05

 3 11  0.0
 3 12  0.0         3 22  0.0
 3 13  1.784E-07   3 23  6.310E-08   3 33  0.0
 3 14  3.832E-08   3 24 -1.909E-09   3 34  0.0         3 44  0.0
 3 16  0.0         3 26  0.0         3 36 -4.725E-07   3 46 -1.831E-07
 3 66  0.0

 4 11  0.0
 4 12  0.0         4 22  0.0
 4 13  1.547E-06   4 23  1.367E-07   4 33  0.0
 4 14 -1.715E-07   4 24 -6.592E-08   4 34  0.0         4 44  0.0
 4 16  0.0         4 26  0.0         4 36 -4.322E-06   4 46  3.502E-07
 4 66  0.0

*LENGTH*         236.09679 CM
```

(TRANSPORT INPUT FOR EXAMPLE 3)

```
'A 6PI ACHROMAT USING SEPARATED FUNCTION MAGNETS  2/12/79'
0
15 1 'MM';
1 4.666 .1286 4.666 .1286 0 5 10;
17;
5 'QX' .375 9.08898 100;
18 'SX1' .2 8.90763 100;
3 .3125;
2 .93464; 4 'BX' 5.4 2.015253; 2 .93464;
3 .5125;
5 'QY' .75 -9.08002 100;
3 .2;
3 .3125;
2 .93464; 4 'BX' 5.4 2.015253; 2 .93464;
```

```
3 .5125;
5 'QX' .75 9.08898 100;
3 .2;
3 .3125;
2 .93464; 4 'BX' 5.4 2.015253; 2 .93464;
3 .5125;
5 'QY' .75 -9.08002 100;
3 .2;
3 .3125;
2 .93464; 4 'BX' 5.4 2.015253; 2 .93464;
3 .5125;
5 'QX' .75 9.08898 100;
18 'SX1' .2 8.90763 100;
3 .3125;
2 .93464; 4 'BX' 5.4 2.015253; 2 .93464;
3 .5125;
5 'QY' .75 -9.08002 100;
18 'SY1' .2 -17.32425 100;
3 .3125;
2 .93464; 4 'BX' 5.4 2.015253; 2 .93464;
3 .5125;
5 'QX' .75 9.08898 100;
3 .2;
3 .3125;
2 .93464; 4 'BX' 5.4 2.015253; 2 .93464;
3 .5125;
5 'QY' .75 -9.08002 100;
3 .2;
3 .3125;
2 .93464; 4 'BX' 5.4 2.015253; 2 .93464;
3 .5125;
5 'QX' .375 9.08898 100;
5 'QX' .375 9.08898 100;
3 .2;
3 .3125;
2 .93464; 4 'BX' 5.4 2.015253; 2 .93464;
3 .5125;
5 'QY' .75 -9.08002 100;
18 'SY1' .2 -17.32425 100;
3 .3125;
2 .93464; 4 'BX' 5.4 2.015253; 2 .93464;
3 .5125;
5 'QX' .75 9.08898 100;
18 'SX2' .2 8.90763 100;
3 .3125;
2 .93464; 4 'BX' 5.4 2.015253; 2 .93464;
3 .5125;
5 'QY' .75 -9.08002 100;
3 .2;
3 .3125;
2 .93464; 4 'BX' 5.4 2.015253; 2 .93464;
3 .5125;
5 'QX' .75 9.08898 100;
```

```
3 .2;
3 .3125;
2 .93464; 4 'BX' 5.4 2.015253; 2 .93464;
3 .5125;
5 'QY' .75 -9.08002 100;
3 .2;
3 .3125;
2 .93464; 4 'BX' 5.4 2.015253; 2 .93464;
3 .5125;
5 'QX' .75 9.08898 100;
18 'SX2' .2 8.90763 100;
3 .3125;
2 .93464; 4 'BX' 5.4 2.015253; 2 .93464;
3 .5125;
5 'QY' .75 -9.08002 100;
18 'SY2' .2 -17.32425 100;
3 .3125;
2 .93464; 4 'BX' 5.4 2.015253; 2 .93464;
3 .5125;
5 'QX' .375 9.08898 100;
5 'QX' .375 9.08898 100;
3 .2;
3 .3125;
2 .93464; 4 'BX' 5.4 2.015253; 2 .93464;
3 .5125;
5 'QY' .75 -9.08002 100;
3 .2;
3 .3125;
2 .93464; 4 'BX' 5.4 2.015253; 2 .93464;
3 .5125;
5 'QX' .75 9.08898 100;
3 .2;
3 .3125;
2 .93464; 4 'BX' 5.4 2.015253; 2 .93464;
3 .5125;
5 'QY' .75 -9.08002 100;
18 'SY2' .2 -17.32425 100;
3 .3125;
2 .93464; 4 'BX' 5.4 2.015253; 2 .93464;
3 .5125;
5 'QX' .75 9.08898 100;
3 .2;
3 .3125;
2 .93464; 4 'BX' 5.4 2.015253; 2 .93464;
3 .5125;
5 'QY' .75 -9.08002 100;
3 .2;
3 .3125;
2 .93464; 4 'BX' 5.4 2.015253; 2 .93464;
3 .5125;
5 'QX' .75 9.08898 100;
3 .2;
3 .3125;
```

```
2 .93464; 4 'BX' 5.4 2.015253; 2 .93464;
3 .5125;
5 'QY' .75 -9.08002 100;
3 .2;
3 .3125;
2 .93464; 4 'BX' 5.4 2.015253; 2 .93464;
3 .5125;
5 'QX' .375 9.08898 100; 13 1; 13 4;
SENTINEL
```

(TRANSPORT MATRIX OUTPUT FOR EXAMPLE 3)

```
*TRANSFORM 1*
            0.99998    0.00060    0.0        0.0        0.0        0.00021
           -0.00000    0.99999    0.0        0.0        0.0        0.00001
            0.0        0.0        0.99996   -0.00010    0.0        0.0
            0.0        0.0        0.00000    0.99996    0.0        0.0
           -0.00000   -0.00000    0.0        0.0        1.00000   -0.34682
            0.0        0.0        0.0        0.0        0.0        1.00000
*2ND ORDER TRANSFORM*
  1 11 -9.304E-08
  1 12  2.496E-06    1 22 -9.062E-06
  1 13  0.0          1 23  0.0          1 33 -2.243E-07
  1 14  0.0          1 24  0.0          1 34  1.719E-06    1 44 -8.287E-06
  1 16 -1.746E-06    1 26 -3.278E-05    1 36  0.0          1 46  0.0
  1 66  4.833E-06

  2 11 -1.779E-09
  2 12  8.865E-08    2 22 -1.210E-06
  2 13  0.0          2 23  0.0          2 33 -1.157E-08
  2 14  0.0          2 24  0.0          2 34  2.724E-08    2 44  3.561E-07
  2 16  1.927E-07    2 26  2.476E-06    2 36  0.0          2 46  0.0
  2 66 -1.012E-06

  3 11  0.0
  3 12  0.0          3 22  0.0
  3 13 -5.786E-08    3 23  3.647E-06    3 33  0.0
  3 14 -7.715E-07    3 24 -9.055E-06    3 34  0.0          3 44  0.0
  3 16  0.0          3 26  0.0          3 36  9.359E-07    3 46 -6.128E-06
  3 66  0.0

  4 11  0.0
  4 12  0.0          4 22  0.0
  4 13 -4.041E-09    4 23  9.750E-08    4 33  0.0
  4 14  7.415E-08    4 24 -1.047E-06    4 34  0.0          4 44  0.0
  4 16  0.0          4 26  0.0          4 36  1.315E-06    4 46 -3.094E-07
  4 66  0.0

*LENGTH*        172.19859 M
```

REFERENCES

1. A preliminary report on this work was given at the Heavy Ion Fusion Workshop held at BNL, October 17-21, 1977. BNL 50769, pp. 107-108, and in a private communication to E. Keil at CERN in September 1977.
2. K. L. Brown, "A first- and second-order matrix theory for the design of beam transport systems and charged particle spectrometers," SLAC Report No. 75, or Advances in Particle Physics, 1,71-134 (1967).
3. K. L. Brown, D. C. Carey, Ch. Iselin and F. Rothacker, "TRANSPORT a computer program for designing charged particle beam transport systems," SLAC 91 (1973 rev.), NAL 91, and CERN 73-16.
4. K. L. Brown, "A systematic procedure for designing high resolving power beam transport systems or charged particle spectrometers," Proc. 3rd. Int. Conf. on Magnet Technology, Hamburg, Germany, May 1970, p.348.
5. D. C. Carey, K. L. Brown and Ch. Iselin, "TURTLE a computer program for simulating charged particle beam transport systems, including decay calculations," FERMILAB Report NAL-64, December 1971, and CERN 74-2, February 1974.
6. R. V. Servranckx and K. L. Brown, "Chromatic corrections for large storage rings," 1979 Particle Accelerator Conference and SLAC-PUB-2270.
7. D. C. Carey, FERMILAB report on the redesign of the M1 beamline, to be published 1979.

FIGURE CAPTIONS

Fig. 1. A typical separated function unit cell for a second-order achromat. The lenses represent quadrupoles, the triangles dipoles, and the hexagons sextupoles.

Fig. 2. An example of a combined function unit cell for a second-order achromat.

Fig. 3. A typical lattice arrangement for an extended, 6π phase shift, second-order achromat using non-interlaced sextupole pairs.

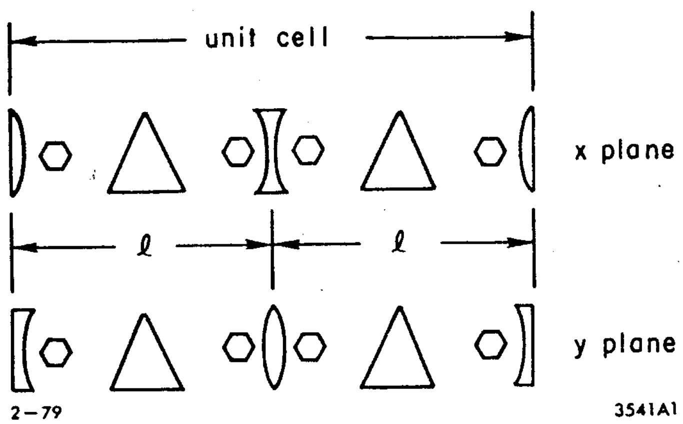
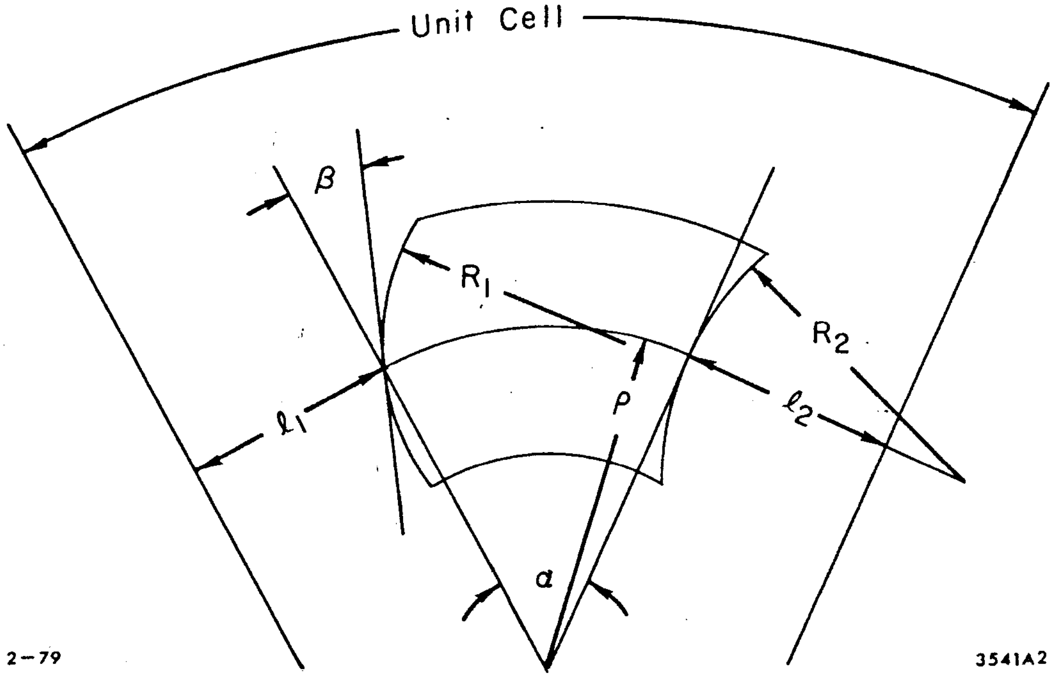


Fig. 1



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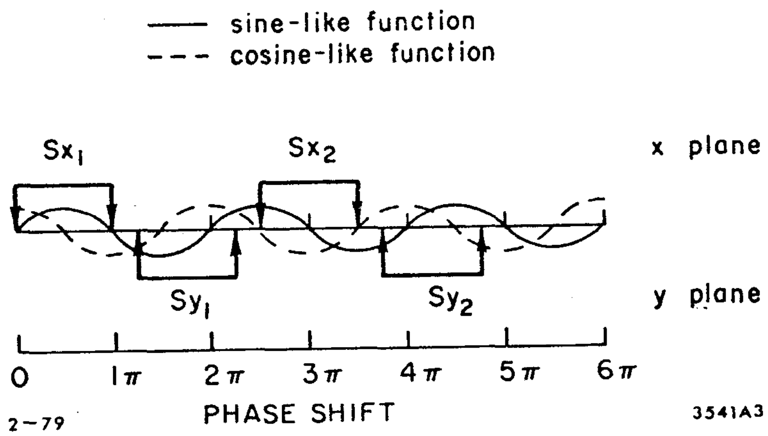
3541A2

Fig. 2



sine-like function
cosine-like function
Sx1
Sx2
x plane
Sy1
Sy2
y plane
0
1π
2π
3π
4π
5π
6π
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PHASE SHIFT
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Fig. 3