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MOMENTS OF STRUCTURE FUNCTIONS AND EXPERIMENTAL TESTS OF QCD*

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ABSTRACT

Simple assumptions which have little or no connection to Quantum Chromo Dynamics, lead to general upper and lower bounds for the slope of the graph for $\log M_{N'}$ versus $\log M_N$, where M_N is the N-th moment of the deep inelastic structure function $xF_3(x,q^2)$. The published results of the CDHS and BEBC collaborations cover the entire range allowed by our bounds and therefore cannot be considered as evidence for the validity of QCD.

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It is extremely important to confront experimental data with unambiguous predictions of Quantum-Chromo-Dynamics (QCD). One such prediction [1] concerns the large-q^2 behaviour of the moments of the structure function $xF_3(x,q^2)$ for deep inelastic neutrino-nucleon scattering. In a recent analysis of their neutrino experiments, the BEBC [2] and the CDHS [3] collaborations have plotted the logarithm of one such moment versus the logarithm of another moment. The slope of the obtained graph is then compared with the predicted ratio of anomalous dimensions in QCD. Agreement with QCD is good, and the analysis allegedly confirms the vector nature of the gluons and provides strong support for QCD.

In the present note we examine this analysis. We show that under quite general assumptions, having little or no connection to QCD, we can derive non-trivial bounds for the ratios of anomalous dimensions of the xF_3 moments. Needless to say, the QCD prediction as well as all available data, obey our bounds. However, the data cover the entire range between the two bounds rather uniformly, and it cannot be viewed as confirming QCD. Thus the ratios of anomalous dimensions do not yet provide a convincing test of QCD.

We first ignore scaling violations, and try to "guess" the form of the structure function $xF_3(x)$. Near $x=1$ a $(1-x)^3$ behaviour is recommended by several arguments such as the Drell-Yan connection [4], Bloom-Gilman duality [5] or dimensional counting [6]. Near $x=0$ an approximate $x^{\frac{1}{2}}$ behaviour is suggested by the leading Regge asymptotic term for a non-singlet function. We therefore "guess" that for some value $q^2=q_o^2$ within the presently available experimental region:

$$xF_3(x,q_o^2) = Cx^{\frac{1}{2}}(1-x)^3 \tag{1}$$

We further assume the Gross-Llewellyn-Smith (GLS) [7] sum rule:

$$\int_0^1 F_3(x,q^2)\,dx = 3 \tag{2}$$

Hence

$$C = \frac{3}{B(\frac{1}{2},4)} \tag{3}$$

where B(x,y) is an Euler beta function.

Having assumed an explicit x-dependence for some accessible value of q^2, we now introduce scaling violations. It is very likely that any field theory would yield scaling violations and that for larger q^2, the function xF_3 will shift towards $x=0$, as a result of the emission of whatever fields (or "partons") which appear in the theory. A simple way to do this is to suggest that as q^2 changes, the two powers (½ and 3) change slowly in opposite directions. We are therefore led to our final form:

$$xF_3(x,q^2) = Cx^{\frac{1}{2f}}(1-x)^{3g} \tag{4}$$

where f and g are arbitrary, slowly varying, functions of q^2, obeying

$$f(q_o^2) = g(q_o^2) = 1 \tag{5}$$

$$f'(q_o^2)\ g'(q_o^2) \geq 0 \tag{6}$$

The coefficient C, as determined by the GLS sum rule [7], will now depend on f and g:

$$C = \frac{3}{B\left(\frac{1}{2f}, 3g+1\right)} \tag{7}$$

Our complete equation for xF_3 is therefore:

$$xF_3(x,q^2) = \frac{3x^{\frac{1}{2f}}(1-x)^{3g}}{B\left(\frac{1}{2f}, 3g+1\right)} \tag{8}$$

The N-th moment $M_N(q^2)$ is defined as [8]:

$$M_N(q^2) = \int_0^1 x^{N-2} \cdot xF_3(x,q^2)\,dx \tag{9}$$

It is easy to see that by inserting eq. (8) into eq. (9) we get:

$$M_N(q^2) = \frac{3B\left(\frac{1}{2f}+N-1,\ 3g+1\right)}{B\left(\frac{1}{2f}, 3g+1\right)} \tag{10}$$

<u>Notice that all of our assumptions have little or no relation to QCD.</u> Most of them are likely to be true in any field theory. Others (such as the $x^{\frac{1}{2}}$ behaviour near $x=0$) are more suspect, but are supported by empirical evidence and are unrelated to any known field theory.

We are now ready to state our main result. Given our expression (8), we can calculate upper and lower numerical bounds for the slope of the graph of log $M_N(q^2)$ versus log $M_{N'}(q^2)$. Denoting such a slope by $p_{N'}/p_N$ we find (for $N' > N$):

$$\frac{\psi(N'+3.5)-\psi(4.5)-\psi(N'-0.5)+\psi(0.5)}{\psi(N+3.5)-\psi(4.5)-\psi(N-0.5)+\psi(0.5)} \le \frac{p_{N'}}{p_N} \le \frac{\psi(N'+3.5)-\psi(4.5)}{\psi(N+3.5)-\psi(4.5)} \tag{11}$$

where $\psi(x)$ is the logarithmic derivative of the gamma function:

$$\psi(x) = \frac{d}{dx}\log\Gamma(x) \tag{12}$$

An equivalent form of the bound (11) utilized the recursion formula for the function $\psi(x)$. We get (for $N' > N$):

$$\frac{\sum_{k=1}^{N'-1}\left(\frac{1}{k+3.5}-\frac{1}{k-0.5}\right)}{\sum_{k=1}^{N-1}\left(\frac{1}{k+3.5}-\frac{1}{k-0.5}\right)} \leq \frac{p_{N'}}{p_N} \leq \frac{\sum_{k=1}^{N'-1}\left(\frac{1}{k+3.5}\right)}{\sum_{k=1}^{N-1}\left(\frac{1}{k+3.5}\right)} \tag{13}$$

The numerical values of the bounds for some N', N values are listed in table I together with the corresponding predictions of QCD and the results of the CDHS and BEBC analysis.

The derivation of the bounds is straightforward. Using eq. (10) we find:

$$\left.\frac{d}{dq^2}\log M_N(q^2)\right|_{q^2=q_o^2} = -\frac{f'}{2}[\psi(N-0.5)-\psi(0.5)] - \left(3g'-\frac{f'}{2}\right)[\psi(N+3.5)-\psi(4.5)] \tag{14}$$

where f', g' are the derivatives of f,g at q_o^2, obeying eq. (6). Remembering that f and g are slowly varying functions of q^2, the slope $p_{N'}/p_N$ is then given by:

$$\frac{p_{N'}}{p_N} = \frac{[\psi(N'-0.5)-\psi(0.5)]+\left(6\frac{g'}{f'}-1\right)[\psi(N'+3.5)-\psi(4.5)]}{[\psi(N-0.5)-\psi(0.5)]+\left(6\frac{g'}{f'}-1\right)[\psi(N+3.5)-\psi(4.5)]} \tag{15}$$

The slope is a monotonic function of the non-negative ratio g'/f' and its bounds are found by substituting $g'/f'=0$ and $f'/g'=0$, respectively, leading to eq. (11).

The assumptions which lead to our form (eq. (8)) of xF_3 are quite general (and are consistent with the gross features of the existing data).

It is therefore apriori guarranteed that the slope of log $M_{N'}$ versus log M_N will obey our bounds (eq. (11)). In order to claim that the experimental slope provides a valid test of QCD (or that it prefers QCD over some other reasonable possibility) we have to show that within the domain defined by our bounds, the QCD value is preferred. Figure 1 indicates that this is not the case. The CDHS data [2] for the Cornwall-Norton moments [8] and Nachtmann moments [8] cover the entire range between our bounds. The BEBC points [2] for the Nachtmann moments [8] are slightly more convincing, but they lean heavily on low-q^2 data. In fact, a reanalysis of the BEBC data with a higher q^2-cutoff yields higher values [9] for $p_{N'}/p_N$.

An extremely puzzling numerical coincidence emerges from our analysis. We have no good reason to assume that $f' = g'$ in our expression (15) for $p_{N'}/p_N$. However, if we arbitrarily make this assumption, we obtain $p_{N'}/p_N$ values which are incredibly close to the QCD predictions (e.g., we obtain $p_5/p_3 = 1.453$ compared with the QCD value 1.456; $p_6/p_4 = 1.301$ compared with 1.291, etc.). We do not understand this peculiar coincidence, which does depend on the powers ½ and 3 in our original formula for xF_3.

We conclude with a few comments:

(a) The widely publicized QCD test [2,3] of plotting log $M_{N'}$ versus log M_N has the advantage that it does not depend on the two free parameters N_F (number of quark flavours) and Λ (the scale parameter). However, in integrating the data and eliminating the q^2-dependence, important additional information is lost. The explicit q^2-dependence predicted by QCD is not probed by the test, and, as we have showed in this note - the present data do not allow us to reach definite conclusions.

(b) We have no criticism of more direct QCD tests such as plotting $M_N(q^2)$ versus log q^2. Such tests are valid, but they depend, of course, on N_F and Λ.

(c) The numerical values of our bounds are not very sensitive to a variation of the powers ½ and 3, assumed in eq. (8). Any value of these powers which is not far from the experimental x-dependence in the present q^2-domain, would yield bounds similar to those listed in table I and shown in fig. 1. For instance, an $x^{\frac{1}{2}}(1-x)^4$ behaviour gives $1.19 \leq p_5/p_3 \leq 1.75$ instead of $1.18 \leq p_5/p_3 \leq 1.71$.

(d) A technical side remark: Our general expression eq. (8) has the property that if one slope $p_{N'}/p_N$ agrees with QCD, all other slopes (for $N', N \leq 8$) must be in good agreement with QCD. Hence, when and if data are sufficiently accurate to select a small numerical range within our bounds, thus providing a meaningful test of QCD, one slope will tell us the whole story. Other slopes will not provide independent tests.

Until such time, we believe that the practice of using the slope of log $M_{N'}$ versus log M_N as a convincing test of QCD is unjustified. Direct plots of q^2-dependence over wide q^2-ranges are more meaningful, and will hopefully support QCD.

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	New Lower Bound	New Upper Bound	QCD Prediction	"Experimental Data"		
				CDHS (Nachtmann)	CDHS (C – N)	BEBC (Nachtmann)
p_5/p_3	1.18	1.71	1.46	1.34 ± 0.12	1.58 ± 0.12	1.50 ± 0.08
p_6/p_4	1.10	1.45	1.29	1.18 ± 0.09	1.34 ± 0.07	1.29 ± 0.06
p_6/p_3	1.22	2.00	1.62	1.38 ± 0.15	1.76 ± 0.15	---
p_7/p_3	1.26	2.26	1.76	---	---	1.84 ± 0.20

TABLE I

Lower and upper bounds for $p_{N'}/p_N$ are listed together with QCD predictions and the results of the CDHS [3] and BEBC [2] analysis. The bounds follow from eq. (8) and are quite general. A graphic display of the contents of the table is given in fig. 1. The numbers quoted under "experimental data" represent the analysis of the experimental groups, using Nachtmann or Cornwall-Norton (C-N) moments.

FIGURE CAPTION

Fig. 1. For each published slope of log $M_{N'}$ versus log M_N we show our lower and upper bounds together with the results of the BEBC (ref. 2) and CDHS (ref. 3) analysis. We show only results which were published by the experimental groups themselves. We therefore do not have the p_7/p_3 values for CDHS, the p_6/p_3 value for BEBC and the Cornwall-Norton moments for BEBC. The figure indicates that the data do not clearly prefer any particular slope within the guaranteed bounds.

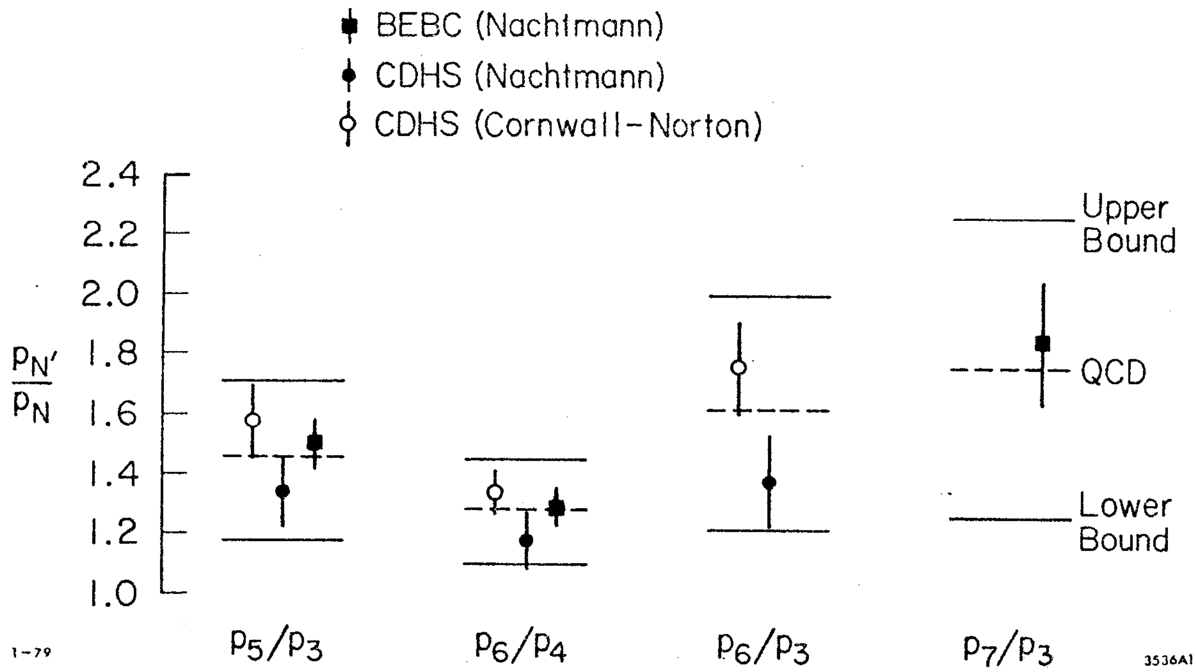
BEBC (Nachtmann)
CDHS (Nachtmann)
CDHS (Cornwall-Norton)
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
$\frac{p_{N'}}{p_N}$
Upper Bound
QCD
Lower Bound
p_5/p_3
p_6/p_4
p_6/p_3
p_7/p_3
1-79
3536A1

Fig. 1