SLAC-PUB-2234
November 1978
(T/E)

SIMPLE UPPER AND LOWER BOUNDS ON THE LIFETIME OF THE b-QUARK*

Haim Harari**
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

We show that, within the standard left-handed six-quark model, the lifetime of the B-meson (lightest meson containing a b-quark) must be between 10^{-11} sec and 10^{-14} sec. The upper bound is new, and may be useful to experimentalists who consider measurements of the B-lifetime. The derivation is based on a trivial calculation, using an expression for the ε parameter in the CP-violating decay $K_L^0 \to 2\pi$.

(Submitted for Publication)

* Work supported by the Department of Energy under contract number EY-76-C-03-0515.

** On leave of absence from the Weizmann Institute of Science, Rehovot, Israel.

In the standard left-handed six-quark model,[1] all charged weak transitions as well as some CP-violating amplitudes[2,3] can be expressed in terms of four parameters: Three Cabibbo-like angles θ_1, θ_2, θ_3 and one phase δ. The lifetime of the lowest lying meson containing a b-quark (the B-meson) depends on these parameters. One of the angles (θ_1) is known. One constraint between θ_2, θ_3, and δ is provided by the ε-parameters of CP-violating K_L^o decay.[3] In addition, we have upper bounds for θ_2 and θ_3. It is a simple matter to derive the upper limit and the lower limit for the B-lifetime, subject to these constraints. We do so in this note, and find that for $m_t \sim 15$ GeV, 10^{-14} sec $\lesssim \tau_b \lesssim 10^{-11}$ sec.

The parameters θ_1, θ_2, θ_3, are defined[1] by the charged current:

$$J^- = (\bar{u}\ \bar{c}\ \bar{t})\ \gamma_\mu (1 - \gamma_5)\ A \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{1}$$

A is a 3×3 matrix given by:

$$A = \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & -c_1 s_2 c_3 - c_2 s_3 e^{i\delta} & -c_1 s_2 s_3 + c_2 c_3 e^{i\delta} \end{pmatrix} \tag{2}$$

where

$$c_j \equiv \cos\theta_j \quad ; \quad s_j \equiv \sin\theta_j \quad ; \quad j = 1,2,3 \ .$$

We know that:[1,4]

$$\theta_1 = 13.2 \pm 0.5^o \tag{3}$$

$$\theta_3 \lesssim 16^o \tag{4}$$

$$\tan^2\theta_2 \lesssim \frac{m_c}{m_t} \tag{5}$$

Assuming $m_t > m_b$, the b-quark can decay only into u or c quarks. We assume that there is no significant enhancement of nonleptonic B-decays. Consequently, we may write the following expression for the total width:[5]

$$\Gamma_b = \Gamma_o\left(K_u|A_{ub}|^2 + K_c|A_{cb}|^2\right) \qquad (6)$$

where

$$\Gamma_o = \Gamma_\mu\left(\frac{m_b}{m_\mu}\right)^5 = \frac{G_F^2 m_b^5}{192\pi^3} \qquad (7)$$

$$K_u = \left[1 + 1 + f\left(\frac{m_\tau}{m_b}\right) + 3 + 3f\left(\frac{m_c}{m_b}\right)\right] \qquad (8)$$

$$K_c = \left[f\left(\frac{m_c}{m_b}\right) + f\left(\frac{m_c}{m_b}\right) + \phi(m_c,m_\tau;m_b) + 3f\left(\frac{m_c}{m_b}\right) + 3\phi(m_c,m_c;m_b)\right] \qquad (9)$$

$$|A_{ub}|^2 = (s_1s_3)^2 \qquad (10)$$

$$|A_{cb}|^2 = (c_1c_2s_3)^2 + (s_2c_3)^2 + 2c_1c_2s_2c_3s_3\cos\delta \qquad (11)$$

Each of the factors K_u and K_c contain 5 terms, representing decays into $e^-\bar{\nu}_e$, $\mu^-\bar{\nu}_\mu$, $\tau^-\bar{\nu}_\tau$, $d\bar{u}$, $s\bar{c}$, respectively. We neglect m_e, m_μ, m_u, m_d, m_s with respect to m_b. $f\left(\frac{m}{M}\right)$ and $\phi(m_1,m_2;M)$ are kinematic factors[5] relevant to the decay of a fermion with mass M into one massive and two massless fermions, or two massive and one massless fermions, respectively. We have:

$$f(x) = (1-x^4)(1-8x^2+x^4) - 12x^4\,\ln(x^2) \qquad (12)$$

The function $\phi(m_1,m_2;M)$ is simplified for $m_1 = m_2$:

$$\phi(m,m;M) \equiv g\left(\frac{4m^2}{M^2}\right) \tag{13}$$

where

$$g(x) = \left(1 - \frac{7}{2}x - \frac{1}{8}x^2 - \frac{3}{16}x^3\right)(1-x)^{\frac{1}{2}} + 3x^2\left(1 - \frac{1}{16}x^2\right)\log\left(\frac{1+\sqrt{1-x}}{\sqrt{x}}\right) \tag{14}$$

Assuming:

$$m_b = 4.5 \text{ GeV} \quad ; \quad m_c = 1.4 \text{ GeV} \quad ; \quad m_\tau = 1.8 \text{ GeV} \tag{15}$$

We obtain

$$\Gamma_o = 6.3 \times 10^{13} \text{ sec}^{-1} \quad ; \quad K_u = 6.8 \quad ; \quad K_c = 3.1 \tag{16}$$

We can therefore write

$$\Gamma_b = 4.3 \times 10^{14} \times G(\theta_1, \theta_2, \theta_3, \delta)\text{sec}^{-1} \tag{17}$$

where

$$G(\theta_1,\theta_2,\theta_3,\delta) = (s_1s_3)^2 + 0.45\left[(c_1c_2s_3)^2 + (s_2c_3)^2 + 2c_1c_2s_2c_3s_3\cos\delta\right] \tag{18}$$

Since θ_1 is known, G depends only on three unknown parameters. One relation between θ_2, θ_3, and δ can be obtained from the CP-violating decay $K_L^o \to 2\pi$:[3]

$$\varepsilon \approx \sqrt{2}\; s_2c_2s_3 \sin\delta \; P(\theta_2,\eta) \tag{19}$$

where

$$P(\theta_2,\eta) = \frac{s_2^2\left(1+\frac{\eta\,\ell n\,\eta}{1-\eta}\right) - c_2^2\left(\eta+\frac{\eta\,\ell n\,\eta}{1-\eta}\right)}{c_2^4\eta + s_2^4 - 2s_2^2c_2^2\,\frac{\eta\,\ell n\,\eta}{1-\eta}} \qquad (20)$$

and $\eta \equiv m_c^2/m_t^2$.

This expression[3] for ϵ is probably valid only within a factor of 2 or so. It depends on the assumption that the "vacuum" intermediate state dominates the K^o-$\bar{K}^o$ mass matrix. We can now substitute Eq. (19) into Eq. (18), obtaining:

$$G = (s_1s_3)^2 + 0.45\left[(c_1c_2s_3)^2 + (s_2c_3)^2 \pm 2c_1c_3\sqrt{(s_2c_2s_3)^2 - A^2}\,\right] \qquad (21)$$

where

$$A = \frac{\varepsilon}{P(\theta_2,\eta)\sqrt{2}}\ ; \qquad \varepsilon \sim 2\times 10^{-3} \qquad (22)$$

For any given value of m_t we can compute the allowed range of $P(\theta_2,\eta)$, and using that range, find the upper limit and lower limit of the expression (21), for all θ_2,θ_3 values subject to the bounds of Eqs. (4) and (5). Using the obtained value of G we can then compute Γ_b and τ_b.

The results of the calculation are given in Fig. 1. They depend on m_t and, to a lesser extent, on m_b. Any change in the expression for ϵ (Eq. (19)) would yield a similar change (by approximately the same multiplicative factor) in our <u>upper</u> limit, but no substantial change of the lower limit. For most reasonable values of these parameters, the allowed range of values is around:

$$10^{-14}\ \text{sec} \lesssim \tau_b \lesssim 10^{-11}\ \text{sec} \qquad (23)$$

For $m_t \sim 15$ GeV, the longest allowed lifetime is achieved for:

$$|\theta_2| \sim 2^o \quad ; \quad |\theta_3| \sim 2^o \quad ; \quad \theta_2 \sim -\theta_3 \quad ; \quad \delta \sim 17^o \qquad (24)$$

while the shortest lifetime is obtained for

$$\theta_3 \sim 16^o \quad ; \quad \theta_2 \sim \text{arc tan} \sqrt{\frac{m_c}{m_t}} \qquad (25)$$

and is almost independent of δ.

It might be tempting to try a "best guess" for τ_b. Several authors have tried to express the angles θ_1, θ_2, θ_3 in terms of the quark masses. While all of these attempts make many unjustified assumptions and suffer from many defficiencies, one of the obtained estimates[6] for the three angles is aesthetically appealing to us:

$$\tan^2\theta_1 \sim \frac{m_d}{m_s} \quad ; \quad \tan^2\theta_2 \sim \frac{m_c}{m_t} \quad ; \quad \tan^2\theta_3 \sim \frac{m_s}{m_b} \qquad (26)$$

Using these values we may calculate τ_b, producing an "uneducated guess." It turns out, however, that the result is very sensitive to the (unknown) relative sign of θ_2 and θ_3. We therefore have two such guesses, both shown in Fig. 1. Note that both guesses are far below the predicted upper limit of τ_b.

Our upper and lower bounds are calculated entirely within the framework of the standard model. Any of the following variations of the model may eliminate our upper limit:

(i) It is possible that CP-violation is partly or entirely due to other effects such as Higgs couplings or right-handed currents. Eq. (19) would then be inapplicable.

(ii) Additional quarks may exist, leading to a proliferation of angles and phases.

(iii) Right-handed currents mediated by heavy vector mesons may exist, contributing to b-decays and to $K_L^o \to 2\pi$.

For any of these variants, other bounds may be obtained, but the number of possibilities is so large that we do not find it useful to pursue them here.

If our bounds are valid, the lifetime of the b-quark (or B-meson) can be measured only by techniques which, at present energies, can detect tracks (or gaps) whose length is of the order of .1 cm or less. The only presently available experimental upper limit[7] on τ_b is 5×10^{-8} sec, several orders of magnitudes above our upper limit.

We have learned that crude estimates of the upper bound for τ_b were mentioned by Glashow[8] and by Ellis.[9] Our results, based on detailed calculations are different, but are essentially based on the same simple procedure.

I thank J. D. Bjorken and L. Wolfenstein for discussions, and Mark Wise for pointing out two algebraic errors in the manuscript.

REFERENCES

1. For a review and earlier references see e.g. H. Harari, Phys. Rep. 42C, 235 (1978).
2. M. Kobayashi and K. Maskawa, Prog. Theor. Phys. 49, 652 (1973).
   S. Pakvasa and H. Sugawara, Phys. Rev. D14, 305 (1976).
   L. Maiani, Phys. Lett. 62B, 183 (1976).
3. J. Ellis, M. K. Gaillard and D. V. Nanopoulos, Nucl. Phys. B109, 213 (1976).
4. J. Ellis, M. K. Gaillard, D. V. Nanopoulos and S. Rudaz, Nucl. Phys. B131, 285 (1977).
5. See e.g. R. N. Cahn and S. D. Ellis, Phys. Rev. D16, 1485 (1977); C. Quigg and J. L. Rosner, LBL-7961 preprint (1978). The general treatment followed here was first proposed by J. D. Bjorken and C. H. Llewellyn-Smith, Phys. Rev. D7, 887 (1973).
6. H. Fritzsch, Phys. Lett. 73B, 317 (1978). See also T. Kitazoe and K. Tanaka, Ohio State preprint (1978); M. A. de Crombrugghe, Leuven preprint (1978).
7. D. Cutts *et al.*, Phys. Rev. Lett. 41, 363 (1978).
   R. Vidal *et al.*, Fermilab preprint (1978).
8. S. L. Glashow, to be published in the Proc. of the 1978 Oxford Neutrino Conf., quoted: $\tau_b < 10^{-12}$ sec. We disagree with this limit.
9. J. Ellis, to be published in the Proc. of the 1978 SLAC Summer Institute, quoted: $10^{-13} < \tau_b < 10^{-8}$ sec. We disagree with these limits.

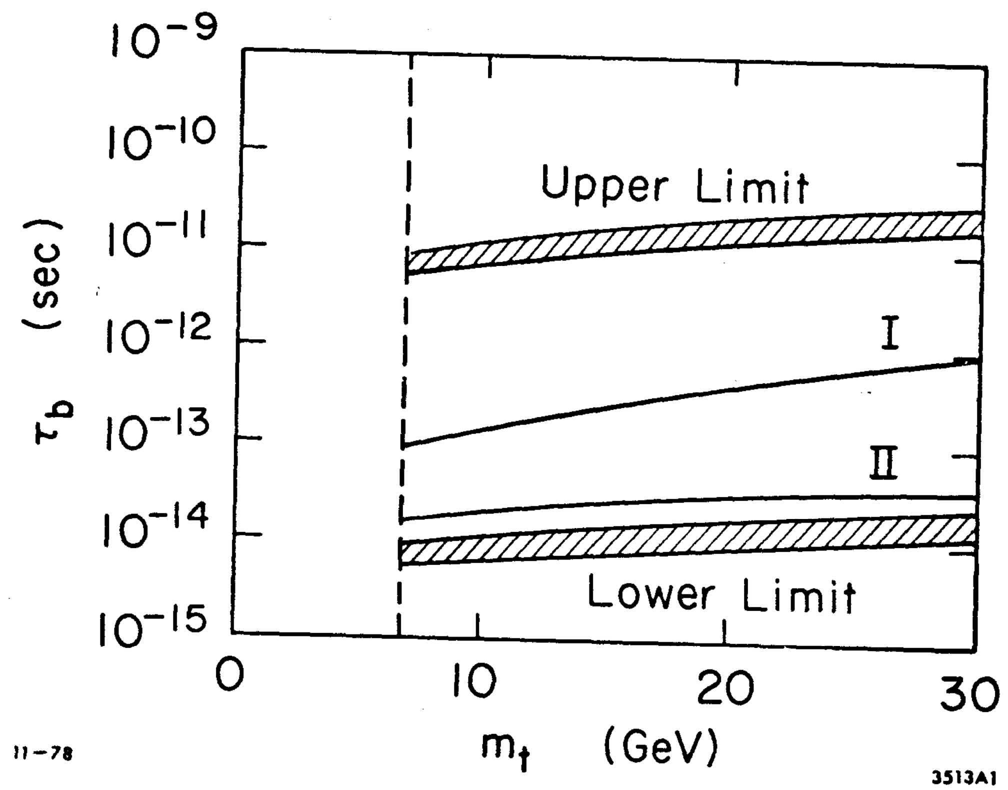

Fig. 1. Upper and lower limits for the lifetime of the lightest meson containing a b-quark. The bounds are plotted as a function of m_t. The shaded area for each bound represents its sensitivity to the variation of m_b between 4.5 and 5 GeV. The lines marked I and II represent "guesses" based on Eq. (26) in the text: (I) $\theta_2\theta_3 < 0$; and (II) $\theta_2\theta_3 > 0$.