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ERRATA

ON THE PHYSICAL INTERPRETATION OF THE COMBINATORIAL HIERARCHY

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p. 13 2nd paragraph, line 4: for "entology" read "ontology"

p. 16 1st paragraph of Sect. II, line 4: for "0+1=0" read "0+1=1"

p. 19 2nd paragraph, line 3: for "con" read "can"

p. 20 Table 2.1, row N=4, column ℓ=2: insert " 16
15
$2^{15}-1$"

Table 2.1, row N=4, last column:

for "Mapping not possible (Amson) even though $4^2 > 15$"

read "$(16)^2 < 2^{(2^{15}-1)}$"

p. 24 line 8 from botton: for "above" read "have"

p. 25 line 5: for "10^{38}" read "$\sim 10^{-38}$"

line 7: for "10^{44}" read "$\sim 10^{-44}$"

p. 33 is a repeat of p. 32 and should be deleted

p. 41 equation 4.3: for " $\frac{c}{137}$ " read " $\frac{\hbar c}{137}$ "

equation 4.4: requires a solidus between the two integrals

p. 44 2nd paragraph, line 9: for "photon" read "proton"

p. 49 ref. 1: for "(Bastin, 1976)" read "(Bastin, 1976, rev. 08-xii-1978)."

p. 50 add reference:

"14. J. H. M. Whiteman, "The Phenomenology of Observations and Explanation in Quantum Theory", in Quantum Theory and Beyond (T. Bastin, ed.), Cambridge, (1971), p. 71."

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ON THE PHYSICAL INTERPRETATION OF THE COMBINATORIAL HIERARCHY*

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ABSTRACT

The combinatorial hierarchy model for basic particle processes is compared and contrasted with the Ur-theory as developed at the Tutzing Conferences. While agreeing with Ur-theory about a finite basis, the "fixed past - uncertain future" aspect of physics, and the necessity of dropping Bohr's requirement of reduction to the haptic language of commonsense and classical physics, we part company at the point of introducing continuous groups. We retain a constructive, hierarchical approach which can yield only an approximate and discrete "space time", and introduces the observation metaphysic at the start. Concrete interpretation of the four levels of the hierarchy (with cardinals 3, 7, 127, $2^{127}-1 \approx 10^{38}$) associates the three levels which map up and down with the three absolute conservation laws (charge, baryon number, lepton number) and the spin dichotomy. The first level represents +, -, and ± unit charge. The second has the quantum numbers of a baryon-antibaryon pair and associated charged meson (e.g., $n\bar{n}$, $p\bar{n}$, $p\bar{p}$, $n\bar{p}$, π^+, π^o, π^-). The third level associates this pair, now including four spin states as well as four charge states, with a neutral lepton-antilepton pair ($e\bar{e}$ or $\nu\bar{\nu}$) in four spin states (total, 64 states)- three charged spinless, three charged spin 1, and neutral spin 1 mesons (15 states), and a neutral vector boson associated with the leptons; this gives $3+15 + 3\times 15 = 63$ possible boson states, so a total correct count of $63+64 = 127$ states. Something like $SU_2 \times SU_3$ and other indications of quark quantum numbers can occur as substructures at the fourth (unstable) level. Breaking into the (bose) hierarchy by structures with the quantum numbers of

a fermion, if this is an electron, allows us to understand Parker-Rhodes calculation of $m_p/m_e = 1836.1516$ in terms of this interpretation of the hierarchy. A slight extension gives us the usual static approximation to the binding energy of the hydrogen atom, $\alpha^2 m_e c^2$. We also show that the cosmological implications of the theory are in accord with current experience. We conclude that we have made a promising beginning in the physical interpretation of a theory which could eventually encompass all branches of physics.

I. INTRODUCTION: GENERAL PRINCIPLES OF THE COMBINATORIAL HIERARCHY IN THE TUTZING CONTEXT

In this section we explain the basic principles of a theory which has been presented in two successive Tutzing conferences, as a contribution to the corpus of thinking on the most basic issues in physics and in the philosophy of physics which has emerged from these conferences. Our theory consists in the use of a combinatorial hierarchy model of basic physical interactions.

The background to the Tutzing conferences has been one of finitism. The theory of *Ur*'s — basic, discrete, two-valued entities, proposed by von Weizsäcker and accepted as an adequate starting point by some of his associates — has served to give the Tutzing conferences a particular orientation and framework within which discussion could take place. The claim made in effect by the *Ur*-theorists (see particularly von Weizsäcker's 1978 paper) has been that if descreteism is firmly and clearly enough embraced, then the something very like the usual quantum theoretical formalism can be sustained as a consistent theory, and the paradoxes and other perplexities avoided.

A different position has been maintained by Finkelstein, who accepts the finitist part of the Ur program but considers that further innovation in basic principles is necessary. He adopts a process philosophy, thinking that the elementary discrete constituents of nature must have a principle of concatenation, and that this principle, whatever it may be, must tell us a good deal about the interrelations of the classical and the quantum worlds.

Our theory accords with Finkelstein's demand for innovation beyond the finitist assumption; we adopt the general direction of his "process," or sequential concatenating conjecture. We present a specific model within the class specified by his conjecture, and can claim experimental backing for our model. Our model is distinct from quantum mechanics; it might become equivalent to the latter under special conditions. Some results which would normally be thought to be dependent upon quantum mechanics as a complete theory appear in our model at a more general stage than that at which we make contact with the special case of quantum mechanics.

We think that one of the very great difficulties in physical science at the moment — particularly at any rate from the point of view of the philosopher of physics — is the extremely monolithic character of technical physics. We are accustomed to hear lip-service fulsomely paid by physicists and philosophers to the importance of having alternative theories between which the facts may be allowed to decide. In practice it is almost impossible to suggest a serious (as distinct from a science fiction) modification to basic physical theory. If you change one piece you change the monolith in every particular. Fruitful change seems nearly impossible. The practical physicist behaves as though it is inconceivable. This schizophrenia between the theory of the philosophy of science and practice seems to us very serious: we believe it an advantage that we have to present an "alternative" model. We are surprised that in the Tutzing milieu so few of the contributors have penetrated into these areas of discussion.

The historical origins of the quantum theory concerned the experimental discovery of discreteness and an attempt to explain it using a continuum conceptual framework (we may consider that the Planck radiation formula was a striking experimental ratification of theoretically arbitrary mathematical imposition of discreteness). Early quantum theory hardly claimed to be explanatory; the modern form of the theory has usually been seen as a successful reconciliation of the continuous and the discrete, and therefore as a satisfactory explanation of the latter. However in view of the continuing unease with the conceptual foundations of the theory, it seems as appropriate today as ever it was to enquire a) wherein the explanation lay, and b) how successful it was. It is sensible to carry on our enquiry in the context of any of the traditional Gedankenexperimente (two slit experiments, photon splitting experiments, photon correlation experiments such as have been imagined by a sequence of theorists going back to Einstein, Podolsky and Rosen).

As everybody knows, quantum theory has maintained that there is a distinct class of things in the universe called measurements or observations and that different rules apply to these from those that apply to interactions in which the acquisition of knowledge is not involved. In one way or another use is made of this principle to justify the importation into the formalism of a discrete principle. As everybody also knows, this principle has never produced peace of mind, even though the great thinkers of the quantum theory have concentrated their attention upon it. (Consider, for example, the essay by John Wheeler in "An Encyclopaedia of Ignorance" p1, 1977, Pergamon, Oxford).

Let us review the position of this principle in the Tutzing milieu. When we have the assumption of discreteness to start with, do we need no observation metaphysic, or is the boot simply on the other foot with our having no continuum (instead of no discretum) without appeal to some observation metaphysic? We are at one with the Ur theorists in thinking that the provision of a discrete base for theory is a sine quo non for understanding the observation metaphysic, but how far does agreement go when we raise the question of what further principles dictate the shape of the quantum mechanical formalism properly required by the observation metaphysic? As we understand the Ur theorists (and again this remark is related particularly to von Weizsäcker, 1978) the Ur theorists expect the finitism of quantum theory to follow from the existence of a finite number of Urs at any time. They also appear to expect the existing mathematical formalism of quantum theory to provide an adequate encapsulation of the observation metaphysic. Thus the ideas connected with indeterminacy (traditionally the point where the observer metaphysic has its central impact) are derived as matters of principle from the existence of only a finite number of finite alternatives (themselves derived from the binary alternatives embodied by the Urs). However, the traditional view of the uncertainty principle as a direct consequence of the observation metaphysic would seem to demand an explicit connexion between the finiteness of the number of Urs and the existence of Planck's constant (expressed in terms of dimensionless ratios of other fundamental constants, of course), if we are to regard the Ur theory as having such a consequence. If one didn't demand an actual value for it to be derived, at least one would expect an existence proof for it to have some value rather than any other.

Thus, in answer to our question we find that the Ur theory claims that it embraces the observation metaphysic and provides an explication of its appearance in the current quantum mechanical formalism. Again we agree. We find, however, that the Ur theory fails to exhibit the detailed working of the observation metaphysic at the vital point, and it is at this point that we find ourselves parting company with the Ur theorists at the level of practical thinking and theorizing, though perhaps not so vitally in basic philosophy. We want to incorporate an "observer - metaphysical logic" in our basic description of the individual Urs, and believe that in this way we shall fill the gap we find in the Ur theory.

Our point of departure from the Ur theory is that at which they follow a specific mathematical course of development. The specific point at which the Ur theorists seem to us to go wrong is in the introduction of continuous groups which need no operational justification since they are pure mathematical entities. We would feel that the only correct way to proceed would be to build the discrete theory of Urs so as to define the necessary continuous groups to a sufficient degree of approximation instead of to introduce a dualism of operational and formal mathematical entities. To pursue this argument in detail would take vast space and for the present purpose, we are content to observe that in current ideas on the foundations of mathematics, both points of view exist. That is to say, there are schools of thought which hold that a distinction can be made between formal mathematical structures and interpreted structures, and there are schools which deny that such a distinction can be made except on an arbitrary and ephemeral basis. So there is no unambiguous answer to be had from the study of the foundations of mathematics.

Before describing our model, we clear up a couple of matters in which we are in agreement with the Ur theorists. First, the Ur theorists have departed from the position of Bohr in one vital partiular. In Bohr's attempt to achieve an understanding of the observer metaphysic, an absolutely central part was played by his (Bohr's) insistence that all theoretical formulations had to be interpreted through the massively consistent and pervasive haptic language which was at once classical physics and the common sense world. Bohr thought it inconceivable that any underpinning or revision of this language using conceptual entities less evident to the senses was conceivable, practicable or desirable. Indeed his philosophy made a virtue of the necessity of this position.

But the Ur theorists do propose just such an underpinning as Bohr thought inconceivable. So do we, though a different one.

A second point concerns probability and its place in a discretely-based quantum physics. There we find ourselves in complete agreement with the analysis of the use of probability that von Weizsäcker has undertaken (Tutzing, 1978), and need therefore to make few remarks on this topic. Probability is closely related to the concept of time in the quantum physics context. The concept of time which is commonplace in modern philosophical writing and which owes more to Hume than to any other thinker, seems to be in conflict with a good deal of the thinking of physicists. Starting with Galileo, the time of physicists is based primarily upon the analogy between time "displacement" and displacement in space. Our model has developed partly from discussion which was designed to show that in a discrete approach one might have the advantage of adopting the Humean point of view without outrage to physical theory. Then one could take the past simply as the fixed domain and the future as the domain of uncertainty and of probabilistic inference. This point of view can be tagged "Fixed Past, Uncertain Future" (Noyes, 1974, 1975, 1977).

We now return to our suggestion of how Ur theory might have been developed in such a way as to avoid a dualism of formal and contingent entities respectively which we find a defect. This suggestion affords a convenient introduction to our theory, which we can regard as having developed out of an attempt to avoid this defect. It also enables us to state the main argument in favour of our theory at the foundational level (as distinct from successful calculations and formal deductions). Our claim is that in reaching a monistic picture we discover what lies beneath what, for want of a better term, we have called the observation metaphysics.

Let us imagine a universe containing elementary entities which we may think of as our counterparts of the <u>Urs</u>. To avoid confusion we will amend the terminology and call them <u>Schnurs</u> — a term which appropriately suggests computing concepts, in a way that represents their most fundamental aspect of concatenating strings. The Schnurs are discrete, and any representation they may have is two valued. Consider a definite small number of them. Consider an elementary creation act as a result of which two different Schnurs generate a new Schnur but which is again different. We speak of this process as "discrimination". By this process, and by concatenations of this process, can the complexity of the universe be explored. It is also necessary that a record of these discriminations and resulting creations be kept as a part of the structure defined by the Schnurs, otherwise there is no sense in saying that they have or have not, been carried out. Hence we consider a new lot of Schnurs which consist of concatenations of creation processes preserving the discriminate structure explored by the original Schnurs. The members of the new class are themselves constituents of the universe and are also free to take part in the creation or discrimination process, and to map up to higher or down to lower levels. This last requirement is the stage at which the necessity becomes clear for a reflexive or recursive aspect to our model which in current quantum theory takes the form of the "observation metaphysics". The construction of a hierarchy of new levels of Schnurs is necessary to obtain an approximation to a physical continuum, and by means of it we can ultimately speak of a physical entity in a background of other physical entities in accordance with the requirements of common sense. However, it makes no sense to speak of the individual entities except in terms of the part they play in the construction. Everything plays a dual role, as a constituent in a developing process, where something comes in from outside to interact, and as a synopsis or concatenation of such a process where the external interaction becomes subsumed in one new entity.

Now can a thing be both aspects at once? I do not think we are able at present to say clearly how it can, and we must let our model, which incorporates this duality, lead us forward without having a complete insight as earlier theorists had to do in quantum theory. However we are in better case than current quantum theory, for we can adopt a strictly process view and insist that we always view the process from one viewpoint — albeit a viewpoint which can, and must, change. Then we are freed from conceptual confusion, and we progress by considering stability conditions under which the limitations of our way of approaching the inescapable duality are compensated. Indeed we find in the stability of the hierarchy levels a profound condition under which we can be sure of a sort of automatic self-consistency which reflects itself in the properties of quantum objects, and which is the basis of our interpretation of our model.

We do not think it impossible that a mathematical way of thinking will emerge in which the dual function can be comprehended without the device of considering the structure of the universe from one point at which the decision making is occurring. One might revert to a more classical or synoptic mathematics. However we do not think we can do it at present (though Parker - Rhodes, whose work has played such an important part in our model, and who feels uncomfortable with a process philosophy, is trying to formulate something very similar in terms of a "mathematics of indistinguishables" which transcends the process aspect). We would conjecture that if such a conceptual framework ever is discovered, its proper field of application would be wider than physics, and that the restricted process view would probably be adequate for physics.

Our view of space time is constructive in the sense that there is one set of principles which gets us from the Schnurs to whatever approximation to the continuum of space we decide we need. It is also constructive in the sense that we requre that any mathematical constructions that are needed to specify the attributes of any physical things, including the space continuum, shall also be so derived. In this sense the Ur theory is not constructive, and we have found our vital objection to it in this lack of constructivity. This use of the term "constructive" is stringent. We are, however, using it as in its *locus classicus*, Brouwer's theory of mathematical intuition (which also stimulated the development of intuitionist logic.)

Brouwer's basic concept was that of the free choice sequence. The formal need for the free choice sequence was to construct the continuum adequately. For Brouwer, the constructions of mathematics had no absolute quality, but were creations of the intellect whose validity was relative to the state of mathematical understanding at a given epoch. They play a part in guiding the development of the free choice sequences. So do other considerations which we should normally regard as contingent. (An example of Brouwer's was to make the development of a free choice sequence depend upon whether, at the particular time in question, four successive sevens were known to occur in the expansion of π.) It would be possible (and Brouwer was quite open to this suggestion) to regard the totality of considerations which could influence free choice sequences as including the contingent begaviour of physical systems, in which case the similarity of the processes in our constructive model and the basic entities with which Brouwer constructed his universe would be quite close.

It would be fascinating to pursue this connexion with Brouwer's thought but this cannot be the place. We introduce it at all here only because it may be felt by some readers that our theory requires a mathematical entology which is just wrong, and it may reassure such persons to know that something very like what we propose has been authoritatively put forward for analogous reasons in the literature of the foundations of mathematics. The connexion is also relevant to our present discussion, because Brouwer's constructivism has no separate world of mathematical entities; we recall the difficulty we encountered with the Ur theorists in their allowing themselves the use of continuous mathematical constructions where we felt that a constructive development should include mathematical entities used in the theory.

This last point leads on to another difference between our Schnur theory and the Ur theory. This concerns the question whether we locate the reflexive character in the individual Ur processes or in statistical assemblages of them. We hold the former view, the Ur theorists the latter. The tradition is on our side, even though one is stretching a point in arguing as we have done that traditional quantum theory fails crucially at the point where it has to appeal to an observation metaphysic to introduce the reflexive character of quantum processes, and yet claim support from that quarter. Still, the traditional argument that the essential character of quantum processes have to be defined for individual processes is very strong. One is accustomed to having to refute various facile approaches to the foundations of quantum theory by pointing out that the characteristic quantum - observation effect is individual and therefore cannot depend upon a statistical effect. For example, in the photon - splitting experiment, the incident beam can be attenuated to such a degree that the incident photons would have to be treated individually, and therefore could not interfere. Yet interference does take place. This piece of experimental evidence provides a very sharp refutation of any view whose attribution of simple atomic properties to the photons is subject to the restriction that one may consider only statistical distributions of these. Presumably von Weizsäcker's distinction between possibility and probability would be brought in to explain why the Ur theory is not in this class. (J.H.M. Whiteman, 1971, introduced a concept which he called potentiality to achieve a similar end.) However, this matter is crucial and one feels that the detailed mechanics which makes a statistical effect appear as an individual one should be presented. Certainly we feel that our model, in which individual effects appear directly, has a crucial advantage, and that this advantage is a direct consequence of our constructive approach.

It is obviously tempting to identify the duality of function of our elementary discriminators or Schnurs with the duality of description in complementarity. Certainly the two are connected, but the connexion is not simple, as must be clear from the foregoing discussion of the differences between our view and current quantum theory. Bohr's view of complementary descriptions seem to be very much a special form of a more general philosophy and to have had its special form dictated by the special form in which quantum physics has developed. It is probably safe to say that if one could state the general philosophy without such special reference, it would contain the reflexive or recursive character which has concerned our discussions so much. However, Bohr's philosophy has proved notoriously difficult to state in this bare form in spite of the best efforts of fifty years. We conclude this section by stating what we feel to be the reason for this recalcitrance.

In a discrete or finite theory it is not too perplexing to introduce a reflexive philosophy by using a recursive mathematical model which is what we do. The really perplexing difficulties seem to appear if we associate this reflexive character with an observation imagined against an objectively existing background, as is done in so-called "measurement theory". Two incompatible principles are being appealed to. One principle requires entities in the universe to be constructed using the observation process; the other takes a realist view of them. Not surprisingly, no reconciliation of the resulting perplexities is achieved by studies at a technical level where fundamental principles tend to be assumed rather than discussed.

One question has been avoided till now. In our model the elementary entities have a dual function. One of the dual aspects is analogous to that of an observing system. Do we imagine that this aspect of its dual role would correspond to the quantum theoretical "observation", and if so how would we react to those writers on quantum theory who wish to see something irreducably mentalist in the observation? In reply, we would first observe that we are not compelled to answer this question before we can use our model. We have a model for interactions which are elementary (Ur) in the sense that all we know is built up from them, and we have an interpretation for the model in terms of scattering processes. This interpretation does not have to be the only one. We have tacitly assumed that the conditions of high energy are favourable for exhibiting the simplicity of the model and hence the scattering situation. However, under other conditions the interacting entities might even be living organisms with consciousness. The model should still apply. What we absolutely are not either compelled or allowed to say is that the phenomenon of consciousness as a separable ingredient is necessary for the interaction.

II. CONSTRUCTION OF THE HIERARCHY

In this section we develop the specific formalism by which we are implementing the program discussed above. Our basic elements are the existence symbols "0" and "1", and our basic mathematical operation is symmetric difference or addition, modulo 2: (0+0=0, 1+0=1, 0+1=0, 1+1=0). The symbols are grouped as ordered sets (vectors) of height (if thought of as columns) n. The comparison between two such vectors is called "discrimination". If a vector x, whose height n we can indicate by writing $(x)_n$, has elements ("discriminators") x_i, $i=1,\ldots,n$, the basic binary operation of discrimination between two vectors x, y is defined by

$$D_n(x,y) = (x+y)_n = (x_i + y_i)_n \tag{2.1}$$

The concept of such discriminators is abstracted from the more familiar idea of discrete quantum numbers, while the discrimination operation itself can be viewed, as we will discuss in another section, as an abstract model of a general scattering ("production") process in which the result of scattering two different systems is a third system that differs from either. Our mathematical model thus describes chains of atomic or elementary processes. Our policy for presenting the theory is first to establish a correspondence between the mathematical model which describes these chains of processes and the familiar structure of quantum numbers. In this way we can first view the mathematical model as providing a classification scheme. The basic dynamics of our theory is represented during the construction of this classification scheme by the concept of discriminate closure. We introduce this concept by the following argument.

Starting with vectors of a given height, we imagine new vectors formed by concatenating a sequence of them. Entities corresponding to the new vectors are said to constitute a new level in the hierarchy. There is no difference between the new and the old in logical type; the only difference is that the boundary between the observing system and that which is observed has changed. The great conceptual and mathematical difficulties of such an idea can be handled in one special case, which is therefore of great importance. This case is that in which the entities at the new level represent all statistically possible concatenations of entities at the previous level, starting with a given set. Hence we get a discriminately closed subset.

A "discriminately closed subset" or DCsS consists of one or more non-null vectors. If the set contains more than one vector, it is said to be "discriminately closed" if discrimination between any two distinct vectors in the set yields a third member of the set. Assume that we start from a basis of j linearly independent vectors, that is vectors for which no sum of two or more different vectors is null. Then there will be $2^j - 1$ distinct discriminately closed subsets. Symbolizing a DCsS by $\{\ \}$, a basis of two vectors a, b, gives the three DCsS $\{a\}$, $\{b\}$, $\{a, b, a+b\}$; a basis of three vectors a, b, c gives the seven DCsS $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b, a+b\}$, $\{b, c, b+c\}$, $\{c, a, c+a\}$, $\{a, b, c, a+b, b+c, c+a, a+b+c\}$. Proof of the general result is immediate either by noting that the number of DCsS is simply the number of ways we can combine j things $1, 2, \ldots, j$ at a time, or by induction. The first step in constructing the hierarchy is then to consider the $2^j - 1$ DCsS so formed as the basic entities of a new level.

The reason for seeking a constructive process of hierarchical nature that yields levels of rapidly increasing (in our case exponentiating) complexity is again abstracted from experience. We have detailed in the first section the reasons why we start from an elementary process (discrimination) which already implicitly contains the "observation metaphysic". There we also explained why, in our view, we adopt a constructive, process-oriented approach. The further requirement that the hierarchy so generated terminate is a basic requirement if we are to retain the principle of finitism. We defer the discussion of the reflexive character of the scheme until it is further developed. That the combinatorial hierarchy obtained by starting with vectors of height $n = 2$ yields levels of interesting physical structure and sufficient complexity, and terminates at the appropriate level has been shown previously (Bastin, 1966). We summarize the construction here.

We have seen that, given j linearly independent vectors, we can always construct 2^j-1 DCsS at that level. For them to form the basis of a new level, however, they must themselves be representable by linearly independent entities which contain the same information about discriminate closure as the sets themselves. For this purpose we introduce multiplication (modulo 2) and matrices because these linear operators preserve discrimination. We look for 2^j-1 matrices which a) map each vector in one of the subsets onto itself, and no other vectors, b) map only the null vector onto the null vector, and hence are non-singular, and c) are linearly independent. Provided this can be done, and the original basis consists of columns of height n, then the matrices themselves can be rearranged as columns (e. g., by putting one row on top of another by some consistent rule), and will then provide a linearly independent basis of 2^j-1 vectors of height n^2. Such mapping matrices are easy to find for $n=2$ (see below). Explicit examples have been found for $n=3$, 4, and 16 (Noyes, 1978) proving the existence of the hierarchy. A formal existence proof has also been provided (Kilmister, 1978) based on unpublished work (Amson, 1976) which formed the appendix to Bastin's paper for the 1976 Tutzing Conference that unfortunately has yet to appear (Bastin, 1976).

We can now present the general situation. We have seen that if at some level ℓ there are $j(\ell)$ linearly independent vectors of height $n(\ell)$, we con construct immediately $d(\ell)=2^{j(\ell)}-1$ DCsS. Provided these can be mapped according to the restrictions given above, they form the basis for a new level with $j(\ell+1)=d(\ell)$ and $n(\ell+1)=n^2(\ell)$. The process will terminate if $n^2(\ell)<2^{j(\ell)}-1$ since at level ℓ there are only $n^2(\ell)$ linearly independent matrices available (and not all non-singular);

clearly this will always happen for some finite n. The situation for $n(1) = j(1) = N$, i.e., when the vectors at the lowest level which span the space are used as the basis, is exhibited in Table 2.1.

TABLE 2.1

The Possible Hierarchies Starting from $n(1) = j(1) = N$

ℓ		1	2	3	4	Hierarchy terminates because
$N=2$	$n(\ell)$	2	4	16	256	
	$j(\ell)$	2	3	7	127	$(256)^2 < 2^{127}-1$
	$d(\ell) = 2^{j(\ell)}-1$	3	7	127	$2^{127}-1 \approx 10^{38}$	
$N=3$	$n(\ell)$	3	9			
	$j(\ell)$	3	7			$9^2 < 127$
	$d(\ell)$	7	127			
$N=4$	$n(\ell)$	4				Mapping not possible
	$j(\ell)$	4				(Amson) even though
	$d(\ell)$	15				$4^2 > 15$
$N>4$						$n^2(\ell) < 2^{n(\ell)}-1$

It is clear that the case $N=3$ is in some sense immersed in $N=2$, and that this immersion is necessary if we are to reach an interesting level of complexity. Thus, perhaps surprisingly considering the simplicity of the assumptions, the hierarchy turns out to be unique.

Although the cardinal numbers given by the hierarchy are unique, the specific representations used in the construction are not. It is important to understand this clearly because it is a complication in making any simple interpretation of the discriminators as representing the presence or absence of particular conventional quantum numbers in an isolated system. This ambibuity is present at the lowest level since for the two basis vectors we have three choices: $a = \begin{pmatrix}1\\0\end{pmatrix}$, $b = \begin{pmatrix}0\\1\end{pmatrix}$; $a' = \begin{pmatrix}1\\0\end{pmatrix}$, $b' = \begin{pmatrix}1\\1\end{pmatrix}$; $a'' = \begin{pmatrix}1\\1\end{pmatrix}$, $b'' = \begin{pmatrix}0\\1\end{pmatrix}$. Corresponding to these three possible choices of basis, there are three different sets of mapping matrices. When, as here, the number of independent vectors is equal to the height of the vectors ($n = j$), the maximal discriminately closed set (MDCS) contains all the non-null vectors in the space (here it is $\left\{\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}, \begin{pmatrix}1\\1\end{pmatrix}\right\}$) independent of the choice of basis; further, the only possible mapping matrix for the MDCS is then the unit matrix. For the first basis, the mapping matrices for $\{a\}$ and $\{b\}$ are $\begin{pmatrix}11\\01\end{pmatrix}$ and $\begin{pmatrix}10\\11\end{pmatrix}$ respectively. For the second $a = a'$, so that matrix is the same but the mapping matrix for b' is $\begin{pmatrix}01\\10\end{pmatrix}$; for the third we note that $a'' = b'$ and $b'' = b$. Rearranging the matrices as columns then give three different possible bases for the second level of the hierarchy, namely

$$a_2 = \begin{pmatrix}1\\0\\0\\1\end{pmatrix} \quad b_2 = \begin{pmatrix}1\\0\\1\\1\end{pmatrix} \quad c_2 = \begin{pmatrix}1\\1\\0\\1\end{pmatrix}; \quad a_2' = \begin{pmatrix}1\\0\\0\\1\end{pmatrix} \quad b_2' = \begin{pmatrix}1\\0\\1\\1\end{pmatrix} \quad c_2' = \begin{pmatrix}0\\1\\1\\0\end{pmatrix}$$

$$a_2'' = \begin{pmatrix}1\\0\\0\\1\end{pmatrix} \quad b_2'' = \begin{pmatrix}0\\1\\1\\0\end{pmatrix} \quad c_2'' = \begin{pmatrix}1\\1\\0\\1\end{pmatrix}. \qquad (2.2)$$

In addition to this ambiguity, there is the further problem that we could have used any other rule for converting the matrices into column vectors, provided only the same rule is used for all three matrices. Thus the rows in the representation have no significance, and <u>within</u> a level the properties of the system under discrimination are unaltered by a permutation of rows in the basis. What the construction <u>does</u> guarantee, however, is that instead of the basis of three unit vectors such as (1000), (0100), (0010) or any linearly independent set constructable on such a basis, two of the vectors in the basis always contain two "1"s in the same two rows. This guarantees that the MDCS (up to a permutation of rows) at the second level will always be

$$\left\{ \begin{pmatrix}1\\0\\0\\1\end{pmatrix}, \begin{pmatrix}1\\1\\0\\1\end{pmatrix}, \begin{pmatrix}1\\0\\1\\1\end{pmatrix}, \begin{pmatrix}1\\1\\1\\1\end{pmatrix}, \begin{pmatrix}0\\1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\1\\1\\0\end{pmatrix} \right\} \tag{2.3}$$

This is important, because if we could end up with the minimal basis given above, the fourth row in the vectors could be ignored and we would be in the $j = n = 3$ situation; but this starting point terminates at the next level. Thus the doubling of one discriminator is a structural requirement which comes from the representations and could not be guessed from the cardinal numbers. We will find it significant as a clue to physical interpretation.

When it comes to constructing mapping matrices for the second level, we cannot use the unit matrix to represent the MDCS given in Eq. 2.3 because it maps all 15 possible non-null vectors of height four onto themselves, and not just the required seven. The eight vectors which must be excluded are of the form (1 x y 0) or (0 x y 1). A non-singular matrix which has none of these as eigenvectors, but all the vectors of Eq. 2.3 as eigenvectors is given below.

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ x \\ y \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ x \\ y \\ 1 \end{pmatrix} ; \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ x \\ y \\ 0 \end{pmatrix} \qquad (2.4)$$

The remaining six mapping matrices for the remaining six DCsS can then be constructed by adding "1"s to the first three rows in such a way as to eliminate all but three or all but one of the vectors from the set of eigenvectors. Care must be taken to keep the matrices non-singular and linearly independent. Leaving the last row as (1000) guarantees that none of the unwanted eight vectors will be eigenvectors.

If we carry out the construction in this way, it is clear that one of the basis vectors at the third level will have a quadrupled discriminator. Sixty-four of the 127 vectors in the MDCS carry this discriminator and 63 carry four nulls in those four rows. If the discriminators of the remaining six linearly independent vectors were non-null only in six rows, then there would be only $(6 + 4)^2 = 100$ linearly independent positions in the mapping matrices, and the fourth level could not be constructed. Hence, at least two additional descriminators must be doubled, providing 144 significant positions in the mapping matrices. This structural feature will again be used as a clue in constructing the physical interpretation, but we have as yet no proof that it is the only way in which the hierarchy can be constructed. An analog of the matrix given in Eq. 2.4 which has the 127 members of the MDCS as eigenvectors and excludes the remaining

126 mapping matrices which are non-singular and linearly independent have been constructed in several ways. Details will be published elsewhere.

III. PHYSICAL INTERPRETATION OF THE HIERARCHY AS A CLASSIFICATION SCHEME

In this section we attempt to correlate the mathematical structure developed above with some facts known from elementary particle physics. Because any physical process requires development of the hierarchy through the levels successively, the significant physical magnitude is not the cardinal of each level separately, but also their cumulative sum, which gives the sequence 3, 10, 137, $137+2^{127}-1 \approx 10^{38}$. Obviously this can be interpreted immediately as the inverse of the superstrong, strong, electromagnetic and gravitational coupling constants and suggests that in some sense the cumulative levels refer to systems of bosons with increasingly refined definitions of their possible interactions. One way to make this more specific would be to assume that the various systems at each cumulative level all above equal <u>a priori</u> probability, and that the probability of coupling into any one of them by the characteristic described at that level is therefore the inverse of the corresponding number. We give this vague idea more specific content in the next section. Further, the fact that the first three levels can be mapped up or down freely, but that any attempt to construct a linearly independent representation of the fourth level with $2^{127}-1$ DCsS must fail after $(256)^2$ linearly independent matrices have been selected suggests that the destabilization of particle systems due to weak decay processes with coupling constant $10^{-5}m_p$ might also emerge from the scheme since $1/(256)^2$ has approximately this value (Bastin 1966). This requires us to assume that the unit of mass in the scheme is the proton mass, but this is already clear from the initial sequence, since 10^{38} is the gravitational coupling between two protons; the gravitational coupling between two electrons is 10^{44}. Thus we can hope to derive the ratio of the electron mass to the proton mass once the scheme is sufficiently developed. How this might be done is discussed in the next section.

The lowest level with the three non-null vectors $\begin{pmatrix}1\\0\end{pmatrix}$, $\begin{pmatrix}1\\1\end{pmatrix}$, $\begin{pmatrix}0\\1\end{pmatrix}$, and the null $\begin{pmatrix}0\\0\end{pmatrix}$ is suggestive of the triplet-singlet system obtained from two dichotomic variables (spinors). This analogy becomes even closer if we double the discriminators to obtain the same system in a four row notation — a step we have already seen will be required at the third level of the hierarchy — as shown below, in Table 3.1.

TABLE 3.1

Triplet - Singlet System from Two Dichotomic Vectors

Conventional Notation				Hierarchy Notation			
S = 1			S = 0	S = 1			S = 0
$S_z = 1$	0	-1	$S_z = 0$	$S_z = 1$	0	-1	$S_z = 0$
$\begin{pmatrix}1\\0\\1\\0\end{pmatrix}$	$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\\1\\1\end{pmatrix}$	$\begin{pmatrix}0\\1\\0\\1\end{pmatrix}$	$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\\1\\-1\end{pmatrix}$	$\begin{pmatrix}1\\0\\1\\0\end{pmatrix}$	$\begin{pmatrix}1\\1\\1\\1\end{pmatrix}$	$\begin{pmatrix}0\\1\\0\\1\end{pmatrix}$	$\begin{pmatrix}0\\0\\0\\0\end{pmatrix}$

This should make it clear that we can view the lowest level as representing the triplet states of such a system in the minimal (i.e., undoubled) notation $\begin{pmatrix}1\\0\end{pmatrix}$, $\begin{pmatrix}1\\1\end{pmatrix}$, $\begin{pmatrix}0\\1\end{pmatrix}$. The question is what dichomic variable to choose. Here we are guided by the idea that the three stable levels which close under discrimination and map that closure up and down might represent the three absolute conservation laws, namely baryon number, lepton number and charge (Noyes 1977). We, also, as discussed in the next section, want in our dynamical scheme to be able to interpret $1+1=0$ as the exclusion principle at a single (prespacial) locus, so also can include the z-component of spin.

After a number of attempts to find a reasonable interpretation which accomplishes these goals, we now believe we have achieved our objective. Rather than spin, we find it more natural to interpret the lowest level as representing the three possible non-null states obtained from positive and negative unit charge, namely +, - and ±. Equally well, if we wish to stick closer to the idea of dichotomic variables, we could take these to be the three triplet isospin states described by the z - component, since the z - component of isospin is also exactly conserved. We summarize both identifications below.

TABLE 3.2

Interpretation of the lowest level	
+ or T = 1, $T_z = +1$	$\begin{pmatrix}1\\0\end{pmatrix}$
± or T = 1, $T_z = 0$	$\begin{pmatrix}1\\1\end{pmatrix}$
- or T = 1, $T_z = -1$	$\begin{pmatrix}0\\1\end{pmatrix}$

We have already seen that, up to a permutation of rows, the seven vectors in the MDCS of the second level are uniquely given in Eq. 2.3. Looking at these we see that the last three are simply a repetition of the first level with zero's added in the first and last row, while the remaining four have one's in these two rows — and hence can allow the second and third row to be null, accounting for the fourth possibility. In line with our general attempt to relate the classification to the absolute conservation laws, we interpret this doubled discriminator as referring to the presence of a baryon - antibaryon pair, and the remaining two discriminators as referring to the charge state. Thus the interpretation as well as the notation map up from the lowest level. Naturally, the three states where the $b\bar{b}$ discriminator is null are the three charge states of the associated meson. The lowest mass exemplar of seven bose systems with these quantum numbers would therefore be $n\bar{n}$, $p\bar{n}$, $n\bar{p}$, $p\bar{p}$, π^+, π^-, π^0 .

When we go to the third level, again we want this interpretation to map up. The richer system of quantum numbers that leads to 127 vectors in the MDCS is now natural to interpret as coming from lepton number and spin. Just as the doubled discriminator was interpreted as representing $b\bar{b}$, when this becomes quadrupled it is easy to assume that we have added a lepton - antilepton pair, and that we have at this level the charge and spin states of a $b\bar{b}\,\ell\bar{\ell}$ system with associated mesons. This works out very neatly. We have seen that in addition to one quadrupled discriminator, we have to have at least two doubled discriminators in order to be able to construct the mapping and the fourth level. These are easily interpreted as the spins of the two particle - antiparticle pairs, which together give 16 spin states. If charge were treated in the same way we would get too many states, and they would include doubly charged states which would not map down to the lower levels. Thus we have the interesting asymmetry between the leptons and baryons at this level that while all charge states of the baryons are allowed, the leptons must be either $e\bar{e}$ or $\nu\bar{\nu}$. Then we have 16 spin states times 4 charge states, or 64 in all. The mesons associated with the baryons, in terms of their lowest mass exemplars are the spin zero π (3 charge states), the spin 1 ρ (9 states, 3 spin $\times$ 3 charge), and the spin 1 ω_0 (3 spin states). Depending on whether the lepton pair is $e\bar{e}$ or $\nu\bar{\nu}$ the neutral vector boson associated with them will be either the two transverse states of the photon plus coulomb field in the radiation gauge, or the weak neutral vector boson (3 states in either case). Thus the remaining states add up to

$3 + 15 + 3 \times 15 = 63$, which together with the 64 particle-antiparticle states make exactly the required 127. The ambiguity between γ and W_0 in interpretation looks promising as a route toward understanding weak-electromagnetic unification at the fourth level, since only there can it be resolved. Table 3.3 summarizes our classification scheme for the third level. In it we use the shorthand notation of $(0)_n$ for a null column of height n; $(1)_n$ for a column of "1"s; $(3)_2$ for the three vectors $\begin{pmatrix}1\\0\end{pmatrix}$, $\begin{pmatrix}1\\1\end{pmatrix}$, $\begin{pmatrix}0\\1\end{pmatrix}$; $(4)_2$ when null vector is added to the set, and $(3)_4$ and $(4)_4$ for the corresponding doubled notation given in Table 3.1.

TABLE 3.3

Physical Interpretation of the Second and Third Levels

Second Level

$b\bar{b}$	Charged Meson
$(1)_2$	$(0)_2$
$(4)_2$	$(3)_2$

$(4 + 3 = 7 = 2^3-1)$

Third Level

	$\bar{b}\bar{b}\ell\ell$	π	ρ	ω_0	γ, Coulomb or W_0
$b\bar{b}$, $\ell\ell$	$(1)_4$	$(0)_4$	$(0)_4$	$(0)_4$	$(0)_4$
$S_{b\bar{b}}$	$(4)_4$	$(0)_4$	$(3)_4$	$(3)_4$	$(0)_4$
$S_{\ell\bar{\ell}}$	$(4)_4$	$(0)_4$	$(0)_4$	$(0)_4$	$(3)_4$
$C_{b\bar{b}}$	$(4)_2$	$(3)_2$	$(3)_2$	$(0)_2$	$(0)_2$
$C_{\ell\bar{\ell}}$	$(1)_2$ or $(0)_2$	$(0)_2$	$(0)_2$	$(0)_2$	$(0)_2$
	64	15			3

$(64 + 3 + 15 + 3\times15 = 127 = 2^7-1)$

Note that again, if we consider the zero spin state for the baryons and leptons, the second level recurs as a subsystem of the third level.

It is to be emphasized that while the doubling of some discriminators is a necessary requirement as discussed in the last section, the precise form even this takes has yet to be worked out in detail. Even then, the specific interpretation given in Table 3.2 still requires developing a dynamics in which the specific row assignments for certain quantum numbers can be justified either by these columns occuring as decay products from the weak instabilities occurring at the fourth level or by some specific dynamical way in which we can "break-in" to the stable levels of the hierarchy by a single particle (or system of particles) not contained in the hierarchy. We show one way this might happen in the next section.

Before continuing on to this dynamical analysis, however, we first show how the quantum number assignments given above can arise from putting together two columns outside the hierarchy. For the second level this is demonstrated in Table 3.4.

TABLE 3.4

Second Level of the Hierarchy as a Representation of the Lowest Mass States of SU_2

$$\begin{array}{cccccccccccc}
p & , & n & & p\bar{n} & p\bar{p} & n\bar{p} & n\bar{n} & & \bar{p} & , & \bar{n} \\
\begin{pmatrix}1\\1\\0\\0\end{pmatrix} & & \begin{pmatrix}1\\0\\0\\0\end{pmatrix} & \Longrightarrow & \begin{pmatrix}1\\1\\0\\1\end{pmatrix} & \begin{pmatrix}1\\1\\1\\1\end{pmatrix} & \begin{pmatrix}1\\0\\1\\1\end{pmatrix} & \begin{pmatrix}1\\0\\0\\1\end{pmatrix} & \Longleftarrow & \begin{pmatrix}0\\0\\1\\1\end{pmatrix} & & \begin{pmatrix}0\\0\\0\\1\end{pmatrix}
\end{array}$$

$$\Downarrow$$

$$\begin{array}{ccc}
\begin{pmatrix}0\\1\\0\\0\end{pmatrix} & \begin{pmatrix}0\\1\\1\\0\end{pmatrix} & \begin{pmatrix}0\\0\\1\\0\end{pmatrix} \\
\pi^{+} & \pi^{o} & \pi^{-}
\end{array}$$

Note that the isosinglet mesonic state is missing. This state is, of course, very important in the nuclear force, but is due to the composite 2π system and not to any simple structure which can be described at the level of abstraction we are dealing with here.

We have noted above that the two level hierarchy, starting from $n(1) = j(1) = 3$ with seven DCsS in the first level and 127 in the level which cannot be mapped, is immersed in the combinatorial hierarchy as a subsystem. Following the line suggested by Table 3.4, we interpret the seven DCsS as the seven lowest mass mesonic states of SU_3 with the η-meson excluded, and then find that we can relate these to baryon-antibaryon states as shown in Table 3.5.

TABLE 3.5

The Seven DCsS for N=3 Interpreted as the Lowest Mass Mesonic States of SU_3 and the corresponding Baryon-Antibaryon Composition

$$
\begin{array}{cccccccccccc}
n & p & \Lambda_0 & \Sigma^+ & \Sigma_0 & \Sigma^- & & \bar{\Sigma}^- & \bar{\Sigma}_0 & \bar{\Sigma}^+ & \bar{\Lambda}_0 & \bar{p} & \bar{n} \\
\begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}1\\1\\0\\0\\0\end{pmatrix} & \begin{pmatrix}1\\0\\1\\0\\0\end{pmatrix} & \begin{pmatrix}1\\1\\1\\0\\0\end{pmatrix} & \begin{pmatrix}1\\1\\0\\1\\0\end{pmatrix} & \begin{pmatrix}1\\0\\1\\1\\0\end{pmatrix} & \Downarrow & \begin{pmatrix}0\\1\\1\\0\\1\end{pmatrix} & \begin{pmatrix}0\\1\\0\\1\\1\end{pmatrix} & \begin{pmatrix}0\\0\\1\\1\\1\end{pmatrix} & \begin{pmatrix}0\\0\\1\\0\\1\end{pmatrix} & \begin{pmatrix}0\\0\\0\\1\\1\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\\1\end{pmatrix}
\end{array}
$$

$$
\begin{array}{cccccccccc}
p\bar{n} & p\bar{p} & n\bar{p} & n\bar{n} & p\bar{\Lambda}_0 & \Lambda_0\bar{p} & n\bar{\Sigma}_0 & n\bar{\Lambda}_0 & & \\
\begin{pmatrix}1\\1\\0\\0\\1\end{pmatrix} & \begin{pmatrix}1\\1\\0\\1\\1\end{pmatrix} & \begin{pmatrix}1\\0\\0\\1\\1\end{pmatrix} & \begin{pmatrix}1\\0\\0\\0\\1\end{pmatrix} & \begin{pmatrix}1\\1\\1\\0\\1\end{pmatrix} & \begin{pmatrix}1\\0\\1\\1\\1\end{pmatrix} & \begin{pmatrix}1\\1\\0\\1\\0\end{pmatrix} & \begin{pmatrix}1\\0\\1\\0\\1\end{pmatrix} & = & \begin{matrix}(1)_2\\ \\ (7)_3\end{matrix}
\end{array}
$$

$$\Downarrow$$

$$
\begin{array}{cccccccccc}
\begin{pmatrix}0\\1\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\1\\0\\1\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\1\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\1\\1\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\1\\1\\0\end{pmatrix} & \begin{pmatrix}0\\1\\0\\1\\0\end{pmatrix} & \begin{pmatrix}0\\0\\1\\0\\1\end{pmatrix} & = & \begin{matrix}(0)_2\\ \\ (7)_3\end{matrix} \\
\pi^+ & \pi^0 & \pi^- & (\eta_0) & K^+ & K^- & K_S & K_L & &
\end{array}
$$

The fact that the η_0 has to be missing in our notation, and hence that we do not have an exact representation of the SU_3 symmetry is, in our opinion, a strength rather than a weakness. This "symmetry" is broken in nature, so we had best not find it precisely obeyed at the level where we have absolute conservation laws to maintain.

We have found several representations of the three by three mapping matrices for the seven DCsS of this lowest $N = 3$ level. One of them gives at the second level vectors which always have two null rows, and hence can be represented by seven unit vectors, all of which are on an equal footing. Another representation contains one tripled discriminator, so leads at the second level to vectors of the form $(1)_3(7)_3(7)_3$. The tripled discriminator is very suggestive of three quark indices, while the doubling of the "7" that can represent SU_3 is suggestive of $SU_2 \times SU_3$. Pursuing this idea, we see that if we can indeed see how to imbed this in our basic hierarchy, at the next level where we have 127 basis vectors of height 256, we could represent $7 \times 9 = 63$ of them as three quarks with three colors, and in some sense an $SU_2 \times SU_3$ structure, and 63 more as the corresponding anti-particles. These would form substructures among the $2^{127}-1$ DCsS which should prove to be quite stable against a random background dynamics. But, fortunately, this gives only 126 of the 127 basis vectors, and cannot be the full story. This again is promising since the quark quantum number assignments, and corresponding symmetries are badly broken in nature, and must be destabilized by the weak interactions if we are to reach agreement with well extablished experimental facts. Thus, although a three-quark, three-color theory might come out of the hierarchy as an

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IV. DYNAMICAL AND COSMOLOGICAL EXTENSIONS

In going beyond a classification scheme, it is necessary to go from the consideration of the quantum numbers (discriminators) in a single vector to sequential comparisons between successive pairs of vectors. To do this properly requires that we go behind our ideas of statistical averaging that enable us to make the simplification of discriminate closure, and consider detailed sequences with detailed memory (in the computer sense). Indeed the development of dynamics, for us, has to come from the fundamental concept of computer memory. This work has not been done yet, and we have the task of inventing some suggestive short cuts.

If, as we would like to do, we wish to view the binary comparison between two vectors called discrimination as a primitive scattering process described only in terms of quantum numbers, then the requirement that $x+x=0$ is in apparent conflict with any attempt to interpret these quantum numbers as <u>conserved</u>. There are two ways we have considered which might avoid this difficulty. One is deliberately to introduce an idea of spacial separation, and say that the discrimination process defines the quantum numbers which can remain at a (pre-spaciotemporal) <u>locus</u>, when two systems are compared at that locus. Thus, all quantum numbers which both systems have in common are removed and only those which differ remain. This has a clear interpretation as representing an "exclusion principle". However, if we are <u>also</u> to interpret these quantum numbers as conserved, the two identical sets of discriminators which disappear from the columns must reappear at two additional distinct loci. We can insure this possibility by extending the original discrimination operation to account for this in a simple way by defining the operator

$$T_n(x,y) = (x+y)_n,\ (x\cdot y)_n,\ (x\cdot y)_n \qquad (4.1)$$

where "$\cdot$" is the usual binary multiplication ($0\cdot 0=1\cdot 0=0\cdot 1=0$; $1\cdot 1=1$) and the operation acts row by row as does the "+". If the two vectors have no quantum numbers in common, this coincides with D_n; if the two vectors are the same, we end up with two identical columns at two new loci; in the general case we get a third and different vector as well. This approach has the disadvantage that we have added a new postulate to the theory in order to insure both conservation and the irreversible multiplication of loci as we consider more and more T-operations, rather than trying to derive these results from an interpretation of the original scheme. But it may well be that introducing dynamics does indeed require a new postulate.

An alternative approach is to assume that the case $x+x = 0$ represents genuine annihilation such as occurs between particle and antiparticle in elementary particle theory. Then a discrimination which leads to a null result has to be interpreted now as saying that one was the antiparticle of the other. This then allows us to interpret our original binary discrimination graphically as a primitive Feynman diagram. Consider for simplicity the graphical representation of the discrimination between any two systems at the lowest level given in Fig. 4.1.

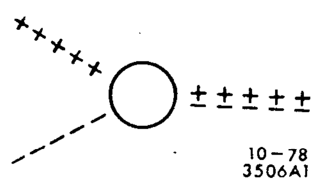

Figure 4.1 Graphical representation of the lowest level.

Read as a discrimination diagram, all this is intended to convey is that if we discriminate between any two legs of the diagram, we get the third leg. To read this as an actual vertex in space time is more subtle. Read from left to right, this represents a positive and a negative system coming together to form an externally neutral system. Read from right to left, it represents an externally neutral system coming apart into a positive and a negative system. But if, for example, we read the "-" leg as coming in and the two others as going out, the process so interpreted does not conserve charge. Hence, to maintain charge conservation, we assume (as Feynman would) that all three legs are <u>incoming</u> and that a particle moving backward in time is an antiparticle. Hence, if we keep to the D operation, interpreted as representing (when quantum numbers disappear) annihilations, we also are forced to introduce a dynamical postulate in order to preserve the conservation law interpretation of the discriminators.

At present it is unclear which of these routes will be the most fruitful to pursue, or whether they may not be in some sense already included in the sequential constructive process from which we start. It will also be important to understand dynamically and not just formally the process by which one goes from a level of lower complexity to a level of higher complexity <u>or visa versa</u>. In spite of the vagueness of our dynamical ideas at this stage, we can still present a dynamical calculation here which is of considerable interest. It is not as yet a direct consequence of the combinatorial hierarchy; rather we view it as a clue as to how we should extend our theory dynamically.

The calculation was originally achieved by A. F. Parker-Rhodes who justifies his physical interpretation of the hierarchy, and of more extended structures, on the basis of his theory of indistinguishables (Parker-Rhodes 1978). Unfortunately, this theory requires considerable logical development for consistent presentation since objects which can be counted as two when together, but which are truly indistinguishable when separate (called "twins"), cannot be grouped in ordered sets; they can, however, be grouped in such a way as to define a unique cardinal for the group or "sort". Thus a "sort theory" dealing with this possibility has to be developed, based on the three parity relations "identical", "distinguishable", and "twins" — together with their negations. This requires a semantic theory, using two-valued logic, for discussion of the object theory, and an implication language, again using two-valued logic, for the statement and proof of theorems. However important the theory of indistinguishables may be, Parker-Rhodes' ideas of interpretation are inconsistent with those developed in this paper, and we give his deductions in an amended form. We expect that before very long a consistent presentation on our own principles will have been reached, but the form we give below is to some extent a compromise with conventional thinking. Our excuse for (in a sense) premature publication is the astonishing accuracy of the result. We believe that the presentation we give here is believable in terms that are closer to ordinary quantum mechanical usage — once one is willing to make the conceptual leap that allows the discussion of quantum ideas <u>prior</u> to any mention of space time.

We have seen that the three stable levels of the hierarchy can be viewed as systems carrying the quantum numbers of baryon - antibaryon pairs and lepton - antilepton pairs and the associated bosons. Since comparison between any two such systems leads to a third, and all three levels map up or down, it seems appropriate to think of the hierarchy as containing all 137 possibilities with equal <u>a priori</u> probability. But to discover the actual structure, we must somehow "break-in" to this closed system, which necessarily requires a vector that is not one of the members of the hierarchy. The example we pick is the electron, which we can obtain in the notation of Table 3.3 by stripping out the baryon - antibaryon pair and the antilepton, leaving only the electron number, charge, and spin discriminators, in that notation, the vector:

$$\begin{array}{l}
\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \begin{array}{l} b \\ \bar{b} \\ e \\ \bar{e} \end{array} \\[2ex]
(0)_4 \; S_{b\bar{b}} \\[2ex]
\begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ or } \begin{pmatrix} 0 \\ 1 \end{pmatrix} \; S_e \qquad = \text{an electron} \\[2ex]
(0)_2 \; S_{\bar{e}} \\[2ex]
(0)_2 \; C_{b\bar{b}} \\[2ex]
\begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{array}{l} C_e \\ C_{\bar{e}} \end{array}
\end{array} \tag{4.2}$$

In order to couple this column into the hierarchy, we have to introduce some new sort of vertex which does conserve quantum numbers; just how does not have to be specified for our current purpose. The only member of the 137 vectors in the hierarchy which does not change the electron spin or charge, or refer to irrelevant quantum numbers, is the coulomb case. So we assume that the electron couples to this with a probility of 1/137. This member of the hierarchy then communicates with all the others in a random fashion, eventually ending up again with the coulomb case and back to the electron. In this respect we view the hierarchy as resembling something like the "vacuum fluctuations" of quantum field theory. The reason that this can lead to a result is that the electron cannot coincide with those members of the hierarchy which contain electron-positron pairs while this process is taking place, thanks to the exclusion principle. Thus the process necessarily involves some space time separation or interval between the electron and the hierarchy, which we will estimate statistically. Further, since we have no reference frame to refer this distance to, the resulting charge distribution relative to this space time interval must also be distributed statistically, subject only to charge conservation. The calculation we present is of the ratio of the square of this charge to the space time interval equated, as is often assumed, to the electron rest energy $m_e c^2$. Schematically, the process we are computing is shown in Fig. 4.2.

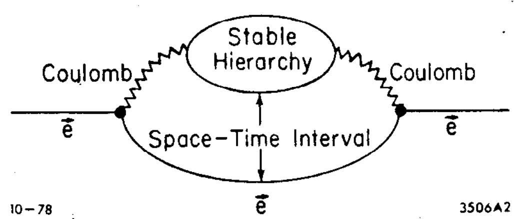

Figure 4.2 Schematic representation of the electron self-energy.

Our first step is to take out the dimensional factors and thus reduce the statistical part of the calculation to dimensionless form. The square of the charge is e^2; it is smeared out into two (or more) parts over some distance r. We introduce a random variable x to represent the charge in one part, and in order to conserve charge between two parts, write the square of the charge as $e^2 x\,(1-x)$. As we have already argued, the coupling we should use at this stage in the development of the theory is 1/137, not the empirical value of the fine structure constant α, so $e^2 = \hbar c\,/\,137$.

Because of the exclusion principle, there will be some distance of closest approach d, which acts as a cutoff in the distance r. Since the only stable mass other than the m_e we are computing is the proton mass m_p, and proton-antiproton pairs occur in the levels of the hierarchy, it seems reasonable to take this shortest distance we can define to be the Compton wavelength of a proton-antiproton pair $d = h/2m_p c$; our second random variable y is then defined by $r = yd$. Alternatively, we might assume that because of the uncertainty principle we cannot ascribe coulomb energy to charge separations in regions of linear dimension smaller than $d = h/2m_p c$. This introduces Planck's constant directly into the theory as the measure of the statistical uncertainty that can only subsequently be reduced by successive hierarchical stabilizations. Either assumption leads to the same result (i.e., $r = yd$, $y \geq 1$) for the calculation at hand.

The random variable x represents the charge in a system with three degrees of freedom smeared out statistically and interacting with the remaining charge $1 - x$. If we could cut the charge into two pieces, like a hunk of butter, x would vary between 0 and 1. But in our interpretation the hierarchy contains pieces with both positive charge ($p\bar{n}, \pi^+, \rho^+$, ...) and negative charge ($\bar{p}n, \pi^-, \rho^-$, ...) as well as neutral and internally neutralized systems, all of which communicate with each other in the stabilization process. Hence, if we look at all the possibilities, and maintain overall charge conservation, x can have any value between $-\infty$ and $+\infty$. Once we have gone beyond the first separation, we have no way of knowing whether the coulomb energy we are evaluating is attractive (unlike charges) or repulsive (like charges) outside of the interval $0 < x < 1$. Statistically the positive and negative effects outside this interval must cancel. However, since we are required to carry out this statistical averaging over all real values of x, we have to require the weight-function $P(x(1-x))$ to be positive over the same range, even though, after we have recognized the cancellation, we need norm it only between 0 and 1. The simplest such weight function is $x^2(1-x)^2$. Taking this argument from simplicity without further physical justification is, in our opinion, the weakest point in the calculation.

Putting this together, we see that

$$m_e c^2 = \langle q^2 \rangle \langle \frac{1}{r} \rangle = \frac{c}{137} \langle x(1-x) \rangle \frac{2m_p c}{h} \langle \frac{1}{y} \rangle$$

or

$$m_p / m_e = \frac{137 \pi}{\langle x(1-x) \rangle \langle \frac{1}{y} \rangle} \tag{4.3}$$

To calculate the expectation value of $1/y$ we need some probability weighting factor $P(1/y)$. We have seen above that the hierarchy has three distinct levels with different interpretations, each carrying charge, so we assume that the distribution of charge in the statistical system has three degrees of freedom, each of which brings in its own random $1/y$. Thus we assume $P(1/y) = 1/y \cdot 1/y \cdot 1/y$ and find that

$$\langle \frac{1}{y} \rangle = \int_1^\infty \left(\frac{1}{y}\right) P\left(\frac{1}{y}\right) \frac{dy}{y^2} \Big/ \int_1^\infty P\left(\frac{1}{y}\right) \frac{dy}{y^2} = \frac{4}{5} \tag{4.4}$$

Although the random variable x representing the charge can vary from minus infinity to plus infinity, the probability $P(x(1-x))$ must be positive. As the simplest choice we take $P(x(1-x) = x^2(1-x)^2$. If we had only one degree of freedom, the expectation value would then be

$$K_1 = \langle x(1-x) \rangle_1 = \int_0^1 x(1-x)\, P\,(x(1-x))\, dx \Big/ \int_0^1 P\,(x(1-x))\, dx = \frac{3}{14} \quad (4.5)$$

Actually, as already noted, we have three degrees of freedom coming from the three levels of the hierarchy. Once the distribution has separated into x and $1-x$ the effective squared charge of each piece is x^2 or $(1-x)^2$, so we can write the recursion relation

$$K_n = \int_0^1 \left[x^3 (1-x)^3 + K_{n-1}\, x^2 (1-x)^4 \right] \Big/ \int_0^1 x^2 (1-x)^2\, dx$$

$$= \int_0^1 \left[x^3 (1-x)^3 + K_{n-1}\, x^4 (1-x)^2 \right] \Big/ \int_0^1 x^2 (1-x)^2\, dx \quad (4.6)$$

$$= \frac{3}{14} + \frac{2}{7} K_{n-1} = \frac{3}{14} \sum_{i=0}^{n-1} \left(\frac{2}{7}\right)^i$$

Putting this back into our formula, using the three degrees of freedom of the hierarchy as before, we have

$$m_p / m_e = \frac{137\,\pi}{\frac{3}{14} \times \left[1 + \left(\frac{2}{7}\right) + \left(\frac{2}{7}\right)^2 \right] \times \frac{4}{5}} = 1836\,.\,1516 \quad (4.7)$$

as compared with the latest empirical result 1836.15152 ± 0.00070, (Barash-Schmidt, 1978).

Clearly, in presenting our calculation in this way, we have leaped ahead of what we are justified in doing as an explicit dynamical calculation. But the calculation illustrates one way in which two algebraic quantities can be introduced into the theory in the form of the square of one divided by the other. The specific interpretation is compelling because of the high quality of the numerical result; the critical integer 3 which enters both the charge distribution and the separation as three degrees of freedom is, we are confident, correctly identified as the three levels of the hierarchy. That we should be able to interpret this calculation within our framework is evident. This fact alone puts us in a strong position.

The quality of the result makes it important to discuss corrections which might destroy it. To begin with, we have used the value 137 for $1/\alpha$ rather than the empirical value. As discussed above, the $\gamma - W_0$ ambiguity encountered in our interpretation suggests we should strive for weak - electromagnetic unification at the fourth level; independent of that, we can anticipate corrections to $1/\alpha$ of order $1/256^2$, which is of the correct order of magnitude. The second correction we can anticipate is in the cutoff parameter d. Our first estimate is almost certainly approximately correct, but does not account for the fact that electrons in the hierarchy are sometimes present and sometimes absent. Hence, we can anticipate a correction to d of order $m_e/2m_p$ as well as in the calculation of the correction to $1/\alpha$. Thus we anticipate something like the empirical result for $1/\alpha$ and must hope that the correction to d will almost exactly compensate for it in our formula. Looked at this way, the calculation can be viewed as a guide to <u>how</u> to construct the dynamics, rather than as a prediction of our theory. It has already proved of great value in setting up the classification scheme given in the last section.

Since the language we use for justifying the calculation when exhibited pictorially as in Figure 4.1 makes the stable hierarchy look like a photon, we can try to extend this analogy. To begin with, if we look at coupling into the hierarchy through transverse photons, these will flip the spin of the electron. But again, for a specified spin of the electron, this can happen in only 1 of the 137 possible cases, so the coupling constant is the same as we used in the coulomb calculation (and including this in our "self-energy" calculation does not alter the result), which is encouraging. So consider an electron and a photon which exchange a "photon" so described. Making the static, non-relativistic assumption that the mass of the proton does not change with velocity and that its motion does not effect the energy of the system, the additional effect we must consider is that the electron must acquire its own mass both before and after the exchange by the process already considered. This leads to the diagram given in Figure 4.2.

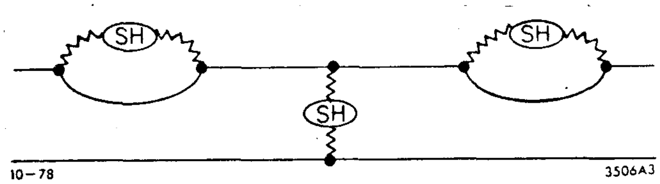

Figure 4.3 Single "photon" exchange between electron and proton.

If the "photon" exchanged in the figure carries any momentum, the diagram cannot represent the whole story, since there will also be the emission of "bremstrahlung" in the final state. So we consider the diagram only for the case when both electron and proton are at rest, but as far apart as we like. This is to be interpreted as an electron and proton bound in the ground state of hydrogen, and contrasted with a free electron and proton with the coulomb effect shielded out. The second case then is the one already considered except that an inert proton has been added, and the first can be calculated as before, provided we multiply the coupling by the two additional powers of α shown in Fig. 4.2; the statistical calculation remains unaltered. We conclude that the binding energy of the ground state of hydrogen is given by $\alpha^2 m_e c^2 = m_e e^4/\hbar^2$ which is indeed the correct result, in the static case. To obtain the center of mass correction we must allow for the motion of the proton, which takes more dynamics than we have developed. Further, to get the excited states, we must be able to describe unstable systems which decay via photon emission, for which we are as yet unready.

We already have seen that in going beyond the three stable levels of the combinatorial base hierarchy, we encounter $2^{127}-1$ discriminately closed subsets in addition to the 137 already discussed. Thus the complete scheme contains $\sim 10^{38}$ discriminable entities. Just as we interpret 1/137 as an approximation to α, we interpret 10^{-38} as an estimate of the gravitational coupling constant between two protons — protons rather than electrons, since we have already accounted for the rest mass of the electron in terms of this unit. At this point a more conventional argument, adapted from a remark of Dyson's (1952) becomes relevant. If we try to count N_e charged particle - antiparticle pairs within a volume whose radius is their compton wavelength, their electrostatic energy is

$$N_e \, e^2/(\hbar/2mc) = N_e \, (e^2/\hbar c) \, 2 \, mc^2 \qquad (4.8)$$

We interpret this result as saying that if we try to determine the number

N_e for a system with more than 137 pairs by electromagnetic means, we are unable to do so because the energy has become so large that additional pairs could be present, and the counting breaks down. Hence, $N_e = 137$ is the maximum meaningful number of charged particle pairs we can discuss electromagnetically in such a volume.

Extending the argument to gravitation, we see that, since

$$N_G \, G \, m_p^2/(\hbar/m_p c) = N_G \, (G \, m_p^2/\hbar c) \, m_p c^2 \tag{4.9}$$

the maximum number of gravitating protons we can discuss within the compton wavelength of any one of them is $N_G \cong 10^{38}$. In this case, the gravitational field at the surface is so intense that light cannot escape, so this system forms a Laplacian "black hole" (Laplace, 1795). Hence, just as failure of the "fourth level" of the hierarchy to posses linearly independent mappings gives us an estimate of instability to weak decay, the upper limit $2^{127}-1 \approx 10^{38}$ represents a gravitational instability for systems with large numbers of particles.

Since we have $\sim 10^{38}$ discriminate entities in the scheme, we are logically justified in starting our discussion with the $(10^{38})^2$ possible discriminations between them. For stability, these systems should contain lepton number and baryon number $(10^{38})^2$, although we cannot as yet prove such a conjecture. Given it, the initial discriminations will create all sorts of ephemeral forms of the type already discussed, and a historical system of loci that provides an initial space time mesh. Once the decays and scattering have proceeded a while, these will settle down to protons, electrons, photons, hydrogen atoms,... and we have started the "big bang". The radiation soon breaks away from the matter, and provides a unique space time framework, locally defined in terms of the cosmic background radiation. Since this "black body spectrum" can be measured locally, it provides us both a cosmic time scale from the temperature, and an absolute frame for measuring particle velocities. Our hope is that we can use this idea to define space time frameworks more easily connected to laboratory observation than abstract definitions. In particular, since our W boson - photon coupling is discrete, and defined at proto - spacetime loci, we should be able to use our dynamic scheme to explain what we mean by a local discrete coordinate system for physical measurement. Only when this task is complete can we tackle the question of what we might mean by a "wave function", and how we are to relate our particular formalism to the successful results obtained by conventional quantum mechanics.

V. CONCLUSION

In this paper we have sketched a physical interpretation of the combinatorial hierarchy which, if the program can be carried through, should provide a finitist conceptual frame for that fundamental revision of physics which we seek. Our philosophical reasons for adapting this approach are discussed in detail in the opening section. Here we stress that the contact with experiment already established in this paper, together with the indications of structural contact with the classification schemes used in elementary particle physics, and conceptual contact with the fundamental ideas underlying current cosmology, make it clear that no field of physics need be omitted in this synthesis. The original coincidence between the cardinals of the hierarchy and the inverse boson field coupling constants allow us to believe that we have indeed unified strong, electromagnetic and gravitational phenomena in one framework. The weak decay instability is also indicated. Our proposed classification scheme brings in the absolute conservation laws at the correct level, and points toward a weak-electromagnetic unification at the next level. Structural contact exists between SU_2, SU_3 and SU_6 (quark) classifications, including an approximate three color-three flavor option flexible enough to allow for new flavors and new heavy leptons. The cosmology should yield the conserved quantum numbers of the universe, some sort of "big bang" and hence the cosmic background radiation as a unique reference system. Since this background is not time reversal invariant, it might even lead ultimately to the explanation of the $K_L - K_S$ decay. So far as we see, no major area of physics has been omitted as potentially outside the reach of a scheme of this structure.

ACKNOWLEDGMENTS

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