

THE $\Delta I = \frac{1}{2}$ RULE AND VIOLATION OF CP
IN THE SIX QUARK MODEL*

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ABSTRACT

The consequences for K^0 decay of the six quark model with its natural possibility of incorporating a CP violating phase, δ , are investigated when a particular mechanism to give the $\Delta I = \frac{1}{2}$ rule is operative. The most important result is that the CP violation parameter ϵ' is much larger than in previous analyses, and hence the theory has predictions for $K^0 \rightarrow \pi\pi$ decay which are experimentally distinguishable from those of the superweak model.

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In models of the weak interactions employing the gauge group [1] $SU(2) \times U(1)$ and with three or more left-handed doublets, CP violation arises automatically: In addition to real Cabibbo-like mixing angles, there are one or more complex phases which cannot be swept into the fermion fields and which violate CP. Such is the case in a six quark model, proposed for precisely this reason by Kobayashi and Maskawa [2], where there are three real angles, θ_i with $i = 1, 2, 3$, and one CP violating phase, δ . In this model the left-handed doublets are

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L, \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L, \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L$$

where, with a standard choice of the quark fields [2],

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 c_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \quad (1)$$

and $c_i = \cos \theta_i$, $s_i = \sin \theta_i$.

Such a model has become popular with the increasing evidence for a fifth lepton [3], τ , and more recently for a fifth quark, b , as a constituent for the upsilon family [4] of particles. A sixth quark, t , is expected, being necessary for the (generalized) GIM mechanism [5] as well as the cancellation of anomalies [6]. The six quark model with $\delta \neq 0$ has been systematically studied by Ellis et al. [7] as the origin of CP violating effects in K^0 decay and elsewhere. They found that the contribution to the $K^0 - \bar{K}^0$ mass matrix corresponding to fig. 1, with the mixing expressed in eq. (1), leads to CP violation in K^0 decay in accord

with experiment and in close approximation to predictions of the super-weak model [8] for common K decays.

There is another feature of strange particle decays, and in particular of K decays, which has detectable consequences for the pattern of CP violation, but whose effects so far have not been taken adequately into account. This is the $\Delta I = \frac{1}{2}$ rule. It had been hoped that strong interaction effects at short distances would sufficiently enhance the $\Delta I = \frac{1}{2}$ portion of the non-leptonic weak interaction to explain the $\Delta I = \frac{1}{2}$ rule. However, detailed calculations using quantum chromodynamics (QCD) do not lead to a large enough enhancement [9].

It is claimed [10] that the answer lies in the amplitudes shown in fig. 2, sometimes called "Penguin diagrams". Since the gluon (g) carries no isospin, these amplitudes are pure $\Delta I = \frac{1}{2}$. Although they appear at first glance to give a smaller contribution to the effective weak Hamiltonian than the lowest order current-current term, when their matrix elements in strange particle decays are evaluated using light current-quark masses very large contributions to decay amplitudes result. An extensive analysis [10,11,12] of both strange meson and baryon decays supports the hypothesis that the magnitude of the amplitudes and the $\Delta I = \frac{1}{2}$ rule are understandable on this basis.

However, when the full six quark model weak current with the mixing of eq. (1) is used to calculate the "Penguin diagrams" in fig. 2, they too will give a CP violating amplitude. In this paper we evaluate this contribution to K^0 decay and show it is of the same order as that arising from the standard analysis of fig. 1 for the CP violation parameter ϵ . More importantly, the parameter ϵ' is predicted to be much larger than

previously in the six quark model -- sufficiently large so as to permit distinguishing the predictions from those of the superweak model.

To be quantitative we first recall the contribution to the mass matrix given by fig. 1. Using Eq. (1) with s_1 and s_3 treated as small quantities [13], direct calculation gives [7]

$$\frac{\text{Im } M_{12}}{\text{Re } M_{12}} = 2s_2 c_2 s_3 \sin \delta P(\theta_2, \eta) = \epsilon_m \quad (2a)$$

with

$$P(\theta_2, \eta) = \frac{s_2^2 \left(1 + \frac{\eta \ln \eta}{1-\eta}\right) - c_2^2 \left(\eta + \frac{\eta \ln \eta}{1-\eta}\right)}{c_2^4 \eta + s_2^4 - 2s_2^2 c_2^2 \frac{\eta \ln \eta}{1-\eta}}, \quad (2b)$$

where $\eta = m_c^2/m_t^2$ and M_{12} is the element of the $K^0 - \bar{K}^0$ mass matrix defined by [14]

$$M_{12} = \langle K^0 | H_W | \bar{K}^0 \rangle + \sum_n \frac{\langle K^0 | H_W | n \rangle \langle n | H_W | \bar{K}^0 \rangle}{m_{K^0} - m_n} + \dots \quad (3)$$

On the other hand, direct calculation of fig. 2 gives a contribution to the effective weak Hamiltonian density of the form

$$\begin{aligned} \mathcal{H}_W^{(\text{Penguin})} = & \frac{G}{\sqrt{2}} \frac{\alpha_s}{12\pi} \left[s_1 c_2^2 \ln \left(\frac{m_c^2}{\mu^2} \right) + i s_1 s_2 c_2 s_3 \sin \delta \ln \left(\frac{m_c^2}{\mu^2} \right) \right. \\ & \left. + s_1 s_2^2 \ln \left(\frac{m_t^2}{\mu^2} \right) - i s_1 s_2 c_2 s_3 \sin \delta \ln \left(\frac{m_t^2}{\mu^2} \right) \right] \\ & \cdot \left(\bar{s} \gamma_\mu (1 - \gamma_5) \lambda^\alpha d \right) \cdot \left(\bar{u} \gamma^\mu \lambda^\alpha u + \bar{d} \gamma^\mu \lambda^\alpha d + \dots \right) \\ & + \text{h.c.} \end{aligned} \quad (4)$$

In eq. (4), μ is a typical hadronic mass, α_s the strong interaction

fine structure constant, and u, d, \dots represent the corresponding quark fields in both ordinary space-time and in color space, where the matrices λ^α operate. The first two terms in square brackets arise from fig. 2 with a c -quark loop, whereas the last two involve a t -quark. Terms involving m_s and m_d , as well as higher powers of the sines of the small Cabibbo-like angles θ_1 and θ_3 have been neglected. Some effects due to higher order QCD diagrams may be treated by renormalization group techniques. Inasmuch as we will only be interested in the relative size of the real and imaginary terms in eq. (4), only differences in the magnitude of these multiplicative corrections for different terms need concern us. In the following we shall ignore differences between the leading logarithmic corrections to the terms in square brackets in eq. (4), just as they have also been ignored for the terms in the numerator and denominator of eq. (2). These are not expected to effect the character of our results.

When $\delta = 0$ the effective "Penguin" Hamiltonian is responsible for a fraction f of the real decay amplitude $A_0^{(\delta=0)}$ for $K^0 \rightarrow 2\pi$ ($I=0$) defined by

$$\langle 2\pi (I=0) | H_W (\delta=0) | K^0 \rangle = A_0^{(\delta=0)} e^{i\delta_0}, \quad (5)$$

where δ_0 is the $I=0$ strong interaction $\pi\pi$ phase shift. The claim [10,11,12] from detailed analysis of K^0 decays is that f is not far from unity. The total amplitude (apart from final state $\pi\pi$ strong interactions) when $\delta \neq 0$ is then

$$A_0 = A_0^{(\delta=0)} - ifA_0^{(\delta=0)} s_2 c_2 s_3 \sin\delta \frac{\left(\ln\left(\frac{m_t^2}{\mu^2}\right) - \ln\left(\frac{m_c^2}{\mu^2}\right) \right)}{\left(c_2^2 \ln\left(\frac{m_c^2}{\mu^2}\right) + s_2^2 \ln\left(\frac{m_t^2}{\mu^2}\right) \right)}$$

$$\approx A_0^{(\delta=0)} e^{i\xi} \quad (6)$$

with the small quantity

$$\xi = f s_2 c_2 s_3 \sin \delta \left(\frac{\ln \eta}{\ln \left(\frac{m_c^2}{\mu^2} \right) - s_2^2 \ln \eta} \right) . \quad (7)$$

However, the standard convention in analyzing CP violation in K^0 decays is to take A_0 real [14]. This may be accomplished by redefining the phases of the K^0 and \bar{K}^0 states [15]:

$$\begin{aligned} |K^0\rangle &\rightarrow e^{-i\xi} |K^0\rangle \\ |\bar{K}^0\rangle &\rightarrow e^{i\xi} |\bar{K}^0\rangle \end{aligned} ,$$

so that $A_0^{(\delta=0)} e^{i\xi} \rightarrow A_0^{(\delta=0)}$ and $\text{Im}\langle K^0|M|\bar{K}^0\rangle \rightarrow \text{Im}\{e^{2i\xi}\langle K^0|M|\bar{K}^0\rangle\} \approx \text{Im}\langle K^0|M|\bar{K}^0\rangle + 2\xi \text{Re}\langle K^0|M|\bar{K}^0\rangle$. Therefore as a result

$$\frac{\text{Im } M_{12}}{\text{Re } M_{12}} \rightarrow \frac{\text{Im } M_{12}}{\text{Re } M_{12}} + 2\xi = \epsilon_m + 2\xi . \quad (8)$$

The presence of the 2ξ term is not new -- even the notation here is that of ref. [7]. What is new is that ξ as given in eq. (7) is not negligible compared to ϵ_m , as given in eq. (2). The whole "Penguin" contribution previously was neglected because it was guessed that the $\ln(m^2/\mu^2)$ terms in eq. (4) were instead of the form m^2/M_W^2 . Indeed, the "Penguin diagram" appears as fig. 2c in ref. [7], but its contribution to ξ is taken as small compared to ϵ_m .

How does all this effect the parameters of CP violation in $K^0 \rightarrow \pi\pi$ decay? In standard notation [14],

$$\epsilon = i \frac{\text{Im } \Gamma_{12} + i \text{Im } M_{12}}{\frac{\Gamma_S - \Gamma_L}{2} + i(m_S - m_L)} , \quad (9)$$

and since experimentally [16] $|\frac{1}{2}(\Gamma_S - \Gamma_L)| \approx |m_S - m_L|$ and $|\text{Im } \Gamma_{12}/\text{Im } M_{12}| \lesssim 1/10$, we neglect $\text{Im } \Gamma_{12}$ (due to CP violating phase differences between the $K^0 \rightarrow \pi\pi$ ($I=0$) decay and other modes) to obtain

$$|\epsilon| \approx \frac{|\text{Im } M_{12}|}{\sqrt{2} |m_S - m_L|} . \quad (10)$$

Recalling the result of our phase redefinition, and using $2\text{Re } M_{12} = m_S - m_L$, it follows that [17]

$$|\epsilon| = \frac{|\epsilon_m + 2\xi|}{2\sqrt{2}} = \left| \frac{s_2 c_2 s_3 \sin \delta}{\sqrt{2}} \right| \cdot \left| P(\theta_2, \eta) + f \frac{\ln \eta}{\ln\left(\frac{m^2}{\mu^2}\right) - s_2^2 \ln \eta} \right| . \quad (11)$$

The CP violation parameter [14]

$$\epsilon' \equiv \frac{i}{\sqrt{2}} e^{i(\delta_2 - \delta_0)} \frac{\text{Im } A_2}{A_0} , \quad (12)$$

where A_0 and A_2 are the $K^0 \rightarrow \pi\pi$ decay amplitudes and δ_0 and δ_2 the strong interaction $\pi\pi$ phase shifts for final isospin 0 and 2, respectively.

Neither the CP violation arising from the mass matrix (eq. (2)) nor that from the "Penguin diagram" (eq. (4)) can contribute to the $\Delta I = 3/2$ transition involved in the amplitude A_2 . However, by virtue of the redefinition of the K^0 and \bar{K}^0 phases to make A_0 real, A_2 picks up a phase $e^{-i\xi}$ relative to A_0 . Hence,

$$|\epsilon'| \approx \frac{1}{\sqrt{2}} |\xi| \left| \frac{A_2}{A_0} \right| . \quad (13)$$

Experimental information on K^+ and K^0 decays yields [14] $|A_2/A_0| \approx 1/20$.

Numerically, we find that the two contributions to ϵ from the mass matrix and "Penguin diagrams" are comparable. For example, with $m_c = 1.5$ GeV, $m_t = 15$ GeV, $\mu = 1$ GeV, $\theta_2 = 15^\circ$, and $f = .75$, the two terms in the second set of absolute value signs on the right-hand-side of eq. (11) are 5.2 (from ϵ_m) and -3.1 (from 2ξ). The difference in sign holds for all reasonable quark masses, mixing angles, and values of f . Since $\sin \delta$ is adjustable to fit the experimental value of $|\epsilon|$, there is no critical test of the theory at this point.

What is critical, and changed from previous estimates [7], is ϵ'/ϵ , which is independent of δ and θ_3 . From eqs. (11) and (13),

$$\left| \frac{\epsilon'}{\epsilon} \right| \approx \frac{1}{20} \left| \frac{2\xi}{\epsilon_m + 2\xi} \right| \quad . \quad (14)$$

With the quark masses and other parameters used above, $|\epsilon'/\epsilon| \approx 1/13$. Bigger values of θ_2 , m_c/μ , or m_t can lead to smaller values of $|\epsilon'/\epsilon|$: for example, with $m_t = 30$ GeV and all other parameters left the same $|\epsilon'/\epsilon| \approx 1/30$, while changing μ to 0.5 GeV but leaving $m_t = 15$ GeV gives $|\epsilon'/\epsilon| \approx 1/55$. The present experimental limit [16] is $|\epsilon'/\epsilon| \lesssim 1/50$.

Given the uncertainties in θ_2 , m_t , μ , and the fraction, f , of the $K \rightarrow 2\pi$ amplitude given by the "Penguin diagram," we conclude that the model is not yet ruled out by experiment. However, a quantitative increase in the accuracy of experimental determination of the CP violating $K^0 \rightarrow \pi\pi$ decay parameter ϵ' , which now seems possible [18], would be capable of distinguishing the six quark model with a CP violating phase in the mixing matrix plus any sizable fraction of the $K^0 \rightarrow 2\pi$ amplitude being due to "Penguin diagrams" from the superweak model [8] where all the CP violation in the K^0 system arises from the mass matrix and $\epsilon' = 0$.

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- [13] The signs of the quark fields may be chosen so that θ_1 , θ_2 , and θ_3 (but not necessarily δ) lie in the first quadrant and all sines and cosines of them are positive.
- [14] We assume CPT invariance and adopt the conventions of T. D. Lee and C. S. Wu, Ann. Rev. Nucl. Sci. 16, 511 (1966), except that we label the K^0 eigenstates by S and L.
- [15] We have also checked explicitly that if the phase convention on A_0 is dropped the predictions for physical observables in $K^0 \rightarrow 2\pi$ decay remain exactly the same.
- [16] See, for example, K. Kleinknecht in Proceedings of the XVII International Conference on High Energy Physics, London, 1974, edited by J. R. Smith (Science Research Council, Rutherford Laboratory, 1974), p. III-23.
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FIGURE CAPTIONS

1. Diagram contributing to the $K^0 - \bar{K}^0$ mass matrix.
2. "Penguin diagram" contributing to weak decays.

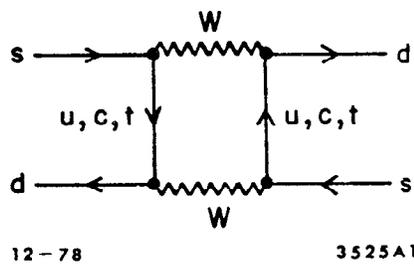


Fig. 1

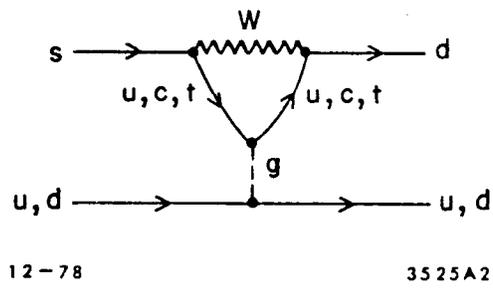


Fig. 2