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HEAVY LEPTON PHENOMENOLOGY *

Martin L. Perl

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

Table of Contents

Lecture I

What is a Lepton?

Lecture II

The τ Heavy Lepton

Lecture III

Past and Future Searches for Other Heavy Leptons

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HEAVY LEPTON PHENOMENOLOGY

These three lectures on heavy lepton phenomenology were presented at the Advanced Summer Institute at the University of Karlsruhe in September, 1978. They are written primarily from an experimenter's point of view. Thus in the first lecture, I present an empirical definition of a charged lepton using the properties of the electron and muon. To do this, I survey the evidence that the electron and muon are elementary particles without constituents; and I discuss some of the evidence that these particles do not partake of the strong interactions. The discussion of neutrinos is similarly empirical. Only a small amount of theory is presented in the first lecture; just enough to provide a basis for discussion of the tau lepton in the second lecture. Much more complete discussions of lepton theory and of the relations between leptons and quarks have been presented by T. Walsh at this Institute[1] and by other writers previously.[2-4]

The second lecture, when presented, contained a full discussion of the history of the work on the tau and the evidence that it is a lepton. However, the history and evidence have been fully reviewed in several recent papers.[5-8] Therefore, in this written version, I only discuss properties of the tau and areas for future tau studies.

If we believe the tau has a unique neutrino and if we believe some current theories which hold that there are only six lepton types and six quark types; then there is no need for further lepton searches. However, from the experimenter's point of view again, as long as we have no evidence against the existence of a new particle, we are free to search for it.

Therefore, in the third lecture, I review limits from past searches on the existence of other types of heavy leptons; and I discuss the extensions of such searches.

LECTURE I: WHAT IS A LEPTON?

1. Experimental Definition of a Charged Lepton

1.A. Basic Requirements: A charged particle can be classified as a lepton if it satisfies three basic requirements:

a. the particle does not interact through the strong interactions;

b. the particle has no internal structure and no constituents; and

c. its electromagnetic interactions are described by conventional quantum electrodynamic theory. In section 2 we discuss if there are additional basic requirements.

These requirements were of course developed by experimenting with the electron and muon; and hence discovering what properties separate the charged leptons from the hadrons. I will discuss in sections 1C - 1E relevant experiments in three areas: the e^+e^- colliding beams reactions $e^+ + e^- \to e^+ + e^-$ and $e^+ + e^- \to \mu^+ + \mu^-$; the precision measurements of g_e-2 and $g_\mu-2$; and comparisons of $e+p$ and $\mu+p$ inelastic scattering.

1.B. The Spin of the Lepton: Although the spins of the e and the μ are 1/2, there is no need to restrict the empirical definition of a charged lepton to spin 1/2 particles. We can conceive of leptons with spin 0, 1, 3/2, and so forth; they need only obey the three requirements in section 1.A. This raises the question of how to distinguish a spin 1 charged lepton from the proposed charged intermediate bosons--the $W^\pm$. The only way that I can see to do this is to impose the concept of lepton conservation on all leptons. In that case a spin 1 lepton $L^{\pm}$ could not decay unless there existed a related smaller mass charged or neutral lepton L' such that

$$L \rightarrow L' + \text{other particles} \tag{1}$$

The W which decays through channels such that

$$W^{+} \rightarrow e^{+} + \nu_{e},\ \mu^{+} + \nu_{\mu} \tag{2}$$

does not have this lepton conservation property. I will continue this discussion of lepton conservation in section 2.

<u>1.C. The Reactions $e^{+} + e^{-} \rightarrow e^{+} + e^{-},\ \mu^{+} + \mu^{-}$</u>: The reactions

$$e^{+} + e^{-} \rightarrow e^{+} + e^{-} \tag{3a}$$

$$e^{+} + e^{-} \rightarrow \mu^{+} + \mu^{-} \tag{3b}$$

have been used for many years to test the correctness of the three requirements listed in section 1.A. To carry out these tests we use quantum electrodynamics, the single photon exchange diagrams of Fig. 1a - 1c, and the assumption that the leptons are point particles to calculate the barycentric differential cross sections

$$d\sigma(e^{+}e^{-} \rightarrow e^{+}e^{-}, \text{ theor })/d\Omega$$

and

$$d\sigma(e^{+}e^{-} \rightarrow \mu^{+}\mu^{-}, \text{ theor })/d\Omega\ .$$

We do not calculate the contributions of two photon exchange diagrams, Fig. 1d, or of weak interactions because existing measurements are not sufficiently precise to detect these contributions. We must, however, correct the data for the radiative effects coming from diagrams such as Fig. 1e. We then study the ratios of the experimental results to these theoretical cross sections; namely,

$$\rho_{ee} = \frac{d\sigma(e^+ e^- \rightarrow e^+ e^-, \text{exp})/d\Omega}{d\sigma(e^+ e^- \rightarrow e^+ e^-, \text{theor})/d\Omega} \tag{4a}$$

$$\rho_{\mu\mu} = \frac{d\sigma(e^+ e^- \rightarrow \mu^+ \mu^-, \text{exp})/d\Omega}{d\sigma(e^+ e^- \rightarrow \mu^+ \mu^-, \text{theor})/d\Omega} . \tag{4b}$$

No statistically significant deviations from

$$\rho_{ee} = 1, \quad \rho_{\mu\mu} = 1 \tag{5}$$

have been found; the most recent experiments[9,10] having been carried at the SPEAR e^+e^- colliding beams facility in the energy range $3.0 \leq E_{cm} \leq 5.2$ GeV.

There are various ways to interpret the limits placed by these experiments on possible deviations of the e or μ from the basic requirements in section 1.A. If we are looking for deviations from the third requirement--conventional quantum electrodynamics--then we modify the photon propagator

$$\frac{1}{q^2} \rightarrow \frac{1}{q^2} \cdot \left[\frac{1}{1 \pm q^2/\Lambda_\pm^2}\right] \tag{6}$$

Here q^2 is the square of the four-momentum carried by the virtual photon; and Λ is a parameter equal to infinity for conventional quantum electrodynamics. Hence, the smaller Λ, the greater the deviation from conventional quantum electrodynamics. The effect on the ρ ratios is, for example,

$$\rho_{ee} = \left[1 \mp q^2/\Lambda_\pm^2\right]^2 \simeq 1 \mp 2q^2/\Lambda_\pm^2 \tag{7}$$

The approximation on the right hand side is for $\Lambda^2 >> |q^2|$.

Alternatively, there could be an additional anomalous interaction between charged leptons or perhaps there is some residue of the strong interactions between charged leptons. A model for an anomalous interaction, Fig. 1f, assumes there is the exchange of a vector particle x of mas M_x and coupling constant g. Then the total amplitude for the diagram in Fig. 1f is

$$\frac{e^2}{q^2} \pm \frac{g^2}{q^2 + M_x^2} \tag{8}$$

Then, for example,

$$\rho_{ee} = 1 \pm \left[\frac{(g/e)^2 q^2}{q^2 + M_x^2}\right]^2 \simeq 1 \pm 2\left(\frac{g}{eM_x}\right)^2 q^2 \tag{9}$$

The right hand approximation is for $M_x^2 >> |q^2|$.

To test the basic requirement that the charged leptons have no structure, we attach a q^2 dependent form factor

$$F_{e,\mu}(q^2) = \frac{1}{1 \pm q^2/\Lambda_{e,\mu}^2} \simeq 1 \mp q^2/\Lambda_{e,\mu}^2 \tag{10}$$

to the $ee\gamma$ or $\mu\mu\gamma$ vertices in Figs. 1a - 1c. Then, for example,

$$\rho_{\mu\mu} = \left[1 \pm q^2/\Lambda_e^2\right]^{-2}\left[1 \pm q^2/\Lambda_\mu^2\right]^{-2} \simeq 1 \mp 2q^2/\Lambda_e^2 \mp 2q^2/\Lambda_\mu^2 \tag{11}$$

We see that all these interpretations can be expressed in the approximate linear form

$$\rho = 1 + aq^2/\Lambda^2 \tag{12}$$

where a is a coefficient depending upon the type of interpretation and Λ^{-1} gives the size of the deviation. Since all measurements are consistant with $\rho = 1$, we can only deduce upper limits on Λ^{-1} and hence lower limits on Λ. Table I gives some of these limits.

TABLE I

Lower Limits on Λ at the 95% Confidence Level

(The range for Λ is between Λ_+ and Λ_-)

Reaction Used	Lower Limit on Λ (GeV)	Relevant Equation in Text	Reference
$e^+ e^- \to e^+ e^-$	$\Lambda > 15$ to 19	6	9
$e^+ e^- \to e^+ e^-$	$\Lambda > 14$ to 23	6	10
$e^+ e^- \to e^+ e^-$ and $e^+ e^- \to \mu^+ \mu^-$	$\Lambda > 35$ to 47	6	9
$e^+ e^- \to e^+ e^-$ and $e^+ e^- \to \mu^+ \mu^-$	$\Lambda_e > 19$ to 21 $\Lambda_\mu > 16$ to 21	10	9

The significance of these limits depends upon the model used for the deviation. Thus if we use the anomalous interaction model, Eq. 8 and 9, then $(e/g)M = \Lambda$. Hence, $\Lambda > 20$ GeV means $M > (g/e)\ 20\ \mathrm{GeV}/c^2$, and either M is very massive or the coupling constant g is much less than the electromagnetic coupling constant e . If we use the form factor model, Eq.10, then the uncertainty principle says that the upper limit on the size of possible structure in the e or μ is given by

$$r_{e,\mu,\text{upper limit}} = \hbar c/\Lambda_{\text{lower limit}} \qquad (13)$$

For $\Lambda = 20$ GeV,

$$r_{\text{upper limit}} = 10^{-15}\text{cm} .$$

1.D. Precision Measurements of g_e-2 and g_μ-2: The foregoing discussion concerned high energy, large q^2, moderate precision test of the basic requirements. We now turn to an example of low energy, very high precision tests of these requirements--the very beautiful measurements of the e and μ gyromagnetic ratios g_e and g_μ. We recall that g is defined by

$$\mu/s = g\,\frac{e\hbar}{2mc}$$

where μ, s, e, and m are respectively the magnetic moment, spin, charge, and mass of the particle. For Dirac particles $g = 2$; however, there are quantum electrodynamic corrections due to diagrams such as those in Fig. 2a - 2c. Therefore, it is conventional to write

$$a_\ell = \frac{g_\ell - 2}{2} = \sum_{n=1}^{\infty} A_n^{(\ell)} \left(\frac{\alpha}{\pi}\right)^n + \sum_{n=2}^{\infty} B_n^{(\mu)} \left(\frac{\alpha}{\pi}\right)^n \qquad (14)$$

Here ℓ means e or μ; $\alpha = 1/137$ is the five structure constant; and the B_n terms only apply to the μ.

Table II shows that there is excellent agreement between experiment[11-13] and theory. The theoretical values are taken from a recent review[14] of the theory. Incidently the calculations of a_μ include a hadronic contribution $a_\mu(\text{hadronic}) = 66.7 \pm 9.4 \times 10^{-9}$ from Fig. 2d, and a weak interaction contribution $a_\mu(\text{weak}) = 2.2 \pm 0.2 \times 10^{-9}$ from diagrams such as Fig. 2e.

TABLE II

Comparison of Theory and Experiment for a_ℓ

Parameter	Theoretical Value × 10^9	Experimental Value × 10^9	Experimental Reference
a_e	1,159,652.4±0.4	1,159,652.41±0.2	12
a_e	1,159,652.4±0.4	1,159,656.7±3.5	11
a_μ	1,165,920.6±12.9	1,165,922.±9.	13

1.E. Lepton Interactions in the Presence of Hadrons: We have reviewed some examples of how the basic requirements in section 1.A. are satisfied in purely electromagnetic interactions. We might speculate, however, that the basic requirements are not satisfied in the presence of hadrons. Consider three tests of such speculations.

a. Atomic Spectroscopy: The complete explanation of atomic spectra by quantum electrodynamics shows that electrons at atomic distances from nucleons have no anomalous interactions.

b. Mu-Mesic Atoms: Similarly, in mu-mesic atoms[15] there are no proven deviations from conventional quantum electrodynamics.

c. High-Energy Comparison of e+p and μ+p Reactions: Since there are no established low energy deviations, we look at high energy reactions. For example, using the inelastic reactions

$$e + p \rightarrow e + \text{hadrons}, \qquad (15a)$$

$$\mu + p \rightarrow \mu + \text{hadrons}; \qquad (15b)$$

we measure the ratio

$$\frac{\sigma(\mu + p \rightarrow \mu + \text{hadrons})}{\sigma(e + p \rightarrow e + \text{hadrons})} = \frac{N^2}{\left[1 + |q^2|/\Lambda^2\right]^2} \tag{16}$$

to look for deviations of either the e or μ from the basic requirements. In Eq. 16, N is a measure of the relative strengths of the two reactions; and $\Lambda \neq \infty$ would mean that the reactions differ in their dependence on q2--the square of the four-momentum transferred to the hadrons. Unfortunately, there are no recent tests of the ratio, and older tests suffer from unknown systematic errors caused by the differences between the experimental techniques used to study Eq. 15a and those used to study Eq. 15b. An old analysis[16] found that N = 1 to ±10% and

$$\Lambda_{\text{lower limit}} \simeq 10 \text{ GeV} \tag{17}$$

Summarizing all of section 1, all experiments show that the e and μ:

a. do not interact through the strong interactions;

b. have no internal structure;

c. obey conventional quantum electrodynamics.

We take these properties as part of the basic experimental definition of a charged lepton.

2. Lepton Conservation and Leptonic Weak Interactions

The e and μ, and their associated neutrinos ν_e and ν_μ, have two other properties: (a) there is lepton conservation in all known reactions; and (b) all these particles interact through the weak interactions. We do not know if these two properties are associated in some direct way with the three basic charged lepton properties discussed in the previous section. The reason for this ignorance is that we obviously can conceive of a point charged particle which interacts only through the electromagnetic force, but not through the strong or weak force. Wouldn't we still want to call this particle a lepton? We also can conceive of a charged point particle which does not interact through the strong force, which obeys conventional quantum electrodynamics, which interacts through the weak force; but which does not have a conserved property in reactions. We might also want to classify such particles as leptons. Although, as mentioned in section 1.B., we then have a problem as to how to classify the proposed $W^{\pm}$ intermediate boson. There is no way at present to resolve these questions: is lepton conservation or interacting through the weak force an intrinsic property of leptons? It is from one point of view only an issue of nomenclature. This issue can be avoided by simply realizing that in searches for a new, non-hadron-like, non-photon-like, particle one need <u>not</u> impose the requirement that there be a conservation property analogous to lepton conservation, <u>nor</u> need one impose the requirement that the particle interact through the weak force.

I shall not discuss in these talks the weak interaction properties of the e, μ, ν_e, or ν_μ because there have been several recent reviews of this subject;[17] however, I shall discuss a few examples of the tests of lepton conservation, namely: the search for the decay $\mu^{\pm} \to e^{\pm} + \gamma$; μ to e conversion in nuclei; and the reaction $e^- + e^- \to \mu^- + \mu^-$.

<u>2.A. Search for the Decay $\mu^{\pm} \to e^{\pm} + \gamma$</u>: If the decay

$$\mu^{\pm} \to e^{\pm} + \gamma \tag{18}$$

existed, it would be the simplest example of the violation of separate e and μ lepton number conservation. Upper limits on this decay rate, expressed as the ratio $\Gamma(\mu \to e\gamma)/\Gamma(\mu \to e\nu\nu)$, are given in Table III. Thus the experiments[18-21] are already extraordinarily precise; and still no violation of lepton conservation has been seen. These upper limits can be reduced in future experiments by at least a factor of 100.

TABLE III

Upper Limits on $\Gamma(\mu \to e\gamma)/\Gamma(\mu \to e\nu\nu)$ at the 90% Confidence Level

Year	Upper Limit	Reference
1964	2.2×10^{-8}	18
1977	3.6×10^{-9}	19
1977	1.1×10^{-9}	20
1978	2.0×10^{-10}	21

2.B. μ to e Conversion in Nuclei: The reaction

$$\mu^- + \text{Nucleus} \to e^- + \text{Nucleus} \tag{19}$$

can only take place if separate e and μ lepton number conservation is violated. A recent experiment using sulfur measured the ratio

$$R_{\mu e} = \Gamma(\mu^- S \to e^- S)/\Gamma(\mu^- S \to \nu_\mu P^*) \tag{20}$$

They found

$$R_{\mu e} < 4 \times 10^{-10}, \text{ 90\% confidence limit} \tag{21}$$

Again, lepton conservation is extraordinarily precisely obeyed. Another type of test involving nuclei is the search for neutrinoless double beta decay of a nucleus

$$_Z N^A \to {}_{Z+2}N^A + e^- + e^- \tag{22}$$

No evidence for such decays have been found.[23]

1.C. The Reaction $e^- + e^- \rightarrow \mu^- + \mu^+$: The foregoing tests of lepton conservation involve relatively small energies—less than the $\mu - e$ mass difference. Perhaps violations of lepton conservation only occur at high energies or large momentum transfers? Unfortunately, it is difficult to devise precise high energy tests, because in high energy reactions there is a relatively large background of e's and μ's from the decays of hadrons. For example, it is difficult to put a relatively small upper limit on the reaction

$$\mu + p \rightarrow e + \text{hadrons}, \tag{23}$$

the high energy analogy to the reaction in Eq. 19.

A somewhat high energy test carried out about ten years ago[24] looked for the reaction

$$\sigma(e^-e^- \rightarrow \mu^-\mu^-) < 0.67 \times 10^{-32}\text{cm}^2 \tag{25}$$

at the same energy. Modern techniques could substantially improve the precision of this test; however, I don't know of any plans to repeat this experiment. It could be done using the two rings of the DORIS or DCI colliding beams facilities, but those facilities have so far only been used for e^+e^- collisions.

3. Neutrinos

3.A. General Properties: Just as we used the e and μ to develope an emperical definition for charged leptons; se we can use the ν_μ and ν_e to develop an emperical definition for neutral leptons. There is no need to repeat that elaborate discussion, and I will only summarize the

the properties of the neutrinos:

a. no interactions through the strong force;

b. no structure or constituents;

c. obey conventional weak interactions;

d. spin 1/2;

e. obey lepton conservation.

Once again properties a and b are essential to the deffinition of a neutral lepton; whereas, properties c, d, and e are a matter of taste.

3.B. Neutrino Masses: A fascinating property of the neutrinos is that they may have zero mass, Table IV. We only know upper limits.

TABLE IV

Upper Limits on Neutrino Masses at the 90% Confidence Level

Neutrino	Upper Limit On Mass	Method	Reference
ν_e	55 to 60 eV/c^2	measure p_e in $_1H^3 \to {_2H^3} + e^- + \bar{\nu}_e$	25
ν_μ	0.51 to 0.65 MeV/c^2	measure p_μ in $\pi^+ \to \mu^+ + \nu_\mu$	26
ν_τ	250 MeV/c^2	measure p_μ in $\tau^- \to e^- + \bar{\nu}_e + \nu_\mu$	27

(I have broken my rule of not discussing τ physics in this part of the talk to strengthen this point). It would be useful to substantially lower these upper limit measurements; however, it is very difficult to improve on these measurements[25,26] (except for ν_τ where experiments have just begun.[27]

There are two related questions associated with the neutrino masses for which there are no answers at present:

(1) Are the masses of the known neutrinos different from zero, or is zero mass an intrinsic property of a neutral lepton?

(2) Are there neutrinos with relatively large masses; that is, do heavy neutral leptons exist?

I will return to this subject in section 9.

4. Simple Lepton Models

The last three sections have presented a very general discussion of lepton phenomenology based on the e, μ, ν_e, and ν_μ . I will now discuss three well known models for heavy leptons based on a conservative extension of this phenomenology. I mean by the term conservative that these models retain the following properties:

a. the particles have no strong interactions or anomalous interactions;

b. the particles have no structure;

c. the particles obey conventional quantum electrodynamic theory;

d. the particles obey conventional weak interaction theory;

e. there is some form of lepton conservation;

f. the particles have spin 1/2.

4.A. The Sequential Heavy Lepton Model:

In this model[16] one assumes there is a mass sequence of charged leptons, each lepton type having a separately conserved lepton number and a unique associated neutrino of smaller mass. We visualize a sequence:

$$\begin{array}{cc} \text{Charged Lepton} & \text{Associated Neutrino} \\ e^{\pm} & \nu_e, \bar{\nu}_e \\ \mu^{\pm} & \nu_\mu, \bar{\nu}_\mu \\ \ell^{\pm} & \nu_\ell, \bar{\nu}_\ell \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{array} \tag{27}$$

The radiative decays $\ell^{\pm} \to e^{\pm} + \gamma$, $\ell^{\pm} \to \mu^{\pm} + \gamma$ are then forbidden; and only weak decays such as

$$\ell^- \to \nu_\ell + e^- + \bar{\nu}_e \tag{28a}$$

$$\ell^- \to \nu_\ell + \mu^- + \bar{\nu}_\mu \tag{28b}$$

$$\ell^- \to \nu_\ell + \pi^- \tag{29}$$

$$\ell^- \to \nu_\ell + K^- \tag{30}$$

$$\ell^- \to \nu_\ell + \rho^- \tag{31}$$

$$\ell^- \to \nu_\ell + \text{2-or-more hadrons} \tag{32}$$

occur.

4.B. Ortholepton Model:[28] In this model the charged lepton is assigned the same lepton number as the same sign e or μ; and there is no additional neutrino. For example, we conceive of a particle $e^{*\pm}$ which may be thought of as an excited e.

Then

$$e^{*\pm} \rightarrow e^{\pm} + \gamma \tag{33}$$

will be the dominant decay mode unless we suppress the $e^* - \gamma - e$ coupling. I will continue this discussion in section 8.

4.C. Paralepton Model:[28-30] In this model we conceive of a particle E^+ or M^+ which has the lepton number of the e^- or μ^- respectively. This association of lepton number with opposite electric charge prevents radiative decays such as $E^+ \not\rightarrow e^+ + \gamma$. Hence, such charged leptons decay through the weak interactions. For example

$$E^- \rightarrow \bar{\nu}_e + e^- + \bar{\nu}_e \tag{34}$$

$$E^- \rightarrow \bar{\nu}_e + \mu^- + \bar{\nu}_\mu \tag{35}$$

$$E^- \rightarrow \bar{\nu}_e + \text{hadrons} \tag{36}$$

4.D. Decay Rates and Branching Ratios in Charged Sequential Heavy Lepton Decays: There have been several papers[31-34] on the theoretical decay rates (Γ) for decays such as those in Eqs. 28 through 32, and I will only summarize a few results here. In all these results conventional weak interaction theory is assumed with V-A coupling for the ℓ-ν_ℓ vertex and zero mass for the ν_ℓ.

a. $\ell^- \rightarrow \nu_\ell + e^- + \bar{\nu}_e$, $\ell^- \rightarrow \nu_\ell + \mu^- + \bar{\nu}_\mu$: This decay occurs through the diagram in Fig. 3a. Neglecting the masses of the e and μ

$$\Gamma(\ell^- \rightarrow \nu_\ell e^- \bar{\nu}_e) = \Gamma(\ell^- \rightarrow \nu_\ell \mu^- \bar{\nu}_\mu) = \frac{G^2 m_\ell^5}{192\pi^3} \tag{37}$$

Here G is the Fermi coupling constant and m_ℓ is the heavy lepton mass.

b. $\ell^- \rightarrow \nu_\ell + \pi^-$: Using Fig. 3b,

$$\Gamma(\ell^- \rightarrow \nu_\ell \pi^-) = \frac{G^2 f_\pi^2 \cos^2\theta_c m_\ell^3}{16\pi}\left(1 - \frac{m_\pi^2}{m_\ell^2}\right)^2 \qquad (38)$$

Here $f_\pi \approx 0.137 m_p$, θ_c is the Cabibbo angle, and m_π is π mass.

c. $\ell^- \rightarrow \nu_\ell + \rho^-$: Using Fig. 3c:

$$\Gamma(\ell^- \rightarrow \nu_\ell \rho -) = \frac{G^2 m_\ell^3 \cos^2\theta_c m_\rho^2}{64\pi^2}\left(1 - \frac{m_\rho^2}{m_\ell^2}\right)^2 \left(1 + \frac{2m_\rho^2}{m_\ell^2}\right) \qquad (39)$$

Here m_ρ is the ρ mass.

d. $\ell^- \rightarrow \nu_\ell$ + hadrons: Fig. 3d illustrates a simple way to estimate the ratio of the decay rate to hadrons compared to the total decay rate. In Fig. 3d we note that given sufficient energy, the virtual W^- has equal probability to decay to an $e^- \bar{\nu}_e$ pair, or to a $\mu^- \bar{\nu}_\mu$ pair; or if quarks did not have color to a $\bar{\mu}d$ pair or $\bar{c}s$ pair. However, color makes decay to a $\bar{\mu}d$ pair or $\bar{c}s$ pair three times as probable. We neglect Cabbibo suppressed decay modes of the ℓ.

Hence

$$\frac{\Gamma(\ell^- \rightarrow \nu_\ell + \text{hadrons})}{\Gamma(\ell^- \rightarrow \text{all})} = \frac{3}{5}, \quad m_\ell < m_c + m_s \text{ and } m_\ell < m_\tau$$

$$\frac{\Gamma(\ell^- \rightarrow \nu_\ell + \text{hadrons})}{\Gamma(\ell^- \rightarrow \text{all})} = \frac{6}{9}, \quad m_\ell > m_c + m_s \text{ and } m_\ell > m_\tau \qquad (40)$$

Here m_c and m_s are the masses of the c and s quarks. (For very high mass leptons, heavier quarks such as the b quark must be included).

e. Branching Ratios and Lifetime: Figs. 4 and 5 give an overall view of the theoretical predictions for the branching ratios and lifetimes as a function of the lepton mass. Figure 5 assumes that the coupling constant has its conventional value $G = 1.02 \times 10^{-5}/m_p^2$, where m_p is the proton mass.

LECTURE II: THE τ HEAVY LEPTON

This lecture, when presented, contained a full discussion of the history of the work on the τ and the evidence that it is a lepton. Several recent papers[5-8] have reviewed these subjects; therefore, I will not present these subjects here. I only note here that:

a. there is very substantial evidence that the τ is a lepton;

b. all measured properties of the τ are consistent with it being a lepton;

c. no other hypotheses explains the measured properties of the τ.

5. Properties of the τ

5.A. Mass of the τ : Table V gives three recent mass measurements.

TABLE V

The τ Mass

Mass in MeV/c^2	Group	Reference
1807 ± 20	DASP	35
1782^{+2}_{-7}	DELCO	36
1787^{+10}_{-18}	DESY-Heidelberg	37

The special significance of these mass values is that they prove that the τ mass is less than the mass of the lightest charmed meson — the D. Hence, the τ and D are different particles, and there is an energy region in e^+e^- annihilation physics where one can study τ's without charmed particles being present.

5.B. Mass of the ν_τ: The smallest upper limit[27] on the ν_τ mass is 250 MeV/c^2, determined by the DELCO collaboration.

5.C. $\tau - \nu_\tau$ Coupling: The nature of the τ-ν_τ coupling is investigated by studying the e or μ momentum spectrum from the decay mode $\tau \to e\nu\nu$ or $\tau \to \mu\nu\nu$. The basic question is whether the coupling is V - A, as it is for the $e - \nu_e$ and $\mu - \nu_\mu$ coupling. In that case, the ν_μ would be left-handed. There was some interest as to whether the coupling could be V + A and hence the ν_τ be right-handed. Measurements by the SLAC - LBL collaboration[38] and by the DELCO collaboration[27,39] have excluded V + A coupling, and the latter measurement finds a preliminary value for the Michel parameter

$$\rho_{exp} = 0.83 \pm 0.19 \qquad (41a)$$

Excluding radiative corrections, the theoretical values are

$$\begin{aligned} \rho_{theor} &= 0.75 \text{ for V - A coupling} \\ \rho_{theor} &= 0.375 \text{ for pure V or pure A coupling} \qquad (41b) \\ \rho_{theor} &= 0.0 \text{ for V + A coupling} \end{aligned}$$

5.D. The τ Lifetime: Two 95% confidence upper limits have been measured for the τ lifetime τ_τ. The SLAC - LBL collaboration finds[40]

$$\tau_\tau < 1.0 \times 10^{-11} \text{sec} \qquad (42a)$$

The PLUTO collaboration finds[41]

$$\tau_\tau < 3.5 \times 10^{-12} \text{ sec} \tag{42b}$$

The theory discussed in section 4.D. predicts

$$\tau_\tau \approx 2.5 \times 10^{-13} \text{ sec} \tag{42c}$$

for the mass of about 1800 MeV/c^2 ; and assuming

$$g_{\tau w \nu_\tau} = g_{e w \nu_e} = g_{\mu w \nu_\mu} = g \tag{43}$$

Here $g_{\tau w \nu_\tau}$ is the weak interaction coupling constant at the τ-W-ν_τ vertex in Fig. 3. From Eqs. 42b and 42c

$$\frac{g^2_{\tau w \nu_\tau}}{g^2} > 0.071, \text{ 95\% confidence limit} \tag{44}$$

<u>5.E. Type of Lepton</u> : Muon neutrino experiments have shown[42] that the τ is not a μ-related ortholepton or μ-related paralepton. The argument goes as follows. If the τ^+ or τ^- had the lepton number of the ν_μ, one of the reactions in Eq. 45 should occur in ν_μ-nucleon reactions

$$\begin{array}{l} \nu_\mu + N \longrightarrow \tau^- + \text{hadrons} \\ \qquad\qquad\quad\ \ \llcorner\!\!\longrightarrow e^- + \bar{\nu}_e + \nu_\mu \end{array} \tag{45a}$$

or

$$\begin{array}{l} \nu_\mu + N \longrightarrow \tau^+ + \text{hadrons} \\ \qquad\qquad\quad\ \ \llcorner\!\!\longrightarrow e^+ + \nu_e + \nu_\mu \end{array} \tag{45b}$$

Hence anomalous e^- or e^+ events should appear in ν_μ experiments with a cross section

$$\sigma(\nu_\mu \to e) = \left(\frac{g_{\tau w \nu_\mu}}{g}\right)^2 \sigma(\nu_\mu \to \text{all})\ B(\tau \to e\nu\nu) \qquad (46)$$

Here $g_{\tau w \nu_\mu} = g_{\tau w \nu_\tau}$ in Eq. 43, g is defined in Eq. 43 and $B(\tau \to e\nu\nu)$ is the $\tau \to e\nu\nu$ branching ratio. However, the measured upper limits on $\sigma(\nu_\mu \to e)$ give an upper limit on $(g_{\tau w \nu_\mu}/g)^2$ smaller then the lower limit in Eq. 44. Hence the τ does not have the lepton number of the μ^+ or μ^-.

If the τ were an electron-related paralepton, then the τ^- would have the lepton number of the e^+; and in the decay

$$\tau^- \to \nu_e + e^- + \bar{\nu}_e \qquad (47)$$

there would be two identical particles. This would lead to[43]

$$\frac{\Gamma(\tau^- \to \bar{\nu}_e e^- \bar{\nu}_e)}{\Gamma(\tau^- \to \bar{\nu}_e \mu^- \bar{\nu}_\mu)} \approx 2 \qquad (48)$$

instead of the equality of decay rates predicted by the sequential heavy lepton model. We have shown[44] that the data excludes Eq. 48. Hence, the τ is not an e-related paralepton.

All existing data on the τ is consistant with the τ being a sequential heavy lepton or an e-related ortholepton. The sequential model is certainly more elegant, since the e-related ortholepton model requires that the unobserved decay mode $\tau^- \to e^- + \gamma$ be suppressed by an additional mechanism. Indeed the suppression mechanism must be sufficiently strong to explain why experimentally $\Gamma(\tau \to e\gamma)/\Gamma(\tau \to \text{all}) < 2.6\,\%$.

5.F. Spin of the τ: The cross section for the reaction

$$e^+ + e^- \rightarrow \tau^+ + \tau^- \tag{49}$$

is

$$\sigma_{\tau\tau,\mathrm{theor}} = \frac{2\pi\alpha^2\beta(3-\beta^2)}{3E_{cm}^2} \tag{50}$$

if the τ has spin 1/2. Here E_{cm} is the total energy and $\beta=v/c$ where v is τ velocity. We find

$$\sigma_{\tau\tau,\mathrm{measured}} / \sigma_{\tau\tau,\mathrm{theor}} = 1.00 \pm 0.15 \tag{51}$$

where the error includes systematic errors. Hence the data is consistant with spin 1/2 for the τ. The cross section for $e^+e^- \rightarrow \tau^+\tau^-$ has also been studied carefully near the threshold $E_{cm} = 2m_\tau$ by the DELCO collaboration,[27,39] and has been further analyzed by Tsai.[45] As shown in Fig. 6, the threshold behavior strongly favors a spin of 1/2. Therefore, we believe the τ spin is 1/2.

6. Decay Modes of the τ

6.A. Theoretical Predictions for Branching Ratios: Table VII gives the theoretical predictions[46] for the branching fractions for the various τ decay modes assuming conventional weak interaction theory. A τ mass of 1800 MeV/c^2, a massless ν_τ, and V - A coupling is used. The (ν_τ + several hadrons) decay mode (also called the continuum decay mode) excludes the specific hadronic decay modes listed above it in the table. The calculation of this continuum requires the use of $\sigma(e^+e^- \rightarrow \text{hadrons})$ data in the $E_{cm} < 1800$ MeV region and is somewhat uncertain.

TABLE VI

Theoretical Predictions for τ Decay Branching Ratios

Mode (written for τ^-)	Branching Fraction in Percent
$\nu_\tau + e^- + \bar{\nu}_e$	16.4
$\nu_\tau + \mu^- + \bar{\nu}_\mu$	16.0
$\nu_\tau + \pi^-$	9.8
$\nu_\tau + K^-$	0.6
$\nu_\tau + \rho^-$	23.0
$\nu_\tau + K^{*-}$	1.6
$\nu_\tau + A_1^-$	9.3
ν_τ + several hadrons	23.3
Total	100.0%

6.B. Measurements of Branching Ratios: Table VII presents averaged measured values of the branching ratios for various τ decay modes. These averages were computed by G. Feldman[5] and his paper should be consulted for the individual values, the method of averaging, and the references.

Comparison of Table VII with Table VI shows excellent agreement. This agreement is one of the reasons for accepting the identification of the τ as a lepton. The $(\nu_\tau + A_1)$ decay mode is discussed in Section 8.A.

TABLE VII

Average Measured Values for Branching Ratios in τ Decay

Mode (written for τ^-)	Branching Fraction in Percent
$\nu_\tau + e^- + \bar{\nu}_e$	16.9 ± 1.9
$\nu_\tau + \mu^- + \bar{\nu}_\mu$	18.3 ± 1.9
$\nu_\tau + \pi^-$	8.3 ± 1.4
$\nu_\tau + \rho^-$	24.0 ± 9.0
ν_τ + (1 hadron)$^-$	33.8 ± 4.8
ν_τ + (3 hadrons)$^-$	31.0 ± 3.2

6.C. Upper Limits on Lepton Conservation Violating Decay Modes: The sequential heavy lepton model requires that a neutrino, assummed to be a ν_τ, always be present in a decay mode of the τ. No violations of this requirement have been found;[40] Table VIII lists measured upper limits on branching ratios for decay modes without a neutrino.

TABLE VIII

Upper Limit[40] on Branching Ratios for τ Decay Modes Which Do Not Have a Neutrino

Mode	Upper Limit in Percent	Confidence level	Experiment
τ → 3 charged particles	1.0	95%	PLUTO
τ → 3 charged leptons	0.6	90%	SLAC-LBL
$\tau \to e + \gamma$	2.6	90%	SLAC-LBL
$\tau \to \mu + \gamma$	1.3	90%	SLAC-LBL

7. Future Studies of the τ

There are three areas in which future research on the τ is required. One area concerns further studies of the τ decay modes (section 7A) and the $\tau - W - \nu_\tau$ coupling constant (section 7b). The second area concerns higher energy tests of the leptonic nature of the τ (section 7C). The third area concerns τ production by means other than e^+e^- annihilation (section 7D).

7.A. Further Studies of τ Decay Modes: It would be very useful for checking the conventionality of the weak interactions of the τ, and for other reasons, to obtain more experimental information on various decay modes:

a. $\tau^- \to \nu_\tau + \pi^-$: This decay process has such a simple Feynman diagram, Fig. 3b, that it is worthwhile to make a precise measurement. Incidently, a study of the endpoints of the π momentum spectrum will provide an independent measurement of m_τ.

b. $\tau^- \to \nu_\tau + e^- + \bar{\nu}_e$, $\nu_\tau + \mu^- + \nu_\mu$: A study of the endpoint of the e or μ can provide a smaller upper limit on the mass of the ν_τ.

c. $\tau^- \to \nu_\tau + \rho^-$, ν_τ + several hadrons: Reduced statistical errors are required for the branching ratios for these decay modes.

d. $\tau^- \to \nu_\nu + A_1^-$: Two experiments[47,48] have seen invariant mass enhancements at 1100 MeV/c^2 in the $\pi^-\pi^+\pi^-$ state in the decay mode

$$\tau \to \nu_\tau + \pi^- + \pi^+ + \pi^- \tag{52}$$

This enhancement could be the long sought A_1 meson with $J^P = 1^+$. However much more data is required because there have been many negative searches for the A_1 in hadronic reactions.

e. $\tau^- \to \nu_\tau + K^-$, $\nu^\tau + K^{*-}$: These two Cabibbo suppressed decay modes have not been found. Conventional weak interaction theory predicts

$$\frac{\Gamma(\tau^- \to \nu_\tau K^-)}{\Gamma(\tau^- \to \text{all})} \approx (\tan \theta_c)^2 \frac{\Gamma(\tau^- \to \nu_\tau \pi^-)}{\Gamma(\tau^- \to \text{all})} \approx \frac{0.1}{16} \approx 0.006 \tag{53}$$

The measured upper limit[49] on this branching ratio is 0.016.

7.B. The τ-W-ν_τ Coupling Constant $g_{\tau W \nu_\tau}$: as noted in section 5.D., we only know that

$$g_{\tau W \nu_\tau} > 0.27g \tag{54}$$

where g is the charged lepton-W-neutrino coupling constant for e's or μ's. We would like to know if $g_{\tau W \nu_\tau} = g$ so that $e-\mu$ weak interaction universality extends to the τ. There are two ways to determine $g_{\tau W \nu_\tau}$; both very difficult.

One method, Fig. 7a, requires the direct measurement of the τ lifetime; which would be about 2.5×10^{-13} sec if $g_{\tau W \nu_\tau} = g$.

The other method, Fig. 7b, requires the study of the semi-leptonic cecay modes of a meson M much heavier than the τ, namely

$$M \to \tau + \nu_\tau + \text{hadrons} \tag{55a}$$

compared to

$$M \to e + \nu_e + \text{hadrons} \tag{55b}$$

If the mass is ignored, then for identical hadronic states

$$\frac{\sigma(M\to\tau\nu_\tau\ \text{hadrons})}{\sigma(M\to e\nu_e\ \text{hadrons})} = \left(\frac{g_{\tau w\nu_\tau}}{g}\right)^2 \tag{56}$$

A candidate for M is the proposed meson that contains a b quark and a non-b antiquark. Since the Υ with a mass of about 10 GeV is presumed to be made up of a $b\bar{b}$ pair; this meson would have a mass of about 5 GeV, allowing the decay in Eq. 55a to occur.

7.C. Higher Energy Tests of the Leptonic Nature of the τ : Whenever higher energies become available, we test once more whether the e and μ obey the basic requirements for a lepton listed in section 1.A: does the e or μ still behave as a point particle; do they obey conventional quantum electrodynamics, do they exhibit any anomalous interaction? As higher e^+e^- annihilation energies become available at PETRA and PEP, we should maintain a similar inquiring attitude towards the τ . There are several types of tests:

a. $e^+ + e^- \to \tau^+ + \tau^-$: One is simply whether the reaction

$$e^+ + e^- \to \tau^+ + \tau^- \tag{57}$$

occurs through the diagram in Fig. 8a and hence still obeys

$$\sigma_{\tau\tau} = \frac{2\pi\alpha^2\beta(3-\beta^2)}{3E_{cm}^2} \tag{58}$$

at higher energy. A related question is whether the reaction

$$e^+ + e^- \to \tau^+ + \tau^- + \text{hadrons} \tag{59}$$

occurs more copiously than would be expected from higher order (in α) reactions such as Fig. 8b.

b. Two-Virtual-Photon Production of τ Pairs: The reaction

$$e^+ + e^- \rightarrow e^+ + e^- + \tau^+ + \tau^- \tag{60}$$

occurs through two-virtual-photon diagrams such as Fig. 8c. Unlike the one-virtual-photon cross section, Eq. 58, the two-virtual-photon cross section

$$\sigma_{ee\tau\tau} \approx \frac{112\alpha^4}{977 m_\tau^2} \left[\ln\left(\frac{E_b}{m_e}\right)\right]^2 \left[\ln\left(\frac{E_b}{m_\tau}\right)\right] \tag{61}$$

increases with energy. Here $E_b = E_{cm}/2$ and m_e is the electron mass. Indeed, in the vicinity of $E_b \doteq 15$ GeV the two cross sections become equal[51] (Fig. 8d), Eq. 61 assumes the τ is a conventional lepton, hence the study of the reaction in Eq. 60 constitutes an additional test of the leptonic nature of the τ.

c. $T \rightarrow \tau^+ + \tau^-$: Ignoring mass effects the decay rates for

$$\begin{aligned} T &\rightarrow e^+ + e^- \\ T &\rightarrow \mu^+ + \mu^- \\ T &\rightarrow \tau^+ + \tau^- \end{aligned} \tag{62}$$

should be equal if the τ is a lepton. However, since the decay occurs through a virtual photon, this test is not completely independent of a test using the reaction $e^+e^- \rightarrow \tau^+\tau^-$.

7.D. Other Means of Producing τ's:

a. The Bethe-Heitler process, Fig. 9a, can yield τ pairs through the reaction[52,53]

$$\gamma + \text{Nucleus} \rightarrow \tau^+ + \tau^- + \text{hadrons} \tag{63}$$

The total cross section[52,53] for this reaction on a Be nucleus is about 10^{-33}cm^2 at $E_\gamma = 200$ GeV.

However, the major experimental problem is how to detect the pairs so produced. The most feasible method so far proposed is to look for $e^\pm\mu^\mp$ pairs from

$$\gamma + \text{Nucleus} \rightarrow \underset{\displaystyle e^+\nu\nu}{\underset{\downarrow}{\tau^+}} + \underset{\displaystyle \mu^-\nu\nu}{\underset{\downarrow}{\tau^-}} + \text{hadrons} \tag{64}$$

Unfortunately, the photoproduction of pairs of charmed mesons $(C\bar{C})$ and their subsequent decay

$$\gamma + \text{Nucleus} \rightarrow \underset{\displaystyle e^+\nu\nu}{\underset{\downarrow}{C}} + \underset{\displaystyle \mu^-\nu\nu}{\underset{\downarrow}{\bar{C}}} + \text{hadrons} \tag{65}$$

is roughly 100 to 1000 times more probable. (We assume the total $C\bar{C}$ photoproduction from Be is 10^{-30} to 10^{-31}cm^2). Hence, the reaction in Eq. 64 is very difficult to detect. Nevertheless, it is important to look for the Bethe-Heitler process for τ pairs as a test of the leptonic nature of the τ.

Figure 9b shows a related process using almost real virtual photons from a muon beam. The proposal[54] is to use a calorimetric target, and to select events in which there is a relatively small amount of hadronic energy. This criterion would reduce the background from the production of charmed particle pairs.

b. Hadronic Production of ν_τ's: A similar problem bedevils any so far proposed scheme for making a ν_τ beam by dumping an intense proton beam into a thick target. One might hope to carry out the series of reactions

$$\left.\begin{array}{l} p + \text{Nucleus} \rightarrow \tau + \text{anything} \\ \tau \rightarrow \nu_\tau + \text{anything} \end{array}\right\} \begin{array}{l}\text{occurs in} \\ \text{thick target}\end{array} \qquad (66)$$

$$\left.\begin{array}{l} \nu_\tau + \text{Nucleus} \rightarrow \tau + \text{anything} \\ \tau \rightarrow \text{decay mode} \end{array}\right\} \begin{array}{l}\text{occurs in} \\ \text{neutrino} \\ \text{detector}\end{array} \qquad (67)$$

Unfortunately the ν_e's and ν_μ's from charmed particles produced and decaying in the thick target will overwhelm the $\nu_\tau \rightarrow \tau$ signal.

LECTURE III: PAST AND FUTURE SEARCHES FOR OTHER HEAVY LEPTONS

8. Other Searches For Charged Leptons

8.A. Future Searches for Sequential Heavy Leptons: The only practical method remains e^+e^- annihilation

$$e^+e^- \rightarrow \ell^+ + \ell^- \qquad (68)$$

and as with the τ 6 the best signature is $e^\pm\mu^\mp$ events. PEP and PETRA will allow searches for masses up to $15\text{GeV}/c^2$. The search will be more difficult than the search for the τ for three reasons:

a. the production cross section will be smaller at the higher energy required.

b. the $e^\pm\mu^\mp$ events from the τ will constitute an annoying background.

c. In the case of the τ many of the hadronic decay modes (Table VI) have only one charged particle. These provided the very useful two-prong event signature: electron-hadron and muon-hadron. Larger mass sequential leptons will have much less single hadron decay modes, Fig. 4; hence, these useful signatures will not be available.

However, there is no doubt that a complete search for new charged sequential leptons can be made in the available mass range.

8.B. Electromagnetic Production of Ortholeptons: As discussed in section 4B, an ortholepton would be: e^{*-} with the same lepton number as the e^-, μ^{*-} with the same lepton number as the μ^-, or perhaps τ^{*-} with the same lepton number as the τ. To keep the discussion brief, I shall only give examples using the e^*.

Figures 10a and 10b show some reactions in which an e^* is produced at an electromagnetic vertex e^*-γ-e. Incidently, this same vertex would contribute to g_e-2 (Fig. 10c). Since no ortholeptons have been found, past searches can only put lower limits on the e^* mass. This is done as follows. Current conservation forbids the e^*-γ-e vertex from having the usual form:

$$V^\mu = q\bar{\psi}_{e*}\gamma^\mu\psi_e \tag{69a}$$

It has become conventional[16,55] to use the form

$$V^\mu = q\left(\frac{\lambda}{M^*}\right)\bar{\psi}_{e*}\sigma^{\mu\nu}\psi_e p_\nu \tag{69b}$$

Here q is the electron charge, p is the photon four-momentum, M* is the e* mass and λ is a dimensionless coupling constant. Figures 10c and 11 show current limits on M* and λ^2 for e* and μ*. The use of PEP and PETRA can obviously considerably extend the M* search range. Indeed, no direct use has yet been made of SPEAR or DORIS data. The e^+e^- results in Fig. 10 come from research at ADONE.

8.C. ν_μ Production of Charged Leptons: If a heavy charged lepton couples to the ν_μ, then the reactions in Fig. 12a and 12b would demonstrate its existence through the presence of anomalous $e^\pm$ or μ^+ events.[57] The measured lower limits on the mass depend on the strength of the coupling constant $g_{\nu_\mu WL^\pm}$ in Fig. 12. If we assume $g_{\nu_\mu WL^\pm} = g_{\nu_\mu W\mu^-}$ the lower limits on the $L^\pm$ mass are[57]

$$M_{L^-} \geq 7.5 \text{ GeV}, \quad M_{L^+} \geq 9.0 \text{ GeV} \; ; \quad \text{from Fig. 12a}$$

$$M_{L^+} \geq 12.0 \text{ GeV}; \quad \text{from Fig. 12b} \qquad (70)$$

Obviously such searches will continue at higher energies.

There have also been extensive searches[58] for heavy charged and neutral leptons using the so-called trimuon events which might be produced through reactions such as Fig. 12c. At present almost all trimuon events are explainable by conventional processes;[59] and the few which cannot be so explained appear not to be connected to heavy lepton production reactions.

9. Neutral Heavy Lepton Searches

9.A. Past Searches for Heavy Neutral Searches: Past searches for heavy neutral leptons have usually used hadron reactions[60] or cosmic rays.[61] No definitive evidence for heavy neutral leptons have been found in these searches. However, these searches are not at all restrictive because they usually have depended for their success on either:

a. an unconventional production mechanism, such as an anomalously large cross section for hadronic production of leptons, or

b. an unconventional lepton property, such as large mass combined with a long lifetime.

Searches have also been carried out using muon neutrinos.[58,62,63] There have been some interesting events found,[62,63] but again, there is no evidence for a heavy neutral lepton.

9.B. Future Searches Using e^+e^- Annihilation: The only known search method which is very general and yet uses a reasonably probable interaction is

$$e^+ + e^- \to L^o + \bar{L}^o \tag{71}$$

through s-channel exchange of a neutral current, Fig. 13a. We can even calculate this production cross section if we assume the model[64] in Fig. 13b, where the proposed neutral intermediate boson, Z^o, has a mass M_Z. A rough calculation which ignores vector coupling, assumes the axial coupling constant at both vertices is g_A, ignores the L^o mass, and assumes $M_Z \gg E_{cm}$ yields

$$\sigma_{L^o \bar{L}^o} \approx \frac{g_A^4}{12\pi E_{cm}^2}\left(\frac{E_{cm}}{M_Z}\right)^4 \tag{72}$$

Recall that the cross section (Eq. 58) for the production of a charged heavy lepton via

$$e^+ + e^- \to L^+ + L^- \tag{73}$$

is

$$\sigma_{L^+L^-} = \frac{4\pi\alpha^2}{3E_{cm}^2} = \frac{e^4}{12\pi E_{cm}^2} \tag{74}$$

if we ignore the mass of the $L^{\pm}$.

Hence

$$\frac{\sigma_{L^o\bar{L}^o}}{\sigma_{L^+L^-}} \approx \left(\frac{g_A}{e}\right)^4 \left(\frac{E_{cm}}{M_Z}\right)^4 \tag{75}$$

Conventional weak interaction theory uses

$$g_A/e = 1/\ 4\sin\theta_W\cos\theta_W = 0.6 \tag{76}$$

$$M_Z = 100 \text{ GeV}$$

In the next few years PEP and PETRA will have maximum E_{cm} of about 36 GeV. Hence

$$\sigma_{L^o\bar{L}^o} / \sigma_{L^+L^-} \sim 10^{-2} \quad , E_{cm} = 36 \text{ GeV} \tag{77}$$

and $L^o\bar{L}^o$ production will be much smaller than L^+L^- production. The need for higher energy e^+e^- colliding beams facilities in order to make comprehensive L^o searches is clear.

An L^o search at PEP or PETRA is just on the verge of being possible. At $E_{cm} = 36$ GeV, $\sigma_{L^o\bar{L}^o} \approx 1.4 \times 10^{-37} \text{cm}^2$ assuming an average luminosity of $10^{31}\text{cm}^{-2}\text{sec}^{-1}$; this will yield 0.12 $L^o\bar{L}^o$ pairs per day.

Of course the $L^o\bar{L}^o$ pairs must be detected, and the detection method depends upon how the L^o decays. I will list a few examples:

a. The L^o might be stable. In this case the reaction $e^+e^- \rightarrow L^o\bar{L}^o$ is not detectable at the low production rates calculated here with present detector technology.

b. The L^o could have an associated smaller mass charged lepton L^-. Typical decay modes would be:

$$\begin{aligned} L^o &\rightarrow L^- + e^+ + \nu_e \\ L^o &\rightarrow L^- + (\text{hadrons})^+ \end{aligned} \tag{78}$$

c. The L^o could have the same lepton number as the e^-, μ^-, or τ^-. Typical decay modes would be

$$\begin{aligned} L^o &\rightarrow e^- + e^+ + \nu_e \\ L^o &\rightarrow e^- + (\text{hadrons})^+ \end{aligned} \tag{79}$$

d. The L^o could have an associated smaller mass neutral lepton. For example, a neutrino ν_L. Typical decay modes would be

$$\begin{aligned} L^o &\rightarrow \nu_L + e^+ + e^- \\ L^o &\rightarrow \nu_L + (\text{hadrons})^+ \end{aligned} \tag{80}$$

Incidently, in this case the production reaction

$$e^+ + e^- \rightarrow L^o + \bar{\nu}_L \tag{81}$$

could also occur.[65] This reaction doubles the mass range of the L^o search if the ν_L is massless. However, the $L^o - Z^o - \nu_L$ coupling constant is an unknown quantity.

Returning to the detection of the $e^+e^- \rightarrow L^o \bar{L}^o$ reaction, the decay modes in Eq. 78 - 80 are not all detectable. Indeed probably only 10% to 20% of the $L^o \bar{L}^o$ pairs will decay to obviously anomalous looking, and hence useful, events. Therefore per experiment, only 0.01 to 0.02 detectable $L^o \bar{L}^o$ pairs occur in the model we have been developing here. The pooling of anomalous events by the half dozen experiments likely to be simultaneously looking for $L^o \bar{L}^o$ events in their data tapes might help this very low rate problem.

10. How Many Leptons Exist?

The question of how many leptons exist cannot be answered without a theory about the classification of particles and their properties. However, any theory of particles can be destroyed by finding just one particle which is not predicted by the theory. Thus this question is an experimental one, and as long as new experiments looking for new particles can be devised, this question has no answer. This thought was stated most succinctly by my thesis professor I. I. Rabi, "Physics is an experimental science."

Nevertheless, it is very interesting to ask a more restricted question: how many leptons exist within the framework of existing theories, particularly within the framework of the very successful Weinberg-Salam theory of weak and electromagnetic interaction? We shall also assume:

a. the sequential heavy lepton model;

b. massless neutrinos;

c. all $L^{\pm}$-ν_L pairs couple with the same $W^{\pm}$ or Z^{o} intermediate bosons with the same strength.

10.A. Can There Be an Infinite Number of Massless Neutrino Types:

With these assumptions, it is easy to see that there cannot be an infinite number of lepton pairs. Consider the K decay

$$K^{\pm} \to \pi^{\pm} + \nu_n + \bar{\nu}_n \tag{84}$$

where $n = 1, 2, 3 \ldots N$ represents N different types of massless neutrinos. If N were infinite, the total decay rate $\sum_1^N \Gamma(K \to \pi\nu_n\bar{\nu}_n)$ would be infinite. We know that the $K^{\pm}$ has a finite decay rate; hence, N is finite.

Similarly, the fact that the total cross section for reactions such as

$$e^+ + e^- \to \nu_n + \bar{\nu}_n \qquad (83A)$$

$$e^+ + e^- \to \gamma + \nu_n + \bar{\nu}_n \qquad (83b)$$

$$e^+ + e^- \to \text{hadrons} + \nu_n + \bar{\nu}_n \qquad (83c)$$

is finite, means there can only be a finite number of massless neutrinos. Of course, direct measurements of the total decay rate for Eq. 82, or direct measurements[67] of the total cross section for any of the reactions in Eq. 83, would fix the value of N . Unfortunately, such measurements are exceedingly difficult because the ν's cannot be detected directly, and there are huge backgrounds from much more copious reactions.

10.B Astrophysics Limits on Neutrinos: If one accepts the "big bang" theory of the creation of the universe, then one can calculate some interesting limits on numbers and properties of neutrinos. The calculation most pertinent to the considerations here is that of Steigman et al.[68] They find that total number of different types of massless neutrinos is less than or equal to 7! This includes the ν_e, ν_μ and ν_τ. Several astrophysics calculations[69 - 72] have set limits on the masses and decays of neutrinos and heavy neutral leptons. For example, upper limits of the order of tens of eV have been set on the masses of individual neutrinos, and on the sum over the masses of all types of neutrinos. Thus, further studies of known neutrinos and future searches for new heavy leptons and neutrinos offer us the fascinating possibility of either helping to confirm the "big bang" theory of creation or helping to destroy it.

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FIGURE CAPTIONS

1. Feynman diagrams for: (a) and (b) $e^+e^- \to e^+e^-$ via single photon exchange, (c) $e^+e^- \to \mu^+\mu^-$ via single photon exchange, (d) $e^+e^- \to e^+e^-$ via two photon exchange, (e) a typical radiative correction to $e^+e^- \to e^+e^-$, (f) an anomalous contribution through x exchange to $e^+e^- \to e^+e^-$.

2. Feynman diagrams for calculating g - 2: (a) through (c) lowest order quantum electrodynamic diagrams for the coefficients in Eq. 14, (d) the hadronic contribution, (e) the lowest order weak interaction contribution.

3. Feynman diagrams for calculating some decay modes of a sequential heavy lepton ℓ with associated neutrino ν_ℓ.

4. Branching ratios for a sequential heavy lepton as a function of its mass.

5. The lifetime of a sequential heavy lepton as a function of its mass.

6. Comparison of the energy behavior of the ratio $R^{2P}_{ex} = (e^+e^- \to \tau^+\tau^- \to e^\pm x^\mp)/(e^+e^- \to \mu^+\mu^-)$.

7. Two methods for determining the $g_{\tau W \nu_\tau}$ coupling constant: (a) by measurement of the τ lifetime, (b) by comparing the $\tau\nu_\tau$ and $e\nu_e$ semileptonic decay rates of a heavy meson M.

8. Production of a $\tau^+\tau^-$ pair (a) through one photon exchange, (b) through a higher order reaction, (c) through a two photon exchange process. The production cross section for two photon exchange is compared with one photon exchange in (d) ; taken from Ref. 51 .

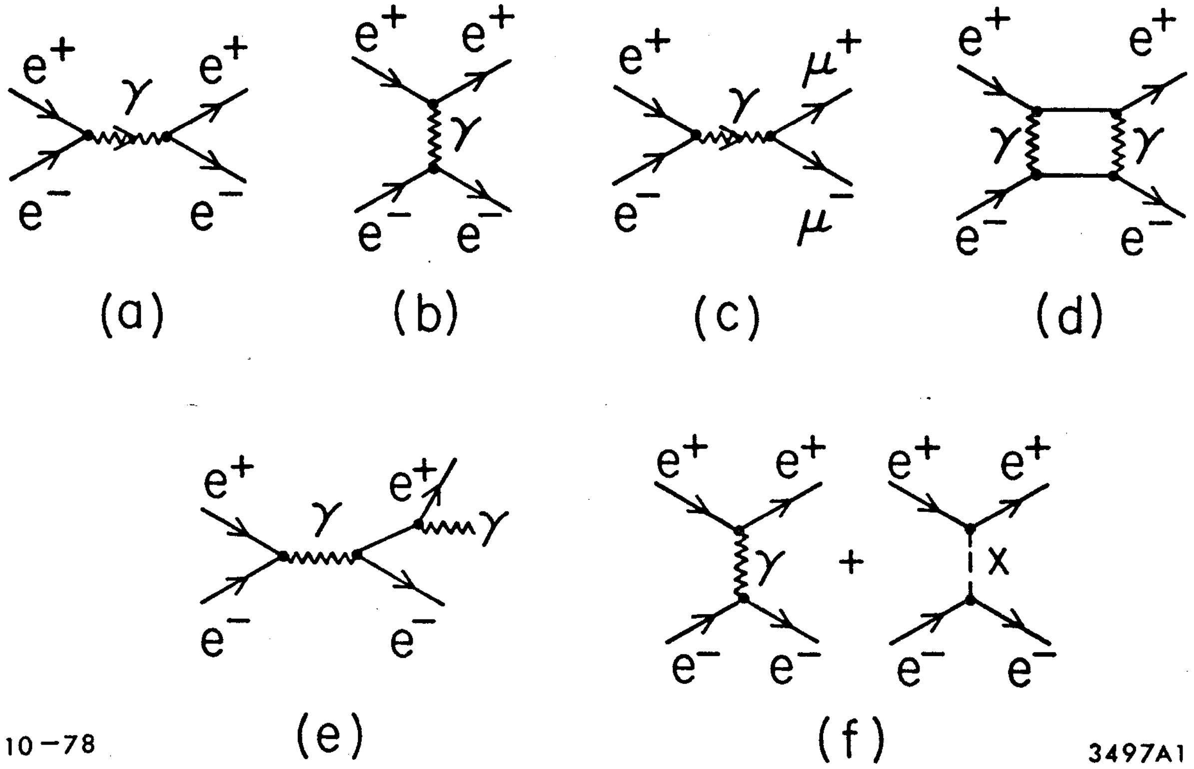
9. Production of a $\tau^+\tau^-$ pair through (a) the Bethe-Heitler process, (b) a virtual photon from a muon.

10. Feynman diagrams for production of an e^* ortholepton through (a) e - p scattering, (b) e^+e^- annihilation, (c) the upper limit on λ^2 versus $M(e^*)$ in Eq. 69b from Ref. 56, (d) the contribution of an e^* to g - 2 of the electron.

11. Comparison of upper limits on the value of λ^2, Eq. 69b, for the e^* and μ^*. References are: 1. H. Gittleson _et_ _al_., Phys. Rev _D10_, 1379 (1974), (μ - p scattering); 2. C. Betorne _et_ _al_., Phys. Letters _17_, 70 (1965), (e - p scattering); 3. A. De Rujula and B. Latrup, Letter Nuovo Cimento _3_, 49 (1972), (g - 2 of electron); 4. R. Budnitz _et_ _al_., Phys. Rev. _141_, 1313 (1966), (e - p scattering); 5. C. D. Boley, _et_ _al_., Phys. Rev. _167_, 1275 (1968), (e - p scattering); 6. A. De Rujula and B. Lautrup, Letter Nuovo Cimento _3_, 49 (1972), (g - 2 of muon); 7. H. J. Behrend, _et_ _al_., Phys Rev. Letters _15_, 900 (1965), (e - p scattering).

12. Possible production processes for heavy leptons using a ν_μ.

13. Production of a pair of heavy neutral leptons through e^+e^- annihilation via the weak interactions.



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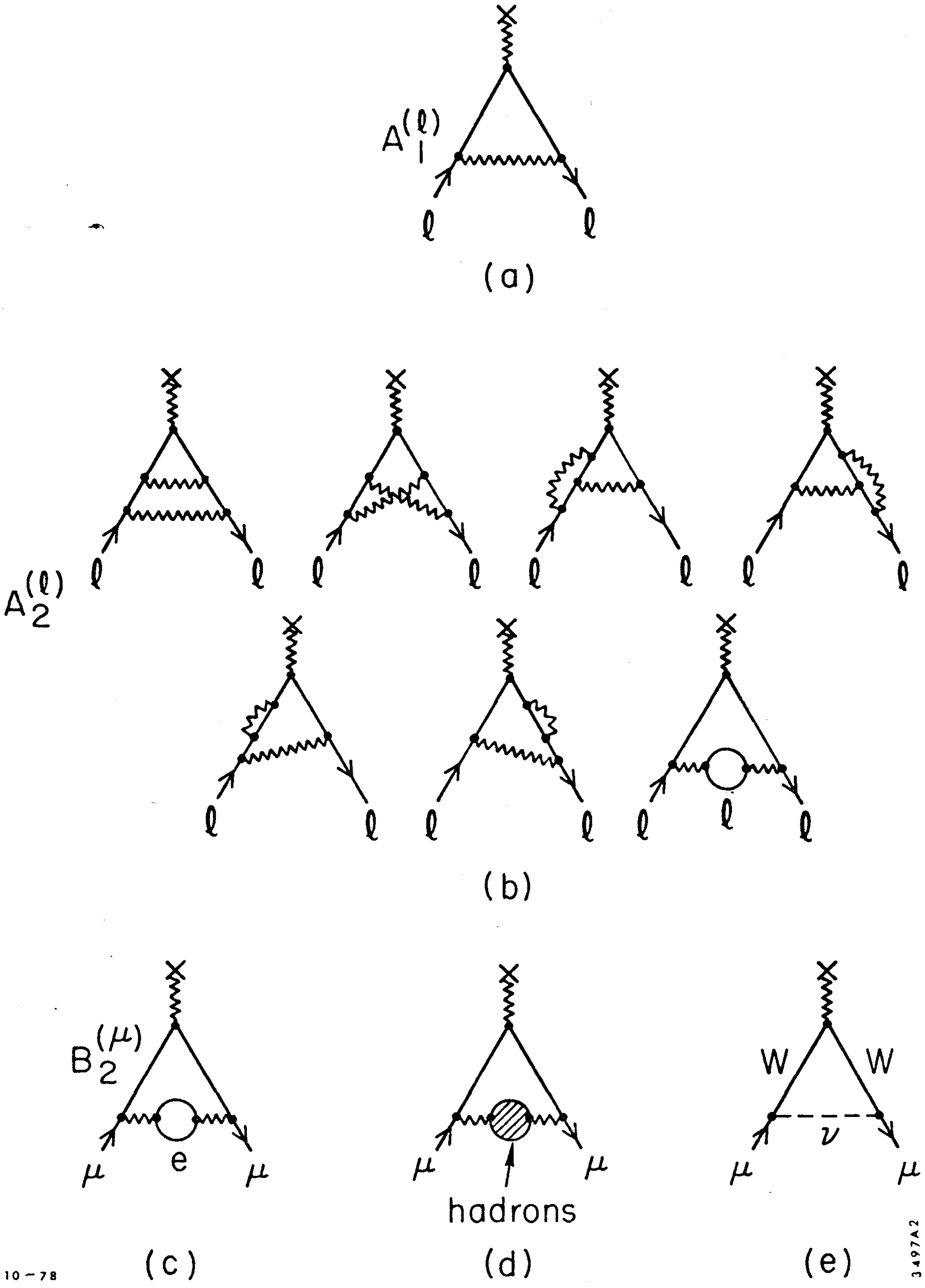

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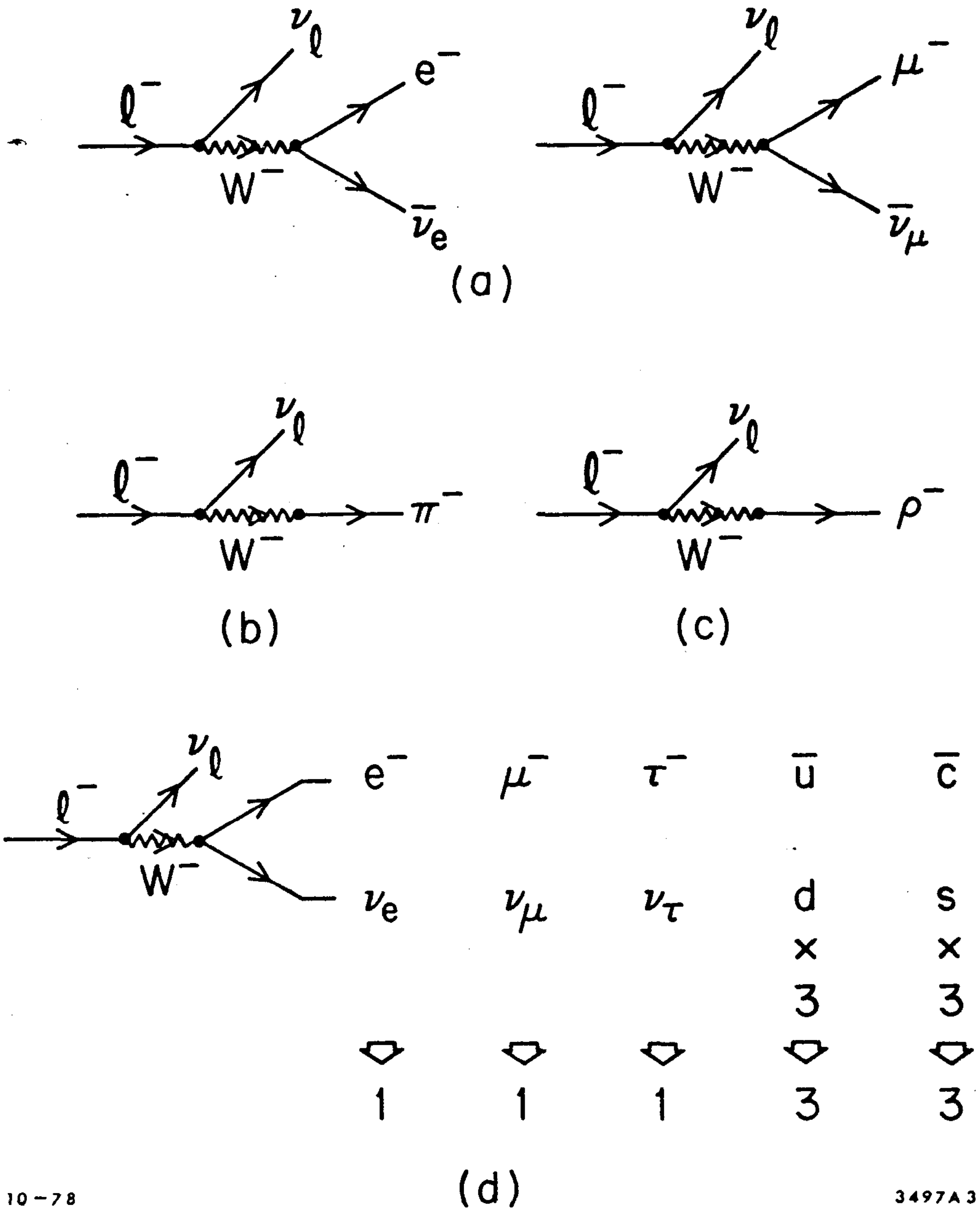

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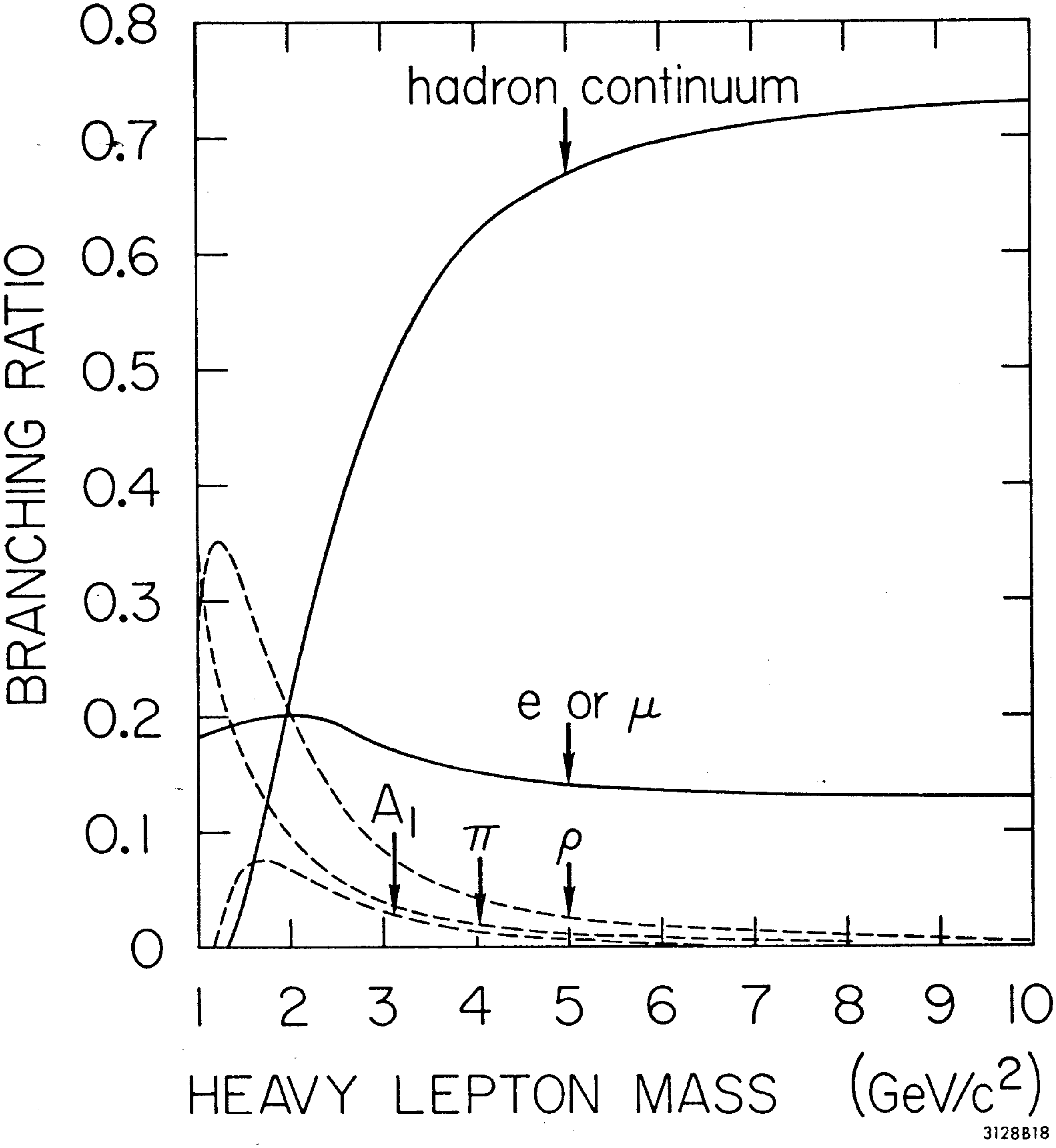

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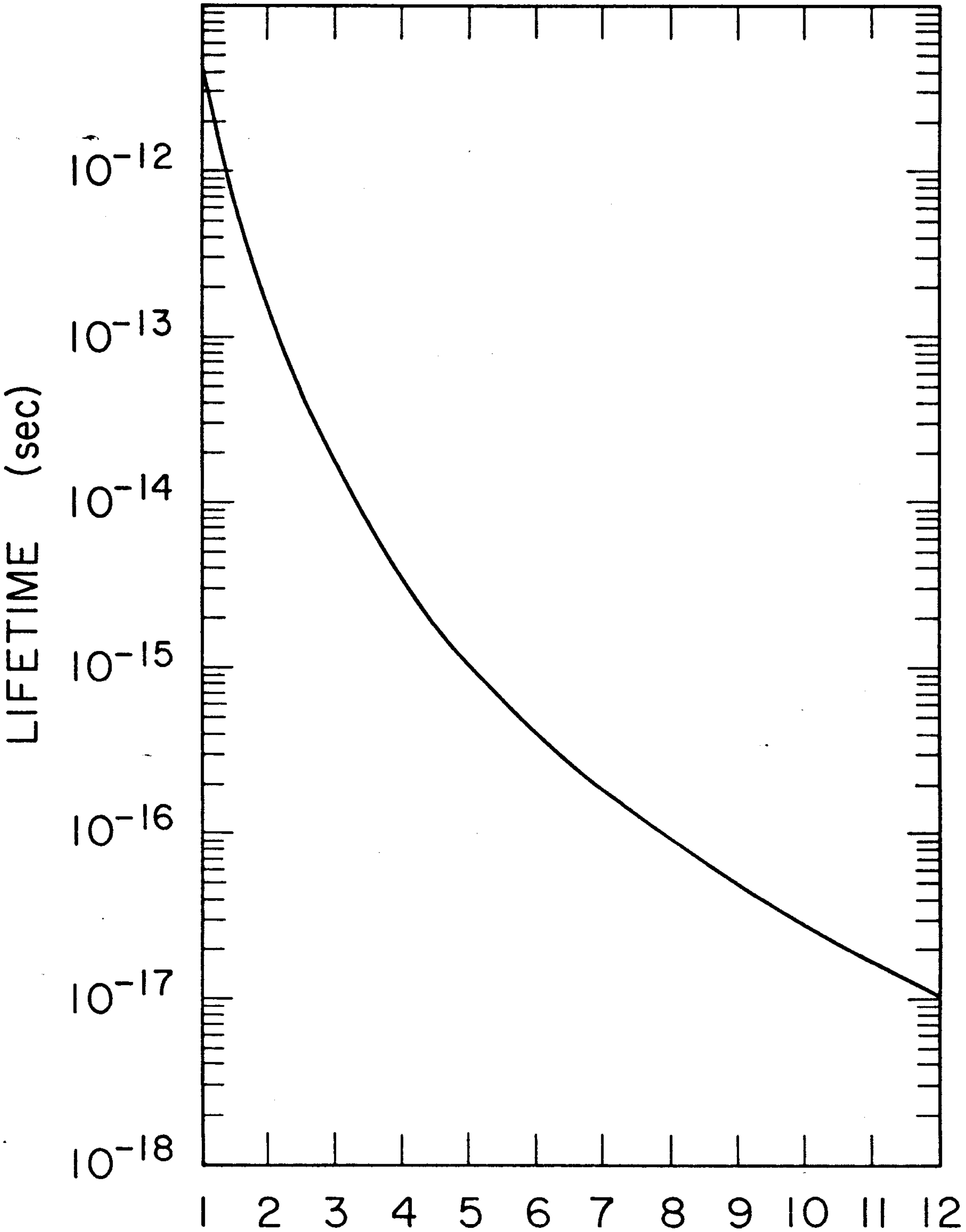

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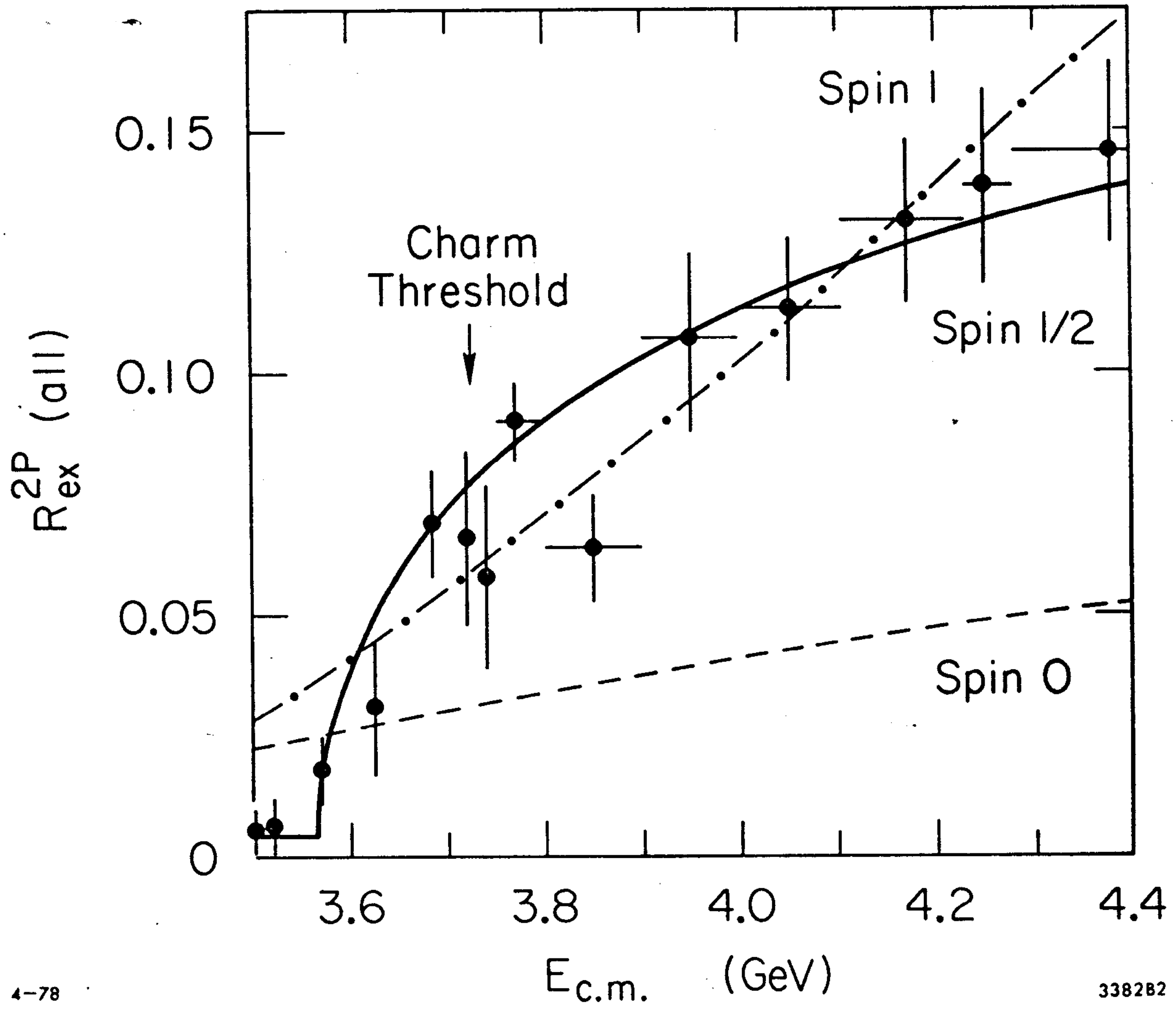

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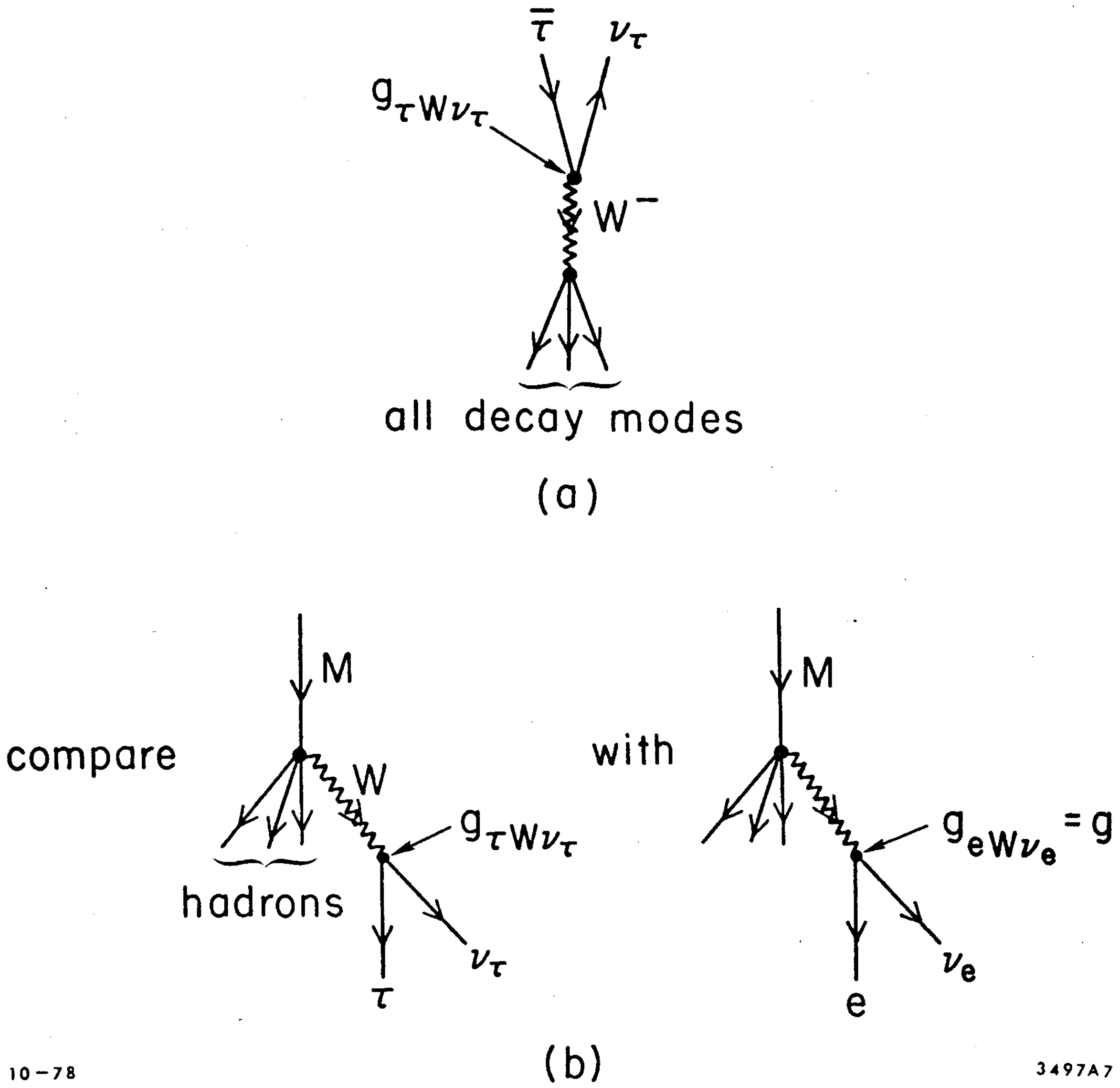

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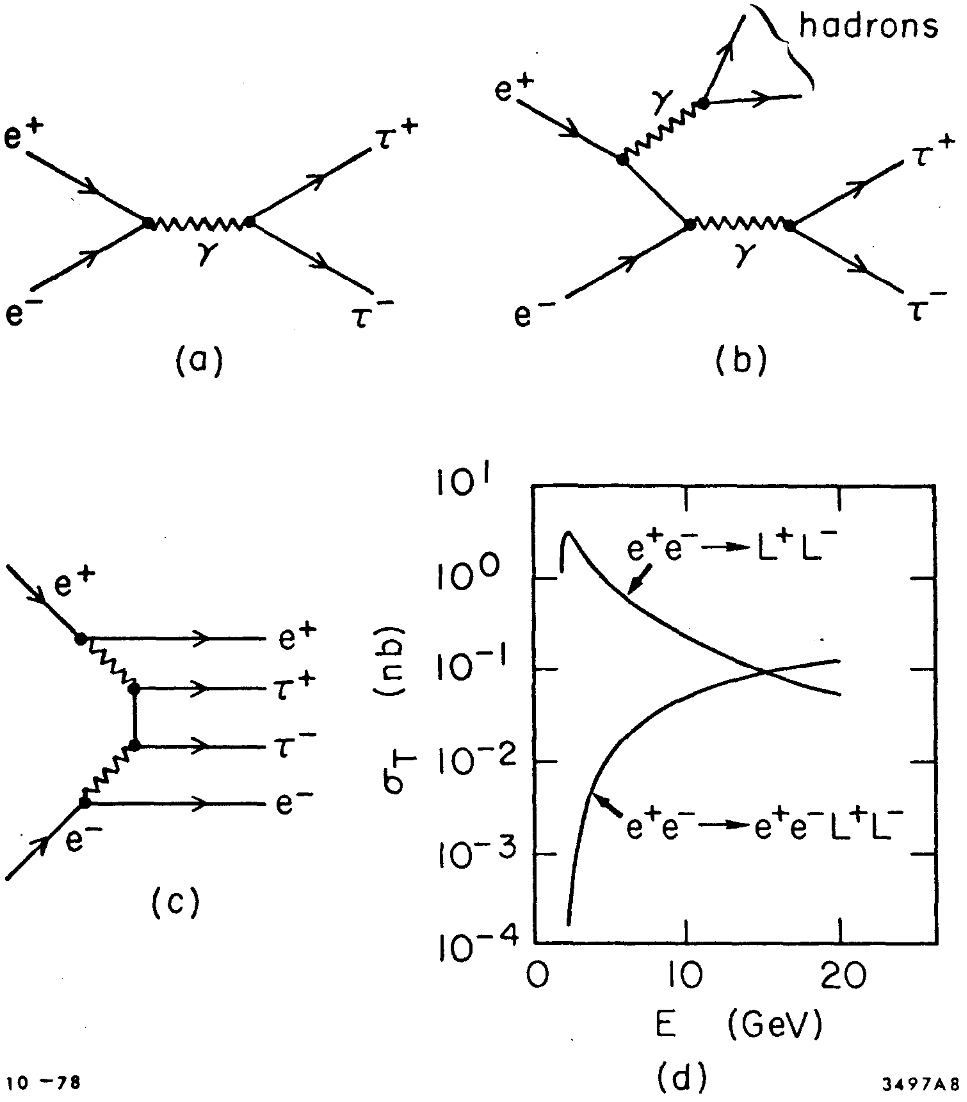

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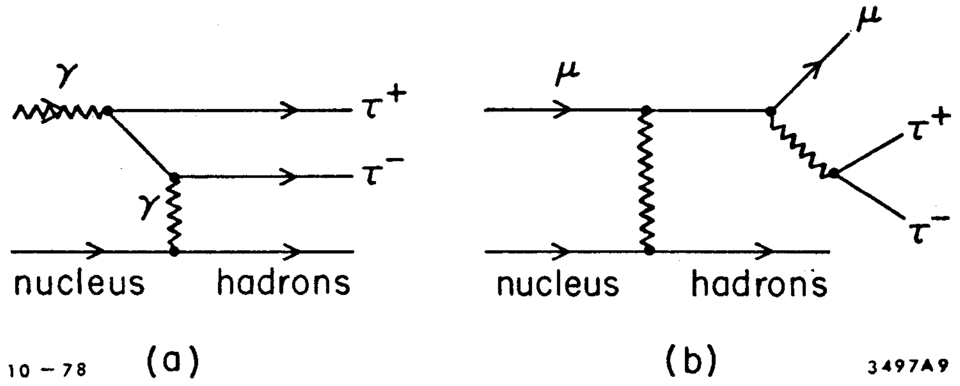

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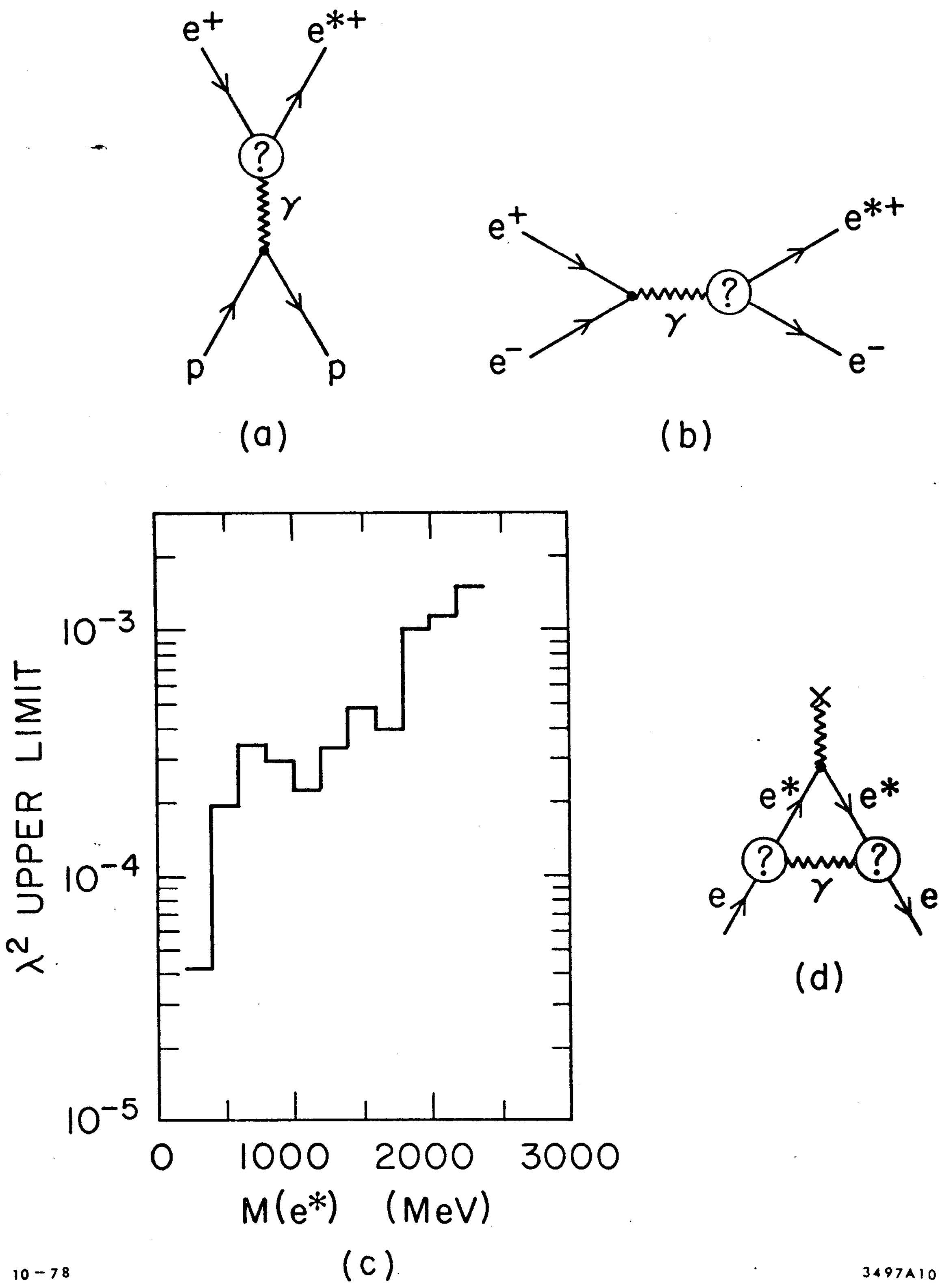

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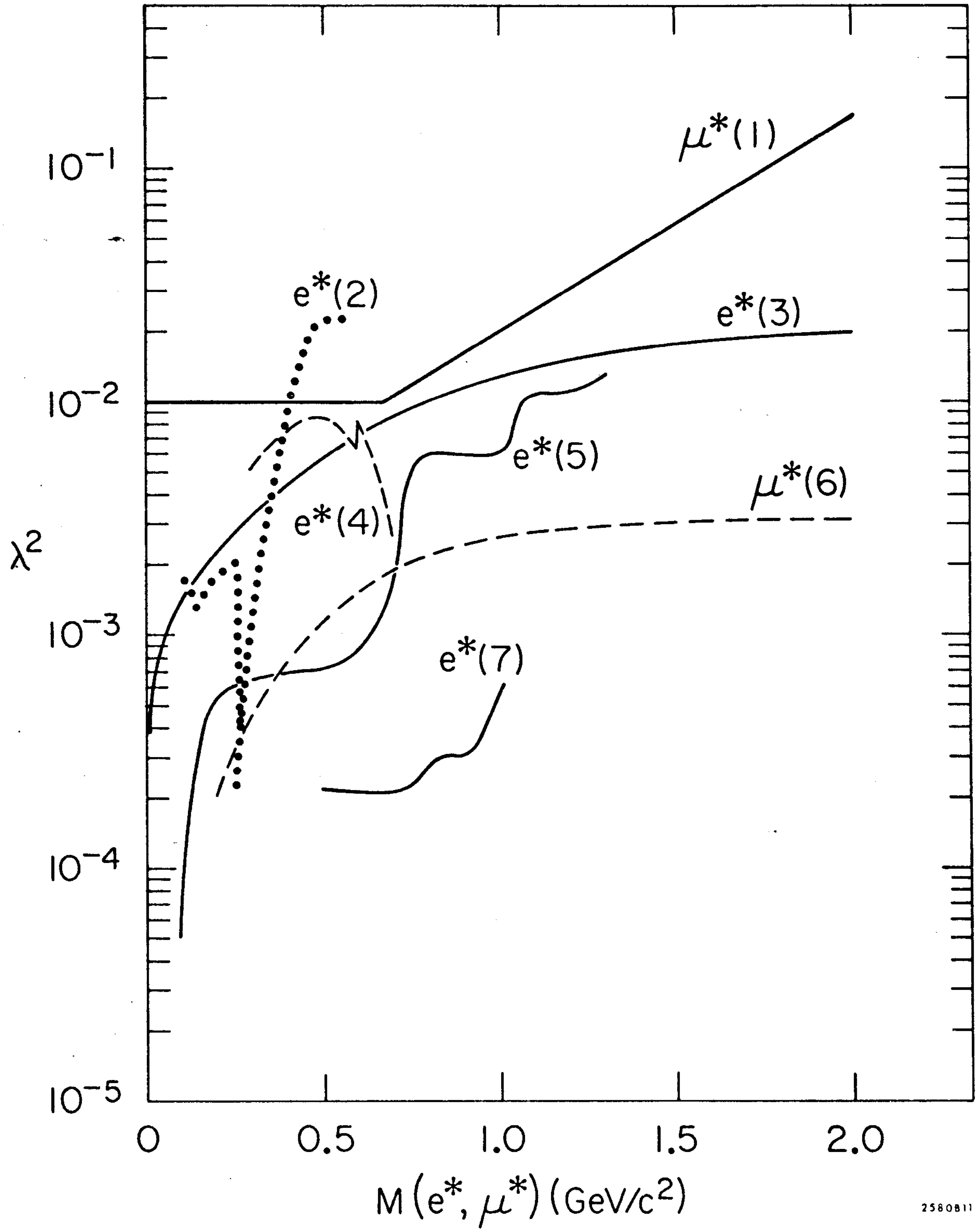

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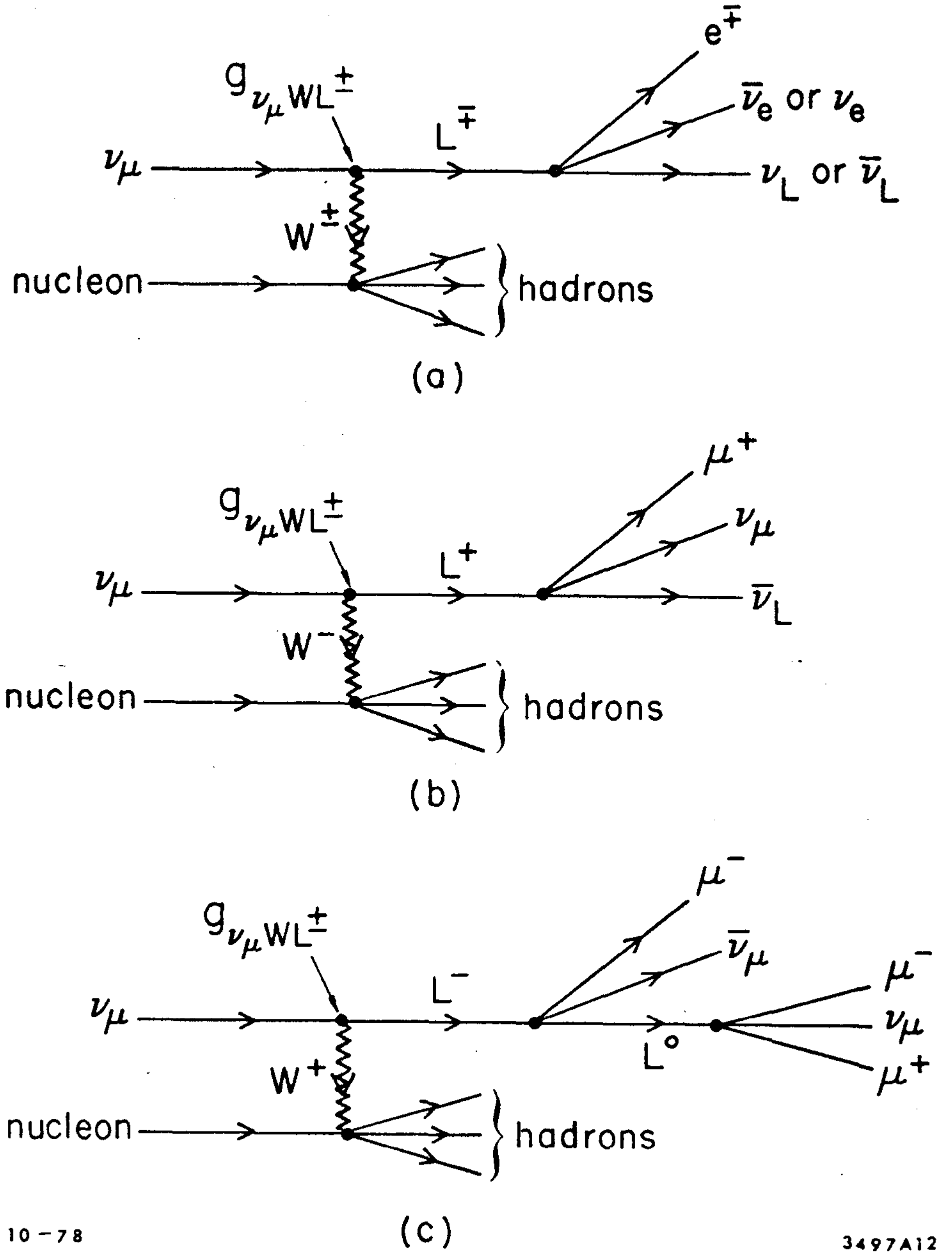

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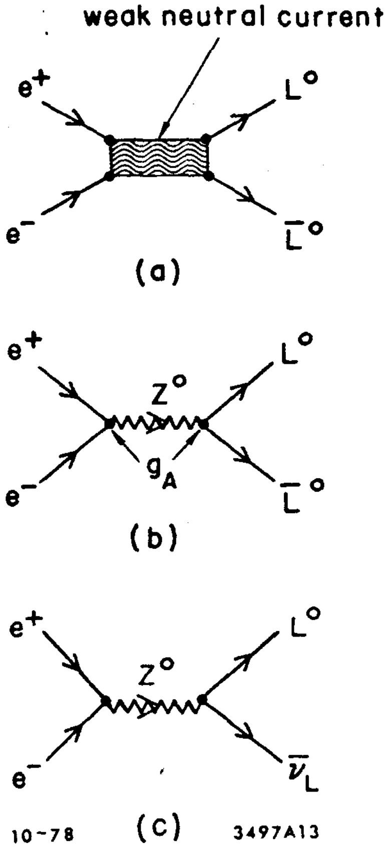

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