

WEAK DECAYS OF NEW PARTICLES*

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I. Introduction

With the discovery of new particles in the last several years has come the opportunity to gain increased insight into the weak interactions through study of the weak decays of both heavy leptons and heavy quarks. Now we have the additional perspective of being able to compare the "new, heavy particles" with the "old, light particles", and many questions come to mind: What are the selection rules? How accurate are they? Are non-leptonic decays of heavy hadrons enhanced to the same degrees as those of strange particles? How much do heavy lepton semi-hadronic decays deviate in rate from the most naive model? etc., etc.

In parallel with work on the new particles, the evidence has been mounting that the weak interactions of the "old" quarks and leptons are described by the "standard model" of weak and electromagnetic interactions.¹ The gauge group is $SU(2) \times U(1)$ with left-handed fermions behaving as doublets under the weak isospin and right-handed fermions as singlets.²

Even for the newest member of the lepton family, the τ , if one restricts oneself to $SU(2) \times U(1)$ as the correct gauge theory, it is rather difficult to avoid assigning the τ to a left-handed doublet (with its own neutrino, ν_τ , as partner) and to a right-handed singlet.³⁻⁵ Thus the pattern of the electron and its neutrino, already duplicated by the muon and muon neutrino, is copied by nature once again. We adopt this standard picture in what follows.

II. Tau Decays

The tau can then decay either purely leptonically or semi-hadronically. In the standard model we have $\tau^- \rightarrow \nu_\tau + e^- \bar{\nu}_e$, $\tau^- \rightarrow \nu_\tau + \mu^- \bar{\nu}_\mu$, and $\tau^- \rightarrow \nu_\tau + d\bar{u}$,

where the $d\bar{u}$ quark system⁶ manifests itself in terms of hadrons equally in overall vector and axial-vector states. A most naive calculation would predict that the rates for these three processes would be in the ratio 1:1:3, where the factor of 3 arises because of the three colors of quarks. A more sophisticated calculation, using QCD corrections, obtains a number slightly larger than three.⁷

For the purely leptonic decays it is straightforward to calculate the width. With zero mass neutrinos:

$$\Gamma(\tau \rightarrow \nu_{\tau} e \bar{\nu}_e) = \frac{G^2 m_{\tau}^5}{192\pi^3}, \quad (1)$$

where the Fermi constant $G = 1.02 \times 10^{-5}/M_N^2$. The width for $\tau \rightarrow \nu_{\tau} \mu \bar{\nu}_{\mu}$ is the same, except for a small correction ($\sim 3\%$) due to the muon mass.

The decays into a neutrino plus hadrons with zero net strangeness which take place through the action of the vector coupling to $d\bar{u}$ can be directly related, invariant mass by invariant mass and multiplicity by multiplicity, to e^+e^- annihilation cross sections using CVC. The precise relation is⁸

$$\Gamma(\tau \rightarrow \nu_{\tau} + (\text{hadrons})_{1-}) = \frac{G^2 \cos^2 \theta_c}{96\pi^3 m_{\tau}^3} \times \int_0^{m_{\tau}^2} dQ^2 (m_{\tau}^2 - Q^2)^2 (m_{\tau}^2 + 2Q^2) \frac{\sigma^{(1)}(Q^2)}{\sigma_{pt}(Q^2)} \quad (2)$$

where θ_c is the Cabibbo angle, $\sigma_{pt}(Q^2) = 4\pi\alpha^2/(3Q^2)$ is the point cross section for $e^+e^- \rightarrow \mu^+\mu^-$ and $\sigma^{(1)}(Q^2)$ is the cross section for e^+e^- annihilation into hadrons through the isovector part of the electromagnetic

current at center-of-mass energy Q . In particular, $\sigma^{(1)}(Q^2)$ involves the channels $e^+e^- \rightarrow \pi^+\pi^-$, 4π , 6π , etc.⁹

The results of substituting the annihilation data into the right-handed side of Eq. (2) are conveniently expressed in terms of the ratio of the predicted τ partial widths to that (in Eq. (1)) for $\tau \rightarrow \nu_\tau + e\bar{\nu}_e$. For a τ mass of 1.8 GeV (1.9 GeV) one finds¹⁰

$$\Gamma(\tau^- \rightarrow \nu_\tau \pi^- \pi^0) / \Gamma(\tau \rightarrow \nu_\tau e\bar{\nu}_e) = 1.22 \quad (1.12) ,$$

$$\Gamma(\tau^- \rightarrow \nu_\tau + \pi^+ \pi^0 \pi^-) / \Gamma(\tau \rightarrow \nu_\tau e\bar{\nu}_e) = .35 \quad (.46) ,$$

and
$$\Gamma(\tau^- \rightarrow \nu_\tau + \pi^0 \pi^0 \pi^-) / \Gamma(\tau \rightarrow \nu_\tau e\bar{\nu}_e) = .08 \quad (.11) .$$

The total width for all hadronic vector modes^{10,11} is 1.65 (1.69) times the width for $\tau \rightarrow \nu_\tau e\bar{\nu}_e$. This is in adequate agreement with the value of 1.5 predicted from the most naive calculation and also with the $\sim 20\%$ upward correction to that number given by the lowest order corrections predicted by QCD.⁷

Of the decays that involves hadrons in an axial-vector state, only one can be predicted precisely from other data. This is $\tau \rightarrow \nu_\tau \pi$, which on the basis of the pion decay rate, has the value⁸⁻¹¹

$$\Gamma(\tau \rightarrow \nu_\tau \pi) / \Gamma(\tau \rightarrow \nu_\tau e\bar{\nu}_e) = 0.60 \quad . \quad (3)$$

This is consistent with the most recent measurements.¹²

Decays of the form $\tau \rightarrow \nu_\tau 3\pi$ proceeding through the axial-vector weak current should exist. In the past year experimental data supporting the existence of this mode has accumulated.¹² The three pion state has a large $\rho\pi$ component and a mass distribution peaking around 1100 MeV. This supports the idea of an A_1 resonance at or near this mass. However, while the data

does not fit pure phase space well it does not unambiguously demand resonant behavior either.

We are now in a position to examine the charged multiplicity distribution in τ decays. Given a branching ratio for $\tau \rightarrow \nu_\tau e \bar{\nu}_e$, we take the relative rates for $\tau^- \rightarrow \nu_\tau \pi^- \pi^0$, $\tau^- \rightarrow \nu_\tau \pi^+ \pi^0 \pi^-$, $\tau^- \rightarrow \nu_\tau \pi^0 \pi^0 \pi^-$, and $\tau \rightarrow \nu_\tau \pi^-$ from the calculations¹⁰ discussed above. This is also confirmed roughly by experimental measurements.¹² After also adding a small contribution from the Cabibbo suppressed decays $\tau \rightarrow \nu_\tau K$ and $\tau \rightarrow \nu_\tau K^*$, we force the remaining decays, which in the standard model are of the form $\tau \rightarrow \nu_\tau + \text{hadrons}$, with the hadrons arising from the axial-vector weak current, to fill up the gap so as to get 100% of the decays from the sum of leptonic and semi-hadronic decays.

The results of this exercise are contained in Table I for a τ mass of 1.8 GeV. The semi-hadronic decays through the axial-vector current (other than $\tau \rightarrow \nu_\tau \pi$) are of the form $\tau \rightarrow \nu_\tau 3\pi$ or possibly $\tau \rightarrow \nu_\tau 5\pi$, given the known value of m_τ . In Table I it is assumed that $\tau \rightarrow \nu_\tau + 3\pi$ dominates, in which case isospin one for the final 3π state demands that $\tau^- \rightarrow \nu_\tau \pi^+ \pi^- \pi^-$ be between 50% and 80% of all $\tau \rightarrow \nu_\tau 3\pi$ decays.

We see from Table I that for $\text{BR}(\tau \rightarrow \nu_\tau e \bar{\nu}_e) = 0.20$ only $\sim 14\%$ of τ decays are of the form $\tau \rightarrow \nu_\tau 3\pi$ (or $\tau \rightarrow \nu_\tau 5\pi$), and total three charged prong decays are altogether at most 18%. The direct measurements¹² of $\tau \rightarrow$ multi-prong give values of 30% or slightly larger. We conclude that in the standard model the branching ratio for $\tau \rightarrow \nu_\tau e \bar{\nu}_e$ must be less than 20%. In particular, the Table shows that when $\text{BR}(\tau \rightarrow \nu_\tau e \bar{\nu}_e) = 0.16$, the three charged prong decays of the τ can be as much as 30%, in agreement with experiment. However, most of this 30% comes from $\tau \rightarrow \nu_\tau 3\pi$ and $\tau \rightarrow \nu_\tau 5\pi$

while direct measurements¹² of the quantity $\tau^- \rightarrow \nu_\tau \pi^+ \pi^- \pi^-$ are much smaller. Whether this is due to $\tau \rightarrow \nu_\tau 5\pi$ decays, wrong measurements, or wrong assumptions in the calculations remains an open question.

TABLE I
Tau Decay Multiplicity

Mode	BR($\tau \rightarrow \nu_\tau e \bar{\nu}_e$) = 0.16		BR($\tau \rightarrow \nu_\tau e \bar{\nu}_e$) = 0.20	
	one charged prong	three charged prongs	one charged prong	three charged prongs
$\tau^- \rightarrow \nu_\tau e \bar{\nu}_e$.16	0	.20	0
$\rightarrow \nu_\tau \mu \bar{\nu}_\mu$.16	0	.20	0
$\rightarrow \nu_\tau \pi^+ \pi^-$.20	0	.24	0
$\rightarrow \nu_\tau (4\pi)^-$.01	.06	.01	.07
$\rightarrow \nu_\tau K^-$.01	0	.02	0
$\rightarrow \nu_\tau K^{*}$				
$\rightarrow \nu_\tau \pi^-$.10	0	.12	0
$\rightarrow \nu_\tau (3\pi)^-$	<u>.15 to .06</u>	<u>.15 to .24</u>	<u>.07 to .03</u>	<u>.07 to .11</u>
Total	.79 to .70	.21 to .30	.86 to .82	.14 to .18

III. Heavier Leptons

As of now there are no indications for leptons heavier than the tau. Measurements of $R = \sigma(Q^2)/\sigma_{pt}(Q^2)$ in e^+e^- annihilation in the upsilon region

(9 to 10 GeV) are consistent off resonance with values (~ 5) measured from 5 to 8 GeV.¹³ However, a rise of one unit, as expected from production of a point fermion of unit charge, could still be accommodated within the present error bars.

For a charged heavy lepton with a mass of 5 GeV or so within the standard model, one expects decays into $\nu e \bar{\nu}_e$, $\nu \mu \bar{\nu}_\mu$, $\nu \tau \bar{\nu}_\tau$, $\nu d \bar{u}$, and $\nu s \bar{c}$ in the ratio 1:1: ~ 0.5 :3:3. The suppression of the decay involving the τ is purely kinematic (and exactly calculable), while the decay into $\nu s \bar{c}$ might be argued to be subject to similar suppression. If the new lepton is heavy enough, and there is an additional weak doublet of quarks, $\begin{pmatrix} t \\ b \end{pmatrix}$, then we must add the decay into $\nu b \bar{t}$.

In any case the branching ratio into $\nu e \bar{\nu}_e$ should be around 10%, a quite respectable level. Individual channels, like $\nu \pi$ or $\nu \rho$ which correspond to major τ branching fractions will be at the 1% level for a lepton with a mass, $M_L = 5$ GeV (such exclusive channels have rates which go down as $1/M_L^2$ compared to that for $\nu e \bar{\nu}_e$). However, some caution is needed here, for if the mass of the new lepton is just right so that a decay like $L \rightarrow \nu + (b \bar{t})$ can barely take place, then all the rates for such channels may be soaked up in a very few $b \bar{t}$ discrete states (say the ground state pseudoscalar and vector). Thus some very interesting individual channels may not have small branching ratios.

The detection of such a heavy charged lepton is relatively easy in e^+e^- annihilation. Production by $e^+e^- \rightarrow L^+L^-$ with a point cross section followed by the $\nu e \bar{\nu}_e$ decay of one and $\nu \mu \bar{\nu}_\mu$ decay of the other with $\sim 10\%$ branching fraction each, should make a replay of the initial τ discovery straightforward.¹⁴ In fact, the τ will likely furnish the main background.

High mass neutral leptons generally are much more difficult to find. Only if they are coupled to electrons does the lowest order weak process $e^+e^- \rightarrow \bar{\nu}_e$ or $N\bar{\nu}_e$ present itself and make the job a little less than impossible.¹⁵

IV. Weak Decays of Heavy Quarks

If we assume $SU(2) \times U(1)$ as the gauge group of the weak and electromagnetic interactions, charged current weak processes lead to the conventional assignment of the left-handed u, d, and to a lesser extent, c, s to weak isospin doublets. Data on neutral current neutrino reactions indicate that the right-handed u and d quarks act as weak isospin singlets. The lack of anomalous behavior in antineutrino deep inelastic scattering prohibits the assignment of the right-handed b quark into a doublet with the u quark.¹

The discovery of the T family of particles¹⁶ and measurement of their leptonic widths strongly suggests the existence of a b quark with charge $-e/3$. With observations supporting five quarks, we are led to assume a sixth, t, with charge $+2e/3$, and group the left-handed quarks in three weak doublets (and the right-handed ones all in singlets).

The only remaining freedom is which linear combinations of d, s, and b (with charge $-e/3$) are coupled to u, c, and t (with charge $+2e/3$), respectively. This freedom may be expressed in terms of a 3×3 unitary matrix which has 9 free parameters. However, five phases may be absorbed in redefined quark fields, so there are only 4 parameters of physical significance: 3 Cabibbo-like angles and a complex phase which gives rise to CP violation. With the conventional left-handed doublets

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L ; \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L ; \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L ,$$

the relation between the primed and unprimed quark fields is¹⁷

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_2 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \quad (4)$$

where $c_i = \cos \theta_i$, $s_i = \sin \theta_i$ for $i=1,2,3$, and $\delta \neq 0$ leads to CP violation.

The angle θ_1 is essentially the Cabibbo angle and $\sin^2 \theta_1 \approx 0.05$.

The other two angles remain to be measured, but one can put upper limits on them.

(a) The sum of the squares of the charged current couplings of $u \rightarrow d(\cos^2 \theta_1)$ from $n \rightarrow p e \bar{\nu}_e$ and of $u \rightarrow s(\sin^2 \theta_1 \cos^2 \theta_3)$ from strange particle decays should deviate from that for $\mu \rightarrow \nu_\mu$ (unit strength) from $\mu \rightarrow \nu_\mu e \bar{\nu}_e$ by $\sin^2 \theta_1 \sin^2 \theta_3$, the square of the $u \rightarrow b$ coupling. With appropriate radiative corrections to the decays the last coupling is consistent with zero within errors and one has the upper limit¹⁸

$$(s_1 s_3)^2 < 0.003 \quad (5a)$$

or

$$s_3^2 < 0.06 \quad (5b)$$

using $\sin^2 \theta_1 \approx 0.05$.

(b) A more theoretical argument is based on the calculation of the $K^0 \leftrightarrow \bar{K}^0$ transition with two W bosons exchanged. If the introduction of the t quark with a mass greater than 7 GeV is not to ruin the usual calculation (involving the c quark) then^{15,18}

$$\sin^2 \theta_2 \lesssim 0.1 \quad (6)$$

Thus all $\sin^2\theta_i$ are less than 0.1.

We are now ready to begin our discussion of weak decays of hadrons with heavy quarks. The most naive model of such decays pictures the heavy quark, say the charm quark, decaying as if free into other quarks and possibly leptons. We then have $c \rightarrow se^+v_e$, $c \rightarrow s\mu^+v_\mu$ and $c \rightarrow sd\bar{u}$ occurring in the ratio of 1:1:3 and neglecting the s quark mass,

$$\Gamma(c \rightarrow s + \dots) \approx 5\Gamma(c \rightarrow se^+v_e) \approx 5 \frac{G_m^2 c^5}{192\pi^3} . \quad (7)$$

The factor of three in non-leptonic decay is, as in heavy lepton decay, because of color. The assumption has been implicitly made that the final quarks materialize as hadrons with unit probability. There is to be no special enhancement (or suppression) or special final states.

This is not true in the decays of strange particles where non-leptonic decays are much enhanced over their naively expected level and further a special part, that with $\Delta I = 1/2$ is the enhanced portion. In an SU(3) context it is the octet part of the weak interaction which is enhanced. This enhancement is thought to come about both because the decaying quark interacts (by W exchange) with other quarks in the initial hadron (modified by gluon exchanges) and because $s \rightarrow ud\bar{u}$ doesn't occur at the naive level, especially when gluon exchanges are taken into account.¹⁹

For charmed meson decays there are two pieces of evidence that a corresponding large enhancement of non-leptonic decays does not occur:

(a) The average D semi-electronic branching ratio²⁰ is $\sim 10\%$. This is close enough to the most naive 20% (or the $\sim 17\%$ in τ decay) to argue that we are talking about less than a factor of two in amplitude.

(b) $D^+ \rightarrow \bar{K}^0 \pi^+$ occurs with a branching ratio²⁰ of 1.5%, about the

same as that for $D^0 \rightarrow K^- \pi^+$. If the total widths of D^0 and D^+ are about the same, $D^+ \rightarrow \bar{K}^0 \pi^+$ and $D^0 \rightarrow K^- \pi^+$ have comparable rates. However $D^0 \rightarrow \bar{K}^0 \pi^+$ is forbidden by the analogue of the $\Delta I = 1/2$ rule in strange particle decay plus SU(3) symmetry, whereas $D^0 \rightarrow K^- \pi^+$ is perfectly allowed.

There is one out.²¹ If $\Gamma(D^0) \gg \Gamma(D^+)$, then $BR(D^+ \rightarrow e^+ \nu_e + \dots) \gg BR(D^0 \rightarrow e^+ \nu_e + \dots)$ and $\Gamma(D^+ \rightarrow \bar{K}^0 \pi^+) \ll \Gamma(D^0 \rightarrow K^- \pi^+)$. We eliminate both (a) and (b) as evidence against enhancement. In fact, D^0 (but not D^+) non-leptonic decays would have to be very much enhanced over the naive model. This possibility can probably already be eliminated by experiment. First, the average D meson semi-leptonic branching ratio measured in $e^+ e^-$ annihilation would vary with center-of-mass energy as the proportion of D^+ and D^0 varies. Second, the ratio of two electron to one electron events from D decays at 3.77 GeV (where D^+ and D^0 are produced almost equally) would be twice as big as expected on the basis of equal D^+ and D^0 semi-leptonic branching ratios and the single electron events.²² Although some difference in D^0 and D^+ semi-electronic branching ratios can't be ruled out, it seems unlikely that an order of magnitude difference is compatible with the data.²² It should be emphasized that calculations based on QCD predict only a small enhancement in charm decays and specific applications²³ to $D \rightarrow K\pi$, for example, appear to be in quantitative agreement with experiment.

For the still heavier hadrons containing b and t quarks, we follow the naive model applied to charm above, with appropriate modifications. Namely we treat the heavy decaying quark as if it were free and write

$$\Gamma \begin{pmatrix} e^+ \nu \\ x \rightarrow y + u \bar{d} \\ \mu^+ \nu \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} \left(\frac{G^2 m_x^5}{192\pi^3} \right) \Theta_{xy} F \left(\frac{m_y}{m_x} \right), \quad (8)$$

where Θ_{xy} is a factor depending on the charged current coupling of x to y in the weak mixing matrix, Eq. (4), and $F(m_y/m_x)$ is a factor which is unity for $m_y = 0$ and less than unity for $m_y > 0$.

In the particular case of the b quark, it can couple to u, c, and t quarks with the last presumably heavier than b. For $m_b \approx 5$ GeV, we have from Eqs. (22) and (18) that¹⁵

$$\begin{aligned} \frac{\Gamma(b \rightarrow c + \dots)}{\Gamma(b \rightarrow u + \dots)} &= \frac{\Theta_{bc} F(m_c/m_b)}{\Theta_{bu} F(m_u/m_b)} \\ &\approx \frac{s_2^2 + s_3^2 + 2s_2 s_3 \cos \delta}{s_1^2 s_3^2} \times \left(\frac{\sim \frac{1}{3}}{1} \right). \end{aligned} \quad (9)$$

The limits on s_2^2 and s_3^2 allow the right-hand side to vary in principle from zero to infinity. But taking $s_2 \approx s_3$ (and δ small) gives values of ~ 25 and only for $s_2 \approx -s_3$ is the ratio less than unity. The lack of observation²⁴ of stable hadrons with mass ≈ 5 GeV in pN collisions at a cross section level about that of the T implies that the b quark decays and either θ_2 or θ_3 is not zero. Thus we expect that hadrons containing b quarks have "generalized Cabibbo angle suppressed" decays (lifetime probably about $10^{-13 \pm 1}$ seconds) with charmed particles usually found among the decay products.

For the t quark we have correspondingly¹⁵ with $m_t \approx 15$ GeV

$$\frac{\Gamma(t \rightarrow b + \dots)}{\Gamma(t \rightarrow s + \dots)} \approx \frac{\theta_{tb} F(m_b/m_t)}{\theta_{ts} F(m_s/m_t)} \approx \frac{1}{s_2^2 + s_3^2 + 2s_2s_3 \cos \delta} \times \left(\frac{\sim \frac{1}{3}}{1} \right) . \quad (10)$$

The right-hand side of Eq. (10) is at least of order unity and could be infinite with $s_2 \approx -s_3$. We would generally expect it to be in the range of 2 to 10. Thus hadrons containing t quarks should have "generalized Cabibbo angle allowed" decays (lifetime about $10^{-17 \pm 1}$ seconds for $M_t \sim 15$ GeV), with hadrons containing b quarks usually in the final state.

The most characteristic feature to emerge from this analysis is the cascading weak decays, $t \rightarrow b \rightarrow c \rightarrow s$ as the likely dominant decay chain. Since at each weak decay the charged W can materialize as a lepton pair, a unique and very characteristic feature of the net decay products of a hadron containing b or t quarks is the possibility of two (or more) charged leptons. This holds forth the possibility of seeing very characteristic events with greater than two leptons (or two leptons of the same sign) in events above the b and t flavor thresholds in e^+e^- annihilation.

REFERENCES

1. S. Weinberg, rapporteur talk at the XIX International Conference on High Energy Physics, August 23-30, 1978, Tokyo, Japan (unpublished) reviews the evidence for the standard model and assignment of quarks and leptons.
2. S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam in Elementary Particle Physics: Relativistic Groups and Analyticity (Nobel Sump. No. 8), edited by N. Svartholm (Almquist and Wiksell, Stockholm, 1968), p. 37.
3. F. J. Gilman, lectures at the Kyoto Summer Institute for Particle Physics, Kyoto, Japan, September 1-5, 1978 and SLAC-PUB-2226, 1978 (unpublished).
4. G. Altarelli, parallel session talk at the XIX International Conference on High Energy Physics, August 23-30, 1978, Tokyo, Japan (unpublished).
5. C. W. Kim, Johns Hopkins University preprint JHU-HET7804, 1978 (unpublished); and invited talk at this symposium.
6. The d quark is Cabibbo rotated to $d \cos \theta_c + s \sin \theta_c$.
7. C. S. Lam and T. M. Yan, Phys. Rev. D16, 703 (1977).
8. Y. S. Tsai, Phys. Rev. D4, 2821 (1971).
9. Earlier calculations based on known couplings to a few resonances plus estimates of other channels were done by H. B. Thacker and J. J. Sskurai, Phys. Lett. 36B, 103 (1971); Y. S. Tsai, Ref. 8; K. Gaemers and R. Raito, Phys. Rev. D14, 1262 (1976); T. Hagiwara et al., Ann. Phys. (N.Y.) 106, 134 (1977).

10. F. J. Gilman and D. H. Miller, Phys. Rev. D17, 1846 (1978) and unpublished work.
11. Related calculations, in substantial agreement with Ref. 10, have been done by N. Kawamoto and A. I. Sanda, DESY preprint DESY 78/14, 1978 (unpublished); T. N. Pham, C. Roiesnel, and T. N. Truong, Ecole Polytechnique preprints, 1978 (unpublished); S. Narison, Marseille preprint 78/P.997, 1978 (unpublished).
12. G. J. Feldman, rapporteur talk at the XIX International Conference on High Energy Physics, August 23-30, 1978, Tokyo, Japan (unpublished) and SLAC-PUB-2224 contains a complete review, compilation, and references to original experiments on the tau lepton.
13. G. Flugge, rapporteur talk at the XIX International Conference on High Energy Physics, August 23-30, 1978, Tokyo, Japan (unpublished).
14. See, for example, J. A. M. Vermaseren, Purdue University preprint, 1978 (unpublished), or almost any PETRA or PEP proposal.
15. See the discussion and references in J. Ellis, lectures at the SLAC Summer Institute on Particle Physics, July 10-21, 1978 (unpublished).
16. For a review of experiments and references to the original work see L. Lederman, rapporteur talk at the XIX International Conference on High Energy Physics, August 23-30, 1978, Tokyo, Japan (unpublished). Also see G. Flugge, Ref. 13.
17. M. Kobayashi and T. Maskawa, Prog. of Theor. Phys. 49, 652 (1973).
18. J. Ellis et al., Nucl. Phys. B131, 285 (1977).
19. See the discussion of M. K. Gaillard, lectures at the SLAC Summer Institute on Particle Physics, July 10-21, 1978 and Fermilab preprint FERMILAB-Conf-78/64-THY, 1978 (unpublished).

20. Particle Data Group, Phys. Lett. 75B, No. 1 (1978).
21. M. Katuya and Y. Koide, Shizuoka University preprint SH-78-05, 1978 (unpublished).
22. I thank J. Kirkby for pointing out the second of these tests to me and for discussions on the relevant data.
23. N. Cabibbo and L. Maiani, Phys. Lett. 73B, 418 (1978).
24. R. Vidal et al., Phys. Lett. 77B, 344 (1978); D. Cutts et al., Phys. Rev. Lett. 41, 363 (1978).