VII. TESTS OF STRUCTURE FUNCTION SCALING

VII.A. Introduction

Experimental tests of structure function scaling are fraught with ambiguity. Apparent deviations from exact scaling may arise from such diverse effects as two-photon exchange, low-Q^2 turn-on[59] of νW_2, s-channel resonance contributions and non-leading terms in the light cone expansion of the current commutator. They may obscure genuine scaling deviations predicted by field theories with parton structure[15, 16], anomalous dimensions[17], and asymptotic freedom[18], or arising from the production of charmed[72] or colored states.[73] Bjorken's original hypothesis[11] was that $2MW_1(\nu,Q^2)$ and $\nu W_2(\nu,Q^2)$ would scale in the variable $\omega = 2M\nu/Q^2$ (i.e., become functions only of ω) in the limit $\nu \to \infty$, $Q^2 \to \infty$, with ν/Q^2 held fixed. Within the experimental errors, the early data for νW_2^p was consistent with scaling in ω for $Q^2 \gtrsim 1$ GeV^2 and $W \gtrsim 2.6$ GeV. In this experiment, use of the scaling variable $\omega' = 1/x' = \omega + M^2/Q^2 = 1 + W^2/Q^2$ extended the range of W for which scaling of νW_2^p was valid down to W = 1.8 GeV.[74] Other scaling variables[61, 62, 75, 76], all of which approach ω as $Q^2 \to \infty$, have been proposed to fit the data; they are examined in section VII.B. In the remaining scaling tests of this section, only the variables ω and ω' are used and deviations from scaling in these variables are examined. Only data for $Q^2 \geq 2\ GeV^2$ and $W \geq 2$ GeV are used in these scaling tests. These restrictions insured that the tests were influenced neither by the prominent electroproduction resonances nor by the low-Q^2 turn-on of νW_2.

The two independent structure functions $F_1 = 2MW_1(x,Q^2)$ and $F_2 = \nu W_2(x,Q^2)$ for the proton and deuteron, as given in Table (XV) and plotted in Figures (36) and (37), were used for the scaling tests reported in section VII.A. This method had the advantage that the extracted structure functions were independent of any assumptions about the Q^2-dependence of R. These "separated" data were best suited for a comparison of the Q^2-dependence of the four structure functions $2MW_1^p$, νW_2^p, and $2MW_1^d$, and νW_2^d in the same range of of kinematics. This method of extracting the structure functions had the disadvantage of limited precision, as the random error in R at each kinematic point was propagated into the error in the two structure functions. The range of Q^2 and the number of data points available at each x were also somewhat limited in this method.

The second method used to extract the structure functions was similar to that used in earlier scaling tests.[7, 20] In this method the structure function νW_2 was extracted from the inelastic cross section data using equation (I.3), and assuming a functional form for R to be valid throughout the kinematic region in which the cross sections had been measured. Whereas the constant value $R_p = 0.18$ was used to extract νW_2^p in the earlier tests, we used the modified spin-1/2 form(70) $R = cQ^2/(Q^2 + d^2)^2$ with proton coefficients taken from Table (XIII). This functional form has the two advantages that it fits the R data better than the constant form, and that it satisfies gauge invariance as $Q^2 \to 0$, i.e., $R \to 0$ in that limit. Inelastic e-p, e-d, and e-n cross sections from experiments A and B only (Table V) were used in this method. Cross sections from experiment B were normalized to those of experiment A by the normalization factor $N_{AB} = 1.010$ discussed in section V.F. The uncertainty in the extracted values of $2MW_1$ owing to our assumptions about R, was deemed too large in this method, and no results are presented for that structure function. The corresponding uncertainty in νW_2 was always less than the statistical error in $2MW_1$. Because of the statistical accuracy of this large body of data for νW_2, this method was particularly appropriate for a study of the possible functional forms of Q^2-dependent scale-breaking terms in νW_2.

A rough test of scaling is provided by plots of all these "extracted" data for νW_2^p, νW_2^d, and νW_2^n versus x, as in Figure (38), or versus x', as in Figure (39). To a fairly good approximation these data describe single functions of x or x', faring

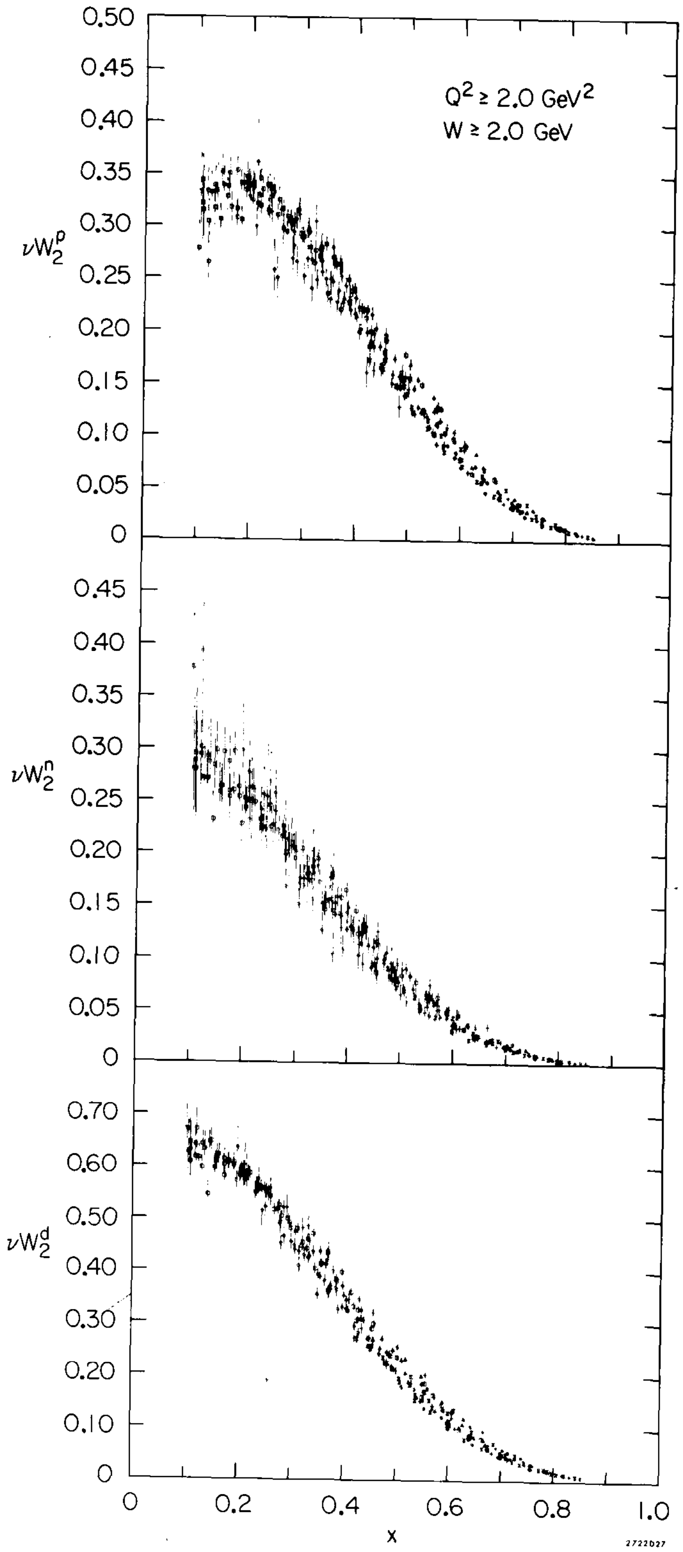

Fig. 38. Values of νW_2^p, νW_2^n, and νW_2^d plotted against x. The errors shown are purely random.

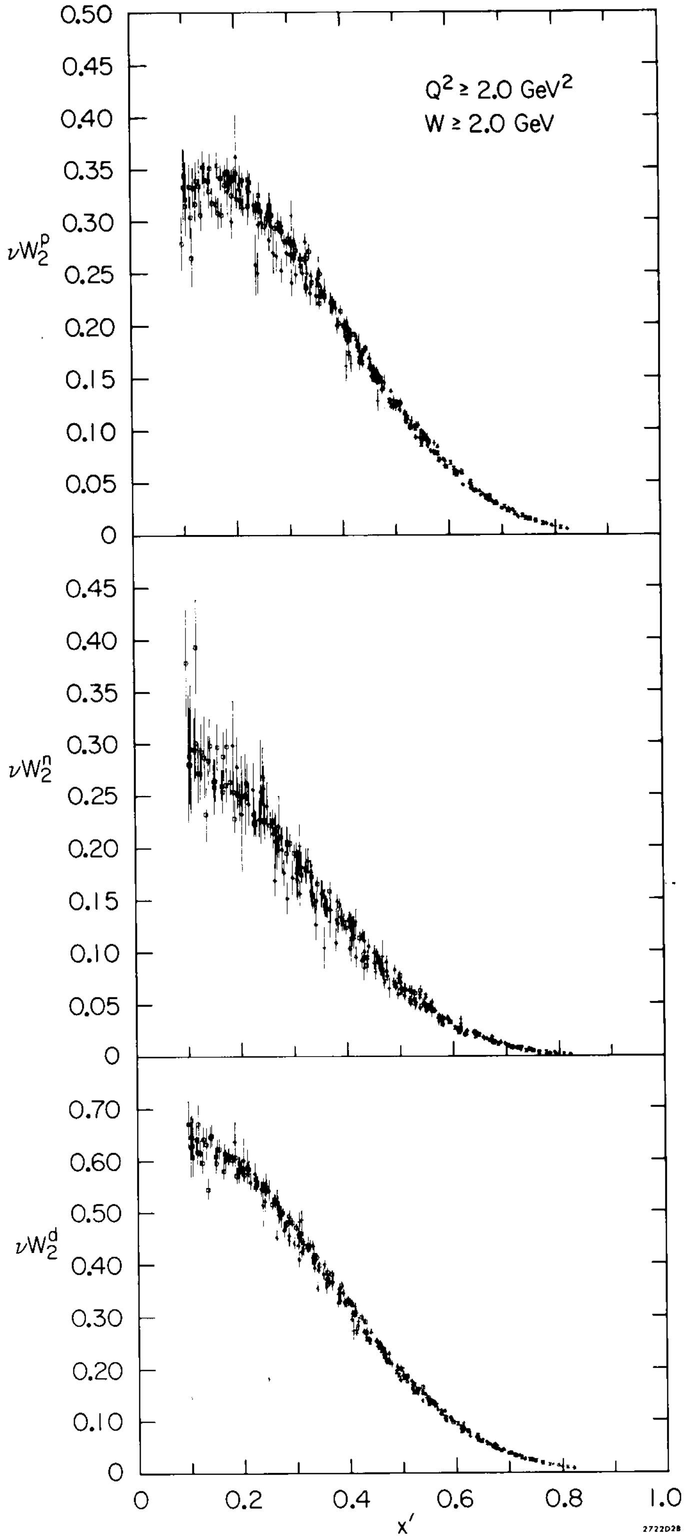

Fig. 39. Values of νW_2^p, νW_2^n, and νW_2^d plotted against x'. The errors shown are purely random.

better in the second variable. The usefulness of this approach is limited, however, as small deviations (on the order of 10-20%) from exact scaling would not be apparent in these plots. More quantitative scaling tests were provided by fits of the form $\nu W_2 = f(\xi)h(Q^2)$, where ξ is one of the proposed scaling variables, and $h(Q^2)$ is either unity or a scale-breaking function. Such is the procedure used in section VII.B., where several proposed scaling variables are compared, and in section VII.C., where deviations from scaling of νW_2^p, νW_2^d, and νW_2^n are compared. The disadvantage of such an approach is that the functional form assumed for the Q^2-dependent term $h_i(Q^2)$ must be the same for all values of ξ. This approach is not compatible with certain field theory models(17, 18, 77) that predict a rise in the structure functions at low x and a fall-off at larger values of x. Deviations from scaling in ω were further examined in section VII.D. by fitting functions with explicit Q^2-dependent terms to F_1 and F_2 for 11 fixed values of $x = 1/\omega$ in the range $0.1 \leq x \leq 0.8$. The separated $2MW_1$ and νW_2 data of Table (XV) were ideally suited to this task, but the accuracy of the results was limited by the accuracy of the separated structure functions and the ranges of Q^2 available at each x. More extensive studies of the Q^2-dependence of νW_2 were possible using values of this structure function that had been extracted from interpolated cross sections using the fit $R = cQ^2/(Q^2 + d^2)^2$. The normalized cross section from experiments A and B were first interpolated at fixed E and θ to values of E' corresponding to the 11 values of x used in the $x - Q^2$ array. These data for $\nu W_2(x,Q^2)$ then permitted extensive tests of the various functional forms proposed for deviations from exact Bjorken scaling.

VII.B. Comparison of $2MW_1$ and νW_2

The two independent structure functions $F_1 = 2MW_1(x,Q^2)$ and $F_2 = \nu W_2(x,Q^2)$ reported in Table (XV) were used in the scaling tests reported here. As mentioned earlier only data for $Q^2 \geq 2$ GeV^2 and $W \geq 2$ GeV were used in these tests. Scaling in the two variables $\xi = \omega$ and $\xi = \omega'$ was tested by fitting functions of the form $F_i(x,Q^2) = f_i(\xi)h_i(Q^2)$ to these proton and deuteron data for F_1 and F_2. Here $f_1(\xi) = \Sigma a_j(1-1/\xi)^j$ and $f_2(\xi) = \Sigma b_j (1-1/\xi)^j$, where j ranges from 3 to 7. Three forms for $h_i(Q^2)$ were tested: a constant $h_i(Q^2) = 1$ for exact scaling; the scale-breaking form $h_i(Q^2) = 1-2Q^2/\Lambda_i^2$ suggested by constituent models[15, 16] wherein $1/\Lambda^2$ is the parton "size" and the propagator form[16, 78] $h_i(Q^2) = (1 + Q^2/\Lambda_i^2)^{-2}$ which is expected in some finite size constituent models[16] as well as in heavy photon theories.[78] Best fit values for Λ_i^2 and for the polynomial coefficients a_j and b_j were obtained simultaneously by least-square fits. Our studies indicated that the results for Λ_1^2 and Λ_2^2 were independent of the functional forms chosen for $f_1(\xi)$ and $f_2(\xi)$. The fits provided a comparison of deviations from scaling in $2MW_1$ and νW_2 for both the proton and the deuteron, independent of assumptions about R. In particular, they permit unbiased tests of models[16] that predict a larger scaling violation for $2MW_1$ than for νW_2.

The best-fit parameters $1/\Lambda_1^2$ and $1/\Lambda_2^2$ of fits in the scaling variable $\xi = \omega$ are presented in Table (XVI). Systematic uncertainties in these quantities arise from the same effects that led to the relative uncertainties in F_1 and F_2 listed in Table (XV). These systematic uncertainties were added in quadrature and included in the errors quoted. For $\xi = \omega$, the two scale-breaking forms listed in Table (XVI) provided much better fits than the exact scaling form $F_i(x,Q^2) = f_i(\omega)$. Over the full range of x, the best-fit values for $1/\Lambda_1^2$ and $1/\Lambda_2^2$ were essentially the same for the proton, but were different by about 2 standard deviations for the case of the deuteron. This difference may well have arisen from smearing effects[20], or resonance contributions[79] at low W, for $1/\Lambda_1^2$ and $1/\Lambda_2^2$ were equal within one standard deviation when the deuteron data were restricted to $W \gtrsim 2.6$ GeV. For $0.3 \lesssim x \lesssim 0.8$, the proton coefficients for the scale-breaking form $h_i(Q^2) = 1 - 2Q^2/\Lambda_i^2$ are in agreement with the values $1/\Lambda_1^2 = 0.0162 \pm 0.0024$ and $1/\Lambda_2^2 = 0.0134 \pm 0.0013$ obtained earlier[24, 26] for $0.33 \lesssim x \lesssim 0.67$ using data from experiments A and C. The results for $1/\Lambda_1^2$ in the propagator scale-breaking form are also in agreement with the results

Table XVI. Deviations from scaling in ω, from least square fits of the form $F_i(x,Q^2) = f_i(\omega)h_i(Q^2)$ to the separated $2MW_1$ and νW_2 data for $W \geq 2.0$ GeV, $Q^2 \geq 2.0$ GeV^2

Fitted data	$h_i(Q^2) = 1-2Q^2/\Lambda_i^2$		$h_i(Q^2) = (1+Q^2/\Lambda_i^2)^{-2}$	
	$1/\Lambda_1^2$	$1/\Lambda_2^2$	$1/\Lambda_1^2$	$1/\Lambda_2^2$
p $0.1 \leq x \leq 0.8$	0.0144 ± 0.0014	0.0141 ± 0.0008	0.0225 ± 0.0038	0.0204 ± 0.0017
p $0.3 \leq x \leq 0.8$	0.0147 ± 0.0013	0.0144 ± 0.0008	0.0245 ± 0.0040	0.0213 ± 0.0009
d $0.1 \leq x \leq 0.8$	0.0162 ± 0.0012	0.0118 ± 0.0008	0.0270 ± 0.0039	0.0155 ± 0.0015
d $0.3 \leq x \leq 0.8$	0.0164 ± 0.0012	0.0125 ± 0.0009	0.0294 ± 0.0043	0.0173 ± 0.0008

of similar fits to recent data[76] for $2MW_1^p$ in the range $0.4 \leq x \leq 0.9$ where a value of $1/\Lambda_1^2 = 0.0233 \pm 0.0008$ was reported. For $x < 0.3$, both the proton and deuteron structure functions differed from scaling behavior in ω by less than two standard deviations. A comparison of these fits with the structure function data is presented in Figures (40) and (41), where ratios $F_i(x,Q^2)/f_i(\omega)$ have been plotted versus Q^2 at fixed x. The polynomial functions f_i correspond to the structure function fits of the form $F_i(x,Q^2) = f_i(\omega)(1-2Q^2/\Lambda_i^2)$ to all the data in the kinematic range $W \geq 2$ GeV, $Q^2 \geq 2$ GeV^2, $0.1 \leq x \leq 0.8$, as listed in Table (XVI). The solid lines represent the best fits to these data of the two scale-breaking forms listed in that table.

The best-fit parameters $1/\Lambda_1^2$ and $1/\Lambda_2^2$ of fits to F_1 and F_2 using the scaling variable $\xi = \omega'$ are presented in Table (XVII). Systematic uncertainties in these quantities were estimated in the same manner as they were for Table (XVI), and are included in the errors quoted in Table (XVII). Except for fits to νW_2^d, the two scale-breaking functions provided better fits to the data than the exact scaling form $F_i(x,Q^2) = f_i(\omega')$. All three functional forms fit the data for νW_2^d with a χ^2 of 0.9 per degree of freedom. The χ^2 for the fits listed in Table (XVII) ranged from 0.7 to 1.1 per degree of freedom.

For data in the range $0.1 \leq x \leq 0.8$ as noted in Table (XVII),

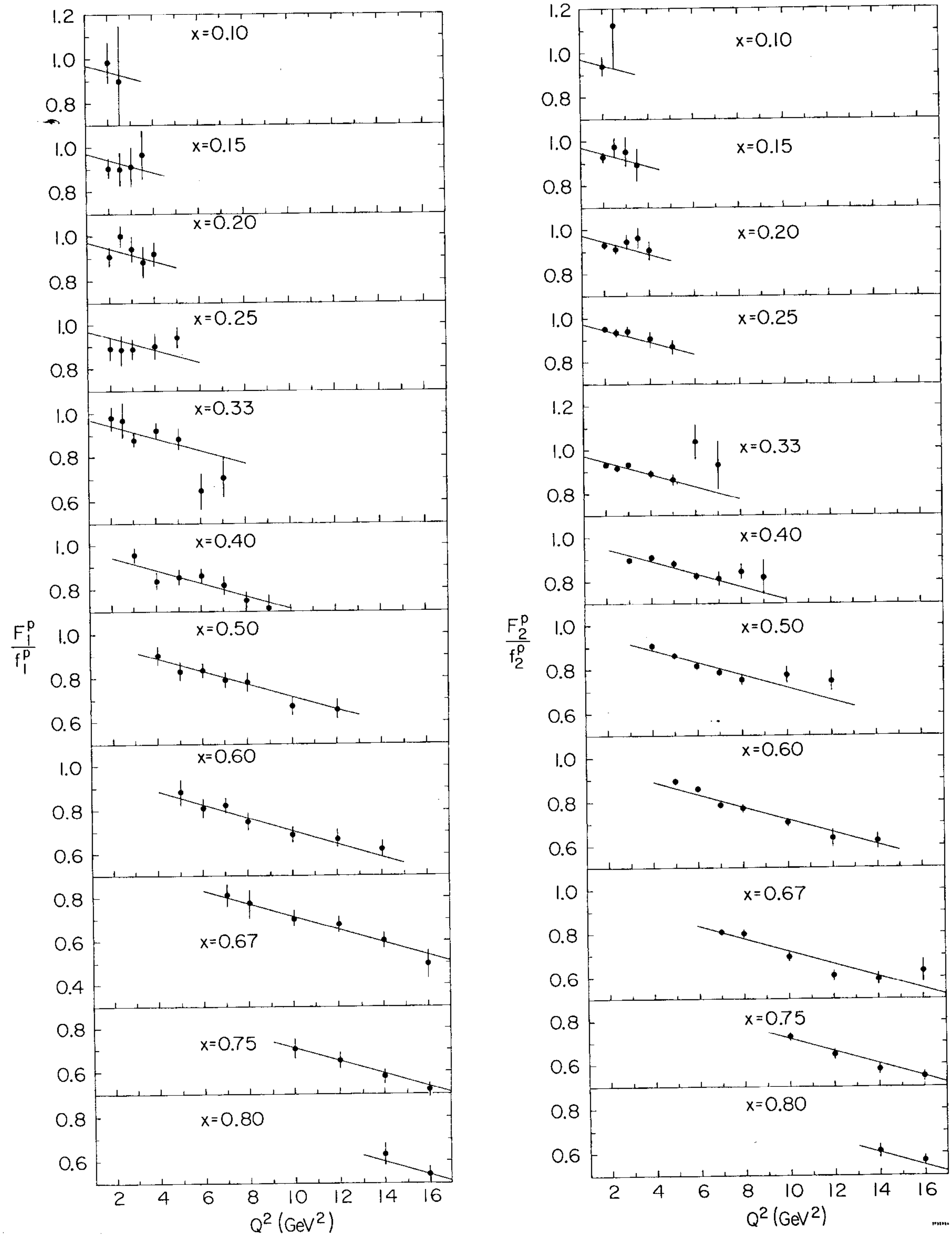

Fig. 40. Ratios of $F_1^p = 2MW_1^p$ and $F_2^p = \nu W_2^p$ to the polynomials $f_1(x)$ and $f_2(x)$ taken from least square fits of the form $F_i(x, Q^2) = f_i(x)(1 - 2Q^2/\Lambda_i^2)$ to all the data for $W \geq 2.0$ GeV and $Q^2 \geq 2.0$ (GeV/c)^2 in Table 15.

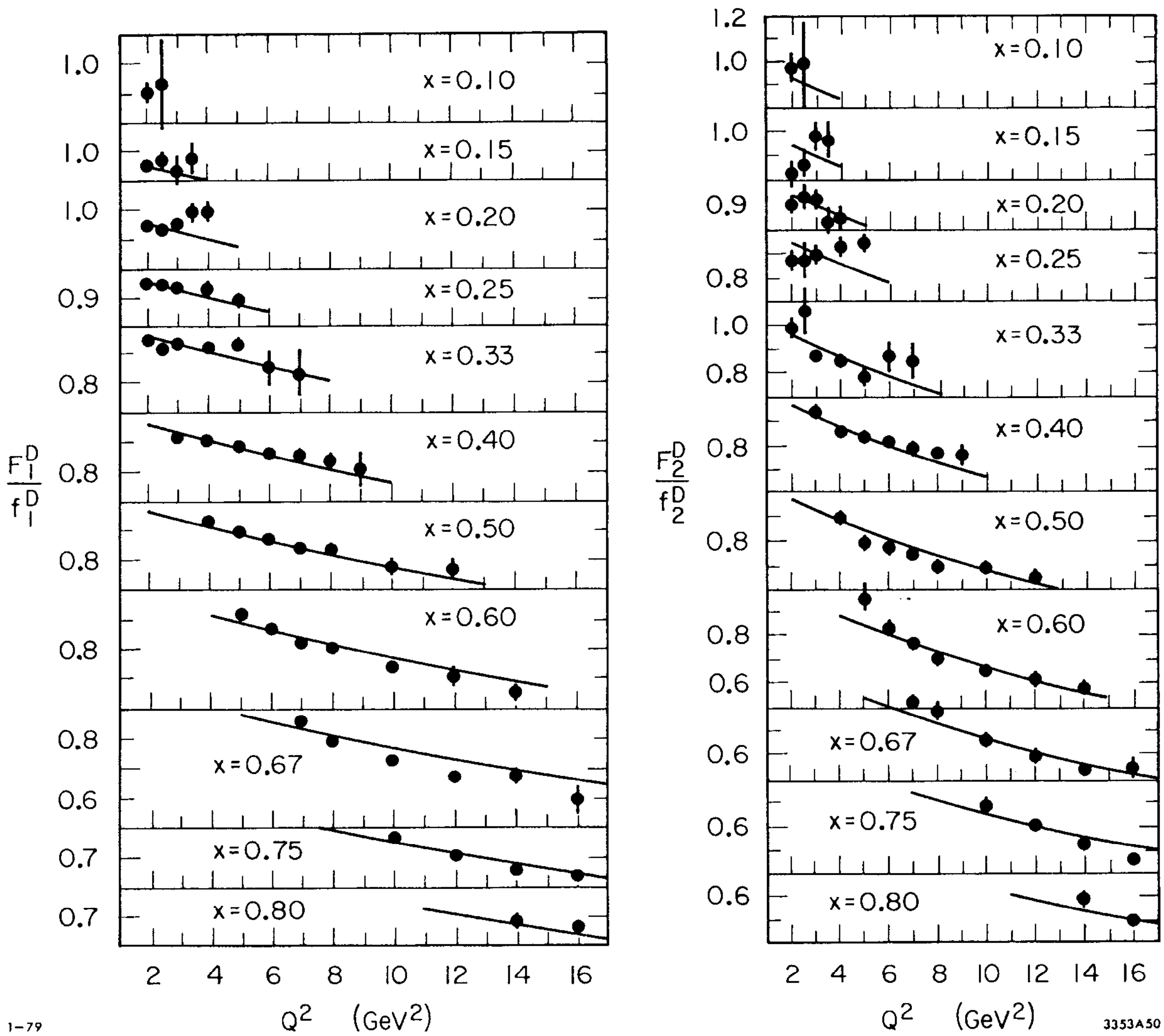

Fig. 41. Ratios of $F_1^d = 2MW_1^d$ and $F_2^d = \nu W_2^p$ to the polynomials $f_1(x)$ and $f_2(x)$ taken from least square fits of the form $F_i(x,Q^2) = f_i(x)(1-2Q^2/\Lambda_i^2)$ to all the data for $W \geq 2.0$ GeV and $Q^2 \geq 2.0\ (\text{GeV}/c)^2$ in Table 15.

Table XVII. Deviations from scaling in ω',

from least square fits of the form $F_i(x,Q^2) = f_i(\omega')h_i(Q^2)$.

fitted data	$h_i(Q^2) = 1-2Q^2/\Lambda_i^2$		$h_i(Q^2) = (1+Q^2/\Lambda_i^2)^{-2}$	
	$1/\Lambda_1^2$	$1/\Lambda_2^2$	$1/\Lambda_1^2$	$1/\Lambda_2^2$
p $0.1 \le x \le 0.8$	0.0044 ± 0.0024	0.0054 ± 0.0012	0.0047 ± 0.0030	0.0059 ± 0.0015
p $0.3 \le x \le 0.8$	0.0052 ± 0.0025	0.0055 ± 0.0013	0.0059 ± 0.0031	0.0061 ± 0.0017
d $0.1 \le x \le 0.8$	0.0069 ± 0.0022	0.0009 ± 0.0013	0.0077 ± 0.0029	0.0009 ± 0.0013
d $0.3 \le x \le 0.8$	0.0077 ± 0.0020	0.0017 ± 0.0015	0.0092 ± 0.0031	0.0017 ± 0.0016

the best-fit parameters $1/\Lambda_1^2$ and $1/\Lambda_2^2$ are equal for the proton, within errors; νW_2^p is inconsistent with scaling in ω', while $2MW_1^p$ is barely consistent, at the two standard deviation level. For the range $0.3 \lesssim x \lesssim 0.8$, the coefficients for the linear scale-breaking form are consistent with the values $1/\Lambda_1^2 = 0.0049 \pm 0.0035$ and $1/\Lambda_2^2 = 0.0020 \pm 0.0018$ reported earlier[26] for $0.33 \lesssim x \lesssim 0.67$ using data from experiments A and C. The results for $1/\Lambda_1^2$ in the propagator form are also consistent with the results of similar fits to the recent data for $2MW_1^p$ in the range $0.4 \lesssim x \lesssim 0.9$, where a value of $1/\Lambda_1^2 = 0.0078 \pm 0.0006$ was reported.[76] For either range of x, νW_2^d is consistent with scaling in ω', but $2MW_1^d$ is not. However, if we restrict the data to $W \gtrsim 2.6$ GeV the best fit parameters $1/\Lambda_1^2$ and $1/\Lambda_2^2$ are equal within one standard deviation and consistent with zero. In the range $0.1 \lesssim x \lesssim 0.3$, no violation of scaling in ω' was observed for either the proton or deuteron structure functions.

For the separated proton structure function data restricted to the kinematic region ($W \geq 2.0$, $Q^2 \geq 2.0$, $x \geq 0.3$), the results of our scaling tests are unambiguous. Both structure functions are inconsistent with scaling in ω and νW_2^p is inconsistent with scaling in ω'. The structure function $2MW_1^p$ shows a violation of scaling in ω' that is equal to that exhibited by νW_2^p with breakdown parameters that are about the same, but the errors are larger and preclude a completely conclusive result. Over the range of Q^2 ($2.0 \leq Q^2 \leq 16.0$ GeV^2) studied in these tests, we see a 40% violation of scaling in ω and a 15% violation of scaling in ω', for $x \geq 0.3$. For either scaling variable, no evidence is seen for different values of $1/\Lambda_1^2$ and $1/\Lambda_2^2$, even when we restrict $W \geq 2.6$ GeV, and we conclude that they are equal, within the present errors. For the range $0.1 \leq x \leq 0.3$, the two proton structure functions are consistent with scaling in both ω and ω'. The lack of any significant Q^2-dependence in this region, when combined with the observed violation of scaling for $x \geq 0.3$, is consistent with field-theoretic models[77] of nucleon structure.

The interpretation of our results for the deuteron structure functions is not so straightforward. For $x \geq 0.3$, both $2MW_1^d$ and νW_2^d are inconsistent with scaling in ω, with 35%-45% scaling violations in the range of Q^2 studied. Over the same range of x, $2MW_1^d$ is inconsistent with scaling in ω', showing a 20% violation, while νW_2^d is consistent with scaling in ω'. For both scaling variables, the apparent difference between $1/\Lambda_1^2$ and $1/\Lambda_2^2$ disappears when the data are restricted to $W \geq 2.6$ GeV, and we can make no firm conclusions about its validity. Uncertainties in the off-mass-shell effects in the smearing corrections are largest at low W, but the magnitudes of these uncertainties, as estimated in Appendix A.3.B., cannot fully account for the observed difference between $1/\Lambda_1^2$ and $1/\Lambda_2^2$.

VII.C. Comparisons of Scaling Variables

In addition to the scaling variable ω originally suggested by Bjorken[11], other scaling variables, all of which approach ω as $Q^2 \to \infty$, have been proposed to fit the low Q^2 structure function data. The variable $\omega_L = M/((Q^2 + \nu^2)^{-1/2} - \nu)$ has been suggested[61] as the scaling variable appropriate to light cone algebras. The previously mentioned scaling variable $\omega' = \omega + M^2/Q^2$, which fit the earlier proton structure function data quite well[7], has been related to finite energy sum rules.[74]

A phenomenological scaling variable $\omega_W = (2M\nu + M_a^2)/(Q^2 + M_b^2)$ (where M_a^2 and M_b^2 are fit parameters) that extends scaling of νW_2^p down to the photoproduction limit $Q^2 = 0$, was first suggested by Rittenberg and Rubinstein.[62] In an analysis[75] of previous electroproduction and photoproduction data, it was concluded the $\omega\nu W_2^p$, not νW_2^p, scaled in ω_W, within the experimental errors. Schwinger[80] has proposed a similar scaling variable ω_S, with $M_a^2 = (3/2)M^2$ and $M_b^2 = (1/2)M^2$, which are close to the best fit values of these parameters in the fits to νW_2^p, νW_2^d, and νW_2^n discussed in section V.3. The scaling variable $\omega_A = \omega + M_A^2/Q^2$, where $M_A^2 = 1.42$ GeV^2, has been used to fit the recent data[76] for $2MW_1^p$.

The quality of scaling in any variable ξ was tested by fitting polynomials of the form $\Sigma a_j (1 - 1/\xi)^j$, where j ranged from 3 to 7, to the extracted data for νW_2^p, νW_2^d or νW_2^n shown in Figures (38) and (39). Only data for $W \geq 2$GeV and $Q^2 \geq 2$ GeV^2 were used in these least-square fits, yielding a total of 274 degrees of freedom for the proton data, and 257 for the deuteron and neutron data. Over the full range of x available here ($0.10 \leq x \leq 0.85$), these five parameter polynomials provided better fits than polynomials with n ranging from 3 to 5. The values χ^2 for these fits, divided by the number of degrees of freedom N_D, are reported in Table (XVIII). In the case of the last two scaling variables, ω_A and ω_W, the parameters M_A^2, M_a^2, and M_b^2 were fit simultaneously with the polynomial coefficients, and the number of degrees of freedom accordingly was smaller. The best fit values of M_A^2 obtained were 1.352 ± 0.032 for the proton, 1.294 ± 0.027 for the deuteron, and 1.109 ± 0.075 for the neutron. None of the scaling variables ω, ω', ω_L, or ω_S could provide even adequate fits to νW_2^p or νW_2^d, and only ω' could fit the neutron data with any degree of success. When only data for $W \geq 2.6$ GeV, $Q^2 \geq 2.0$ GeV^2 were used in similar fits, the values of χ^2/N_D were in general smaller, but only ω_A and ω_W could provide adequate fits to all three sets of data. As no random error from the error in R was included in the errors in the

Table XVIII. χ^2/N_D for various scaling fits to νW_2

Fit data	N_D	ω	ω'	ω_L	ω_S	ω_A	ω_W
νW_2^p	274	10.05	2.30	4.56	4.56	1.43	1.42
νW_2^d	257	12.97	2.60	5.70	5.95	1.62	1.62
νW_2^n	257	2.33	1.32	1.63	1.68	1.29	1.29

structure function νW_2, a χ^2/N_D of 1.3 is judged a "good" fit and χ^2/N_D of 1.5 is judged an "adequate" fit.

Polynomial fits in ω_A, wherein only the fourth and fifth powers of $(1-1/\omega_A)$ were used to fit the structure function data, were also attempted. These fits are identical to those attempted by Atwood[76] for the recent data for $2MW_1$. Results of such fits are presented in Table (XIX), where we list M_A^2 and the polynomial coefficients a_4 and a_5, together with the χ^2/N_D of the fits. Such fits to the νW_2^d data are clearly inadequate, but when νW_2^p and νW_2^n are separated and fit independently, adequate fits are obtained. However the best fit values of M_A^2 for the proton and neutron are significantly different. Similar results[76] were recently obtained for $2MW_1$. When the fit data are restricted to $W \geq 2.6$ GeV, the best-fit values of M_A^2 change to 1.642 ± 0.048 GeV^2 for the proton and 0.861 ± 0.107 GeV^2 for the neutron. For comparison the value of M_A^2 obtained by Atwood <u>et al</u>. in a fit to $2MW_1$ is $M_A^2 = 1.473 \pm 0.042$ GeV^2 for the proton.[76] Our results for νW_2 agree with the results of Atwood <u>et al</u>. for $2MW_1$ in that the neutron structure functions appear to scale in ω', while the proton structure functions scale in $\omega_A = \omega + M_A^2/Q^2$ with M_A^2 about equal to 1.5 GeV^2. Adequate two parameter polynomial fits, for j-values of 4 and 5, can be made to both νW_2^p and νW_2^n using such a scaling variable, but this requires M_A^2 to be <u>different</u> for the proton and neutron.

Table XIX. Fits in the scaling Variable ω_A

Fit data	N_D	χ^2/N_D	M_A^2 (GeV^2)	a_4	a_5
νW_2^p	276	1.55	1.512±0.019	3.371±0.022	-3.218±0.029
νW_2^d	259	2.33	1.469±0.017	4.842±0.031	-4.308±0.040
νW_2^n	259	1.41	0.792±0.048	1.866±0.038	-1.561±0.049

VII.D. Deviations from Scaling in X or X'

Rather than search for new scaling variables that can fit all the data for νW_2, one can parameterize the deviations from scaling in a pre-selected variable, as was done for the F_1 and F_2 in section VII.A. In the same vein, we have made fits of the form $\nu W_2(\nu,Q^2) = f(\xi)h(Q^2)$ to the data for νW_2^p, νW_2^d, and νW_2^n shown in Figures(38) and (39). As in section VII.A., $f(\xi)$ is a five-parameter polynomial in $\xi = \omega$ or $\xi = \omega'$, and $h(Q^2)$ is either the linear scale-breaking form $1 - 2Q^2/\Lambda^2$ or the propagator form $(1 + Q^2/\Lambda^2)^{-2}$. Best fit values of $1/\Lambda^2$ and the polynomial coefficients were obtained simultaneously by least square fits. The results for Λ^2 were independent of the functional form chosen for $f(\xi)$. Although the scale-breaking forms studied cannot vary with ω or ω', this factorization method has the distinct advantage of being a parameterization with greater statistical precision. The same data for νW_2 (with $Q^2 \geq 2$ GeV^2 and $W \geq 2$ GeV) as was used in the previous section are used here, and the following results can be compared directly with those in Tables (XVIII) and (XIX).

For fits in the variable $\xi = \omega$, both linear and propagator scale-breaking forms provide much better fits to νW_2^p and νW_2^d than functions that scale in ω. But the χ^2 for these scale-breaking fits, which ranged from 1.90 to 2.28 per degree of freedom, indicate that the full body of νW_2^p and νW_2^d data cannot be parameterized by either functional form. However, both linear and propagator forms provide good fits to the full body of νW_2^n data, achieving χ^2 of 1.28 and 1.30 per degree of freedom.

The χ^2 for these scale-breaking fits improved markedly when the structure function data were restricted to $W \geq 2.6$ GeV. The χ^2 per degree of freedom ranged from 1.31 to 1.36 for fits to νW_2^p and νW_2^n, while it ranged from 1.60 to 1.71 for fits to νW_2^d. Best-fit parameters of these fits are presented in Table (XX) with quoted errors that include both random errors and systematic uncertainties, added in quadrature. These uncertainties arose from uncertainties in R and the measured differential cross sections that were propagated through the extracted values of νW_2 used in these fits.

The linear and propagator scale-breaking forms fit both the νW_2^p and the νW_2^n data equally well. In both cases, the coefficient $1/\Lambda^2$ is less than two standard deviations larger for the neutron than for the proton. The relatively poor χ^2 obtained for νW_2^d probably reflects the fact that its Q^2-dependence is a composite of proton and neutron behaviors (and smearing).

Best-fit parameters of scale-breaking fits in the variable $\xi = \omega'$ are presented in Table (XXI), along with χ^2/N_D for these fits. The quoted errors are again the quadratic sum of random errors and systematic uncertainties. For both cases

Table XX. Scale-breaking fits to $\nu W_2 = f(\omega)h(Q^2)$ ($W \gtrsim 2.6$ GeV, $Q^2 \gtrsim 2.0$ GeV^2)

data	N_D	$h(Q^2) = 1-2Q^2/\Lambda^2$		$h(Q^2) = (1+Q^2/\Lambda^2)^{-2}$	
		$1/\Lambda^2$	χ^2/N_D	$1/\Lambda^2$	χ^2/N_D
νW_2^p	193	0.0092±0.0004	1.33	0.0122±0.0007	1.36
νW_2^d	183	0.0100±0.0003	1.60	0.0133±0.0006	1.71
νW_2^n	183	0.0110±0.0010	1.31	0.0143±0.0018	1.34

Table XXI. Scale-breaking fits to $\nu W_2 = f(\omega')h(Q^2)$

(a) $W \geq 2.0$ GeV

data	N_D	$h(Q^2) = 1-2Q^2/\Lambda^2$		$h(Q^2) = (1+Q^2/\Lambda^2)^{-2}$	
		$1/\Lambda^2$	χ^2/N_D	$1/\Lambda^2$	χ^2/N_D
νW_2^p	273	0.0053±0.0003	1.30	0.0060±0.0004	1.32
νW_2^d	256	0.0048±0.0002	1.42	0.0055±0.0003	1.44
νW_2^n	256	0.0038±0.0009	1.26	0.0042±0.0011	1.26

(b) $W \geq 2.6$ GeV

data	N_D	$h(Q^2) = 1-2Q^2/\Lambda^2$		$h(Q^2) = (1+Q^2/\Lambda^2)^{-2}$	
		$1/\Lambda^2$	χ^2/N_D	$1/\Lambda^2$	χ^2/N_D
νW_2^p	193	0.0052±0.0005	1.28	0.0061±0.0007	1.28
νW_2^d	183	0.0058±0.0004	1.45	0.0067±0.0006	1.48
νW_2^n	183	0.0062±0.0012	1.28	0.0069±0.0017	1.28

of $W \gtrsim 2.0$ and $W \gtrsim 2.6$, the χ^2 for these fits is better than the χ^2 of the ω-scale-breaking fits represented in Table (XX). Adequate fits were obtained even for the deuteron data. For $W \gtrsim 2$ GeV, the values of $1/\Lambda^2$ for both linear and propagator scale-breaking fits are, for the proton, within one standard deviation of the corresponding values of $1/\Lambda_2^2$ reported in Table (XVII). The extracted quantities νW_2^p, νW_2^d, and νW_2^n clearly do not scale in ω'. Even when the kinematic range for the fits is limited to $W \gtrsim 2.6$ GeV, the coefficients $1/\Lambda^2$ are not consistent with zero. No significant conclusions can be made about the relative degrees of scale-breaking of the present data for νW_2^p and νW_2^n other than that the breaking is similar for both.

In conclusion, we have made fits of the form $\nu W_2 = f(\xi)h(Q^2)$ to the νW_2^p and νW_2^n data using both scaling variables $\xi = \omega$ and $\xi = \omega'$. Both linear and propagator scale-breaking forms allow good fits to these data. In the case of $\xi = \omega$, the proton data must be restricted to $W \gtrsim 2.6$ GeV in order to obtain a good fit. Adequate fits to νW_2^d can be obtained only for $\xi = \omega'$. Statistically significant scaling violations are observed for fits in either scaling variable to νW_2^p, νW_2^d, and νW_2^n. Over the range of Q^2 included in these tests ($2.0 \lesssim Q^2 \lesssim 20.0$ GeV^2), we observe 33-40% deviations from scaling in ω and 14-22% deviations from scaling in ω'. No conclusive evidence can be found for different scaling deviations for the neutron and proton. Scale-breaking fits the variable $\xi = \omega'$ provide better fits to the proton data than fits with exact scaling in ω_A.