IV. DATA ANALYSIS

IV.A. Introduction

After each experiment, the data from the magnetic tapes were reduced in three major steps to differential cross sections for inelastic e-p and e-d scattering. In the first step, or Pass-I, electron events were separated from background events, and histograms of these events were compiled for each experimental run. Cross sections were calculated from the histogram data in Pass-II, and empty target, π^0 decay, and pair-production contributions were subtracted to yield the raw e-p and e-d cross sections. Corrections for radiative effects were applied in the third step, yielding the final inelastic e-p and e-d cross sections. Event and run information were read from magnetic tape in Pass-I, sorted, and stored in condensed form on magnetic disk storage. Subsequent data analysis programs communicated only with this disk.

Much time and effort were devoted to finding and correcting systematic shifts in the data. As the statistical accuracy was frequently better than 2%, systematic errors were often the dominant uncertainty in the cross section. Of particular interest were systematic shifts between measurements at different scattering angles for the same (ν, Q^2), which could seriously affect the structure-function separations and the comparisons of the proton and deuteron measurements.

All known effects of order ±0.1% were consequently included in the analysis.

IV.B. Event Analysis

In the Pass-I analysis of both experiments electron events were distinguished from background events, and were sorted according to scattering angle and momentum, by their signatures in the various detectors. The ideal electron event would show a coincidence between a front and a rear trigger counter, and have at least one counter firing in each row of counters in the two hodoscopes. It would produce a signal in the CT counter that either was above discriminator threshold (in experiment A) or above a cut applied to the CT pulse-height spectrum (in experiment B). The electron usually began to shower in the initial radiator, in which case the signal in each of the three DX counters would at least be above the lower cut on the DX pulse-height spectrum. The signals in the TS and TA counters were also above the electron cuts for those pulse-height spectra. Not all the above criteria were required of every electron event used for cross section calculations. Only those criteria which were necessary to make a clean separation between electron events and background events were required in a given experimental run.

The angle and momentum of a scattered particle were determined from the signatures in the two hodoscopes. The minimum signature required was a single counter fired in each of the two hodoscopes. Events without at least one counter fired in each hodoscope were considered to have fallen outside the spectrometer acceptance and were rejected. For the remaining events, the pattern of counters fired in either hodoscope fell into one of three categories.(20, 40) There were single-track and double-track events, corresponding to one or two clearly definable tracks in either hodoscope and ambiguous events, which had more than two tracks or an undecodable pattern of counters fired. Events were grouped into four classes(20) according to their signatures in the two hodoscopes: class 1 had a single track in each hodoscope, and class 2 had a single track in one hodoscope and a double track in the other; class 3 had a double track in both hodoscopes and class 4 had an ambiguous pattern in either hodoscope. The angle and momentum of events in class 1 were unambiguously calculated from the horizontal and vertical position of the single track. In a typical experimental run, 95% of the events fell into class 1. Electron events in all four classes were accepted in the analysis but only events from class 1 could be used to determine the distribution of events in angle and momentum for a given run. These two-dimensional distributions, which were 20 x 54 arrays called "P-θ planes", were frequently necessary in later analysis for those runs where the cross section varied sharply over the ranges of θ and E' covered by the spectrometer acceptance. For these runs, the events in classes

2-4 provided correction factors (see section IV.C) for the cross sections calculated from the events in class 1. The TA and DX spectra were also found to correlate with the event classes described above.

The event signatures in the remaining detectors provided the basis for discrimination of electron events from background(26):

C1 the event had a front-rear trigger coincidence;

TA the event had a signal above cut in the total absorption counter;

CT the event had a signal above discriminator threshold (experiment A) or above cut (experiment B) in the gas Cerenkov counter;

TS the event had a signal above (low) cut in the truncated shower counter;

DX the event had a signal above (low) cut in all three DX counters.

Two additional event signatures, DXH and TSH, required signals above the high cut in all three DX counters or above the high cut (experiment B only) in the TS counter. Events lacking a few of these signatures (except C1 and TA) could still be classified as electron events, as described below.

Combinations of these event signatures formed the definitions of good electron events used in the analysis of the two experiments. All electron events were required to have a fast trigger coincidence and a signal above cut in the TA counter. Good signatures in the CT, TS, and DX counters were required as necessary to reject pion backgrounds. The ten such definitions, or "tribes", used in the analyses of both experiments are given in Table (II). The tribes are listed roughly in order of increasing efficiency for the rejection of pion backgrounds. The electron-detection efficiency corresponding to each tribe is estimated for E' = 2 GeV to E' = 8 GeV in both experiments. Deadtime losses are included in these estimates. Two additional tribes, which required a TSH signature instead of a TS in tribes 4 and 7, were used in the analysis of experiment B, but not experiment A. For each experimental run, the number of events in each tribe was stored on disk along with TA pulse-height spectra and P-θ planes corresponding to each tribe. Pion contamination of the electron sample from each tribe could be estimated and subtracted by examining visually the appropriate TA spectrum. For the calculation of the cross section in a given run, we used the events in that tribe which had the highest electron-detection efficiency, yet provided a clean separation of the electron and pion peaks in the TA spectrum. Pion contamination of the selected tribe was never more than 1.5% of the total yield of electrons, and the error from the pion subtractions was never more than 0.5% of the cross section. Other pertinent information, such as scaler, charge monitor, and target density data, were stored on disk in Pass-I.

Table II. Event tribes

Tribe #	Tribe signature	Efficiency A (%)	Efficiency B (%)
1	Cl - TA	96 - 99	96 - 99
2	Cl - TA - CT	83 - 88	94 - 97
3	Cl - TA - DX	58 - 80	58 - 80
4	Cl - TA - TS	86 - 89	86 - 97
5	Cl - TA - DXH	37 - 64	37 - 64
6	Cl - TA - CT - DX	51 - 71	57 - 78
7	Cl - TA - CT - TS	75 - 79	85 - 95
8	Cl - TA - CT - DXH	32 - 57	36 - 63
9	Cl - TA - CT - TS - DX	48 - 67	54 - 77
10	Cl - TA - CT - TS - DXH	31 - 55	36 - 62

IV.C. Run Combination

Differential cross sections were calculated (in units of cm^2/steradian-GeV) for each full target run according to the formula

$$\frac{d^2\sigma}{d\Omega\, dE'} = \frac{N_{tr}}{N_{in}(\Delta\Omega\, \Delta P)\ell_t \rho_t} C_A\left(\prod_i C_i\right) \qquad \text{(IV.1)}$$

where N_{in} is the number of electrons incident during the run and N_{tr} is the electron yield in any of the tribes described in the previous section. The appropriate spectrometer acceptance $\Delta\Omega\Delta P$ (in sterad-GeV) was used (see below) while ℓ_t and ρ_t are the target length (in cm) and density (in nuclei/cm^3) at 21.0°K. The averaging correction factor C_A took into account the kinematic variation of the cross section across the spectrometer acceptance and adjusted the cross section to its appropriate value at the quoted central values of E' and θ. The factors C_i correct for electrons lost in measurement or excluded in the analysis. Empty target and positron cross sections were calculated in a similar manner and subtracted in Pass-II to yield the raw cross sections for inelastic e-p and e-d scattering.

Three definitions of the acceptance were used in the calculation of cross sections. For a fraction of runs in both experiments, the cross sections were calculated using the total electron yields and the full spectrometer acceptance. In this case the deviation from unity of the averaging correction factor C_A, calculated from the P-θ planes of single track events in the selected tribe, ranged from 1% to 10%.

Another definition was employed for some runs in experiment B with sufficiently large numbers of events. Here the full acceptance was divided into four segments in E' in order to provide additional information about the E'-dependence of the cross section. The cross section was calculated for each segment using the separate electron yields and the acceptance $\Delta\Omega\Delta P$ of each segment. The averaging correction factor here corrected mainly for the finite angular acceptance of each segment and its deviation from unity was generally less than 3%.

The third definition was used in a subset of runs that overlapped in E' and provided continuous spectra of hydrogen, deuterium, and empty target data for $W \leq 2$ GeV. These were at low E in the 18° measurements of experiment A and at all values of E in the 15°, 19°, and 26° measurements of experiment B. For these runs, events with single tracks in both hodoscopes were binned according to their missing energy $E/\eta - E'$ where

$\eta = 1 + \frac{2E}{M}\sin^2\frac{\theta}{2}$. Fine-mesh cross sections were calculated for each missing energy bin, typically 8-10 MeV wide, using the yields in the 20 x 54 bins of the P-θ plane stored on disk for the selected tribe and the acceptances $\Delta\Omega\Delta P$ of the individual P-θ bins. Here the deviation from unity of the averaging correction factor was also less than 3%. In all three methods of cross section calculation, the experimentally measured variation of the cross section, as determined from the P-θ plane of the selected tribe, was always used in the calculation of the averaging correction factor C_A. Systematic uncertainty in C_A was never more than 1% and usually much less.

In Table (III) are listed the other correction factors that were applied to the measured yields, along with typical values and systematic uncertainties. The correction factors fell into three categories: those that were (a) independent of, (b) related to the individual tribe definitions, or (c) applied only when a subset of the full spectrometer acceptance was used to calculate a cross section.

Correction factors a.1 - a.6 were always applied to the measured yields when calculating a cross section. Correction factor b.1 to b.5 were applied only when the selected tribe required a good signature in the corresponding counter. Embedded in these correction factors are corrections for the singles-rate dependence of the counter efficiencies mentioned in section III. Correction factors c.1 - c.3 were applied only when a limited segment of the P-θ plane was used to calculate a cross section. The factors c.2 and c.3 corrected for an observed variation of the CT and DX counter efficiencies with position in the P-θ plane. More detail about these correction factors may be found in the references. (20, 26)

Contributions to the full target cross sections from electron scattering in the target cell walls were estimated from the electron yields in experimental runs with the "thick"

Table III. Correction factors

Factor	Corrects for	Typical value	Uncertainty (%)
a. 1	Computer deadtime	1.00 - 1.40	0.0
a. 2	Trigger deadtime	1.00 - 1.01	0.0
a. 3	Trigger inefficiency[a]	1.00 - 1.05	0.0 - 0.5
a. 4	Cl deadtime	1.00 - 1.03	0.0 - 0.1
a. 5	TA inefficiency	1.01	0.2
a. 6	Target density fluctuation		
	a) fan on	0.99 - 1.00	0.3
	b) fan off[a]	1.01 - 1.30[a]	0.4 - 0.6[a]
b. 1	CT inefficiency	1.12 - 1.15[a]	0.4
b. 2	DX inefficiency	1.24 - 1.64	0.3 - 0.5
b. 3	DXH inefficiency	1.54 - 2.61	0.3 - 0.5
b. 4	TS inefficiency	1.11[a]	0.2 - 0.3
b. 5	TSH inefficiency[b]	1.08 - 1.38[b]	0.3[b]
b. 6	Residual pion background	0.98 - 1.00	0.0 - 0.5
c. 1	Non-single track events	1.00 - 1.15	0.2
c. 2	CT inhomogeneity	0.95 - 1.20[a]	0.5
c. 3	DX inhomogeneity	0.95 - 1.05[a]	0.5

[a] Experiment A only
[b] Experiment B only

empty replica target in the beamline. Empty target cross sections were calculated from equation (IV.1) using the measured yields and the appropriate full target densities. After suitable normalization for the ratios of cell wall thicknesses, these empty target cross sections were subtracted from the full target cross sections. In those kinematic regions where continuous spectra had been measured, the empty target cross sections were first fit by a polynomial. Empty target cross sections were then computed from this fit for each missing energy bin, and subtracted from the full target cross section for that bin. In experiment A, the empty target cross sections were typically 6-8% of the hydrogen and 4-5% of the deuterium full target cross sections. In experiment B, they were respectively 4-5% and 3-4% of the hydrogen and deuterium full target cross sections.

Electron backgrounds from pair production processes, primarily Dalitz decays of neutral pi-mesons photoproduced or electroproduced in the target, were determined by reversing the spectrometer polarity and measuring the yield of positrons for the same E' and θ. Positron cross sections were calculated from equation (IV.1) using these measured positron yields and subtracting the cell-wall contributions in the manner described above. These positron cross sections were also subtracted from the full target cross sections to yield the raw cross sections for inelastic e-p and e-d scattering. In practice, these positron cross sections had to be measured only at low E', where the pair production processes contributed ∿1% or more to the full target cross section. At the very lowest E' surveyed, particularly below E' = 2 GeV in experiment A, the pair production contribution was as high as 30% of the full target cross section.

The random errors in the correction factors, which were normally a few tenths of one percent, were added in quadrature with the errors from counting statistics to give the random error in the cross section measured in each run. The random errors in the full-target, empty-target, and positron cross sections were then combined in quadrature to yield the random errors in the raw e-p and e-d cross sections.

IV.D. Radiative Corrections

IV.D.1. Introduction

Corrections were applied to the raw e-p and e-d scattering cross sections to account for the effects of radiation by the electrons. This radiation could occur while the electrons were straggling in the material before or after the scattering, or as internal bremsstrahlung during the scattering process itself.(42,43) The entire radiative correction procedure for inelastic scattering can be summarized by the equation

$$\frac{d^2\sigma}{d\Omega\, dE'}(E,E',\theta) = C\left\{\frac{d^2\sigma}{d\Omega\, dE'}(E,E',\theta)_R - I_1 - I_2\right\} \qquad \text{(IV.2)}$$

where $\frac{d^2\sigma}{d\Omega dE'}(E,E',\theta)_R$ is the measured raw cross section and $\frac{d^2\sigma}{d\Omega dE'}(E,E',\theta)$ is the corrected cross section which can then be related to the structure functions according to equation (I.2). For e-p scattering, I_1 is the contribution to the raw cross section from the radiative tail of elastic e-p scattering, and is calculated directly(44) from the well-known proton form factors.(29) For e-d scattering, I_1 is the contribution to the raw cross section from radiative tails of elastic and quasi-elastic e-d scattering. For both e-p and e-d scattering, the quantity I_2 is the contribution to the raw cross section from radiative tails of inelastic scattering processes. A direct calculation of I_2 would presume a knowledge of the structure functions of the proton and deuteron over large kinematic ranges. As these structure functions had not yet been measured and were assumed to be unknown a priori , a model-independent unfolding procedure(43, 45, 46) involving all the measured cross sections at the same angle was used to calculate I_2. The factor C corrects for radiative processes that cause electrons scattered at (E,E',θ) to fall outside the spectrometer momentum acceptance, reducing the measured electron yield. The calculation of I_1, I_2, and C is discussed in the following sections. Exact expressions used in these calculations are given in Appendix 2.

IV.D.2. Radiation Lengths

The quantities I_1, I_2, and C are functions of the amount of material, expressed in radiation lengths, in the path of the incident and scattered electrons. The material before scattering included a thin vacuum separation window, the target cell wall, and, on the average, one half the target liquid. The material after scattering and before the spectrometer vacuum included the remaining target liquid, the target cell wall, the aluminum scattering chamber window, a few mylar windows, and about 54 inches of helium gas at 1 atmosphere. There was a slight angle dependence of the thickness in radiation lengths of target material traversed by the scattered electron. The average thickness in radiation lengths(47) of material before scattering, t_B, and of material after scattering, t_A, used in the radiative corrections of the two experiments are given in table (IV).

IV.D.3. Elastic e-p Radiative Tails

Radiative tails I_{1p} from elastic e-p scattering were calculated according to equation (A2.1) of Appendix 2. This expression uses the exact calculation by Tsai(44) of single photon internal bremsstrahlung. Effects of multiple-photon radiation by the recoiling proton were included in an approximate manner. The proton form factors $G_{Ep}(Q^2)$ and $G_{Mp}(Q^2)$ used in these calculations assumed the dipole form factor modified by a factor due to G. Miller(41) that is given in equation (A2.2). The elastic e-p radiative tails ranged from a minimum of 0.2% of the raw cross section near W = 2.0 GeV to 33% at the lowest E' measured at 18°.

Table IV. Radiation lengths used in radiative corrections[a]

Expt.	θ (deg)	t_B^p (10^{-2}r.l.)	t_A^p (10^{-2}r.l.)	t_B^d (10^{-2}r.l.)	t_A^d (10^{-2}r.l.)
A	18	0.4974	0.9842	0.5875	1.1174
A	26	0.4974	0.9795	0.5875	1.1137
A	34	0.4974	0.9733	0.5875	1.1094
B	15	0.9590	1.3937	1.1223	1.5584
B	19	0.9590	1.3886	1.1223	1.5522
B	26	0.9590	1.3757	1.1223	1.5370
B	34	0.9590	1.3551	1.1223	1.5131

[a] uses radiation lengths for the various materials as given in Ref. 4

IV.D.4. Elastic and Quasi-elastic e-d Radiative Tails

For the case of inelastic e-d scattering, the quantity I_1 contains contributions from the radiative tails from elastic and quasi-elastic e-d scattering. The elastic e-d radiative tail was calculated in a manner identical to the elastic e-p radiative tail using deuteron form factors calculated from the Hamada-Johnston wave function.(48, 49) In general, the elastic e-d radiative tail was a negligible contribution to the raw e-d cross section.

The quasi-elastic e-d radiative tail was calculated in a method that utilized the close connection between quasi-elastic e-d scattering and the sum of elastic e-p and e-n scattering. Due to difficulties in the calculation of radiation by bound target nucleons, an exact calculation of single photon internal bremsstrahlung was not attempted. Rather, an initial approximation of the quasi-elastic tail, I_{1d}^{ER}, was obtained using an equivalent radiator technique(50) to estimate the contribution from internal bremsstrahlung. This technique is a good approximation in the soft photon limit near the quasi-elastic peak, but it is known(43) to be inaccurate at low E', where hard photon radiation dominates. Consequently, the radiative tail calculated in the equivalent radiator method was modified according to

$$I_{1d} = \frac{I_{1d}^{ER}}{I_{1p}^{ER} + I_{1n}^{ER}} \left(I_{1p} + I_{1n} \right) \qquad \text{(IV.3)}$$

The elastic e-p and e-n radiative tails I_{1p} and I_{1n} were calculated from equation (A2.1) which uses the exact formula for single photon emission(44) with the assumption of form factor scaling ($G_{Ep}^2 = G_{Mp}^2/\mu_p^2 = G_{Mn}^2/\mu_n^2$; $G_{En} = 0$) and includes modifications for the effects of multiple soft photon emission from the electrons. Radiation from the hadrons is small and was ignored. The quasi-elastic tail I_{1d}^{ER} and the elastic e-p and e-n tails I_{1p}^{ER} and I_{1n}^{ER} were calculated in the equivalent radiator method as given by equation (A2.3) of Appendix 2 and also include the effects of multiple soft photon emission. Differential cross sections for elastic e-p and e-n scattering used in these calculations were derived from the Rosenbluth equation(51) under the above assumption of form factor scaling. Differential cross sections for quasi-elastic e-d scattering were calculated from the method of Durand(52) using s- and d- state Hamada-Johnston wave functions.(48) The quasi-elastic e-d radiative tails, as calculated from equation(IV.3), ranged from 0.2% of the raw cross section near W = 2.0 GeV to 25% at low E'. They were roughly the same percentage of the raw cross section as the elastic e-p radiative tails(20), except at low E'. Uncertainties in the neutron form factor G_{Mn}, which has been measured(53) only up to a Q^2 of 5 GeV^2, had little effect upon the calculation of the quasi-elastic tail. The neutron contribution to this tail was generally less than 3% of the raw cross section, and most of this contribution arose from low Q^2 e-n scattering, for which the form factors are fairly well known.

IV.D.5. Inelastic Radiative Corrections

The cross sections $\frac{d^2\sigma}{d\Omega dE'}(E,E',\theta)$ remaining after subtraction of the elastic and quasi-elastic radiative tails were subsequently corrected for radiative processes linked to inelastic scattering. The inelastic radiative tails, which were calculated in the same manner for e-p and e-d scattering, may be expressed in the general form,

$$I_2(E,E',\theta,t) = \int_0^t \frac{dt'}{t} \int_{E_{min}}^{E} dE_1 \int_{E'}^{E'_{max}} dE_2 \times S(E,E_1,t') \frac{d^2\sigma}{d\Omega dE'}(E_1,E_2,\theta) S(E_2,E',t-t') \qquad (IV.4)$$

where $S(E_1,E_2,\tau)$ is an appropriate straggling function representing the probability that an electron degrades in energy from E_1 to E_2 in τ radiation lengths, including the effects of internal bremsstrahlung. The cross section $\frac{d^2\sigma}{d\Omega dE'}(E_1,E_2,\theta)$ is the corrected inelastic cross section, whose measurement was the purpose of these experiments. The calculation of $I_2(E,E',\theta,t)$ consequently presumes a knowledge of the corrected inelastic cross sections throughout the kinematic region $E_{min} \leq E_1 \leq E$ and $E'_{max} \geq E_2 \geq E'$. The roughly triangular regions of E-E' space surveyed in the two experiments (see Figures (2) and (3)) permitted us to calculate this integral, using interpolations and an unfolding technique described below.

In the peaking approximation(42, 43), the two dimensional integral in equation (IV.4) reduced to two one-dimensional integrals.

$$I_2(E,E',\theta,t) = I_2^B(E,E',\theta,t_B) + I_2^A(E,E',\theta,t_A) \qquad \text{(IV.5)}$$

The terms I_2^B and I_2^A correspond to radiation before and after scattering and are given explicitly by equations (A2.5) and (A2.6) of Appendix 2. The contribution to I_2 from radiation both before and after scattering is small but not negligible; it was included in these one-dimensional integrals in an approximate manner. Contributions from internal bremsstrahlung were approximated by introducing an equivalent radiator $f(k)\ \frac{\alpha}{\pi}\ (\log\frac{Q^2}{m_e^2} -1)$. The function f(k) is a spectral function in the energy k of the emitted photon; the particular form used, $f(k) = 1 - (k/E) + 0.5(k/E)^2$ is due to Allton(54) and Bjorken.(55) For the case of single photon emission, agreement of the Allton-Bjorken approach with the exact calculations of Tsai(44) was usually better than 1%, and never worse than 5%, of the inelastic radiative tail.(56) Multiple photon emission was treated in an approximate manner, while the effects of radiation by the hadrons were ignored.

At each scattering angle θ, the integrals I_2^B and I_2^A were first computed on a rectangular mesh of points (E_i,E_j') chosen to reflect the distribution of measured data. The mesh spacings ΔE and $\Delta E'$ ranged from a minimum of 10 MeV in the resonance region to a maximum of 100 MeV in the deep inelastic region.

The inelastic radiative corrections were independent of the mesh spacings used.(26) Uncorrected cross sections were calculated at each mesh point by an interpolation scheme(46) applied to the triangles of uncorrected cross sections $\frac{d^2\sigma}{d\Omega dE'}(E,E',\theta)_M$. The six or more spectra measured at each angle in experiment A were sufficient to insure the desired accuracy in the interpolations. Some trouble was encountered in extrapolating to $E < 4$ GeV in experiment A (see Figure (2)) and to $E < 10.0$ GeV at 15° and $E < 12.5$ GeV at 19° in experiment B (see Figure (3)). This difficulty increased the error in the radiative corrections only at the very lowest E' in these triangles. At 26° and 34° in experiment B, additional spectra measured earlier in experiment A were used to obviate this difficulty.

The integral equation that derives from equations (IV.2) and (IV.5)

$$\frac{d^2\sigma}{d\Omega dE'}(E,E',\theta) = C\left\{\frac{d^2\sigma}{d\Omega dE'}(E,E',\theta)_M - I_2^B - I_2^A\right\} \qquad \text{(IV.6)}$$

was solved by an unfolding technique(43, 46) applied to the mesh of uncorrected interpolations $\frac{d^2\sigma}{d\Omega dE'}(E_i,E_j',\theta)_M$ at each angle. The corrected cross section was calculated at each mesh point (E_i, E_j') starting at pion electroproduction threshold and proceding to higher invariant mass W. At threshold the inelastic radiative tail was zero, and the only correction was the factor C (see equation (A2.4)) needed to account for electrons lost from the yield. These corrected cross sections were then used to compute the inelastic radiative tail contributions to the adjacent higher W points according to equations (A2.5) and (A2.6); these were subtracted and the correction factor C applied. This differential unfolding procedure continued until the cross section had been corrected at each mesh point at that angle; these points did not generally correspond to the exact kinematic points at which the measurements were made. The mesh of corrected cross sections $\frac{d^2\sigma}{d\Omega dE'}(E_i,E_j',\theta)$ was then used to correct each uncorrected cross section $\frac{d^2\sigma}{d\Omega dE'}(E,E',\theta)_M$ measured at that angle. The inelastic radiative correction was applied in the manner of equation (IV.6). In experiment A, the total correction factor ranged from 1.18 - 1.78 at electroproduction threshold to 0.60 - 0.96 at the very lowest E' quoted. In experiment B, it ranged from 1.16 - 1.83 at threshold to 0.90 - 1.08 at the lowest E' surveyed. It was generally the same for proton and deuteron data.

IV.D.6. Treatment of Errors

Errors in the elastic and quasi-elastic radiative tails were deemed systematic and were thought not to contribute to the random error in the inelastic e-p and e-d cross sections. Sources of uncertainty in the elastic e-p radiative tails were uncertainties in the proton form factors and approximations in the treatment of multiple photon radiation. The quasi-elastic e-d radiative tails had additional uncertainties arising from uncertainties in the neutron form factors and in the modified equivalent radiator technique, and from the theoretical approximation to the quasi-elastic e-d cross section. Altogether, the uncertainty in the elastic radiative tails was estimated to be 5% of the tail, while the quasi-elastic radiative tails had an estimated 6% uncertainty. Because of the sharp variation with E' of these tails, the resultant systematic uncertainty in the inelastic cross sections ranges from 0 to 4%. For E' greater than about 2.5 GeV, however, this uncertainty is never greater than 1% of the cross section. Most of this uncertainty is not present in the ratio of deuteron to proton cross sections.

Random errors were propagated through the inelastic radiative correction procedure. Because of the interpolations needed to compute the inelastic tails, there was some correlation between the error in the inelastic radiative tail $I_2(E,E',\theta)$ and the error in the cross section $\frac{d^2\sigma}{d\Omega dE'}(E,E',\theta)_M$. In the calculation of the random error in the inelastic cross sections, we accounted for this correlation in an approximate manner.

Systematic uncertainties in the inelastic radiative corrections are believed to arise mainly from the equivalent radiator approximation and from the treatment of multiple photon processes; they are similar for e-p and e-d scattering. A second pass of the radiative corrections was made using another spectral function f(k) at each scattering angle that best approximated the exact tails(44) from single photon radiation(56); the corrected cross sections from this approach were compared with the nominal cross sections which had been calculated using the Allton-Bjorken method. Where the two methods disagreed by more than one half the random error (which only occurred at the E'< 2 GeV), no cross sections are quoted nor were they used in subsequent analyses. The systematic uncertainty in the inelastic radiative tail is believed to vary from 3% near threshold to 10% at low E'. This amounts to at most 5% of the inelastic e-p and e-d cross sections, and that only at E'<2 GeV.

Hadronic radiation was neglected in this formalism. As is known from work comparing elastic e^+p and e^-p scattering(57), the presence of hadron as well as electron radiation makes the radiative corrections for the two processes slightly different, so that, in the absence of two-photon exchange processes the difference between e^+p and e^-p cross sections puts an upper limit on the size of this contribution.

Recently measurements of deep inelastic e^+p and e^-p scattering have been performed.(58) With the above interpretation we can use their results to put an upper limit on two-photon exchange processes plus effects of hadron radiation. Their measurements of the (e^+p) - (e^-p) ratio are consistent with unity within their errors. Assuming no kinematic dependence we can combine their data to yield an overall ratio of e^+p/e^-p = 1.001 ± 0.008. Therefore we have assigned a fractional systematic error of ±0.008 to cover these unknown processes.

An estimate of the systematic uncertainty in the entire radiative correction procedure was obtained from a comparison of the procedure described here and a procedure(7,41), developed by SLAC Group A. Both procedures had been applied to the cross sections measured at 6° and 10° in experiment C. (8, 27) A comparison(27) of the two methods indicated that the cross sections obtained from the SLAC method were typically 3% larger than those obtained from our method. For a few data points at very low E' and low x the discrepancy was as large as 6%. From these comparisons with the SLAC approach we estimate the total systematic uncertainty in the cross section arising from the radiative corrections to vary as

$$\left(\frac{\Delta\sigma}{\sigma}\right)_R = 0.03 + 0.015\left(\frac{E'_{elast}(E,\theta)}{E'}\right) \qquad \text{(IV.7)}$$

Here $E'_{elast}(E,\theta)$ is the scattered electron energy corresponding to elastic e-p scattering, and $(\frac{\Delta\sigma}{\sigma})_R$ is the fractional systematic uncertainty in the corrected cross section.