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ESTIMATE OF SPIN POLARIZATION FOR THE ELECTRON STORAGE RINGS SPEAR AND PEP †

Alexander W. Chao
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

Introduction

It has been found[1,2] that the natural beam polarization produced during the process of synchrotron radiation in a high energy electron storage ring can reach 92.4% in an ideal situation. The question that remains is how strong are the various depolarization effects which, under unfavorable conditions, may significantly reduce the polarization to a lower value. In the following, we will first discuss briefly the origin of some depolarization mechanisms and then describe a matrix method[3] used to calculate the strength of the depolarization effects and obtain an accurate estimate of the expected level of polarization. Applications to SPEAR and PEP are included as examples.

Depolarization Mechanisms

In an electron storage ring the following depolarization mechanisms are considered to be important:

(1) In a storage ring without electromagnetic field errors, the closed orbit lies in the horizontal plane and the equilibrium direction of spin polarization $\hat{n}$ coincides with the direction of bending magnetic field $\hat{y}$, while in the presence of a static EM field error, this is generally not valid. Since the radiative polarization is built up along $\hat{y}$, and only the projection along $\hat{n}$ survives in the storage ring, the net polarization is reduced by a cosine factor $\hat{n}\cdot\hat{y}$. This reduction of polarization[4] is particularly important if the spin precession tune $\nu = \gamma\left(\frac{g-2}{2}\right)$ is close to an integer, where γ is the Lorentz factor and g is the gyromagnetic ratio of an electron.

(2) As an electron emits a photon during synchrotron radiation, it receives a recoil perturbation which excites the horizontal-betatron, vertical-betatron and longitudinal-synchrotron oscillations (with oscillation tunes ν_x, ν_y and ν_s, respectively) in its subsequent orbital motion. The electron then sees a perturbing electromagnetic field, which is modulated by these orbital oscillations, causing its spin to precess accordingly. Summing over the uncorrelated photon-emission events results in a diffusion of spin direction away from the polarization direction $\hat{n}$. This diffusion of polarization due to quantum emissions becomes most serious when one of the sideband resonance conditions $\nu \pm \nu_{x,y,s}$ = integer is satisfied since then the spin motion couples strongly to the orbital motion.

(3) Perturbations induced by nonlinear electromagnetic fields such as the magnetic field in a sextupole magnet or the EM field caused by the beam-beam interaction can drive nonlinear depolarization resonances at $\nu + k_x\nu_x + k_y\nu_y + k_s\nu_s = k$, where $k_{x,y,s}$ and k are integers.

The integer resonances driven by mechanism (1) occur when the beam energy is equal to an integer multiple of $\left(\frac{2}{g-2}\right)m_ec^2 = 440.65$ MeV. As will be seen later, the resonance width around those resonant energies is typically less than 1 MeV. By staying a few MeVs away from multiples of 440.65 MeV, the integer depolarization resonances can be easily avoided. Sideband resonance may have a width of several ten's of MeV. In particular, since ν_s is usually quite small, the two synchrotron sidebands often overlap with the integer resonance to form a single resonance band. Having six sidebands in every 440.65 MeV interval, the energy range within which significant depolarization occurs occupies a sizeable fraction of the total energy range. It is therefore necessary to be able to change the tune values $\nu_{x,y,s}$ during machine operation in order to avoid some of these sideband resonances. The important problem of depolarization due to nonlinear beam-beam collisions has been treated in Ref. 5, so we will restrict attention to the linear mechanisms below.

The Matrix Formalism

The reduction of polarization due to mechanism (1) can be readily evaluated. Knowing the closed orbit of the beam trajectory and the EM field everywhere around it, the equilibrium direction of polarization $\hat{n}$ is given by the spin direction which closes on itself as the electron circulates around one revolution. The reduction factor for polarization is then obtained essentially by averaging the cosine factor $\hat{n}\cdot\hat{y}$ over all bending magnets.

To obtain the depolarization rate of mechanism (2), one needs an accurate description of how the spin and orbital degrees of freedom of an electron couple among themselves.[6] It is well known that in order to fully describe the orbital motion of an electron, one needs six canonical coordinates $(x, x', y, y', z, \delta)$, where x, y and z are the horizontal, vertical and longitudinal displacements of a particle relative to the beam trajectory; $\delta = \Delta E / E_0$ is the relative energy error. In the linear approximation, the transformations of the six-dimensional vector are described by 6×6 matrices.[7] It turns out that spin motion can be conveniently included by adding two more spin coordinates (α,β) to form 8-dimensional vectors, $X = (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) = (x, x', y, y', z, \delta, \alpha, \beta)$, and by generalizing the 6×6 transformations to 8×8. The quantities α and β are the two components of the electron's spin vector perpendicular to $\hat{n}$. The degree of depolarization is specified by $\frac{1}{2}(\alpha^2 + \beta^2)$.

The 8×8 matrix, T, which transforms X for one revolution, has four eigenstates: the x, y,z - states and the spin state; each eigenstate being defined by a complex conjugate pair of eigenvectors of T. Any perturbation to the vector X, such as the recoil perturbation resulted from emitting a photon, is then projected onto the four eigenstates; the projections onto the x,y,z - states give the contribution of this perturbation to the corresponding x,y,z - emittances, while the projection onto the spin state gives the contribution to spin diffusion. Since the very same physical process of quantum emissions drives both the spin diffusion and the beam emittances of the electron beam, the matrix formalism offers the possibility of accurately evaluating the spin diffusion rate and the 21 beam distribution parameters $\langle x_ix_j\rangle$ $i,j = 1,\ldots,6$, in one concise package.

Results

A computer code, SLIM, has been prepared for the polarization and beam distribution calculations. The thin - lens approximation has been used. This is not a fundamental limitation, and is accurate enough for our purpose. The beam - line elements for the ideal SPEAR and PEP latices include horizontal bending magnets, quadrupole magnets, sextupole magnets, rf cavities and drift spaces. The strong quadrupoles in the interaction regions and all bending magnets are split

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in halves to improve accuracy of calculation. Without field errors, the ideal lattice produces an equilibrium polarization of 92.4%. To simulate field errors in these machines, we introduce a distribution of vertical orbit kickers. The resulting vertical closed orbit makes sextupoles behave like skew quadrupoles and quadrupoles behave like additional vertical kickers. In the presence of these field errors, depolarization mechanisms (1) and (2) are driven.

Fig. 1 shows the expected equilibrium polarization

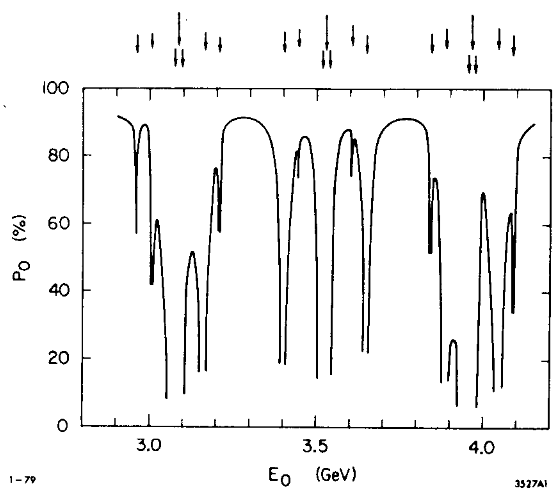

Fig. 1. A typical example for SPEAR showing the expected polarization P_0 versus the beam energy E_0.

P_0 for SPEAR as a function of beam energy E_0 for a given distribution of vertical kickers. The lattice used is specified by the lattice parameters: $\nu_x = 5.28$, $\nu_y = 5.18$, $\nu_s = .022$, $\beta_x^* = 1.2$m, $\beta_y^* = .10$m and $\eta_x^* = 0$, where β_x^*, β_y^* and η_x^* are the horizontal beta-function, vertical beta-function and the energy dispersion function at the interaction points. The strengths of the vertical kickers are normalized such that the rms closed orbit is $\Delta y_{rms} = 1.2$mm. For a different strength of the same kicker distribution, the depolarization strength scales roughly quadratically with Δy_{rms}. Resonance locations are indicated by arrows on the top of Fig. 1. Each integer resonance is surrounded by six sideband resonances, The integer resonances and the two associated synchrotron sideband resonances overlap and are shown as single depolarization dips in Fig. 1. The spin tune width of the region covered by an integer resonance alone is typically less than 10^{-3}. Outside this region, $\hat{n}$ is very close to the magnetic field direction $\hat{y}$ everywhere around the ring. For different distributions of verticle kickers whose strengths are normalized so that the produced orbit distortion has $\Delta y_{rms} = 1.2$mm, the behavoir of P_0 vs E_0 does not change very much from that shown in Fig. 1.

A laser back-scattering experiment has been performed at SPEAR to measure the polarization of a single positron beam. The results are in good agreement with the prediction using the matrix method. It should be mentioned that, for colliding beams, the depolarization mechanism (3) driven by the beam-beam interaction may not be negligible near a nonlinear resonance. According to Ref. 5, however, it is not difficult to avoid those resonances if the beam intensity is below the beam-beam limit.

A similar calculation for PEP is shown in Fig. 2.

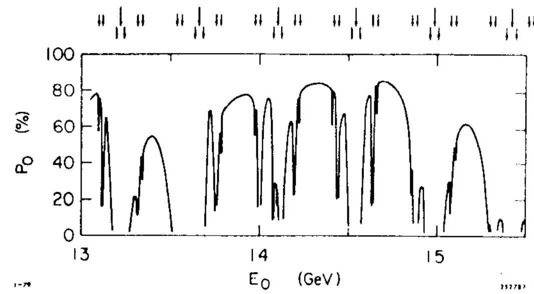

Fig. 2, A typical example for PEP showing the expected polarization P_0 versus the beam energy E_0.

The configuration used has $\nu_x = 21.78$, $\nu_y = 18.72$, $\nu_s = .05$, $\beta_x^* = 3.0$m, $\beta_y^* = .11$m and $\eta_x^* = -.51$m. The field errors are again generated by a distribution of vertical kickers which produce a vertical closed orbit distortion with $\Delta y_{rms} = 1.2$mm. More numerical results for PEP can be found in Ref. 9.

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