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THE HEAVY QUARK POTENTIAL AND THE Υ, J/ψ SYSTEMS*

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Abstract

A new heavy quark potential is proposed which incorporates the two concepts of asymptotic freedom and linear quark confinement in a unified manner. It is shown that this potential reproduces the spectroscopy of the triplet $c\bar{c}$ system charmonium and the triplet $b\bar{b}$ system upsilonium. The only parameters other than a scale size Λ, are the quark masses.

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Many authors[1-6] have proposed various potential models, with varying degrees of success, of the strong quark-antiquark interactions. Here we propose such a model with the added feature of a minimal number of parameters. We impose two restrictions upon such a potential: (1) asymptotic freedom[7] and (2) linear quark confinement.[8] We impose these constraints in such a way that the only parameter which enters into such a potential is Λ, a scale size.

We construct the coordinate space potential $V(r)$ by Fourier transforming the single dressed gluon exchange amplitude which is proportional to $\widetilde{V}(q^2) = \frac{4}{3}\frac{\alpha_s(q^2)}{q^2}$. To impose constraint (1) we recall that asymptotic freedom (with $SU_3(\text{color}) \otimes SU_{n_f}(\text{flavor})$) requires that for large space-like momentum transfers that the strong effective color coupling constant behave as

$$\lim_{-q^2 >> \Lambda^2} \alpha_s(q^2) \sim \frac{12\pi}{33-2n_f} \frac{1}{\ln(-q^2/\Lambda^2)} \quad . \tag{1}$$

Constraint (2) is imposed by requiring that for large distances that

$$\lim_{\Lambda r >> 1} V(r) \sim \text{const} \times r$$

or equivalently

$$\lim_{-q^2 << \Lambda^2} \widetilde{V}(q^2) \to \text{const} \frac{1}{(q^2)^2} \quad . \tag{2}$$

A simple interpolating form which invokes both of these constraints, takes the simple form

$$\widetilde{V}(\vec{q}^{\,2}) = -\frac{4}{3}\frac{12\pi}{33-2n_f}\frac{1}{\vec{q}^{\,2}}\frac{1}{\ln(1+\vec{q}^{\,2}/\Lambda^2)} \quad . \tag{3}$$

Equation (3) forms the basis of this note.

We shall investigate the low-lying spectrum of the effective Hamiltonian

$$H = 2m + \frac{\vec{p}^{\,2}}{m} + V(r) \tag{4}$$

where V(r) is the Fourier transform of Eq. (3). We notice that the Hamiltonian (4) depends on the minimal number of parameters. The two parameters m and Λ which appear in Eq. (4) are the QCD analogs of the two parameters m and α which appear in QED.

Upon performing the Fourier transformation of Eq. (3) we find that V(r) may be written in the form

$$V(r) = \frac{8\pi}{33-2n_f}\,\Lambda\left(\Lambda r - \frac{f(\Lambda r)}{\Lambda r}\right) \tag{5}$$

where

$$f(t) = \frac{4}{\pi}\int_0^\infty dq\,\frac{\sin(qt)}{q}\left[\frac{1}{\ln(1+q^2)} - \frac{1}{q^2}\right] = \left[1 - 4\int_1^\infty \frac{dq}{q}\,\frac{e^{-qt}}{[\ln(q^2-1)]^2+\pi^2}\right] . \tag{6}$$

A graph of V(r) versus the dimensionless variable Λr is shown in Fig. 1. Note that V(r) is softer than a coulomb potential near the origin; that is

$$\lim_{\Lambda r \ll 1} V(r) \sim -\frac{8\pi}{33-2n_f}\,\frac{1}{r\,\ln(1/\Lambda r)} \tag{7}$$

For the purpose of studying the spectra of the Υ, J/ψ systems we shall choose $n_f = 3$, since the effect of heavier quarks should be small at the distances we are studying (using the Appelquist Carazzone theorem[9]). As a consistency check we compute $\langle p^2 \rangle$ to insure that $\langle p^2 \rangle < 4m^2$.

We have computed the spectrum of the Hamiltonian Eq. (4) numerically. m_c and Λ were chosen to obtain $M(J/\psi) = 3095$ MeV and $M(\psi') = 3684$ MeV. We found that $m_c = 1491$ MeV and $\Lambda = 398$ MeV. With this choice of parameters we found that $M(\chi_{c.o.g.}) = 3514$ MeV and $M(\psi'') = 3799$ MeV. These and a few other excited states are shown in Fig. 2 which also compares them with the experimental values as reported in Ref. 10 and references contained therein.

For the purpose of studying the Υ system we use the same value of Λ as obtained from the J/ψ data. The only parameter left is m_b which can be chosen so that $M(\Upsilon) = 9452$ MeV, that is $m_b = 4883$ MeV. We then find that $M(\Upsilon') - M(\Upsilon) = 555$ MeV which is in remarkable agreement with the experimental splitting of 557 ± 5 MeV.[11] We also find that $M(\Upsilon'') - M(\Upsilon) =$ 886 MeV in this model. These and a few other excited states are compared with the experimental values[11] in Fig. 3.

Usually when comparing potential models with experiment, one computes the leptonic decay widths using the Van Royan and Weisskopf formula[12]

$$\Gamma(V \to e^+e^-) = \frac{16\pi\alpha^2}{m_V^2} |\psi(0)|^2 e_Q^2 \tag{8}$$

where m_V is the mass of the vector meson, e_Q is the quark charge and ψ is the $q\bar{q}$ wave function. It has been pointed out by Celmaster[13] and Barbieri <u>et al</u>.[14] that this 0^{th} order expression (in α_s) is subject to QCD radiative corrections and should be replaced (to first order in α_s) by

$$\Gamma(V \to e^+e^-) = \frac{16\pi\alpha^2}{m_V^2} |\psi(0)|^2 \left[1 - \frac{4}{3}\frac{4}{\pi}\alpha_s(m_Q)\right] \quad , \tag{9}$$

which tends to strongly suppress the widths as computed using Eq. (8). Since these corrections are so large, we conclude that we may reliably compute only ratios, such as

$$\frac{\Gamma(V' \to e^+e^-)}{\Gamma(V \to e^+e^-)} = \left|\frac{\psi_{V'}(0)}{M_{V'}}\right|^2 \left|\frac{M_V}{\psi_V(0)}\right|^2 \tag{10}$$

where V and V' are vector mesons of the <u>same</u> $q\bar{q}$ system. Using the previous parameters we find that

$$\frac{\Gamma(\psi' \to e^+e^-)}{\Gamma(\psi \to e^+e^-)} = .45 \quad \text{and} \quad \frac{\Gamma(\Upsilon' \to e^+e^-)}{\Gamma(\Upsilon \to e^+e^-)} = .42$$

to be compared with the experimental values of .4 ± .1 and .3 ± .2 respectively.[10,11]

In summary, we have presented a new quark-antiquark potential which incorporates the concepts of asymptotic freedom and linear quark confinement in a simple manner. This potential has the added feature of a minimal number of parameters. Fairly good agreement has been found between the model and with the experimental measurements for the Υ and ψ systems. We have not treated spin-dependent effects in this simplified treatment but hope to do so in a future discussion.

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REFERENCES

1. E. Eichten et al., Phys. Rev. Lett. 34, 369 (1975).
2. C. Quigg and J. Rosner, Phys. Lett. 66B, 286 (1977).
3. W. Celmaster, H. Georgi and M. Machacek, Phys. Rev. D17, 879 (1978).
   W. Celmaster and F. Henyey, Phys. Rev. D18, 1688 (1978).
4. G. Bhanot and S. Rudaz, Phys. Lett. 78B, 119 (1978).
5. D. Lichtenberg and J. Wills, Indiana Univ. preprint IUHET-27 (1978).
6. B. Margolis, R. Roskies and N. De Takacsy, McGill preprint.
7. H. D. Politzer, Phys. Rev. Lett. 26, 1346 (1973).
   D. Gross and F. Wilczek, Phys. Rev. Lett. 26, 1343 (1973).
8. J. Kogut and L. Susskind, Phys. Rev. D9, 3501 (1974).
   K. Wilson, Phys. Rev. D10, 2445 (1974).
9. T. Appelquist and J. Carazzone, Phys. Rev. D11, 2856 (1975).
10. T. Appelquist, R. M. Barnett and K. Lane, Annu. Rev. Nucl. and Particle Sci. 28, 387 (1978).
11. G. Flügge, XIX International Conf. on High Energy Physics, Tokyo, 1978.
12. R. Van Royan and V. Weisskopf, Nuevo Cimento 50, 617 (1967).
13. W. Celmaster, SLAC-PUB-2151, July 1978.
14. R. Barbieri et al., Nucl. Phys. B105, 125 (1976).

FIGURE CAPTIONS

1. A graph of the potential $V(r)$ versus the dimensionless variable Λr.

2. A comparison of the experimental $c\bar{c}$ spectrum (experimental values[10] are shown in figure with appropriate error bars) versus the potential model prediction (large horizontal lines) with $n_f = 3$, $\Lambda = 398$ MeV, $m_c = 1491$ MeV. The model predictions are M(S-waves) = (3095, 3684, 4096, 4440), M(P-waves) = (3514, 3950, 4308), M(D-waves) = (3799, 4172, 4498) MeV.

3. A comparison of the experimental $b\bar{b}$ spectrum (experimental values[11] are shown in figure with appropriate error bars) versus the potential model prediction (large horizontal lines) with $n_f = 3$, $\Lambda = 398$ MeV, $m_b = 4884$ MeV. The model predictions are M(S-waves) = (9452, 10007, 10338, 10598), M(P-waves) = (9888, 10241, 10512), M(D-waves) = (10137, 10421, 10660) MeV.

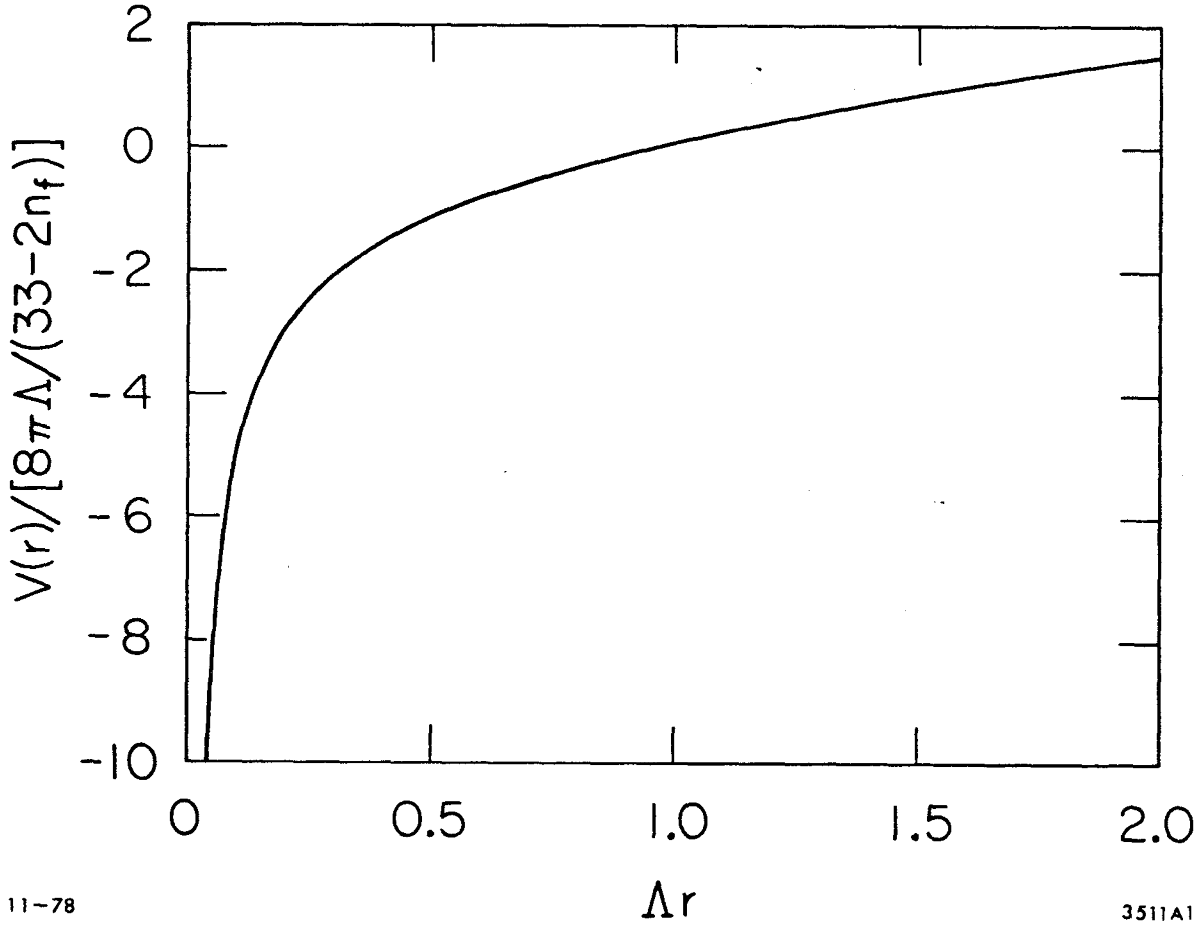
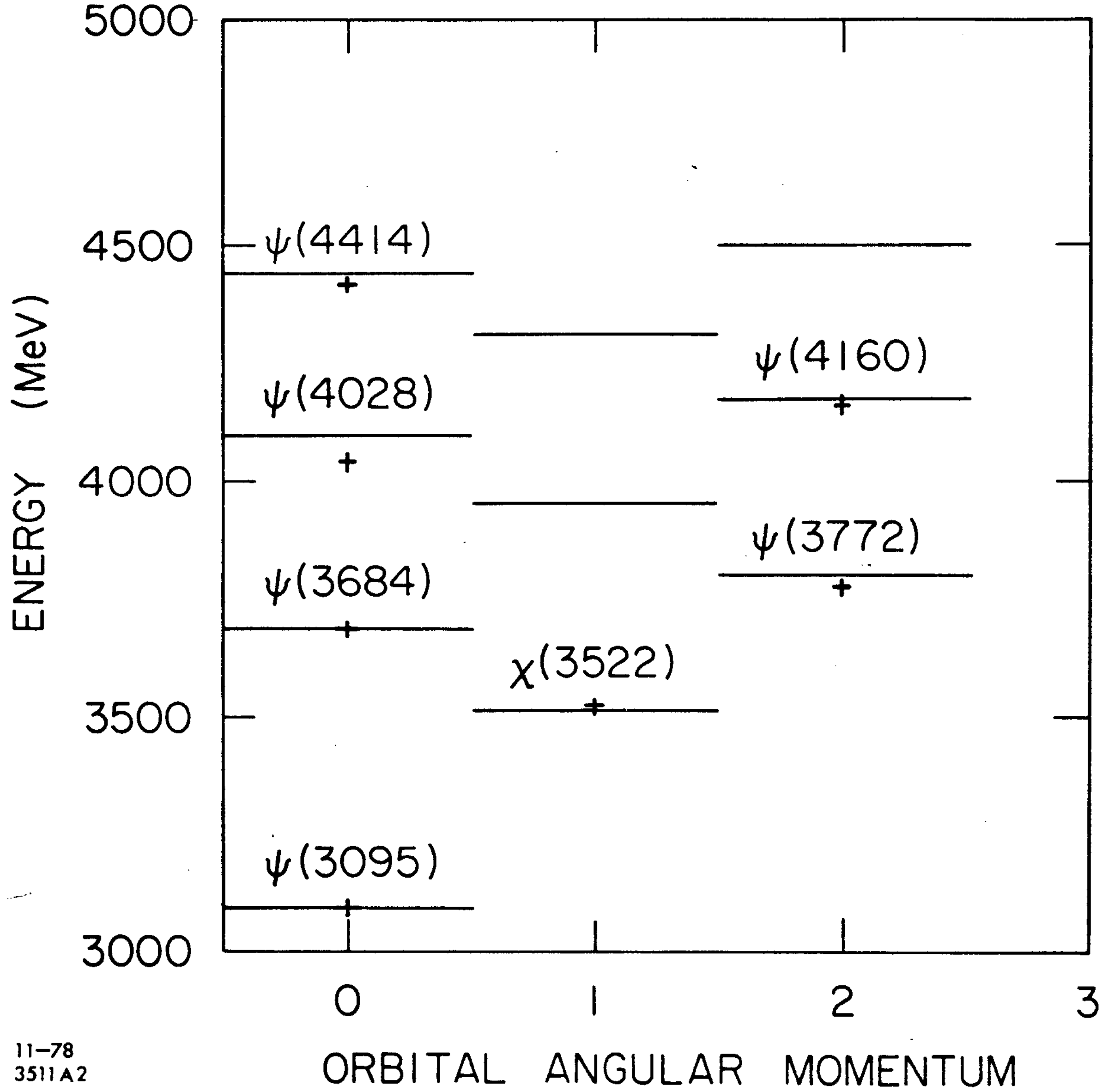
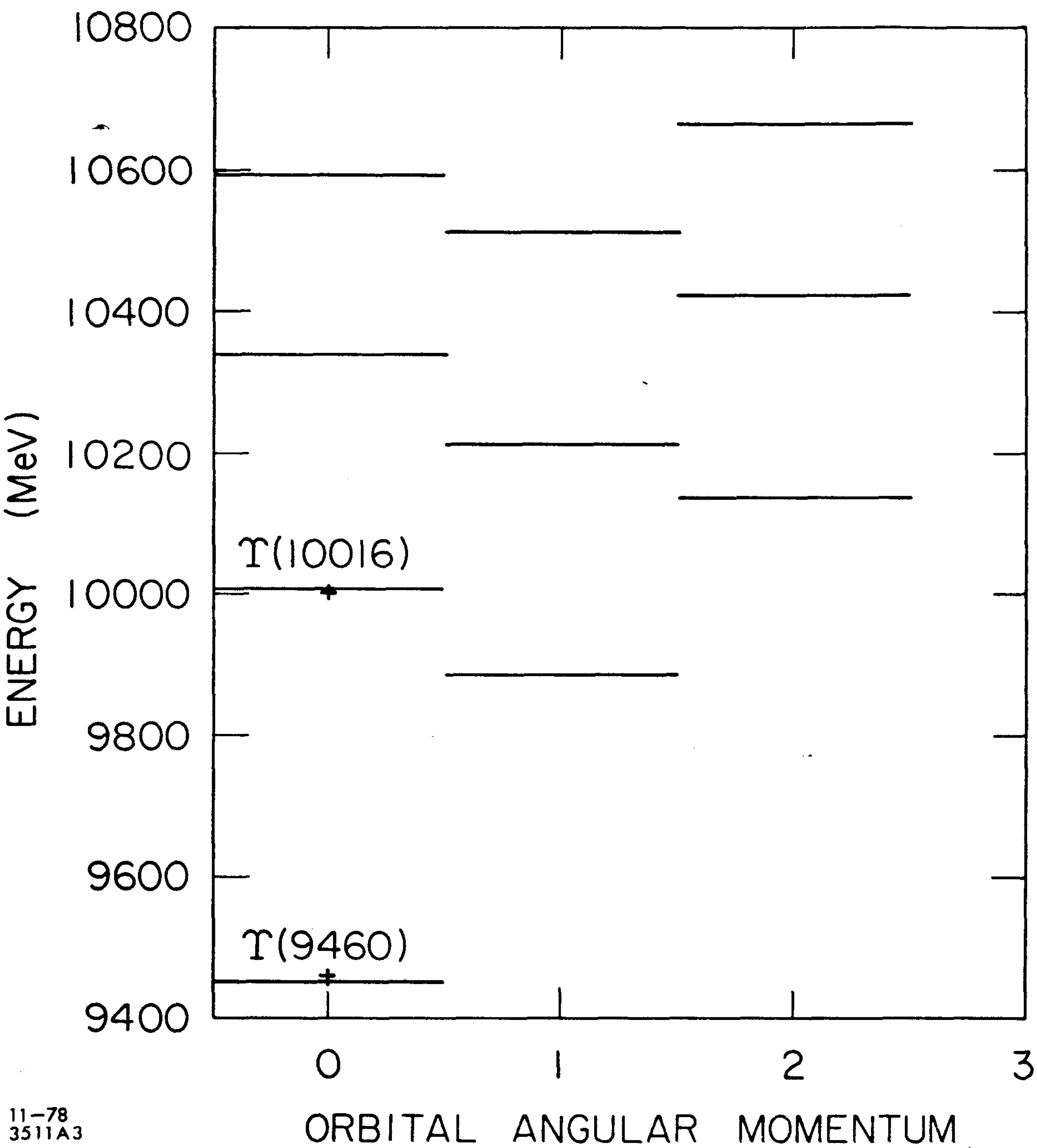


Fig. 1



5000
4500
4000
3500
3000
ENERGY (MeV)
ψ(4414)
ψ(4028)
ψ(3684)
ψ(3095)
χ(3522)
ψ(4160)
ψ(3772)
0
1
2
3
ORBITAL ANGULAR MOMENTUM
11–78
3511A2

Fig. 2



10800
10600
10400
10200
10000
9800
9600
9400
ENERGY (MeV)
ϒ(10016)
ϒ(9460)
0
1
2
3
ORBITAL ANGULAR MOMENTUM
11-78
3511A3

Fig. 3