

PHENOMENOLOGY OF HEAVY LEPTONS AND HEAVY QUARKS*

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I. Introduction

The last few years have brought rather conclusive evidence for a third charged lepton, the τ , and for a fourth and fifth quark. With low statistics, it appears that a new charged lepton is found every forty years, a much longer time than that between "discoveries" of new quarks. Presumably this is due to the large ratio between the masses of successive charged leptons, $m_\mu/m_e \approx 210$ and $m_\tau/m_\mu \approx 17$; while for quarks (using constituent masses), $m_s/m_u \approx m_c/m_s \approx m_b/m_c \approx 3$.

Since we are living in a period when not only the quark but the lepton family is expanding, we review both and start by reviewing the leptons. In the next section the properties of the τ as they are now known are reviewed. We use these data in the sections that then follow on the $SU(2) \times U(1)$ classification of the τ and on τ decays. Section V is devoted to still heavier leptons, after which we turn to quarks and discuss both our understanding of the spectroscopy of heavy hadrons made out of these quarks in Section VI and their strong and electromagnetic decays in Section VII. In the last section we cover the weak interaction properties of the c, b, and t quarks, and the decays of hadrons containing them expected within the context of the standard $SU(2) \times U(1)$ model.

II. Properties of the Tau

For our discussion of the classification and decays of the tau we shall need some of its properties as determined from experiment. These have been

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pinned down to a considerable degree in the past year. We give here only a partial accounting of the most accurate experimental results needed later on. More complete reviews are found elsewhere.¹

The exploration of the threshold region for the reaction $e^+e^- \rightarrow \tau^+\tau^-$ leads to a τ mass¹ of 1782^{+3}_{-4} MeV. The energy dependence of this reaction also establishes that the spin is 1/2. A boson would have to be pair produced in a p-wave, completely contradicting the cross section energy dependence near threshold. Fermions with spins greater than 1/2 have cross sections with divergent high energy behavior and fail to fit the measurements in the 4 GeV region and above.² This latter statement of course assumes a point particle with no form factor.

The lifetime of the τ is not measured. Only upper limits of 3×10^{-12} sec. (DELCO) and 3.5×10^{-12} sec. (PLUTO) have been established.¹

Because of the presence of leptons among its decay products we conclude that the τ decays weakly. The decays $\tau \rightarrow \nu e \bar{\nu}$, $\tau^- \rightarrow \nu \mu \bar{\nu}$, $\tau^- \rightarrow \nu \rho^-$, $\tau^- \rightarrow \nu \pi^-$, and $\tau \rightarrow \nu(n\pi)^-$ are all seen.¹ The world data¹ on $\tau \rightarrow \nu \mu \bar{\nu}$ and $\tau \rightarrow \nu e \bar{\nu}$ allows one to conclude these branching ratios are in the ratio $1.07 \pm .17$ (consistent with being equal) and are each $\sim 18\%$. Decays of the τ involving one charged hadron make up $\sim 35\%$ of all decays, while those with three or more hadrons are $\sim 30\%$.

Almost as important to some theories are the unobserved decay modes. Upper limits exist on the branching ratio for $\tau \rightarrow 3X^\pm$ of 1% (PLUTO) and for the subprocess involving three charged leptons, $\tau \rightarrow 3\ell^\pm$, of 0.6% (SLAC-LBL). The radiative decays $\tau \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ are known to be $<2.6\%$ and $<1.3\%$, respectively.¹

The decay $\tau \rightarrow \nu e \bar{\nu}$ has an electron energy spectrum consistent with there being a V-A interaction at the $\tau \rightarrow \nu$ vertex. V+A is ruled out strongly. Assuming it is V-A, an upper limit of 250 MeV on the ν mass follows.¹

III. SU(2) × U(1) Classification of the τ

Before the discovery of the τ, there were four leptons, the e and the μ and their neutrinos, ν_e and ν_μ. The standard model³ involved the weak-electromagnetic gauge group SU(2) × U(1) and assigning the left-handed leptons to doublets of the weak isospin: $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$ and $\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L$. The right-handed e and μ were taken to be singlets: (e)_R and (μ)_R. We now add the τ and ask what assignment is possible for it as well as the "old" leptons. For the sake of simplicity we restrict the choices to singlets or doublets of weak isospin for either the left- or right-handed leptons.

(A) The "Economy Model"

$$\begin{pmatrix} \nu \\ e' \end{pmatrix}_L ; \begin{pmatrix} \nu_\mu \\ \mu' \end{pmatrix}_L ; (\tau')_L ; (e)_R ; (\mu)_R ; (\tau)_R$$

This assignment^{4,5} is the simplest in the sense that we avoid having a sixth lepton, the τ neutrino, and hence has been referred to as the "economy model."⁶ The primes on the left-handed e, μ, and τ indicate possible mixing. In fact, there must be mixing if the τ is to decay. We write up to second order in small mixing parameters:

$$\begin{aligned} e' &\approx (1 - \epsilon_e^2/2)e + \frac{1}{2}\epsilon_e\epsilon_\mu + \epsilon_e\tau \\ \mu' &\approx \frac{1}{2}\epsilon_e\epsilon_\mu + (1 - \epsilon_\mu^2/2)\mu + \epsilon_\mu\tau \\ \tau' &\approx -\epsilon_e e - \epsilon_\mu\mu + (1 - \epsilon_e^2/2 - \epsilon_\mu^2/2)\tau \end{aligned} \quad (1)$$

Note that this model has "lepton flavor" changing neutral currents, i.e., the Z⁰ connects the e to μ, μ to τ, etc.

However, this model can be ruled out using two pieces of experimental information. First, the upper limit on the τ lifetime implies:

$$\begin{aligned}
3 \times 10^{-12} \text{ sec.} &\geq \frac{\hbar}{\Gamma(\tau \rightarrow \text{all})} \\
&\geq \frac{\hbar [\text{BR}(\tau \rightarrow \nu e \bar{\nu}) + \text{BR}(\tau \rightarrow \nu \mu \bar{\nu})]}{[\Gamma(\tau \rightarrow \nu e \bar{\nu}) + \Gamma(\tau \rightarrow \nu \mu \bar{\nu})]} \\
&\geq \frac{2}{9} \cdot \frac{1.64 \times 10^{-12} \text{ sec.}}{\epsilon_e^2 + \epsilon_\mu^2} \quad (2)
\end{aligned}$$

or

$$\epsilon_e^2 + \epsilon_\mu^2 \gtrsim 0.12 \quad (3)$$

Here we have used the experimental information¹ that

$$\text{BR}(\tau \rightarrow \nu e \bar{\nu}) + \text{BR}(\tau \rightarrow \nu \mu \bar{\nu}) \gtrsim \frac{1}{3}$$

and the result of a calculation⁴ of the widths for $\tau \rightarrow \nu e \bar{\nu}$ and $\tau \rightarrow \nu \mu \bar{\nu}$ (see (ii) below) in terms of the mixing parameters and the width for $\tau \rightarrow \nu_\tau e \bar{\nu}_e$ of $\hbar/(1.64 \times 10^{-12} \text{ sec.})$, corresponding to full strength V-A coupling of the τ to its own neutrino.

Second, the product $\epsilon_e \epsilon_\mu$, which enters all the $\mu \rightarrow e$ neutral current amplitudes can be bounded above⁴ by the lack of observation of $\mu \rightarrow e \gamma$, $\mu \rightarrow 3e$, and $\mu N \rightarrow e N$. The experimental upper limit on any of these processes (all predicted to exist in this model) is good enough for our purposes here, but the best limit comes from the last one:^{6,7}

$$(\epsilon_e \epsilon_\mu)^2 \leq 1.2 \times 10^{-10} \quad (4)$$

The combination of Eqs. (3) and (4) implies that either $\epsilon_e^2 \gg \epsilon_\mu^2$ or $\epsilon_\mu^2 \gg \epsilon_e^2$, by nine orders of magnitude! But then we have as consequences that:

(i) e - μ universality is violated by more than 10%. The strength of the charged current coupling to an electron ($\approx 1 - \epsilon_e^2$) or to a muon ($\approx 1 - \epsilon_\mu^2$) is reduced by at least $\approx 12\%$ from full strength, but not both.⁸

(ii) The ratio $\Gamma(\tau \rightarrow \nu e \bar{\nu})/\Gamma(\tau \rightarrow \nu \mu \bar{\nu}) = [\epsilon_e^2/2 + \epsilon_\mu^2]/[\epsilon_e^2 + \epsilon_\mu^2/2]$ is close to

1/2 or 2 when $\epsilon_e^2 \gg \epsilon_\mu^2$ or $\epsilon_\mu^2 \gg \epsilon_e^2$, respectively.⁴ The peculiar ratio of these purely leptonic decays is a result of having four amplitudes, involving both charged and neutral currents, which contribute to each decay.

(iii) Decays like $\tau \rightarrow e\mu\bar{\nu}$ and $\tau \rightarrow ee\bar{e}$ or $\tau \rightarrow \mu e\bar{e}$ and $\tau \rightarrow \mu\mu\bar{\nu}$, involving neutral "lepton flavor" changing currents, have branching ratios⁵ of order 5%.

Each of (i), (ii), and (iii) is independently completely ruled out by experiment. In particular, (ii) and (iii) contradict the data¹ reviewed in the last section and eliminate the model.

(B) The Heavy Tau Neutrino Model

In this case, we settle for six leptons with left-handed doublets and right-handed singlets;

$$\begin{pmatrix} \nu_e \\ e' \end{pmatrix}_L ; \begin{pmatrix} \nu_\mu \\ \mu' \end{pmatrix}_L ; \begin{pmatrix} N_\tau \\ \tau' \end{pmatrix}_L ; (e)_R ; (\mu)_R ; (\tau)_R ; (N_\tau)_R ,$$

but avoid a light τ neutrino.¹¹ To be specific we assume that N_τ is heavier than the τ so that the latter decays only by mixing. The mixing of e' , μ' , τ' can be described up to second order in small parameters as in Eq. (1). However, as many be verified explicitly, there is a leptonic GIM mechanism⁹ and there are no lepton flavor changing neutral currents, i.e., the Z^0 does not couple e to μ , e to τ , or μ to τ .

However, this model still may be ruled out on the basis of present data.¹⁰

First, we again use the upper limit on the τ lifetime:

$$\begin{aligned} 3 \times 10^{-12} \text{ sec.} &\geq \frac{\hbar}{\Gamma(\tau \rightarrow \text{all})} \\ &\geq \frac{\hbar [\text{BR}(\tau \rightarrow e\nu\bar{\nu})]}{\Gamma(\tau \rightarrow e\nu\bar{\nu})} \\ &\geq \frac{2.6 \times 10^{-13} \text{ sec.}}{\epsilon_e^2 + \epsilon_\mu^2} , \end{aligned} \quad (5)$$

or

$$\epsilon_e^2 + \epsilon_\mu^2 \geq 8.7 \times 10^{-2} . \quad (6)$$

We have used a value for $BR(\tau \rightarrow \nu e \bar{\nu})$ of 0.16 to be on the correct (lower) side of the inequality in Eq. (5).

Second, we can get an upper bound on ϵ_μ^2 from the lack of τ production by (muon) neutrinos:¹²

$$\epsilon_\mu^2 \leq 2.5 \times 10^{-2} . \quad (7)$$

Third, e- μ universality, as tested by the ratio $\Gamma(\pi \rightarrow e\nu)/\Gamma(\pi \rightarrow \mu\nu)$ gives¹³

$$\epsilon_\mu^2 - \epsilon_e^2 = (3.2 \pm 1.9) \times 10^{-2} . \quad (8)$$

Equations (6), (7), and (8) are contradictory: for example, (6) and (8) imply $\epsilon_\mu^2 \geq (6.0 \pm 1.0) \times 10^{-2}$, in contrast to (7). Thus this model is precluded by the data now available.¹⁰

(C) The Extended "Cheng-Li Model"¹⁴

We may go to a model with a tau neutrino and with neutral heavy leptons:

$$\begin{pmatrix} \nu_e' \\ e \end{pmatrix}_L ; \begin{pmatrix} \nu_\mu' \\ \mu \end{pmatrix}_L ; \begin{pmatrix} \nu_\tau' \\ \tau \end{pmatrix}_L ; \begin{pmatrix} N_e' \\ e \end{pmatrix}_R ; \begin{pmatrix} N_\mu' \\ \mu \end{pmatrix}_R ; \begin{pmatrix} N_\tau' \\ \tau \end{pmatrix}_R ; (N_e'')_L ; (N_\mu'')_L ; (N_\tau'')_L$$

The primes denote mixing, and in fact one reason for proposing such a model might be to allow transitions between the leptons (e.g., $\mu \rightarrow e\gamma$) which are allowed with non-zero mixing angles in higher order.^{14,16} The charged leptons have a neutral current with only a vector space-time character.

It is this last aspect of the model which may soon allow it to be ruled out. Forthcoming experiments at SLAC, which extend the observation¹⁵ of parity violation in inelastic polarized electron-deuteron scattering, should permit an unambiguous determination that the neutral current coupling to the electron involves an axial-vector piece. The observation of parity violation in heavy atoms would accomplish the same end. If so, the general class of models involving vector coupling of the Z^0 to electrons will be dead. Otherwise, even though restricted to certain domains of mixing angles and neutral lepton masses by the lack of observation of $\mu \rightarrow e\gamma$, $\mu N \rightarrow eN$, etc., there is a domain of

parameters for which models of the Cheng-Li type are allowed by present data.

(D) Modified "Cheng-Li Model"¹⁷

There is a modification of the model of the last section which can be ruled out already, without waiting for the results of new measurements. This involves eliminating the tau neutrino and replacing it with a neutral heavy lepton:¹⁷

$$\begin{pmatrix} \nu_e'' \\ e \end{pmatrix}_L ; \begin{pmatrix} \nu_\mu'' \\ \mu \end{pmatrix}_L ; \begin{pmatrix} N_\tau'' \\ \tau \end{pmatrix}_L ; \begin{pmatrix} N_e' \\ e \end{pmatrix}_R ; \begin{pmatrix} N_\mu' \\ \mu \end{pmatrix}_R ; \begin{pmatrix} N_\tau' \\ \tau \end{pmatrix}_R ; (N_e'')_L ; (N_\mu'')_L$$

The τ decays in a "normal" manner by the presence of ν_e and ν_μ in the mixed N_τ'' , and also in an "abnormal" way by decay into N_i , if $m_i < m_\tau$.

If we make the standard assumption that the lepton masses arise from couplings to Higgs bosons which lie in singlet or doublet representations of the weak isospin, then a given mixing of the right-handed neutral leptons leads to a particular mixing of the left-handed leptons. Defining the unitary 3×3 mixing matrix U by

$$N_i' = \sum_{j=1}^3 U_{ij} N_j \quad , \quad (9)$$

and neglecting terms of order m_e/m_i , one finds¹⁷

$$\nu_e'' \approx \nu_e \quad , \quad (10a)$$

$$\nu_\mu'' \approx \beta_\mu \nu_\mu + m_\mu \sum_i U_{2i} \frac{1}{m_i} N_i \quad , \quad (10b)$$

$$N_\tau'' \approx \beta_\tau \nu_\tau + m_\tau \sum_i U_{3i} \frac{1}{m_i} N_i \quad . \quad (10c)$$

Unitarity determines β_μ and β_τ . Up to terms of order m_μ^2/m_i^2 one has

$$\beta_\mu^2 \approx 1 \quad , \quad (11a)$$

$$\beta_\tau \approx \beta_\mu \beta_\tau \approx -m_\mu m_\tau \sum_i U_{2i} U_{3i}^* / m_i^2 \quad , \quad (11b)$$

$$m_\tau^2 \sum_i (U_{3i} U_{3i}^* / m_i^2) \approx 1 \quad . \quad (11c)$$

The last equation implies that the smallest m_i is less than m_τ , while the largest m_i is greater than m_τ . Thus "abnormal" decays of the τ into one of the neutral heavy leptons (say N_1) must exist. For masses m_1 fairly close to m_τ the width for these decays behaves as $(m_\tau - m_1)^5$. On the other hand, the "normal" decays which involve mixing of the left-handed partner of the τ with the muon neutrino, are proportional to β_τ^2 . One can show that¹⁷

$$\beta_\tau^2 \leq \left(\frac{m_\mu^2}{m_\tau^2} \right) \left(\frac{m_\tau^2 - m_1^2}{m_1^2} \right) \quad . \quad (12)$$

But then the model can be ruled out in two ways:¹⁸

(i) If $m_1 > (1/2)(m_\tau)$, then Eq. (12) yields $\beta_\tau^2 \leq .04$ and the τ lifetime must be greater than the experimental upper limit.¹ If $m_1 < m_\tau/2 < 900$ MeV, one would have neutral leptons with an important branching ratio being $N_1 \rightarrow \mu^- \pi^+$ or $N_1 \rightarrow e^- \pi^+$. This possibility has been searched for in $e^+ e^-$ annihilation and is ruled out.¹⁹

(ii) Inasmuch as the rate for "normal" decays behaves as $\beta_\tau^2 \propto (m_\tau^2 - m_1^2)$ and that for "abnormal" decays as $(m_\tau - m_1)^5$, a limit on abnormal decays will force m_1 towards m and reduce the rate for both kinds of decays. A (generous) limit of 25% on the branching ratio for the "abnormal" decay $\tau \rightarrow N_1 e \bar{\nu}_e$ forces the τ lifetime to be greater than 2×10^{-10} seconds,¹⁷ orders of magnitude beyond the present limit.

(E) The Standard Model

Except for some very ugly models involving charged leptons in both right-handed singlets and doublets, we are left with what is known as the standard model:

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L ; \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L ; \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L ; \quad (e)_R ; \quad (\mu)_R ; \quad (\tau)_R$$

Nature simply copies the same basic pattern for each charged lepton and its neutrino. The neutrinos need not be massless, or even have equal mass, but that is certainly the simplest possibility. Further, it is consistent with all the data.

It is remarkable that we are able to go so far without any direct experiments involving ν_τ . Rather accurate experiments on electrons and muons, plus a little information on τ decays and an upper limit on its lifetime almost force one within $SU(2) \times U(1)$ to invent a "light" τ neutrino and a classification of the left- and right-handed τ like that in the standard model.

IV. Tau Decays

The tau can decay either purely leptonically or semi-hadronically. In the standard model we have $\tau^- \rightarrow \nu_\tau + e^- \bar{\nu}_e$, $\tau^- \rightarrow \nu_\tau + \mu^- \bar{\nu}_\mu$, and $\tau^- \rightarrow \nu_\tau + d\bar{u}$, where the $d\bar{u}$ quark system²⁰ manifests itself in terms of hadrons equally in overall vector and axial-vector states. A most naive calculation would predict that the rates for these three processes would be in the ratio 1:1:3, where the factor of 3 arises because of the three colors of quarks. A more sophisticated calculation, using QCD corrections, obtains a number slightly larger than three.²¹

For the purely leptonic decays it is straightforward to calculate the width. With zero mass neutrinos:

$$\Gamma(\tau \rightarrow \nu_\tau e \bar{\nu}_e) = \frac{G^2 m_\tau^5}{192 \pi^3}, \quad (13)$$

where the Fermi constant $G = 1.02 \times 10^{-5}/M_N^2$. The width for $\tau \rightarrow \nu_\tau \mu \bar{\nu}_\mu$ is the same, except for a small correction ($\sim 3\%$) due to the muon mass.

The decays into a neutrino plus hadrons with zero net strangeness which take place through the action of the vector coupling to $d\bar{u}$ can be directly related, invariant mass by invariant mass and multiplicity by multiplicity, to e^+e^- annihilation cross sections using CVC. The precise relation is²²

$$\Gamma(\tau \rightarrow \nu_\tau + (\text{hadrons})_{1-}) = \frac{G^2 \cos^2 \theta_c}{96\pi^3 m_\tau^3} \times \int_0^{m_\tau^2} dQ^2 (m_\tau^2 - Q^2)^2 (m_\tau^2 + 2Q^2) \frac{\sigma^{(1)}(Q^2)}{\sigma_{pt}(Q^2)} \quad (14)$$

where θ_c is the Cabibbo angle, $\sigma_{pt}(Q^2) = 4\pi\alpha^2/(3Q^2)$ is the point cross section for $e^+e^- \rightarrow \mu^+\mu^-$ and $\sigma^{(1)}(Q^2)$ is the cross section for e^+e^- annihilation into hadrons through the isovector part of the electromagnetic current at center-of-mass energy Q . In particular, $\sigma^{(1)}(Q^2)$ involves the channels $e^+e^- \rightarrow \pi^+\pi^-$, 4π , 6π , etc.²³

The results of substituting the annihilation data into the right-hand side of Eq. (14) are conveniently expressed in terms of the ratio of the predicted τ partial widths to that (in Eq. (13)) for $\tau \rightarrow \nu_\tau + e\bar{\nu}_e$. For a τ mass of 1.8 GeV (1.9 GeV) one finds²⁴

$$\Gamma(\tau^- \rightarrow \nu_\tau \pi^- \pi^0) / \Gamma(\tau^- \rightarrow \nu_\tau e \bar{\nu}_e) = 1.22 \quad (1.12) ,$$

$$\Gamma(\tau^- \rightarrow \nu_\tau + \pi^+ \pi^0 \pi^- \pi^-) / \Gamma(\tau^- \rightarrow \nu_\tau e \bar{\nu}_e) = .35 \quad (.46) ,$$

and $\Gamma(\tau^- \rightarrow \nu_\tau + \pi^0 \pi^0 \pi^0 \pi^-) / \Gamma(\tau^- \rightarrow \nu_\tau e \bar{\nu}_e) = .08 \quad (.11) .$

The total width for all hadronic vector modes^{24,25} is 1.65 (1.69) times the width for $\tau \rightarrow \nu_\tau e \bar{\nu}_e$. This is in adequate agreement with the value of 1.5 predicted from the most naive calculation and also with the $\sim 20\%$ upward correction to that number given by the lowest order corrections predicted by QCD.²¹

Of the decays that involves hadrons in an axial-vector state, only one can be predicted precisely from other data. This is $\tau \rightarrow \nu_\tau \pi$, which on the basis of the pion decay rate, has the value²²⁻²⁵

$$\Gamma(\tau \rightarrow \nu_\tau \pi) / \Gamma(\tau \rightarrow \nu_\tau e \bar{\nu}_e) = 0.60 \quad . \quad (15)$$

This is consistent with the most recent measurements.¹

Decays of the form $\tau \rightarrow \nu_\tau 3\pi$ proceeding through the axial-vector weak current should exist. In the past year experimental data supporting the existence of this mode has accumulated.¹ The three pion state has a large $\rho\pi$ component and a mass distribution peaking around 1100 MeV. This supports the idea of an A_1 resonance at or near this mass. However, while the data does not fit pure phase space well it does not unambiguously demand resonant behavior either.

We are now in a position to examine the charged multiplicity distribution in τ decays. Given a branching ratio for $\tau \rightarrow \nu_\tau e \bar{\nu}_e$, we take the relative rates for $\tau^- \rightarrow \nu_\tau \pi^- \pi^0$, $\tau^- \rightarrow \nu_\tau \pi^+ \pi^0 \pi^- \pi^-$, $\tau^- \rightarrow \nu_\tau \pi^0 \pi^0 \pi^0 \pi^-$, and $\tau \rightarrow \nu_\tau \pi^-$ from the calculations²⁴ discussed above. This is also confirmed roughly by experimental measurements.¹ After also adding a small contribution from the Cabibbo suppressed decays $\tau \rightarrow \nu_\tau K$ and $\tau \rightarrow \nu_\tau K^*$, we force the remaining decays, which in the standard model are of the form $\tau \rightarrow \nu_\tau + \text{hadrons}$, with the hadrons arising from the axial-vector weak current, to fill up the gap so as to get 100% of the decays from the sum of leptonic and semi-hadronic decays.

The results of this exercise are contained in Table I for a τ mass of 1.8 GeV. The semi-hadronic decays through the axial-vector current (other than $\tau \rightarrow \nu_\tau \pi$) are of the form $\tau \rightarrow \nu_\tau 3\pi$ or possibly $\tau \rightarrow \nu_\tau 5\pi$, given the known value of m_τ . In Table I it is assumed that $\tau \rightarrow \nu_\tau + 3\pi$ dominates, in which case isospin one for the final 3π state demands that $\tau^- \rightarrow \nu_\tau \pi^+ \pi^- \pi^-$ be between 50% and 80% of all $\tau \rightarrow \nu_\tau 3\pi$ decays.

We see from Table I that for $\text{BR}(\tau \rightarrow \nu_\tau e \bar{\nu}_e) = 0.20$ only $\sim 14\%$ of τ decays are of the form $\tau \rightarrow \nu_\tau 3\pi$ (or $\tau \rightarrow \nu_\tau 5\pi$), and total three charged prong decays are altogether at most 18%. The direct measurements¹ of $\tau \rightarrow \text{multi-prong}$ give values of 30% or slightly larger. We conclude that in the standard model the branching ratio for $\tau \rightarrow \nu_\tau e \bar{\nu}_e$ must be less than 20%. In particular, the Table shows that when $\text{BR}(\tau \rightarrow \nu_\tau e \bar{\nu}_e) = 0.16$, the three charged prong decays of the τ can be as much as 30%, in agreement with experiment. However, most of this 30% comes from $\tau \rightarrow \nu_\tau 3\pi$ and $\tau \rightarrow \nu_\tau 5\pi$ while direct measurements¹ of the quantity

$\tau^- \rightarrow \nu_\tau \pi^+ \pi^- \pi^-$ are much smaller. Whether this is due to $\tau \rightarrow \nu_\tau 5\pi$ decays, wrong measurements, or wrong assumptions in the calculations remains an open question.

TABLE I
Tau Decay Multiplicity

Mode	BR($\tau \rightarrow \nu_\tau e \bar{\nu}_e$) = 0.16		BR($\tau \rightarrow \nu_\tau e \bar{\nu}_e$) = 0.20	
	one charged prong	three charged prongs	one charged prong	three charged prongs
$\tau^- \rightarrow \nu_\tau e \bar{\nu}_e$.16	0	.20	0
$\tau^- \rightarrow \nu_\tau \mu \bar{\nu}_\mu$.16	0	.20	0
$\tau^- \rightarrow \nu_\tau \pi^- \pi^0$.20	0	.24	0
$\tau^- \rightarrow \nu_\tau (4\pi)^-$.01	.06	.01	.07
$\tau^- \rightarrow \nu_\tau K^-$.01	0	.02	0
$\tau^- \rightarrow \nu_\tau K^{*-}$				
$\tau^- \rightarrow \nu_\tau \pi^-$.10	0	.12	0
$\tau^- \rightarrow \nu_\tau (3\pi)^-$	<u>.15 to .06</u>	<u>.15 to .24</u>	<u>.07 to .03</u>	<u>.07 to .11</u>
Total	.79 to .70	.21 to .30	.86 to .82	.14 to .18

V. Heavier Leptons

As of now there are no indications for leptons heavier than the tau. Measurements of $R = \sigma(Q^2)/\sigma_{pt}(Q^2)$ in e^+e^- annihilation in the upsilon region (9 to 10 GeV) are consistent off resonance with values (~ 5) measured from 5 to 8 GeV.²⁶ However, a rise of one unit, as expected from production of a point fermion of unit charge, could still be accommodated within the present

error bars.

For a charged heavy lepton with a mass of 5 GeV or so within the standard model, one expects decays into $\nu e \bar{\nu}_e$, $\nu \mu \bar{\nu}_\mu$, $\nu \tau \bar{\nu}_\tau$, $\nu d \bar{u}$, and $\nu s \bar{c}$ in the ratio 1:1:0.5:3:3. The suppression of the decay involving the τ is purely kinematic (and exactly calculable), while the decay into $\nu s \bar{c}$ might be argued to be subject to similar suppression. If the new lepton is heavy enough, and there is an additional weak doublet of quarks, $\begin{pmatrix} t \\ b \end{pmatrix}$, then we must add the decay into $\nu b \bar{t}$.

In any case the branching ratio into $\nu e \bar{\nu}_e$ should be around 10%, a quite respectable level. Individual channels, like $\nu \pi$ or $\nu \rho$ which correspond to major τ branching fractions, will be at the 1% level for a lepton with a mass, $M_L = 5$ GeV (such exclusive channels have rates which go down as $1/M_L^2$ compared to that for $\nu e \bar{\nu}_e$). However, some caution is needed here, for if the mass of the new lepton is just right so that a decay like $L \rightarrow \nu + (b \bar{t})$ can barely take place, then all the rates for such channels may be soaked up in a very few $b \bar{t}$ discrete states (say the ground state pseudoscalar and vector). Thus some very interesting individual channels may not have small branching ratios.

The detection of such a heavy charged lepton is relatively easy in $e^+ e^-$ annihilation. Production by $e^+ e^- \rightarrow L^+ L^-$ with a point cross section followed by the $\nu e \bar{\nu}_e$ decay of one and $\nu \mu \bar{\nu}_\mu$ decay of the other with $\sim 10\%$ branching fraction each, should make a replay of the initial τ discovery straightforward.²⁷ In fact, the τ will likely furnish the main background. High mass neutral leptons generally are much more difficult to find. Only if they are coupled to electrons does the lowest order weak process $e^+ e^- \rightarrow \bar{N} \nu_e$ or $N \bar{\nu}_e$ present itself and make the job a little less than impossible.²⁸

VI. Heavy Quarks and Their Bound States

The last few years have seen first the discovery of hidden charm states (charmonium), then of charmed particles themselves, and in the last year of the first particles, (T, T', \dots) containing yet another hidden quark flavor. We have direct experimental evidence for five flavors of quarks, and good

theoretical reasons for a sixth.

The strong interactions bind the quarks together to form the hadrons actually observed in high energy experiments. When the ψ/J was first discovered it was originally hoped that the charmed quarks inside were sufficiently massive that we finally had a basically non-relativistic bound state problem with which to deal.²⁹

This hope, that $(v/c)^2$ was small, first was taken and applied to calculating energy levels on the basis of simple potentials with a Shrodinger equation. Later the spin dependent forces were taken as arising from the same basic potential in the form of relativistic connections to order v^2/c^2 .³⁰

At short distances one expects the potential to behave as $1/r$ due to single gluon exchange. At large distances there must be quark confinement and some theoretical models lead to the expectation that $V(r) \propto r$. Thus early calculations assumed that:

$$V(r) = -\frac{\kappa}{r} + \frac{r}{a^2}, \quad (16)$$

and assumed as well that the space-time structure was $\gamma_{\mu}^{(1)} \otimes \gamma_{\mu}^{(2)}$ with the superscripts referring to the Dirac spinor spaces of the two quarks.

Potentials of the single term analytic form r^n have also been employed in place of Eq. (16) and are useful in deriving scaling laws, and developing intuition,³¹ but unlike Eq. (16) or similarly motivated forms,³² there is no theoretical reason for their applicability to the real problem at hand. While Eq. (16) gives fair fits to the spectrum of charmonium (as well as ψ leptonic width, electric dipole transitions, etc.), it does not work when examined in detail. This is especially true when compared to the extensive experimental data that have been accumulated on charmonium, and the beginnings made on the upsilon system.

In particular use of a $\gamma_{\mu}^{(1)} \otimes \gamma_{\mu}^{(2)}$ space-time structure gives the wrong absolute magnitude and the wrong relative splitting of the p-wave charmonium states (these are the $\chi(3414)$, $P_c/\chi(3508)$, and $\chi(3550)$, the quark spin one $c\bar{c}$

states with $J^P = 0^+, 1^+, \text{ and } 2^+$ respectively).³³ Furthermore, the spin-spin interaction, which splits the ground state vector from the pseudoscalar (the $\psi(3095)$ from the $X(2830)$) gives much too small a result.³⁰

All this was realized several years ago and ways to fix things up were soon found. First, if the confining potential (the part linear in r) has an effective scalar space-time character, both the relative and absolute p-wave splittings come out much better. The short-distance, $1/r$ part, of the potential remains vector in character (and plays a role in the p-wave splittings as well). Second, the quarks are given a color anomalous magnetic moment. This gives different spin-spin forces in particular, and one can now fit the ψ - X splitting. Typical parameters involved in such an attempt to fit the observations are $\kappa = 0.27$, $a^{-2} = 0.2 \text{ GeV}^2$, and an anomalous magnetic moment of 4.4 quark magnetons.^{33,34}

But what does all this have to do with QCD? It seems we are getting farther from our basic goal of having a "simple" quark system whose properties allow a real test of the parameters and structure of the underlying theory.³⁵ Even as pure phenomenology, there are too many parameters and too few real predictions.

Further, a potential with the parameters given above doesn't do so well on the Υ system; it predicts³⁴ $m(\Upsilon') - m(\Upsilon) \approx 440 \text{ MeV}$, whereas experiment²⁶ now gives $\sim 556 \text{ MeV}$ (which is close to $m(\psi') - m(\psi) = 589 \text{ MeV}$). To get this splitting one must increase the coefficient of the $1/r$ piece of the potential from³⁴ ~ 0.27 to ~ 0.4 . This is not surprising since a $1/r$ potential gives mass splittings which behave as m_{quark} (and hence go up between the ψ and Υ systems), while an r potential gives splittings which behave as $m_{\text{quark}}^{-1/3}$ (and hence go down between the ψ and Υ systems). To get a larger mass splitting one wants to increase the former. With some readjustment of other parameters, one can still get a decent fit to the charmonium spectrum also. However, the cost³⁴ is considerably worse predictions for the leptonic widths of the ψ and ψ' and for the electric dipole transition rates from the ψ' to the χ states.

By increasing the strength of the $1/r$ part of the potential we make the spectrum more Coulomb-like. This will be especially true for even higher mass quarks, where the wave function is pulled in closer to the origin.³⁶ Note also, that the coefficient κ of $1/r$ which is related to the "strong fine structure constant" α_s by

$$\kappa = \frac{4}{3} \alpha_s \quad (17)$$

is already as big (at $Q^2 = M_T^2$) as one would like it to be if it is to be roughly consistent with the value deduced from other applications of QCD.

Finally, note³⁶ that the wave function of the T' samples about the same part of the potential as does that of the ψ . So their wave functions should be very much the same, and any uncertainty in the wave functions will cancel out in the ratio of the leptonic widths. Thus a comparison of $\Gamma(T' \rightarrow e^+e^-)$ and $\Gamma(\psi \rightarrow e^+e^-)$ is a relatively cleaner test of the charge of the quark in the T than is $\Gamma(T \rightarrow e^+e^-)$ vs. $\Gamma(\psi \rightarrow e^+e^-)$. With the recent measurement of $\Gamma(T' \rightarrow e^+e^-)$, it seems that the conclusion of charge $-e/3$ quarks in the T, T', \dots is quite solid.³⁶

As for charmed particles themselves, the charmed pseudoscalar and vector ground state mesons, $D^0(1863)$, $D^+(1868)$ and $D^{*0}(2006)$, $D^{*+}(2009)$, respectively,³⁷ are by now very well established. Evidence²⁶ exists as well for the corresponding charmed-strange states, $F(2030)$ and $F^*(2140)$. Some hints of higher mass excited charmed mesons, D^{**} 's, have been seen,³⁸ but no conclusive evidence or spin-parity assignment has been made.

An interesting consequence³⁹ of the consideration of the quark-antiquark potential discussed above in relation to charmonium occurs for the p-wave charm states. Recall that the κ/r and r/a^2 pieces of the potential contribute with opposite signs to the spin-orbit splitting. Further, these contributions depend on the reduced mass of the quark-antiquark system. The net result is that when one quark mass is much larger than the other, the presumed Lorentz scalar (r/a^2) term in the potential dominates and the coefficient of the $\vec{L} \cdot \vec{S}$ term in the expression

for the mass has opposite sign to the usual (Coulomb) case which holds for charmonium. Therefore, one expects an "inverted" order to the p-wave D^{**} or F^{**} states:³⁹ with increasing mass the spin-parity should be 2^+ , 1^+ , and 0^+ . This should be a dramatic consequence of the quark-antiquark potential deduced from charmonium if it is indeed observed.

The lowest mass charmed baryons involve c, u, and d quarks. If the u and d quarks are in an $I=0$, spin zero state we have a Λ_c^+ , while $I=1$ and spin one can give $\Sigma_c^{0,+}$ and $\Sigma_c^{*,0,+}$ with spin 1/2 and 3/2, respectively. Earlier observations of candidates for Λ_c (and possibly Σ_c or Σ_c^*) in a neutrino induced event,⁴⁰ and in high energy photoproduction,⁴¹ have been bolstered by an improved version of the latter experiment⁴² seeing evidence again for $\bar{\Lambda}_c(2260) \rightarrow \bar{\Lambda}_c^+ \pi^- \pi^-$. Detailed study of charmed baryon spectroscopy and decays remains for the future.

VII. Strong and Electromagnetic Decays of Hadrons Containing Heavy Quarks

In the strong and electromagnetic decays of hadrons all net quark flavor quantum numbers ("upness", "downness", "strangeness", "charm", ...) are conserved. At the quark level both these interactions involve vector bosons coupling a given quark flavor to itself.

In the case of electromagnetism the photon couples to $\bar{q}\gamma_\mu q$. The quark magnetic moment leads to magnetic transitions at the hadron level, while the usual quark convection current leads to electric multipole transitions if the quarks move non-relativistically. Things become more complicated when relativistic effects are included.

As for strong decays, they occur in two distinct classes. Those involving a disconnected quark diagram are referred to as "forbidden" by the Okubo-Zweig-Iizuka (OZI) rule⁴³ and occur at greatly suppressed rates. Even charge conjugation mesons made up of a heavy quark and its corresponding antiquark have forbidden decays into final particles containing only light quarks, which occur through two gluon intermediate states. For odd charge conjugation

states the corresponding decay proceeds through annihilation of the heavy quarks into three gluons. For mesons below the corresponding flavor threshold, only such OZI forbidden strong decays are permitted by phase space. Above the threshold, one has "OZI allowed" decays into pairs of flavorful mesons which occur at typical strong interaction rates.

For charmonium states, the comparison of experiment and theory for strong and electromagnetic decays is a mixed success. Electric dipole transitions, like those from the ψ to the χ states come out relatively well (within a factor of two or better) in absolute rate. Furthermore, the OZI suppressed decays of the even charge conjugation χ states should have larger widths than that of the ψ because they involve two body rather than three body phase space and one less power of α_s (thought to be less than unity). This is borne out by experiment (with the help of some theoretical estimates of absolute radiative widths).^{33,44}

On the other hand, if $X(2830)$ and $\chi(3455)$ are the pseudoscalar partners of the ψ and ψ' , respectively, then the magnetic dipole transitions $\psi \rightarrow \gamma X$ and $\psi' \rightarrow \gamma \chi$ have predicted widths which disagree with the experimental upper limits (by an order of magnitude for $\psi \rightarrow \gamma X$). Further, the ratios $X \rightarrow \text{hadrons}/X \rightarrow \gamma\gamma$ and $\chi \rightarrow \text{hadrons}/\chi \rightarrow \gamma\psi$ disagree with calculations based on the two gluon mechanism for decay into hadrons (by at least an order of magnitude for $\chi(3455)$).³³ A possible new even charge conjugation state²⁶ near 3600 MeV (or 3180 MeV) is not much better in this regard as a replacement for $\chi(3455)$ as the partner of the ψ' , and it also raises the question of its mass splitting from the ψ' relative to the X - ψ mass difference.

The upsilon system should be quite interesting in this regard. All the OZI forbidden strong decays should generally have smaller widths because of the decrease in the gluon coupling with increasing mass. Rules based on a multipole expansion of QCD give definite predictions for the quark mass dependence of hadronic transitions within the ψ vs. T systems.⁴⁵ All spin-spin, spin-orbit, etc. mass splittings and magnetic dipole transition moments should

be reduced by powers of the quark mass.⁴⁶ Electric dipole transition moments can be computed as well once a particular potential is assumed.⁴⁷ The more non-relativistic up-silon system seems to be shaping up as the crucial qualitative as well as quantitative testing ground in the near future for both our ideas on heavy quark-quark forces and strong and electromagnetic decays of hadrons containing heavy quarks.

VIII. Weak Decays of Heavy Quarks

If we assume $SU(2) \times U(1)$ as the gauge group of the weak and electromagnetic interactions, charged current weak processes lead to the conventional assignment of the left-handed u, d, and to a lesser extent, c, s to weak isospin doublets. Data on neutral current neutrino reactions indicates that the right-handed u and d quarks act as weak isospin singlets. The lack of anomalous behavior in antineutrino deep inelastic scattering prohibits the assignment of the right-handed b quark into a doublet with the u quark.⁴⁸

Thus, although not entirely demanded by the data, we are pushed toward the assignment of left-handed quarks to doublets and right-handed ones to singlets, much like the leptons. With observations supporting five quarks, we are led to assume a sixth, t, with charge $+2e/3$, and group the left-handed quarks in three weak doublets (and the right-handed ones all in singlets).

The only remaining freedom is which linear combinations of d, s, and b (with charge $-e/3$) are coupled to u, c, and t (with charge $+2e/3$), respectively. This freedom may be expressed in terms of a 3×3 unitary matrix which has 9 free parameters. However, five phases may be absorbed in redefined quark fields, so there are on 4 parameters of physical significance: 3 Cabibbo-like angles and a complex phase which gives rise to CP violation. With the conventional left-handed doublets

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L; \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L; \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L, \quad ,$$

the relation between the primed and unprimed quark fields is⁴⁹

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \quad (18)$$

where $c_i = \cos \theta_i$, $s_i = \sin \theta_i$ for $i=1,2,3$, and $\delta \neq 0$ leads to CP violation.

The angle θ_1 is essentially the Cabibbo angle and $\sin^2 \theta_1 \approx 0.05$. The other two angles remain to be measured, but one can put upper limits on them.

(a) The sum of the squares of the charged current couplings of $u \rightarrow d$ ($\cos^2 \theta_1$) from $n \rightarrow p e \bar{\nu}_e$ and of $u \rightarrow s$ ($\sin^2 \theta_1 \cos^2 \theta_3$) from strange particle decays should deviate from that for $\mu \rightarrow \nu_\mu$ (unit strength) from $\mu \rightarrow \nu_\mu e \bar{\nu}_e$ by $\sin^2 \theta_1 \sin^2 \theta_3$, the square of the $u \rightarrow b$ coupling. With appropriate radiative corrections to muon decay the last coupling is consistent with zero within errors and one has the upper limit⁵⁰

$$(s_1 s_3)^2 < 0.003 \quad (19a)$$

or

$$s_3^2 < 0.06 \quad (19b)$$

using $\sin^2 \theta_1 \approx 0.05$.

(b) A more theoretical argument is based on the calculation of the $K^0 \leftrightarrow \bar{K}^0$ transition with two W bosons exchanged. If the introduction of the t quark with a mass greater than 7 GeV is not to ruin the usual calculation (involving the c quark) then^{28,50}

$$\sin^2 \theta_2 \lesssim 0.1 \quad (20)$$

Thus all $\sin^2 \theta_i$ are less than 0.1.

We are now ready to begin our discussion of weak decays of hadrons with heavy quarks. The most naive model of such decays pictures the heavy quark, say the charm quark, decaying as if free into other quarks and possibly

leptons. We then have $c \rightarrow s e^+ \nu_e$, $c \rightarrow s \mu^+ \bar{\nu}_\mu$ and $c \rightarrow s \bar{d} u$ occurring in the ratio of 1:1:3 and

$$\Gamma(c \rightarrow s + \dots) \approx 5\Gamma(c \rightarrow s e^+ \nu_e) \approx 5 \frac{G_m^2 m_c^5}{192\pi^3} . \quad (21)$$

The factor of three in non-leptonic decay is, as in heavy lepton decay, because of color. The assumption has been implicitly made that the final quarks materialize as hadrons with unit probability. There is to be no special enhancement (or suppression) of special final states.

This is not true in the decays of strange particles where non-leptonic decays are much enhanced over their naively expected level and further a special part, that with $\Delta I = 1/2$ is the enhanced portion. In an SU(3) context it is the octet part of the weak interaction which is enhanced. This enhancement is thought to come about both because the decaying quark interacts (by W exchange) with other quarks in the initial hadron (modified by gluon exchanges) and because $s \rightarrow u \bar{d}$ doesn't occur at the naive level, especially when gluon exchanges are taken into account.⁵¹

For charmed meson decays there are two pieces of evidence that a corresponding large enhancement of non-leptonic decays does not occur:

(a) The average D semi-electronic branching ratio³⁷ is $\sim 10\%$. This is close enough to the most naive 20% (or the $\sim 17\%$ in τ decay) to argue that we are talking about less than a factor of two in amplitude.

(b) $D^+ \rightarrow \bar{K}^0 \pi^+$ occurs with a branching ratio³⁷ of 1.5%, about the same as that for $D^0 \rightarrow K^- \pi^+$. If the total widths of D^0 and D^+ are about the same, $D^+ \rightarrow \bar{K}^0 \pi^+$ and $D^0 \rightarrow K^- \pi^+$ have comparable rates. However $D^0 \rightarrow \bar{K}^0 \pi^+$ is forbidden by the analogue of the $\Delta I = 1/2$ rule in strange particle decay plus SU(3) symmetry, whereas $D^0 \rightarrow K^- \pi^+$ is perfectly allowed.

There is one out.⁵² If $\Gamma(D^0) \gg \Gamma(D^+)$, then $BR(D^+ \rightarrow e^+ \nu_e + \dots) \gg BR(D^0 \rightarrow e^+ \nu_e + \dots)$ and $\Gamma(D^+ \rightarrow \bar{K}^0 \pi^+) \ll \Gamma(D^0 \rightarrow K^- \pi^+)$.

We eliminate both (a) and (b) as evidence against enhancement. In fact, D^0 (but not D^+) non-leptonic decays would have to be very much enhanced over the

naive model. This possibility can probably already be eliminated by experiment. First, the average D meson semi-leptonic branching ratio measured in e^+e^- annihilation would vary with center-of-mass energy as the proportion of D^+ and D^0 varies. Second, the ratio of two electron to one electron events from D decays at 3.77 GeV (where D^+ and D^0 are produced almost equally) would be twice as big as expected on the basis of equal D^+ and D^0 semi-leptonic branching ratios and the single electron events.⁵³ Although some difference in D^0 and D^+ semi-electronic branching ratios can't be ruled out, it seems unlikely that an order of magnitude difference is compatible with the data.⁵³ It should be emphasized that calculations based on QCD predict only a small enhancement in charm decays and specific applications⁵⁴ to $D \rightarrow K\pi$, for example, appear to be in quantitative agreement with experiment.

For the still heavier hadrons containing b and t quarks, we follow the naive model applied to charm above, with appropriate modifications. Namely we treat the heavy decaying quark as if it were free and write

$$\Gamma \left(\begin{array}{c} e^+ \nu \\ x \rightarrow y + u \bar{d} \\ \mu^+ \nu \end{array} \right) = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} \left(\frac{G^2 m_x^5}{192\pi^3} \right) \Theta_{xy} F \left(\frac{m_y}{m_x} \right), \quad (22)$$

where Θ_{xy} is a factor depending on the charged current coupling of x to y in the weak mixing matrix, Eq. (18), and $F(m_y/m_x)$ is a factor which is unity for $m_y=0$ and less than unity for $m_y>0$.

In the particular case of the b quark, it can couple to u, c, and t quarks with the last presumably heavier than b. For $m_b \approx 5$ GeV, we have from Eqs. (22) and (18) that²⁸

$$\begin{aligned} \frac{\Gamma(b \rightarrow c + \dots)}{\Gamma(b \rightarrow u + \dots)} &= \frac{\Theta_{bc} F(m_c/m_b)}{\Theta_{bu} F(m_u/m_b)} \\ &\approx \frac{s_2^2 + s_3^2 + 2s_2s_3 \cos \delta}{s_1^2 s_3^2} \times \left(\frac{\sim 1/3}{1} \right). \end{aligned} \quad (23)$$

The limits on s_2^2 and s_3^2 allow the right-hand side to vary in principle from

zero to infinity. But taking $s_2 \approx s_3$ (and δ small) gives values of ~ 25 and only for $s_2 \approx -s_3$ is the ratio less than unity. The lack of observation⁵⁵ of stable hadrons with mass ≈ 5 GeV in pN collisions at a cross section level about that of the T implies that the b quark decays and either θ_2 or θ_3 is not zero. Thus we expect that hadrons containing b quarks have "generalized Cabibbo angle suppressed" decays (lifetime probably about 10^{-13} seconds) with charmed particles usually found among the decay products.

For the t quark we have correspondingly²⁸ with $m_t \approx 15$ GeV

$$\frac{\Gamma(t \rightarrow b + \dots)}{\Gamma(t \rightarrow s + \dots)} \approx \frac{\theta_{tb} F(m_b/m_t)}{\theta_{ts} F(m_s/m_t)} \approx \frac{1}{s_2^2 + s_3^2 + 2s_2s_3 \cos \delta} \cdot \left(\frac{1}{3}\right) \quad (24)$$

The right-hand side of Eq. (24) is at least of order unity and could be infinite with $s_2 \approx -s_3$. We would generally expect it to be in the range of 2 to 10. Thus hadrons containing t quarks should have "generalized Cabibbo angle allowed" decays (lifetime about 10^{-17} seconds for $M_t \sim 15$ GeV), with hadrons containing b quarks usually in the final state.

The most characteristic feature to emerge from this analysis is the cascading weak decays, $t \rightarrow b \rightarrow c \rightarrow s$ as the likely dominant decay chain. Since at each weak decay the charged W can materialize as a lepton pair, a unique and very characteristic feature of the net decay products of a hadron containing b or t quarks is the possibility of two (or more) charged leptons. This holds forth the possibility of seeing very characteristic events with greater than two leptons (or two leptons of the same sign) in events above the b and t flavor thresholds in e^+e^- annihilation. Such a signature, in conjunction with other indications of a new flavor threshold, will surely be one of the first things searched for at PEP and PETRA.

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