

PRODUCTION OF JETS AND SINGLE PARTICLES AT LARGE TRANSVERSE  
MOMENTUM IN PHOTON-PHOTON COLLISIONS\*

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ABSTRACT

Photon-photon collisions will be a copious source of jets and single particles produced with large transverse momentum in  $e^+e^-$  storage rings. These jets and single particles have topological properties which allow them to be easily distinguished from  $e^+e^-$  annihilation events. They arise from several sources: a) quark exchange in  $\gamma\gamma \rightarrow q\bar{q}$ , b) QCD induced processes such as  $\gamma_1 \rightarrow q\bar{q}$ ,  $\gamma_2 \rightarrow q\bar{q}$ , with  $qq$  scattering via vector gluon exchange, and c) differential cross sections proportional to  $p_T^{-6}$  and  $p_T^{-8}$  at fixed  $x_T = 2p_T/\sqrt{s}$  and fixed  $\theta_{c.m.}$  characteristic of the constituent interchange (CIM) picture. In particular, we predict  $Ed\sigma/d^3p(e^+e^- \rightarrow e^+e^- \text{ jet } X) \sim 0.03 (1-x_T) p_T^{-4} \text{ nb-GeV}^{-2}$  for  $p_T > 4 \text{ GeV/c}$ , characteristic of the simplest quark exchange process, and  $Ed\sigma/d^3p(e^+e^- \rightarrow e^+e^- \pi X) \sim (1-x_T)^2 p_T^{-6} \text{ nb-GeV}^4$  for  $p_T > 2 \text{ GeV/c}$  characteristic of the CIM subprocess  $\gamma q \rightarrow \pi q$ . Remarkably, the jet trigger cross sections  $Ed\sigma/d^3p(\gamma\gamma \rightarrow \text{jet } +X)$  turn out to be asymptotically scale-free and independent of  $\alpha_c(p_T^2)$  when perturbative QCD contributions to all orders are included.

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## I. INTRODUCTION

It is well known that photon-photon inelastic collisions in  $e^+e^-$  storage rings become an increasingly important source of hadrons as the center-of-mass energy  $\sqrt{s} = 2E_e$  is raised.<sup>1</sup> The dominant part of the cross section for  $e^+e^- \rightarrow e^+e^- + \text{hadrons}$  arises from the annihilation of two nearly on-shell photons emitted at small angles to the beam. The resulting cross section increases logarithmically with energy  $(m_e^2/s \rightarrow 0, s \gg m_H^2)$ <sup>1</sup>:

$$d\sigma_{e^+e^- \rightarrow e^+e^- X}(s)/dm_H^2 \simeq \frac{\alpha^2}{\pi} \log^2 \frac{s}{m_e^2} \frac{\sigma_{\gamma\gamma}(m_H^2)}{m_H^2} \log \frac{s}{m_H^2} \quad (1.1)$$

where  $m_H$  is the invariant mass of the produced hadronic system. In contrast, the  $e^+e^-$  annihilation cross section decreases quadratically with energy. For example, at the beam energy of  $E_e = 15$  GeV, the standard vector dominance estimate for  $\sigma_{\gamma\gamma}(m_H^2)$  gives  $\sigma(e^+e^- \rightarrow e^+e^- \text{ hadrons}) \simeq 15$  nb for  $m_H \geq 1$  GeV, compared to the annihilation cross section

$$\sigma_{e^+e^- \rightarrow \gamma \rightarrow \text{hadrons}} \equiv R\sigma_{e^+e^- \rightarrow \gamma \rightarrow \mu^+\mu^-} \simeq (0.1 \text{ nb})R.$$

If the hadronic interactions of a real photon are the same as those of a vector meson, then high energy photon-photon collisions will be qualitatively similar to high energy hadron-hadron collisions, with most hadrons produced with small momentum transverse to the beam direction. The study of such photon-photon reactions is of considerable interest since the entire range of hadronic phenomena can be studied for two incident spin-one bosons, whose mass and even linear polarization can be individually "tuned" when the  $e^+$  and  $e^-$  are both tagged. Reviews of the various hadronic channels and theoretical predictions which can be probed in  $\gamma\gamma$  collisions are given in Ref. 1.

Although vector-meson-dominance model ideas are useful for low momentum transfer reactions, the production of hadrons at large transverse momentum is sensitive to processes where the photon's point-like coupling to the quark current will dominate. In fact, because of the special role of the photon in quantum chromodynamics (QCD), we find that cross sections expected for jets of hadrons at large  $p_T$  are sizeable, and are much larger than the extrapolation from hadron-hadron collisions.<sup>2</sup> For example, for  $E_e = 15$  GeV, the total cross section for the production of jets with  $p_T^J \geq 3$  GeV/c turns out to be of the order of 0.05 nb; i.e., the equivalent of 1/2 unit of R.

This paper is devoted to a detailed study of high  $p_T$  jet and single particle production processes in  $\gamma\gamma$  collisions, assuming the validity of QCD perturbation theory. In addition to being a pervasive background to  $e^+e^-$  annihilation studies at high energy storage rings such as PEP and PETRA, there are a number of important theoretical reasons why these reactions merit intensive study:

1. The elementary reaction  $\gamma\gamma \rightarrow q\bar{q} \rightarrow \text{hadrons}$ , yields a scale-invariant two-jet cross section at large  $p_T$  which is proportional to the fourth power of the quark charge. The corrections from higher order QCD diagrams are of relative order  $\alpha_c \left(4p_T^2\right)$ . The observation of a two-jet scale-invariant cross section at the level predicted here will directly test the scaling of the quark propagator  $\not{p}^{-1}$  at large  $p^2$  in the  $\gamma\gamma \rightarrow q\bar{q}$  subprocess.<sup>3</sup>

2. QCD processes which involve the large  $p_T$  scattering of quark or gluon constituents are sensitive to the structure functions of nearly on-shell photons. Remarkably, as was first shown by Witten,<sup>4</sup> the photon

structure function has a perturbative component which can be computed from first principles in QCD. At large  $q^2$ ,<sup>4,5</sup>

$$G_{q/\gamma}(x, q^2) = \frac{\alpha}{\alpha_c(q^2)} f(x) + O(\alpha^2) \quad (1.2)$$

Thus, aside from an overall logarithmic factor, the direct  $\gamma \rightarrow q$  distribution displays exact Bjorken scaling. This result is accurate to all orders in  $\alpha_c(q^2)$  (keeping only leading logarithms), and is in striking contrast to the pattern of scaling violations predicted in QCD for hadronic structure functions.

An important consequence of this result first discussed by Llewellyn Smith,<sup>5</sup> is that processes such as  $\gamma_1 \rightarrow q_1 \bar{q}_1$ ,  $\gamma_2 \rightarrow q_2 \bar{q}_2$  with  $q_1 + q_2 \rightarrow q_1 + q_2$  scattering at large  $p_T$  via gluon exchange leads to scale-invariant large  $p_T$  jet cross sections; in fact, we shall show that the dependence on  $\alpha_c(p_T^2)$  cancels out in the asymptotic cross section! This result is also true for the contribution of  $\gamma q \rightarrow gq$  subprocesses.

3. The QCD analysis of large momentum transfer reactions for hadron-hadron collisions is subject to uncertainties due to possible non-perturbative effects, wee interactions, etc.<sup>6</sup> Such corrections are presumably absent when the direct point-like couplings of the incident photons are involved.

4. In the case of single-particle production at large  $p_T$ , we predict that "higher twist" subprocesses such as  $\gamma_1 + q_2 \rightarrow \text{meson} + q$  dominate over fragmentation reactions such as  $\gamma + \gamma \rightarrow q + \bar{q}$  with  $q \rightarrow M + q'$  until very large  $p_T$ . The  $\gamma q \rightarrow Mq$  subprocesses is characteristic of the constituent interchange model (CIM)<sup>7</sup> and breaks scale invariance by a power of  $p_T^{-2}$ .

5. The  $\gamma\gamma$  process allows a study of the interplay between perturbative QCD processes and non-perturbative VDM contributions, and (by utilizing tagged leptons at large angles) the transition between real  $\gamma$  and virtual  $\gamma$  reactions, including deep inelastic scattering on a photon target. Furthermore, the onset of charm and other heavy quark thresholds can be studied once again from the perspective of  $\gamma\gamma$  induced processes.

Thus the study of  $\gamma\gamma$  collisions at large  $p_T$  provides a detailed laboratory for the study of QCD dynamics at short distances. We emphasize that the dominant hard scattering processes in  $\gamma\gamma$  collisions arise from the elementary field nature of the photon and are particular to  $\gamma$ -induced reactions. The observation of such processes will provide unique and important tests of the basic subprocesses which govern large transverse momentum processes. While it is not proven that the perturbative component and couplings of the real photon survive in the presence of confinement, we believe it is very likely. In vector dominance language these contributions are represented by an infinite spectrum of massive vector mesons.<sup>8</sup> Here we shall approximate the hadronic interactions of the photon by combining the contributions of the lowest vector mesons ( $\rho$ ,  $\omega$ ,  $\phi$ ) with the point-like perturbative contribution. No serious double-counting error should occur since the point-like interaction is equivalent to only very heavy vector mesons.

Jets induced by  $\gamma\gamma$  reactions will show unmistakable signatures in colliding beam experiments. Since the probability for  $\gamma$ -emission by an electron into a given rapidity interval is essentially flat, the distribution of jets per rapidity interval will also be approximately flat

and energy-independent away from the edge of phase space. Momentum conservation demands that a large  $p_T$  jet on one side of a reaction must be balanced by one or more large  $p_T$  jets on the away side. For the  $\gamma\gamma \rightarrow q\bar{q}$  process one expects the production of two "SPEAR"-like jets each with total energy less than the c.m. energy  $\sqrt{s}/2$ . Because the rapidity of the  $\gamma\gamma$  system is in general not zero, the jet events range from nearly back-to-back jets to a "V" configuration with a small opening angle along the beam axis. In all the other subprocesses the high  $p_T$  jets are accompanied by additional jets of low  $p_T$  hadrons from spectator particles which do not participate in the hard scattering reaction.

Even without tagging the photon energies, the different contributing hard scattering subprocesses can be distinguished by their topological jet structure; i.e., the number  $n_{\text{jet}}$  of hadronic jets observed in an event reflects the number of quark or gluonic systems initially separated. Specific examples are  $n_{\text{jet}}=2$  for  $\gamma_1+\gamma_2 \rightarrow q+\bar{q}$ ;  $n_{\text{jet}}=3$  for  $\gamma_1+q \rightarrow g+q$ ,  $\gamma_1+g \rightarrow q\bar{q}$  (where the incident quark or gluon is a constituent of  $\gamma_2$ );  $n_{\text{jet}}=4$  for  $qq \rightarrow qq$ ,  $gq \rightarrow gq$ , etc. Gluon jets, quark jets, or hadronic resonances may be distinguishable by their leading particle structure and central region multiplicity density. In the case of  $\gamma\gamma \rightarrow q\bar{q}$ , the multiplicity should be nearly identical to that for  $e^+e^- \rightarrow q\bar{q}$  at the same invariant  $q\bar{q}$  energy.

In practice one will probably wish to reduce the background from the single photon process by demanding that at least one lepton be observed at small angles in the final state. It will also be useful to have jet-tagging capability at small angles to differentiate between reactions which produce jets along the beam axis in addition to large  $p_T$  jets.

It is instructive to compare photon-photon large  $p_T$  processes with the corresponding reactions in hadron-hadron collisions. In the latter case, spectator particles are generally present so  $n_{\text{jet}} \geq 4$ . The dominant contribution to the jet cross section in hadron-hadron collisions is expected to arise from quark or gluon scattering via gluon exchange and to dominate non-scaling CIM (higher twist) contributions for  $p_T > 4 \text{ GeV}/c$ .<sup>7,9</sup> The cross section will be modified by logarithmic scale breaking from both  $\alpha_c(4p_T^2)$  and the hadronic structure functions. In contrast, we find in photon-photon collisions that the jet cross section is dominated by the  $n_{\text{jet}}=2$  subprocess for  $p_T \geq 4 \text{ GeV}/c$ , and is asymptotically scale invariant. In hadron-hadron collisions, the single pion cross section falls as  $p_T^{-8}$  at fixed  $x_T = 2p_T/\sqrt{s}$  and fixed  $\theta_{\text{c.m.}}$  for  $2 \lesssim p_T \lesssim 8 \text{ GeV}/c$ ,  $20 \lesssim \sqrt{s} \lesssim 60 \text{ GeV}^2$ . On the other hand we expect single meson cross sections for photon-photon collisions to behave as  $p_T^{-6} f(x_T, \theta_{\text{c.m.}})$  for  $1 \lesssim p_T \lesssim 10 \text{ GeV}/c$ , at PEP and PETRA energies. The observation of  $p_T^{-6}$  behavior in this domain for the  $\gamma\gamma \rightarrow \pi X$  cross section is of particular interest since it would provide an important confirmation of the magnitude of the CIM contribution.

As the reader might imagine, a large number of different subprocesses are equally a priori important in generating large  $p_T$  jets or mesons. We have had to develop a fairly general machinery for dealing with them, first introducing the parton model as a *modus operandi*, and then employing the hard scattering expansion to group portions of Feynman graphs into hard scattering subprocesses or into fragmentation functions. We then write down all possible hard scatterings and compute their (assumed to be incoherent) contributions to jet and single particle

cross sections. We can normalize all contributions since all coupling constants ( $\alpha$ ,  $\alpha_c$ , and  $\alpha_M$ , the  $q\bar{q}M$  coupling in the CIM) are known or have been determined elsewhere at least to within a factor of 2 or 3.

We conclude the introduction by summarizing the remainder of this paper. In Section II we introduce the theoretical framework which is necessary for describing jet or single particle production at large  $p_T$ . Some details of the calculation are relegated to an Appendix. In Section III we enumerate the many subprocesses which contribute to large  $p_T$  production, and quote both analytical expressions and approximate numerical estimations of their magnitude. In Section IV we combine all these results to present our predictions for jet and single particle cross sections at  $\sqrt{s} = 30$  GeV,  $\theta_{c.m.} = 90^\circ$ . Finally, Section V contains our conclusions. Appendix B contains a detailed discussion on the QCD perturbative contribution to the photon structure function.

## II. FORMALISM

### A. Equivalent Photon Approximation and the Hard Scattering Expansion

We will calculate the differential cross section for the scattering  $e^+e^- \rightarrow e^+e^-\gamma\gamma \rightarrow e^+e^- (\text{jet or } \pi^+)X$  via the equivalent photon approximation.<sup>10</sup> This technique is a useful method for obtaining the leading high energy behavior of electroproduction cross sections where the scattered electron is either undetected or detected only if it is scattered into a small forward angle. The approximation consists of assuming that the emitted photon, which tends to emerge at small angle to the electron and with low invariant mass, is nearly massless, so that one can relate electroproduction cross sections to photoproduction cross sections. The cross section for the process  $ee \rightarrow eeX$  in the equivalent



photon approximation is

$$\begin{aligned} \widehat{d}\sigma_{e^+e^- \rightarrow e^+e^-X}(s,t,u) &= \int_0^1 dx_1 \int_0^1 dx_2 N(x_1)N(x_2) \\ &\cdot d\sigma_{\gamma\gamma \rightarrow X}(\hat{s}=x_1x_2s, \hat{t}=x_1t, \hat{u}=x_2u) \end{aligned} \quad (2.1)$$

where  $N(x)$  is the equivalent photon energy spectrum and  $d\sigma_{\gamma\gamma \rightarrow X}$  is the differential cross section for the scattering of two oppositely directed unpolarized photons (of energy  $x_1 \sqrt{s}/2$ ,  $x_2 \sqrt{s}/2$  in the  $e^+e^-$  c.m. system) into a final state  $X$ . When  $x_1$  and  $x_2$  are computed as light-cone momentum fractions (see below), then Eq. (2.1) is invariant for Lorentz transformations along the beam direction.

The validity of the equivalent photon approximation as compared to exact calculations has recently been examined by Ref. 11. When the full form of the equivalent photon energy spectrum  $N(x)$  is used the double equivalent photon approximation was found to be a very good approximation for the process  $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$ . For our purposes here we use the approximate form

$$N(x) \approx \left( \frac{\alpha}{2\pi} \ln \frac{s}{4m_e^2} \right) \frac{1 + (1-x)^2}{x} \quad (2.2)$$

where only the dominant logarithm is retained. The equivalent photon approximation (2.2) is exact in the limit  $m_e^2/s \rightarrow 0$  to lowest order in  $\alpha$ .

The next ingredient which we will use in our calculations is the hard scattering expansion, which is a technique for evaluating Feynman diagrams in large momentum transfer processes.<sup>12</sup> In a given Feynman graph, when a large  $p_T$  particle is produced, there are generally many paths along which large momentum can flow. The hard scattering expansion consists of isolating the portion of the graph in which the major

part of the hard scattering occurs and labeling it as a scattering sub-process, while lumping the rest of the diagram, where only small momentum transfers occur, into fragmentation functions. This process is carried out while isolating all possible regions where the hard scattering may occur resulting in a sum of terms. For model field theories, the application of this procedure has been shown to reproduce the results of specific Feynman-graph calculations to within about 90 percent accuracy.<sup>12</sup>

Consider a contribution to the hard scattering expansion where the constituents a and b of the initial particles A and B, respectively, undergo a hard scattering to produce a particle or jet C at large transverse momentum. A simple parton formula holds for each such contribution:

$$\begin{aligned} \frac{E d\sigma}{d^3p} (AB \rightarrow CX) &= \iint dx_1 dx_2 G_{a/A}(x_1) G_{b/B}(x_2) \\ &\times \frac{d\sigma}{dt} (ab \rightarrow Cd) \frac{\hat{s}}{\pi} \delta(\hat{s} + \hat{t} + \hat{u}) \quad , \end{aligned} \quad (2.3)$$

where the fragmentation function  $G_{i/I}(x_i)$  gives the probability of finding a parton i of light-cone momentum fraction  $x_i \equiv (p_i^0 + p_i^3) / (P_I^0 + P_I^3)$  in the particle I. The invariants are

$$s \equiv (p_A + p_B)^2, \quad t = (p_A - p_C)^2, \quad u = (p_B - p_C)^2 \quad (2.4)$$

and

$$\begin{aligned} \hat{s} &\equiv (p_a + p_b)^2 \approx x_1 x_2 s \\ \hat{t} &\equiv (p_a - p_c)^2 \approx x_1 t \\ \hat{u} &\equiv (p_b - p_c)^2 \approx x_2 u \end{aligned} \quad (2.5)$$

Here the particles A and B are the initial  $e^+$  and  $e^-$ . In the case that a and b are photons, Eq. (2.3) is identical to Eq. (2.1) where  $G_{\gamma/e}(x) = N(x)$ . However, generally the partons a and b participating in the hard scattering may be constituents of the photons rather than the photons themselves. In this case  $G_{a/e}$  is obtained from the convolution of  $G_{\gamma/e}$  with the probability of finding the given constituent in the photon. Many different fragmentation functions and subprocess differential cross sections may in principle be important. Our task will be to elucidate precisely which of them are, in fact, most important.

#### B. Fragmentation Functions

We now discuss the fragmentation functions which we will use in the convolution integral, Eq. (2.3). This is most easily done by discussing the simplest fragmentation functions separately and building the more complicated ones from them by the convolution formula

$$G_{a/c}(x) = \int_x^1 \frac{dz}{z} G_{b/c}(z) G_{a/b}\left(\frac{x}{z}\right) \quad (2.6)$$

For ease in evaluating the convolution integral it is convenient to normalize a fragmentation function as

$$xG_{a/b}(x) = \sum_i \Lambda_i \left(1 + g_a^{(i)}\right) (1-x)^{g_a^{(i)}} \quad (2.7)$$

and quote values for  $\Lambda$ .

The simplest fragmentation functions are those for photon emission by electrons and for photon annihilation into an electron-positron pair. In the leading-logarithmic approximation to the equivalent photon approximation, the probability for an electron to emit a (transversely

polarized) photon with transverse momentum  $\vec{k}_T$  and longitudinal momentum fraction  $x$  is

$$\frac{dN}{dx d^2k_T} \equiv G_{\gamma/e}(x, \vec{k}_T) = \frac{\alpha}{2\pi^2} \frac{k_T^2}{(k_T^2 + x^2 m_e^2)^2} \frac{1 + (1-x)^2}{x} \quad (2.8)$$

or integrating on  $\vec{k}_T$ ,

$$G_{\gamma/e}(x) = \frac{\alpha}{2\pi} \ln \eta \left( \frac{1 + (1-x)^2}{x} \right) \quad (2.9)$$

where  $\eta = s/4m_e^2$  if the electron is not tagged, and  $\theta_{\max}^2/\theta_{\min}^2$  if the electron is constrained to lie in an angle  $\theta$  between  $\theta_{\max}$  and  $\theta_{\min}$ , in the  $e^+e^-$  c.m. frame. The fragmentation function for a muon emitted from a photon can be obtained from (2.9) by the Gribov-Lipatov relation<sup>13</sup>

$$G_{\mu/\gamma}(x) = x G_{\gamma/\mu}\left(\frac{1}{x}\right) = \frac{\alpha}{2\pi} \ln \left( \frac{s}{4m_\mu^2} \right) \left[ x^2 + (1-x)^2 \right] \quad (2.10)$$

and the distribution for muons from an electron by convolution:

$$G_{\mu/e}(x) \simeq \frac{\alpha}{2\pi} \ln \frac{s}{4m_e^2} \frac{\alpha}{2\pi} \ln \frac{s}{4m_\mu^2} \left[ \frac{1-x}{x} \right] \quad (2.11)$$

If we could rely on only the Born diagrams in QCD, then the distribution of quarks in a photon would be given directly from (2.10)

$$G_{q/\gamma}^{\text{Born}}(x) = \frac{\alpha}{2\pi} e_q^2 \ln \frac{s}{4M^2} \left[ x^2 + (1-x)^2 \right] \quad (2.12)$$

for a quark of each color. We have assumed that the region of  $k_T^2 < M^2$ , where  $M \sim 0$  (300 MeV) is represented by a separate vector meson contribution which we compute below.

As first shown by Witten,<sup>4</sup> the corrections to (2.12) due to gluon bremsstrahlung can be computed to all orders in  $\alpha_c$ , keeping leading

logarithms in each order. The simplest method is to use the convolution formula

$$G_{q/\gamma}(x, Q^2) = \frac{\alpha}{2\pi} e_q^2 \int_{\sqrt{M}^2}^{Q^2} \frac{dk^2}{k^2} \int_x^1 \frac{dz}{z} \left[ z^2 + (1-z)^2 \right] G_{q'/q}(x, Q^2, k^2) \quad (2.13)$$

where  $G_{q'/q}(x, Q^2, k^2)$  is the distribution of quarks (at spacelike mass squared  $Q^2$ ) in a quark (at mass squared  $k^2$ ) allowing for gluon bremsstrahlung to all orders. To the accuracy required here it will be sufficient to only consider  $q'$  and  $q$  of the same flavor and disregard the  $g \rightarrow q'\bar{q}'$  mixing terms. Also, to the leading logarithmic accuracy we can identify  $(k^2)_{\max} = Q^2 = -sx/(1-x)$  with  $s$ . Using the moment analysis discussed in Appendix B, we find for asymptotic  $s$

$$xG_{q/\gamma}(x, s) = \frac{\alpha}{2\pi} e_q^2 \frac{f(x)}{\alpha_c(s)} \quad (2.14)$$

which is to be summed over quark color and flavor. The scaling function  $f(x)$  is plotted in Fig. 1(a) (see also Ref. 5). A comparison with the Born approximation (2.12) is also shown. When  $1-x$  is small, we can use<sup>15</sup>

$$f(x) \simeq \frac{4\pi}{4\pi b - (3-4\gamma_E)C_2 + 4C_2 \ln \frac{1}{1-x}} \quad (2.15)$$

where  $\gamma_E$  is Euler's constant,  $C_2 = (N^2-1)/2N = 4/3$  for SU(3) color, and  $4\pi b = 3(11-2/3 n_f) \simeq 27$  characterizes the asymptotic behavior of the running coupling constant,  $\alpha_c(Q^2) \rightarrow 1/(b \log Q^2/\Lambda^2)$ . A comparison of  $f(x)$  with the asymptotic form (2.15) is also shown in Fig. 1(a).

The convolution of  $G_{q/\gamma}$  and  $G_{\gamma/e}$  then gives for each quark of given color and flavor,

$$xG_{q/e}(x,s) \cong \left(\frac{\alpha}{2\pi}\right)^2 e_q^2 \frac{\log \eta}{\alpha_c(s)} (1-x) \hat{f}_q(x) \quad (2.16)$$

where  $\hat{f}_q(x)$  is shown in Fig. 1(b). For  $1-x$  small,  $\hat{f}_q(x)$  also has the limiting form described by (2.15).

A remarkable fact is that when we compute high  $p_T$  jet cross sections at fixed  $p_T/\sqrt{s}$  from the  $qq \rightarrow qq$  and  $\gamma q \rightarrow gq$  subprocesses using Eq. (2.3), the scale violation from the factors of  $\alpha_c$  in the subprocess cross sections cancel to leading logarithmic order the  $1/\alpha_c$  factors in the  $q/\gamma$  distribution functions. Thus, aside from the log factors in the equivalent photon approximation, the jet cross sections for the  $\gamma\gamma$ -processes actually Bjorken-scale in the asymptotic limit.

For the non-perturbative part of the photon structure functions, we shall use the vector meson dominance model. The distribution of virtual vector mesons in the electron is then given by

$$G_{V/e}(x) = \frac{4\pi\alpha^2}{f_V^2} G_{\gamma/e}(x) \quad (2.17)$$

where  $f_\rho^2/4\pi = 2.2$  for the  $\rho$ . For the distribution of quarks in the meson we take the simple form  $xG_{q/V}(x) = C_V(1-x)$  where  $C_V = 0.5$  corresponds to each quark carrying  $1/4$  of the total momentum. Then for each quark color and flavor

$$\sum_{q=u,d} xG_{q/e}^{\text{non-pert}}(x) \cong \frac{1}{3} \frac{\alpha^2}{f_V^2} C_V \log \eta \cdot \left[ (1-x)^2 + \frac{2}{3} (1-x)^3 + \dots \right] \quad (2.18)$$

(This should be summed over color and vector mesons when the total color and flavor contribution is required.) The distributions (2.16) and (2.18) taken together should give a reasonably accurate representation of the effective quark distributions.

Finally, in order to obtain single particle cross sections from subprocesses involving a large transverse momentum quark we need the probability of finding a meson in a quark. The fragmentation function for  $\pi^+$ 's out of quarks may be fit from  $e^+e^-$  data.<sup>16</sup> Assuming

$G_{\pi^+/u}(x) = G_{\pi^-/\bar{u}}(x) = G_{\pi^+/\bar{d}}(x) = G_{\pi^-/d}(x)$ , a reasonable fit (though not a unique one) is

$$xG_{\pi^+/u}(x) = 0.5 ((1-x) + (1-x)^3) . \quad (2.19)$$

In the calculations to be described below, we include only the  $(1-x)$  piece of (2.19) and neglect all "unfavored" fragmentation ( $G_{\pi^-/u}(x) = 0$ , for instance). All these fragmentation functions are tabulated in Table I.

### C. Generic Forms of Reaction Cross Sections

Next, we shall collect some approximate formulas which will be of help in estimating the sizes of various differential cross sections. Exact formulas and their derivation are given in Appendix A and in Refs. 7 and 17. (The calculations we show later on are the results of numerical integration over these exact formulas.)

If we parametrize a fragmentation function as

$$xG_{a/A}(x) = (1+g_a) \Lambda_{a/A} (1-x)^{g_a} \quad (2.20)$$

and the differential cross section for the subprocess as

$$\frac{d\sigma}{dt} = \pi D s^{-(N-T-U)} (-t)^{-T} (-u)^{-U} \quad (2.21)$$

then Eq. (2.3) reduces to

$$\begin{aligned}
 \frac{Ed\sigma}{d^3p} (AB \rightarrow CX) &= \frac{1}{s} \left(\frac{-t}{s}\right)^{-T} \left(\frac{-u}{s}\right)^{-U} \\
 &\times (1+g_a)(1+g_b) \Lambda_{a/A} \Lambda_{b/B} D \\
 &\times \int dx_1 dx_2 (1-x_1)^{g_a} (1-x_2)^{g_b} \\
 &\times (x_1)^{U-N-1} (x_2)^{T-N-1} \\
 &\times \delta\left(1 + \frac{t}{sx_1} + \frac{u}{sx_2}\right) .
 \end{aligned} \tag{2.22}$$

Now we write

$$\begin{aligned}
 x_R &\equiv 1-\epsilon = 2|p^c|/\sqrt{s} \\
 x_F &\equiv \frac{2p_{\parallel}^c}{\sqrt{s}} \\
 x_T &= \frac{2p_T^c}{\sqrt{s}}
 \end{aligned} \tag{2.23}$$

so that

$$\frac{-t}{s} = \frac{1}{2}(1-\epsilon-x_F) \quad \frac{-u}{s} = \frac{1}{2}(1-\epsilon+x_F) \tag{2.24}$$

and

$$s = \frac{4p_T^2}{x_T} = \frac{4p_T^2}{(1-\epsilon-x_F)(1-\epsilon+x_F)} . \tag{2.25}$$



We can then perform the integrals in (2.22) and write<sup>2</sup>

$$\begin{aligned}
 \frac{Ed\sigma}{d^3p}(AB \rightarrow CX) &= \frac{D\epsilon^F}{\left(\frac{2}{p_T}\right)^N} \Lambda_{a/A} \Lambda_{b/B} 2^{f_+ + f_-} \\
 &\times \frac{\Gamma(2+g_a) \Gamma(2+g_b)}{\Gamma(2+g_a+g_b)} \\
 &\times (1-\epsilon-x_F)^{-f_+} (1-\epsilon+x_F)^{-f_-} \tilde{J}
 \end{aligned} \tag{2.26}$$

where

$$\begin{aligned}
 F &= 1 + g_a + g_b \\
 f_+ &= T + 1 + g_b - N \\
 f_- &= U + 1 + g_a - N
 \end{aligned} \tag{2.27}$$

and

$$\begin{aligned}
 \tilde{J} &= \frac{1}{J_0} \int_{-1}^1 dw (1+w)^{g_a} (1-w)^{g_b} \left[ \frac{1+x_F+\epsilon w}{1+x_F-\epsilon} \right]^{-f_-} \left[ \frac{1-x_F-\epsilon w}{1-x_F-\epsilon} \right]^{-f_+} \\
 J_0 &= \int_{-1}^1 dw (1+w)^{g_a} (1-w)^{g_b}
 \end{aligned} \tag{2.28}$$

Similar expressions are obtained for the case of a final state fragmentation as well, and are given in Appendix A.

These expressions are still an exact representation of Eq. (2.22), but the approximate evaluation of  $\tilde{J}$  by its value at  $\epsilon=0$ ,  $\tilde{J}(\epsilon=0)=1$ , provides a useful approximation valid for small  $\epsilon$ , near the edge of phase space.

#### D. Some Counting Factors

We must now face the task of enumerating all the different contributions to the jet or single-particle cross section. In order to

facilitate matters, we will discuss some multiplicative factors which are common to all subprocesses. To do this, we define

$$F(a,b,d\sigma/dt) \equiv \int_0^1 dx_a dx_b G_{a/A}(x_a) G_{b/B}(x_b) \frac{d\sigma}{dt} \frac{\hat{s}}{\pi} \delta(\hat{s} + \hat{t} + \hat{u}) \quad (2.29)$$

and (see Eq. (A9))

$$F(a,b,c,d\sigma/dt) \equiv \int_0^1 dx_a dx_b \frac{dx_c}{x_c} G_{a/A}(x_a) G_{b/B}(x_b) G_{c/C}(x_c) \frac{d\sigma}{dt} \times \frac{\hat{s}}{\pi} \delta(\hat{s} + \hat{t} + \hat{u}) \quad (2.30)$$

the "bare" convolution formulas for the differential cross sections.

Multiplicative factors which we shall frequently encounter are:

1. Beam-target symmetrization. A process in which the electron emits a parton of type (a) which scatters from a parton of type (b) emitted by the positron is as likely as a process in which "b" came from the electron and "a" from the positron. To include both these reactions, we introduce a symmetrized differential cross section

$$F_S\left(a,b,\frac{d\sigma}{dt}\right) = \frac{1}{2}\left(F\left(a,b,\frac{d\sigma}{dt}\right) + F\left(b,a,\frac{d\sigma}{du}\right)\right) \quad (2.31)$$

(and similarly for  $F_S(a,b,c,d\sigma/dt)$ ) where if  $d\sigma/dt = f(t,u)$ ,  $d\sigma/du = f(u,t)$ .

Our normalization is such that if  $a \neq b$ , a given process will be weighted by an additional factor of 2, which we will refer to in the next section as a "beam-target symmetrized factor."

2. In the case of jet cross sections, the jet can be either of the two groups of particles at large  $p_T$ . To include this we write a jet-symmetrized cross section

$$F_J\left(a,b,\frac{d\sigma}{dt}\right) = F\left(a,b,\frac{1}{2}\left(\frac{d\sigma}{dt} + \frac{d\sigma}{du}\right)\right) \quad (2.32)$$

(normalized so that jet cross sections are given by  $2F_J$ ). We may combine jet and beam-target symmetrization to write

$$F_{SJ}\left(a, b, \frac{d\sigma}{dt}\right) = \left[ \frac{1}{2} F\left(a, b, \frac{1}{2} \left(\frac{d\sigma}{dt} + \frac{d\sigma}{du}\right)\right) + F\left(b, a, \frac{1}{2} \left(\frac{d\sigma}{dt} + \frac{d\sigma}{du}\right)\right) \right]. \quad (2.33)$$

The total jet cross section is then given by  $4F_{SJ}$ .

3. Finally, we will show explicitly all color and charge factors, wave function normalizations, and the like.

### III. HARD-SCATTERING SUBPROCESSES FOR $\gamma\gamma$ -COLLISIONS

The number of different scattering subprocesses which may, in principle, contribute to jet or single particle production is large. To facilitate our discussion of them in a coherent way, we will classify them according to their  $p_T$ -behavior at large  $p_T$ . It happens that all of the important subprocesses which we consider vary as  $p_T^{-N}$ , where  $N = 4, 6, \text{ or } 8$ . We discuss each of these classes separately in Sections III.A, III.B, and III.C.

#### A. $p_T^{-4}$ Subprocesses

These reactions exhibit the underlying structure of a scale-invariant point-like coupling and from that standpoint are the most interesting and fundamental of any of the interactions which may be studied in  $\gamma\gamma$  collisions.

1.  $\gamma\gamma \rightarrow q\bar{q}$ . The simplest reaction we may consider is shown in Fig. 2(a), the production of a single pion or a hadronic jet via quark fragmentation.<sup>3</sup> The subprocess cross section is

$$\frac{1}{2} \left( \frac{d\sigma}{dt} + \frac{d\sigma}{du} \right) = \frac{2\pi\alpha^2}{s} \left( \frac{t}{u} + \frac{u}{t} \right) \quad (3.1)$$

whence

$$\frac{Ed\sigma(\text{jet})}{d^3p} = 2 \cdot 3 \sum_{i=u,d,s,c} e_i^4 F_J\left(\gamma, \gamma, \frac{1}{2} \left(\frac{d\sigma}{dt} + \frac{d\sigma}{du}\right)\right) \quad (3.2)$$

$$\frac{Ed\sigma(\pi^+)}{d^3p} = 3 \sum_{i=u,\bar{d}} e_i^4 F\left(\gamma, \gamma, \pi, \frac{1}{2} \left(\frac{d\sigma}{dt} + \frac{d\sigma}{du}\right)\right) \quad (3.3)$$

The factor of 2 is from jet symmetrization and the factor of 3 is from color. This reaction admits to a new constant of proportionality with relation to the reaction  $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$ : Eq. (3.2) implies

$$\frac{Ed\sigma}{d^3p}(e^+e^- \rightarrow e^+e^- \text{ jet } X) = 2R_{\gamma\gamma} \frac{Ed\sigma}{d^3p}(e^+e^- \rightarrow e^+e^-\mu^+\mu^-) \quad (3.4)$$

where

$$R_{\gamma\gamma} = 3 \sum_{i=u,d,s,c} e_i^4 = \frac{34}{27} \quad (3.5)$$

Evaluation of (3.2) and (3.3) yields the approximate forms

$$\frac{Ed\sigma(\text{jet})}{d^3p} \sim .03 \frac{(1-x_R)}{p_T^4} \text{ nb-GeV}^2 \quad (3.6)$$

$$\frac{Ed\sigma(\pi^+)}{d^3p} \sim 6.3 \cdot 10^{-4} \frac{(1-x_R)^3}{p_T^4} \text{ nb-GeV}^2 \quad (3.7)$$

at  $\sqrt{s} = 30 \text{ GeV}$ . These results are accurate for  $x_R \rightarrow 1$  at all  $\theta_{\text{c.m.}}$ .

There will also be calculable corrections of order  $\alpha_c(p_T^2)$  due to the  $R_{\gamma\gamma}$  from higher QCD corrections to the  $\gamma\gamma \rightarrow \text{jet} + X$  scaling cross section. There are no corrections of order  $\alpha_c(p_T^2) \log p_T^2$  since

- a) infrared gluon mass singularities cancel for the inclusive jet cross section, and
- b) there is no quark mass singularity since the gluon corrections to the  $\gamma\gamma \rightarrow q\bar{q}$  amplitude (not already

canceled by renormalization terms) connect quark lines of different momentum. (This is a sufficient condition for absence of mass singularities if one uses an axial or Coulomb gauge.)

The perturbative corrections to  $\gamma\gamma \rightarrow q\bar{q}$  are thus of the same form as that for  $e^+e^- \rightarrow q\bar{q}$  and are expressible as a power series in  $\alpha_c(p_T^2)$ .

The observation of a  $p_T^{-4}$  distribution in the jet or single particle cross section will also be an important confirmation of the idea that the quark propagator has a  $1/\not{p}$  component which may be seen at large momentum transfer--and the reaction  $\gamma\gamma \rightarrow q\bar{q}$  may be the first one in which that part of the quark propagator will be clearly exposed.

The value of  $R_{\gamma\gamma}$  given by Eq. (3.5) is an asymptotic form, as  $\alpha_c(p_T^2)$  and  $m_q^2/p_T^2 \rightarrow 0$ . It will be interesting to study the approach of  $R_{\gamma\gamma}$  from  $3 \sum_{i=uds} e_i^4$  to  $3 \sum_{i=udsc} e_i^4$  as a function of  $s$  and  $p_T$ .

2.  $\gamma\gamma \rightarrow q\bar{q}q\bar{q}$ . These reactions, shown in Fig. 2(b), involve explicit gluon exchange (or  $q\bar{q}$  annihilation into gluons) as the source of the high- $p_T$  events. They are four-jet events, since one of the quarks in each photon is a spectator to the large- $p_T$  reaction, and forms its own jet along the beam axis, following the photon's original line of motion.

The various subprocesses ( $qq \rightarrow qq$ ,  $q\bar{q} \rightarrow gg$ , etc.) which are to be included in the large- $p_T$  reaction have been computed by many authors<sup>18</sup> and it is most convenient to adapt their results to our needs. The various symmetrizations which must be performed make this a delicate process, however. For instance, a jet trigger occurs whenever one detects any high- $p_T$  particle produced in the reaction. Thus, for jet

triggers, one must (generally) include a particular subprocess plus the  $t \leftrightarrow u$  crossed subprocess, corresponding to triggering on the other jet. For example, in  $\gamma_1 \rightarrow q_1 \bar{q}_1$ ,  $\gamma_2 \rightarrow q_2 \bar{q}_2$ , followed by  $q_1 q_2$  scattering ( $q_1 \neq q_2$ ), we compute  $E d\sigma/d^3p$  ( $e^+ e^- \rightarrow q_1 X$ ) using the subprocess  $d\sigma/dt$  ( $q_1 q_2 \rightarrow q_1 q_2$ ) and add it to  $E d\sigma/d^3p$  ( $e^+ e^- \rightarrow q_2 X$ ) using the subprocess  $d\sigma/dt$  ( $q_1 q_2 \rightarrow q_2 q_1$ ). If non-identical particles scatter, one must also explicitly include cases  $\gamma_1 \rightarrow q_2 \bar{q}_2 + \gamma_2 \rightarrow q_1 \bar{q}_1$  followed by  $q_2 q_1$  scattering. If identical particles scatter, however, one has already "included the crossed terms" in the full expression (including interference as required by quantum mechanics) for the scattering subprocess. Similarly for the  $\pi^+$  spectrum, we assume that only a  $u$  or a  $\bar{d}$  quark can fragment into a fast  $\pi^+$ . But the  $u(\bar{d})$  quark may be produced via reactions where the  $u(\bar{d})$  quark originates from  $\gamma_1$  (say),  $d\sigma/dt(e^+ e^- \rightarrow u(\bar{d})X)$ , or by the crossed reactions in which the  $u(\bar{d})$  originates from  $\gamma_2$ ,  $d\sigma/du(e^+ e^- \rightarrow u(\bar{d})\bar{X})$ . The relevant quark-quark scattering terms are shown in Table II.

After summing over all possible contributions (using the fact that all initial  $q$ 's and  $\bar{q}$ 's from each photon  $\gamma_1$  and  $\gamma_2$  are equally likely) we may express our result in terms of an "effective" subprocess cross section

$$\begin{aligned} \frac{d\sigma}{dt} (\Sigma \text{ all } q \text{ or } \bar{q} + \Sigma \text{ all } q \text{ or } \bar{q} \rightarrow \text{any trigger } q \text{ or } \bar{q} \\ + \text{balancing } q \text{ or } \bar{q}) \end{aligned} \quad (3.8)$$

for jet production and

$$\frac{d\sigma}{dt} (\Sigma \text{ all } q \text{ or } \bar{q} + \Sigma \text{ all } q \text{ or } \bar{q} \rightarrow u(\bar{d}) + \text{balancing } q \text{ or } \bar{q}) \quad (3.9)$$

for  $\pi^+$  production. These "subprocesses" include all crossing and other factors and are to be folded with the  $G_{q/e}$  forms of Section II, where we again recall that  $G_{q/e} = G_{\bar{q}/e}$  for all quark types to first approximation. The effective cross section forms appear in Table III. The color sums are already included in these cross sections.

In order to obtain simple analytic expressions for the inclusive cross sections, let us first approximate  $G_{q/e}$  in Eq. (2.16) by the simple form

$$xG_{q/e}(x) = e_q^2 \left( \frac{\alpha}{2\pi} \ln \eta \right) \left( \frac{\alpha}{2\pi} F_Q \right) (1-x) \quad (3.10)$$

for each quark flavor and color, where roughly  $F_Q \simeq \ln \frac{s}{4M^2}$  if we use the simple Born graphs for  $\gamma \rightarrow q\bar{q}$ , and  $F_Q \simeq (1/\alpha_c(s)) \langle f(x) \rangle^q$  if we use the full QCD result. For a rough estimate we shall take  $\alpha_c(p_T^2) \cdot F_Q \simeq 1.2$ . The vector-meson dominated contributions will be considered separately.

Using (3.10) we then find

$$\begin{aligned} \frac{Ed\sigma}{d^3p}(e^+e^- \rightarrow e^+e^- \text{ jet } X) &= F_J \left( q, q, \frac{d\sigma^{\text{eff}}}{dt} \right) \\ &\simeq \alpha_c^2(p_T^2) \left( \frac{\alpha}{2\pi} \ln \eta \right)^2 \left( \frac{\alpha}{2\pi} F_Q \right)^2 \left( \frac{80}{3} \left( \sum_{i=u,d,s,c} e_i^2 \right)^2 + \frac{52}{9} \sum_{i=u,d,s,c} e_i^4 \right) \frac{(1-x_R)^3}{p_T^4} \text{GeV}^4 \\ &\underset{\substack{x_R \rightarrow 1 \\ \theta_{\text{c.m.}} = 90^\circ}}{\sim} 1.7 \cdot 10^{-2} (1-x_R)^3 p_T^{-4} \text{nb-GeV}^2 \end{aligned} \quad (3.11)$$

$$\begin{aligned} \frac{Ed\sigma}{d^3p}(e^+e^- \rightarrow e^+e^-\pi^+ X) &= F \left( q, q, \pi, \frac{d\sigma^{\text{eff}}}{dt} \right) \\ &\underset{\substack{x_R \rightarrow 1 \\ \theta_{\text{c.m.}} = 90^\circ}}{\sim} 1.3 \cdot 10^{-4} \frac{(1-x_R)^5}{p_T^4} \text{nb-GeV}^2 \end{aligned} \quad (3.12)$$

at  $\sqrt{s} = 30$  GeV. The charge structure of the various contributing terms are displayed in Eq. (3.11). The numerical coefficients in (3.11) and (3.12) are the result of a numerical integration of Eq. (2.3). Equations (3.11) and (3.12) are useful as phenomenological approximations in that they display the approximate analytic and quark charge dependence of the inclusive cross sections.

We have also directly computed the  $e^+e^- \rightarrow e^+e^- \text{jet} + X$  cross section from (2.25) using the full QCD form (2.16) for the quark distribution in the electron  $G_{q/e}(x,s)$ , allowing for gluon bremsstrahlung to all orders. Again we note the asymptotic cancellation of the  $\alpha_c^2 \left(\frac{1}{p_T^2}\right)$  factor from  $d\sigma/dt$  ( $qq \rightarrow qq$ ) and the  $\alpha_c^{-1} \left(\frac{1}{p_T^2}\right)$  factors in  $G_{q/e}(x,s)$  at fixed  $x_T$ .

The resulting (four-jet) cross section has the asymptotic scaling form

$$\frac{E d\sigma}{d^3 p} (ee \rightarrow \text{jet} + X) \Rightarrow \frac{\left(\frac{\alpha}{\pi} \ln \eta\right)^2}{p_T^2} \left[ f(x_T, \theta_{\text{c.m.}}) + O\left(\frac{1}{\log p_T^2}\right) \right] \quad (3.13)$$

A comparison of cross sections using the full QCD and Born contributions for  $G_{q/e}$ , and specific results for PETRA and PEP energies are given in Section IV.

3.  $\gamma\rho \rightarrow q\bar{q}q\bar{q}$  and  $\rho\rho \rightarrow q\bar{q}q\bar{q}$  (Fig. 2(c-d)). The latter vector dominance contribution is the  $\gamma\gamma$  analog of  $\rho\rho$  scattering mediated by gluon exchange.<sup>19</sup> It is quite small with respect to either  $\gamma\gamma \rightarrow q\bar{q}$  or  $\gamma\gamma \rightarrow q\bar{q}q\bar{q}$  perturbative contributions, the change in size being due mainly to the small probability for the sequential transition  $\gamma \rightarrow \rho \rightarrow q\bar{q}$ . We can obtain an estimate of the contribution by considering only  $qq$  scattering via gluon exchange (this neglects the small identical particle



and annihilation effects)

$$\frac{d\sigma}{dt} = \frac{4\pi}{9} \alpha_c^2 \left( \frac{s^2 + u^2}{t^2} \right) \quad (3.14)$$

whence

$$\begin{aligned} \frac{Ed\sigma(\text{jet})}{d^3p} &= \left( \frac{1}{2} \cdot 4 \right)^2 \cdot 3^2 \cdot 2 F_J(q/\rho, q/\rho, \frac{d\sigma}{dt}) \\ &\simeq 1.1 \cdot 10^{-5} \frac{(1-x_R)^5}{P_T^4} \text{ nb-GeV}^2 \end{aligned} \quad (3.15)$$

at  $\sqrt{s} = 30$  GeV,  $\theta_{\text{c.m.}} = 90^\circ$ ,  $1-x_R$  small. The 3's are color traces and 4's and 1/2's the meson flavor sums and wave function normalizations. The inclusive  $\pi^+$  cross section is also quite small.

The  $\gamma\rho \rightarrow q\bar{q}q\bar{q}$  cross sections are somewhat larger. Using the same technique (i.e., neglecting small identical particle and annihilation contributions by using Eq. (3.14)), we have

$$\frac{Ed\sigma(\text{jet})}{d^3p} = \left( \frac{1}{2} \times 4 \right) \left[ 3^2 \cdot 2 \cdot \sum_{i=udsc} e_i^2 (4) F_{JS}(q/e, q/\rho, \frac{d\sigma}{dt}) \right] \quad (3.16)$$

$$= 4.6 \cdot 10^{-4} \frac{(1-x_R)^4}{P_T^4} \text{ nb-GeV}^2 \quad (3.17)$$

at  $\sqrt{s} = 30$  GeV,  $\theta_{\text{c.m.}} = 90^\circ$ , small  $1-x_R$ . The  $1/2 \cdot 4$  corresponds to the  $\rho$  wave function factor and 4 flavor choices; the  $3^2$  is the sum over colors of colliding quarks; the  $2 \sum_i e_i^2$  is the choice of any  $q$  or  $\bar{q}$  from the photon distribution; the 4 is the standard jet beam target symmetrization. The result is less than 3% of the perturbative  $\gamma\gamma \rightarrow q\bar{q}q\bar{q}$  contribution and thus is also negligible. One might also consider

interference between, say,  $\gamma\rho \rightarrow q\bar{q}q\bar{q}$  and  $\gamma\gamma \rightarrow q\bar{q}q\bar{q}$ ; this corresponds to a non-diagonal quark distribution function where in the square, the scattering quark arises from a  $\rho$  but is reabsorbed by a  $\gamma$ . Inclusion of such diagrams correspond to off-diagonal VDM in that it presumes overlap of the low mass  $\rho$  region with the high mass bare region corresponding to high mass vector mesons. We neglect this possibility. Note that we have implicitly assumed that it is not necessary to include explicitly the contributions of massive vector mesons as these are to a great extent already included in the perturbative contribution.

4. Other  $p_T^{-4}$  reactions. In addition to the subprocesses of Section III.A.1-3, one may consider more complicated QCD processes. These processes may be categorized into two classes. First one may consider radiative corrections to already-calculated processes. If QCD perturbation theory calculations have any meaning, higher order in  $\alpha_c$  contributions to cross sections which have the same topological (number of jets produced) and charge structure will not appreciably alter the size and shape of the lowest order cross section.

We must, however, consider higher-order in  $\alpha_c$  contributions when they lead to configurations with a topology different from that of lower order results. An example of this criterion is the inclusion of the  $\alpha_c^2$  four-jet events of Section III.A.2. Another example of a new contribution is the  $O(\alpha_c)$  process of Fig. 2(c),  $\gamma\gamma \rightarrow q\bar{q}g$ , where we require that the hard scattering be  $\gamma q \rightarrow gq$ . These events have  $n_J=3$ , one of the

large  $p_T$  jets being a gluon jet. We find, using (3.10)

$$\frac{E d\sigma}{3 d^3 p} (e^+ e^- \rightarrow e^+ e^- \text{ jet X}) = 3 \cdot 4 \cdot 2 \cdot F_{SJ} \left( \gamma, q, \frac{8}{3} e_q^2 \frac{\pi \alpha_c}{s} \left( \frac{t}{s} + \frac{s}{t} \right) \right) \quad (3.18)$$

$$\simeq \alpha_c \alpha \left( \frac{\alpha}{2\pi} \ln \eta \right)^2 \frac{\alpha}{2\pi} F_Q \frac{(1-x_R)^2}{4 p_T} \sum e_i^4(40) \simeq .038 \text{ nb GeV}^2 \frac{(1-x_R)^2}{4 p_T} \quad (3.19)$$

at  $\sqrt{s} = 30 \text{ GeV}$   $\theta_{\text{c.m.}} = 90^\circ$  and  $1-x_R$  small. In (3.18) in addition to the factor of 3 for color, one factor of 4 is the standard total jet factor (including beam target symmetrization) and the factor of 2 counts

$\gamma q \rightarrow gq + \gamma \bar{q} \rightarrow g\bar{q}$ . This rate is  $O(1-x_R) (\alpha_c/\pi) F_Q$  times the rate for  $\gamma\gamma \rightarrow q\bar{q}$ .

We also have computed the  $\gamma q \rightarrow gq$  subprocess contribution to the 3 jet cross section using the full QCD structure  $G_{q/e}(x,s)$  given in (2.16). In the asymptotic limit the factor of  $\alpha_c (p_T^2)$  in  $d\sigma/dt$  ( $\gamma q \rightarrow gq$ ) is canceled by the  $\alpha_s^{-1}(s)$  in  $G_{q/e}$  and asymptotic scale invariance at fixed  $x_T$  is again obtained for the  $\gamma\gamma \rightarrow \text{jet} + X$  cross section. A comparison of the Born term calculation and the full QCD contributions to  $ee \rightarrow \text{jet} + X$  at PETRA and PEP energies is given in Section IV.

Finally, we note that diagrams such as that of Fig. 2(f) where the hard subprocess is  $\gamma q \rightarrow gq$  and the other gluon is at low transverse momentum relative to the  $\bar{q}$  spectrum are already included in our calculations when the full QCD structure function (2.16) is used.

### B. $p_T^{-6}$ Reactions

We now turn to classes of reactions in which the underlying point-like constituents of the initial or final state particles are not completely exposed. Such reactions should not be expected to exhibit canonical  $p_T^{-4}$  scaling.

We analyze such reactions from the standpoint of the constituent interchange model (CIM).<sup>7,9</sup> This approach has the advantage that it allows us to normalize the magnitude of these non-scaling contributions via comparison with cross sections measured in inclusive and exclusive hadron-hadron large  $p_T$  reactions (where the CIM has been shown to provide a consistent parametrization of the data for  $p_T < 8$  GeV). At this stage it is also worth pointing out that it is not inconsistent to include both QCD and CIM contributions; the CIM is contained within QCD on the level of internal bound state systems which participate in subprocesses within the reaction and can be identified as "higher twist" QCD contributions. The normalization of CIM processes is related to the analysis of Bethe-Salpeter wave functions at short distances in QCD, and is determined empirically from elastic large  $p_T$  cross sections.<sup>7,9</sup>

The CIM contributions of importance here naturally divide themselves into two classes: those in which  $Ed\sigma/d^3p \sim p_T^{-6}$ , where either one photon vector dominates or where the final state hard scattering involves the production of one hadronic resonance at large  $p_T$ , and those in which  $Ed\sigma/d^3p \sim p_T^{-8}$ , where both photons vector-dominate before scattering or several resonances at large  $p_T$  are produced, or some combination of processes occurs.

We discuss first the  $p_T^{-6}$  subprocesses. As we noted in the introduction, the  $p_T^{-6}$  contributions are distinctive features of photon-induced processes predicted by the CIM. Their observation at the levels we calculate below would provide important confirmation of the CIM. The existence of such unique contributions for processes involving photons is supported by model calculations in two-dimensional QCD. Backward

scattering in cases where Regge exchanges are absent provides a two-dimensional analog of wide angle scattering and it was found that meson-meson elastic scattering decreases with a power very similar to that predicted by the CIM ( $A \sim s^{-2}$ ) (slight modification for light mesons of the integral power law was found due to confinement).<sup>20</sup> We have studied photon + photon  $\rightarrow$  meson + meson in this model and find again a power very similar to that predicted by the CIM due to the elementary nature of the photon but again with slight modifications for light mesons due to confinement.

Here the relevant subprocess is  $\gamma q \rightarrow Mq$ , where M is a pseudoscalar meson resonance. With the direction of particle flow as indicated in Fig. 3(a), one finds<sup>7,17</sup> using the minimally-connected Born diagrams in QCD,

$$\frac{d\sigma}{dt}(\gamma q \rightarrow Mq) = \pi\alpha \left( \frac{1}{3} \frac{g^2}{4\pi} \right) \frac{s^2 + u^2}{t} A ;$$

$$A = \left( \frac{e_i}{u} - \frac{e_j}{s} \right)^2 . \quad (3.20)$$

No wave function or color sums have been performed.  $e_i$  and  $e_j$  are defined in Fig. 3.  $\frac{1}{3} g^2/4\pi$  is the CIM  $q\bar{q}M$  coupling constant  $\alpha_M$  which has been determined phenomenologically<sup>7</sup> to be  $\sim 2 \text{ GeV}^2$ . We will treat the production of a pseudoscalar M as the spin-averaged rate for production of a vector meson of the same flavor assignment.

To compute  $\gamma p \rightarrow q\bar{q}$  we cross (3.20) and find (Fig. 3(b))

$$\frac{d\sigma}{dt}(\gamma p \rightarrow q\bar{q}) = 2\pi\alpha \left( \frac{1}{3} \frac{g^2}{4\pi} \right) \frac{t^2 + u^2}{s} \left( \frac{e_i}{t} - \frac{e_j}{u} \right)^2 \quad (3.21)$$

The factor of 2 converts an average over quark j's spin into a sum.

We have found three important types of  $p_T^{-6}$  events.

1.  $\gamma\rho \rightarrow q\bar{q}$  (Fig. 4(a)). There is a finite probability that one photon will vector-dominate and that the other photon will scatter off the quarks in the bound state wave function of the  $\rho$ . Of course, these events form two hadronic jets. The jet cross section is

$$\frac{Ed\sigma(\text{jet})}{d^3p} = 2 \cdot \frac{1}{2} \cdot 3 \cdot 2 \sum_{i=u,\bar{d}} e_i^2 F_{JS}\left(\rho, \gamma, \frac{d\sigma}{dt}\right) \quad (3.22)$$

with

$$\frac{d\sigma}{dt} = 2\pi\alpha \left(\frac{1}{3} \frac{g^2}{4\pi}\right) \frac{t^2 + u^2}{s} \left(\frac{1}{t} + \frac{1}{u}\right)^2 \quad (3.23)$$

In Eq. (3.22) one factor of 2 is for  $\gamma\rho$  vs.  $\rho\gamma$  scattering, the other for triggering on either  $q$  or  $\bar{q}$  jet (no third factor of 2 for  $q$  or  $\bar{q}$  exchange since both pieces are included in the gauge invariant cross section). There is also a factor of 1/2 for the  $\rho$  wave function, and a factor of 3 for the color sum. Similarly, the single  $\pi^+$  cross section is

$$\frac{Ed\sigma(\pi^+)}{d^3p} = 2 \cdot \frac{1}{2} \cdot 3 \sum_{i=u,\bar{d}} e_i^2 F_S\left(\rho, \gamma, \pi^+, \frac{d\sigma}{dt}\right) \quad (3.24)$$

with  $d\sigma/dt$  given above. At  $\sqrt{s} = 30$  GeV,  $\theta_{c.m.} = 90^\circ$ , we obtain

$$\frac{Ed\sigma(\text{jet})}{d^3p} \sim 4.5 \cdot 10^{-2} \frac{(1-x_R)}{6} \text{ nb-GeV}^4 \quad (3.25)$$

$$\frac{Ed\sigma(\pi^+)}{d^3p} \sim 1.06 \cdot 10^{-3} \frac{(1-x_R)^3}{6} \text{ nb-GeV}^4 \quad (3.26)$$

At this point one must also note the possibility of interference between the processes  $\gamma\gamma \rightarrow q\bar{q}$  and  $\gamma\rho \rightarrow q\bar{q}$  since both have identical jet structure (including quantum numbers for a given  $q\bar{q}$  choice). We have not explicitly calculated this interference but expect it is close to

being maximal. Thus we estimate

$$\frac{Ed\sigma^{\text{interference}}(\text{jet})}{d^3p} = \sqrt{\frac{Ed\sigma(\gamma\gamma)}{d^3p} \frac{Ed\sigma(\gamma\rho)}{d^3p}} \quad (3.27)$$

$$\approx \frac{.036}{5} \frac{(1-x_R)}{p_T}$$

Hence

$$\frac{\frac{Ed\sigma^{\text{interference}}(\text{jet})}{d^3p}}{\frac{Ed\sigma^{\gamma\gamma}(\text{jet})}{d^3p}} = \frac{1.2}{p_T} \text{ GeV} \quad (3.28)$$

or approximately 30 percent at  $p_T = 4$  GeV. It will turn out that the third interfering process  $\left(\frac{-8}{p_T}\right) \rho\rho - q\bar{q}$  is sufficiently small that we need not even consider its interference with those larger amplitudes.

2.  $\gamma\gamma \rightarrow Mq\bar{q}$  (Fig. 4(b)). This is a reaction with  $n_J=3$ , one jet being a meson resonance. To calculate the jet cross section, we sum over all  $q_i$  and  $\bar{q}_j$  (or  $\bar{q}_i$  and  $q_j$ ) to find (including charge factors for both the bremsstrahlung and interacting M quark)

$$\frac{d\sigma}{dt} = \pi\alpha \left(\frac{1}{3} \frac{g^2}{4\pi}\right) \frac{s^2 + u^2}{s^2 t} \left(\frac{8}{9} \frac{1}{u^2} + \frac{4}{3} \frac{1}{s^2}\right) \quad (3.29)$$

which neglects charm containing mesons.

$$\frac{Ed\sigma(\text{jet})}{d^3p} = 3 \cdot 4 \cdot 4 F_{JS} \left(\gamma, q, \frac{d\sigma}{dt}\right)$$

$$= 1.1 \frac{(1-x_R)^2}{6} \text{ nb-GeV}^4 \quad (3.30)$$

at  $\sqrt{s} = 30$  GeV,  $\theta_{\text{c.m.}} = 90^\circ$ . One factor of 4 is a beam target + either jet symmetrization, the factor of 3 is from the color sum, and the other factor of 4 is due to the spin sum over M. The sum over all quarks and

and their spins is equivalent to allowing the whole  $\underline{35} + \underline{1}$  of pseudo-scalar and vector mesons to contribute to M. The  $\pi^+$  cross section is found to be

$$\begin{aligned} \frac{E d\sigma(\text{prompt } \pi^+)}{d^3 p} &= 2 \cdot 3 F_S(\gamma, q, \frac{d\sigma}{dt}) \\ &\sim .031 \frac{(1-x_R)^2}{P_T} \text{ nb-GeV}^2 \end{aligned} \quad (3.31)$$

with

$$\frac{d\sigma}{dt} = \frac{\pi\alpha}{81} \left( \frac{1}{3} \frac{g^2}{4\pi} \right) \frac{s^2 + u^2}{s^2 t} \left( \frac{8}{u^2} - \frac{20}{us} + \frac{17}{s^2} \right) \quad (3.32)$$

Inclusion of "non-prompt"  $\pi$ 's from resonance decays can double this answer.

3.  $\gamma\rho \rightarrow Mq\bar{q}$ . Finally, one photon may vector dominate, decay into quarks, and scatter from the other photon (Fig. 4(c)). The calculation is identical to Section III.B.2 except for the  $\rho$  wave function and different charge structure.

For jets (neglecting charm meson production) we have

$$\frac{d\sigma}{dt} = \pi\alpha \left[ \frac{1}{3} \frac{g^2}{4\pi} \right] \frac{s^2 + u^2}{s^2 t} \frac{2}{3} \left( \frac{4}{u^2} + \frac{5}{s^2} \right) \quad (3.33)$$

$$\begin{aligned} \frac{E d\sigma(\text{jet})}{d^3 p} &= 4 \cdot 4 \cdot 3 F_{JS}(\gamma, q/\rho, \frac{d\sigma}{dt}) \\ &= .13 \frac{(1-x_R)^3}{P_T} \text{ nb-GeV}^4 \end{aligned} \quad (3.34)$$



For  $\pi^+$ 's

$$\frac{d\sigma}{dt} = \frac{1}{9} \pi\alpha \left[ \frac{1}{3} \frac{g^2}{4\pi} \right] \frac{s^2 + u^2}{s^2 t} \left[ \frac{5}{u^2} - \frac{8}{us} + \frac{5}{s^2} \right] \quad (3.35)$$

$$\frac{Ed\sigma(\text{prompt } \pi^+)}{d^3 p} = .0051 \frac{(1-x_R)^3}{p_T^6} \text{ nb-GeV}^4 \quad (3.36)$$

These  $p_T^{-6}$  processes  $\gamma\gamma \rightarrow Mq\bar{q}$  and  $\gamma\rho \rightarrow Mq\bar{q}$  again do not interfere in our diagonal approximation in which there is no overlap between distribution function quarks coming from a  $\rho$  versus the bare  $\gamma$ . Without such interference it is clear that the  $\gamma\rho$  contributions are of order 10 percent of the  $\gamma\gamma p_T^{-6}$  contribution.

### C. $p_T^{-8}$ Contributions

Finally, we turn to the class of large- $p_T$  scatterings which most nearly resemble those of purely hadronic reactions. The relevant cross section is that for quark-meson scattering, as displayed in Fig. 5.

1.  $\rho\rho \rightarrow q\bar{q}$ . (Fig. 5(a).) Both photons may vector dominate before scattering via quark exchange.

$$\frac{Ed\sigma(\text{jet})}{d^3 p} = 3 \cdot 2 \cdot \left(\frac{1}{2}\right)^2 \cdot 2 \cdot 2 \cdot F_J\left(\rho, \rho, 2\left(\frac{1}{3} \frac{g^2}{4\pi}\right) \frac{u}{s^2 t}\right)^2, \quad (3.37)$$

Which is valid neglecting interference effects. One factor of 3 in (3.37) is from the color trace; one factor of 2 corresponds to looking at either  $q$  or  $\bar{q}$  jet; one factor of 2 is a flavor sum ( $u$  or  $d$  exchange; one factor of 2 for  $q$  or  $\bar{q}$  exchange; and factors of 1/2 for each  $\rho$  wave function.

$$\frac{Ed\sigma(\text{jet})}{d^3 p} \sim 2_{\text{coh}} \times 10^{-2} \frac{(1-x_R)^3}{p_T^8} \text{ nb-GeV}^6 \quad (3.38)$$

The extra factor  $2_{\text{coh}}$  comes from the fact that there are really two coherently adding amplitudes in the  $\rho\rho \rightarrow q\bar{q}$  cross section (as in Compton  $\gamma\gamma \rightarrow q\bar{q}$ ). At  $90^\circ$  the net result is an increase of (3.37) by a factor of 2.

The  $\pi^+$  cross section is given by a similar formula, except that no flavor sum on the exchanged quark is allowed, and (as in  $\gamma\gamma \rightarrow q\bar{q}$ ),  $d\sigma/du$  must be included to make sure that the "away" quark can fragment into a  $\pi$ .

$$\frac{Ed\sigma}{d^3p} = 3.9 \cdot 10^{-4} \times 2_{\text{coh}} \frac{(1-x_R)^3}{P_T} \text{ nb-GeV}^6 \quad . \quad (3.39)$$

Since these contributions are so very small compared to  $\gamma\gamma \rightarrow q\bar{q}$  their interference with the latter is also negligible.

2.  $\gamma\gamma \rightarrow q\bar{q}M\bar{M}$  (Fig. 5(b)). Here the quarks scatter into meson jets in the final state. Two additional jets along the beam directions also accompany these large- $p_T$  jets. We must sum over flavors u,d,s on quark (antiquark) i and over antiquark (quark) j.

$$\frac{Ed\sigma(\text{jet})}{d^3p} = 2 \cdot 2 \left( \sum_{i=uds} e_i \right)^2 \cdot 3 \cdot 3 \cdot S F_{JS} \left( q, q, \frac{d\sigma}{dt} \right) \quad (3.40)$$

$$\frac{Ed\sigma(\text{prompt } \pi^+)}{d^3p} = 2 \left( e_u^2 + e_d^2 \right) \left( \sum_{uds} e_j^2 \right) \cdot 3S' F_S \left( q, q, \frac{d\sigma}{dt} + \frac{d\sigma}{du} \right) \quad (3.41)$$

In the jet cross section (3.40), one factor of 2 counts  $q_i\bar{q}_j + q_i\bar{q}_j$ , another factor of 2 is for observing a meson or quark jet, one factor of 3 is from a flavor sum over k, and S(S') are spin sums over i, j, and k. The easiest way to compute S and S' is to simply count 2 spin states per quark: S=8 and S'=4 (since in that case two spins are constrained to be antiparallel for  $\pi^+$  production).

At  $\theta_{\text{c.m.}} = 90^\circ$ ,  $1-x_R$  small,

$$\frac{E d\sigma(\text{jet})}{d^3 p} = .24 \frac{(1-x_R)^3}{P_T} \text{ nb-GeV}^6$$

$$\frac{E d\sigma(\text{prompt } \pi^+)}{d^3 p} = 1.7 \cdot 10^{-2} \frac{(1-x_R)^3}{P_T} \text{ nb-GeV}^6 \quad . \quad (3.42)$$

3.  $\rho\rho \rightarrow Mq\bar{q}$  (Fig. 5(c)). This reaction is the one which is most similar to the processes which occur in  $pp \rightarrow \pi X$ . Both u- and s-channel quark exchange are possible: counting flavors, one finds

$$\frac{d\sigma(\text{jet})}{dt} = 8 \times (\text{u-channel exchange}) + 12 \times (\text{s-channel exchange})$$

$$\frac{d\sigma(\pi^+)}{dt} = 2 \times (\text{u-channel exchange}) + 2 \times (\text{s-channel exchange})$$

(3.43)

where  $d\sigma/dt (qM \rightarrow qM) = \pi\alpha_M^2 / su^3$  for the u-channel exchange diagram, whence,

$$\frac{E d\sigma(\text{jet})}{d^3 p} = 4 \cdot \left(\frac{1}{2}\right)^2 \cdot 3 \cdot 4 F_{JS}(\rho, q/\rho, \frac{d\sigma}{dt})$$

$$\approx 1.63 \cdot 10^{-2} \frac{(1-x_R)^3}{P_T} \text{ nb-GeV}^6 \quad , \quad (3.44)$$

where the factor of 4 is a beam target jet symmetrization factor, the factor of  $(1/2)^2$  is from the  $\rho$  wave function, the factor of 3 is from the color trace, and the factor of 4 is due to a spin sum.

$$\frac{E d\sigma(\text{prompt } \pi^+)}{d^3 p} = 2 \cdot \left(\frac{1}{2}\right)^2 \cdot 3 \cdot F_S(\rho, q/\rho, \frac{d\sigma}{dt})$$

$$= 6.7 \cdot 10^{-4} \frac{(1-x_R)^3}{P_T} \text{ nb-GeV}^6 \quad . \quad (3.45)$$

These reactions produce three jets, two at large  $p_T$  and a third along the beam direction. Both cross sections are much smaller than, and do not interfere (in the distribution function diagonal approximation) with, the following  $\rho\gamma \rightarrow Mq\bar{q}$  contributions. In each case we take

$$\alpha_M \equiv 1/3 g^2/4\pi = 2 \text{ GeV}^2.$$

4.  $\rho\gamma \rightarrow Mq\bar{q}$  (Fig. 5(d)). The Feynman graph for this process is identical to that for the reaction of Fig. 4(c). However, we assume that the hard scattering takes place in a different place in the graph when in that reaction. If we are to follow the precepts of the hard scattering expansion we must include this contribution in addition to the former one. It is the sum over all hard scatterings which saturates the Feynman graph.

The calculation is similar to that for the contribution just shown above. We find

$$\begin{aligned} \frac{d\sigma(\text{jet})}{dt} &= 4 \sum_{i=u,d,s} e_i^2 \quad (\text{u-channel exchange}) \\ &+ 6 \sum_{i=u,\bar{d}} e_i^2 \quad (\text{s-channel exchange}) \\ \frac{d\sigma(\pi^+)}{dt} &= \sum_{i=u,\bar{d}} e_i^2 \quad (\text{u-channel exchange} + \text{s-channel exchange}) \end{aligned} \tag{3.46}$$

$$\begin{aligned} \frac{Ed\sigma(\text{q or M jet})}{d^3p} &= 4 \times \left(\frac{1}{2}\right) \times 3 \times 4 F_{JS}(\rho, q, \frac{d\sigma}{dt}) \\ &= .18 \frac{(1-x_R)^2}{8 P_T} \text{ nb-GeV}^6 \end{aligned} \tag{3.47}$$

$$\begin{aligned} \frac{Ed\sigma(\text{prompt } \pi^+)}{d^3p} &= 2 \times \left(\frac{1}{2}\right) \times 3 F_S(\rho, q, \frac{d\sigma}{dt}) \\ &= 7.2 \cdot 10^{-3} \frac{(1-x_R)^2}{8 P_T} \text{ nb-GeV}^6 \end{aligned} \tag{3.48}$$

where the factor in (3.47) 4 is a beam target jet symmetrization factor, the factor of 1/2 is from the wave function normalization, the factor of 3 is a color trace, and the factor of 4 is from a spin sum.

This contribution, though it has the same Feynman graph structure as the  $p_T^{-6} \gamma\rho \rightarrow Mq\bar{q}$  contribution of Fig. 4(c), does not interfere with the latter in the diagonal distribution function approximation. However it does interfere with the  $p_T^{-6}$  subprocess  $\gamma\gamma \rightarrow Mq\bar{q}$ . The interference is not quite maximal but we may obtain an upper bound by using maximal interference:

$$\frac{E d\sigma(\text{jet interference})}{d^3 p} = 0.5 \frac{(1-x_R)^2}{p_T^7} \text{nb-GeV}^5 \quad (3.49)$$

or

$$\frac{\frac{E d\sigma(\text{interference})}{d^3 p}}{\frac{E d\sigma(\gamma\gamma \rightarrow Mq\bar{q})}{d^3 p}} = \frac{0.5}{p_T} \text{GeV} \quad (3.50)$$

which is small.

#### IV. RESULTS

As we have discussed in Section III, there are many contributions to jet and single-particle production in  $\gamma\gamma$  collisions, even though this is perhaps the simplest hadron-hadron reaction to consider in QCD. We will generally classify contributions according to their topological jet structure and nominal scaling power of  $p_T^{-n}$  at fixed  $x_R$  and  $\theta_{\text{c.m.}}$ . For convenience in making predictions for experiments at PETRA and PEP we will generally give predictions at  $\sqrt{s} = 30 \text{ GeV}$ , and  $\theta_{\text{c.m.}} = 90^\circ$ . The results for other kinematics can be obtained from the scaling laws given

in Sections II and III. When the cross sections for  $ee \rightarrow \text{jet} + X$  or  $ee \rightarrow H + X$  are written in terms of  $p_T$  and  $x_R = 2p_{\text{trigger}}/\sqrt{s}$ , the additional dependence on  $\theta_{\text{c.m.}}$  is not very dramatic. The choice of  $\theta_{\text{c.m.}} = 90^\circ$  is convenient for detectors which surround the interaction region in a  $e^+e^-$  colliding beam experiment but have poor detection efficiency at small angles to the beam. For clarity we will plot  $p_T^4 E d\sigma/d^3p$  for each inclusive reaction.

The jet cross sections which nominally scale as  $p_T^{-4}$  at fixed  $\theta_{\text{c.m.}}$  and  $x_R$  are shown in Fig. 6. The dominant cross section is predicted to be the two-jet  $\gamma\gamma \rightarrow q\bar{q}$  channel. As we have noted, the magnitude and scaling properties of this cross section (curve 6(a)) are crucial tests of QCD at short distances, checking the asymptotic approach to scale-invariance as well as the color and  $e_q^4$  factors in  $R_{\gamma\gamma}$  (see Eq. (3.4)). It is most advantageous to compare the  $e^+e^- \rightarrow \text{jet} + \text{jet} + e^+e^-$  rate with  $e^+e^- \rightarrow \mu^+\mu^-e^+e^-$  since corrections due to the two-photon tagging efficiency and the equivalent photon approximation tend to cancel.

Curves 6(b) and 6(c) give the jet cross sections from the 3-jet and 4-jet QCD subprocesses, respectively, as computed using the naive form of  $G_{q/e}$  given in Eq. (3.10) with an initial normalization choice  $\alpha_c(p_T^2) F_Q = 1.2$ . The full QCD form allowing for gluon bremsstrahlung to all orders along the incident quark directions gives the cross sections shown in 6(e) and 6(f) for the contributions of the 3-jet ( $\gamma q \rightarrow gq$ , etc.) and 4-jet ( $qq \rightarrow qq$ , etc.) subprocesses. We see that the full QCD form and the simplified Born calculations would be in fairly good agreement if we choose  $\alpha_c(p_T^2) F_Q = 0.8$ .

We also show the 4-jet contribution to the jet cross section from the  $\rho$ -dominance contribution,  $\rho\rho \rightarrow q\bar{q}q\bar{q}$  with  $qq \rightarrow qq$  scattering, in Fig. 6(d). This contribution is negligible compared to the perturbative QCD contributions.

The various contributions to  $p_T^4 E d\sigma/d^3p$  ( $ee \rightarrow \pi^+ X$ ) from  $p_T^{-4}$  reactions as computed in Section III are shown in Fig. 7. The  $p_T^{-6}$  scaling CIM contributions are shown in Fig. 8 for jets and Fig. 9 for single  $\pi^+$  triggers. The  $p_T^{-8}$  scaling cross sections for jets and single  $\pi^+$  triggers are graphed in Figs. 10 and 11, respectively.

The single  $\pi^+$  trigger cross sections are more than an order of magnitude smaller than the jet trigger rates. For  $p_T$  smaller than  $\sim 8$  GeV, and  $\sqrt{s} = 30$  GeV,  $p_T^{-6}$  contributions such as  $\gamma q \rightarrow \pi q$  where the pion is produced directly in the subprocess is predicted to dominate over the  $p_T^{-4}$  processes. This is the result of the "trigger bias" effect<sup>21</sup>: the scale-invariant reactions produce fast quarks in the final state which fragment into slower-moving pions. Since the  $G_{\pi/q}$  spectrum falls rapidly, the quark  $p_T$  is significantly larger than the trigger  $p_T$  and the resulting  $p_T^{-4}$  production cross section turns out to be suppressed by about two orders of magnitude. The reaction  $\gamma q \rightarrow \pi q$  produces the pion directly without the need for cascade and this has no trigger bias suppression factor. We present our predictions for the inclusive  $\pi^+$  spectrum in Fig. 12, together with the single- $\gamma$   $\pi^+$  inclusive cross section as a comparison. Note that, in the absence of tagging, the  $\gamma\gamma$  reaction is a dangerous background to the single- $\gamma$  inclusive- $\pi$  spectrum at  $90^\circ$  for  $x_T < 0.2$ ,  $\sqrt{s} = 30$  GeV.

We can now plot differential jet cross sections as they might be seen in various sorts of detection schemes.

1. No tagging, central detector. Detectors of this type will be unable to differentiate between large  $p_T$  events without hadronic jets along the beam direction and those events with accompanying jets. Figure 13 shows the expected jet cross sections which will be observed by such a detector, taken from the largest subprocesses of the preceding figures, and the resulting envelope of the sum of all contributions. The jet spectrum will show a  $p_T^{-6}$  falloff up to about  $p_T = 4.5$  GeV/c at which point it will "bend" into a  $p_T^{-4} (1-x_T)$  shape. Essentially no contributions from gluon jets or gluon exchange are visible.

2. Use of forward photon and hadron tagging with a central detector. Experiments of this type have the great advantage that one can, in principle, trigger on events with a particular number of small- $p_T$  jets in addition to large- $p_T$  jets. This considerably simplifies the problem of differentiating the various types of hard scattering reactions which occur. The useful quantities to present here are inclusive jet cross sections with two, three, or four hadronic jets, shown in Figs. 14, 15, and 16, respectively. (No corrections for tagging efficiency for either electrons or hadrons have been included.)

The ability to differentiate subprocesses is dramatically improved by the inclusion of a forward detector. The two-jet reaction is dominated by the process  $\gamma\gamma \rightarrow q\bar{q}$  for  $p_T > 2$  GeV/c. The four-jet cross section is dominated by the  $p_T^{-4}$  process  $\gamma\gamma \rightarrow q\bar{q}q\bar{q}$  for  $p_T > 2$  GeV. The three-jet cross section is dominated by the  $p_T^{-6}$  reaction  $\gamma\gamma \rightarrow Mq\bar{q}$  out to  $p_T = 2$  GeV, but remains important for the entire range of momenta accessible to experiments at  $\sqrt{s} = 30$  GeV.



The jet cross section integrated over transverse momenta is quite large, even for the single particular subprocess  $\gamma\gamma \rightarrow q\bar{q}$  (which we expect will dominate at large  $p_T$ ). The integrated cross section for jets with trigger momentum larger than  $p_T^{\min}$  is<sup>2</sup>

$$\sigma(p_T > p_T^{\min}) = \int_{p_T > p_T^{\min}} d^2 p_T \int dy \frac{Ed\sigma}{d^3 p}(e^+e^- \rightarrow e^+e^- + q\bar{q} \text{ or } \bar{q}q) \quad (4.1)$$

$$\approx \frac{32\pi}{3} \alpha^2 \left(\frac{\alpha}{2\pi} \ln \eta\right)^2 R_{\gamma\gamma} \frac{\left(\log \frac{s}{(p_T^{\min})^2} - \frac{19}{6}\right)}{(p_T^{\min})^2} \quad (4.2)$$

(which is the result of an analytic integration assuming  $xG_{\gamma/e}(x) \sim \frac{\alpha}{\pi} \ln \eta$ ), or about  $0.5 \text{ nb-GeV}^2 / (p_T^{\min})^2$  at  $\sqrt{s} = 30 \text{ GeV}$ . Jets with  $p_T > 4 \text{ GeV}$  contribute 0.3 more units of R to the  $e^+e^-$  total cross section at  $\sqrt{s} = 30 \text{ GeV}$  from this process alone.

We emphasize that  $\gamma\gamma \rightarrow q\bar{q}$  events will be observed in central detectors even in the absence of electron tagging. These hadronic events will appear as two "short" jets (visible energy much less than  $E_{\text{c.m.}}$ ), which are coplanar, roughly balanced in transverse momentum, and, in general are not back-to-back in angle. As we have noted, single hadrons arising from  $\gamma\gamma$  collisions at high  $p_T$  will be a serious background to single-hadron  $e^+e^-$  annihilation physics for  $x_R$  below about 0.2 but are negligible above that value. A good way to reduce contamination of one- $\gamma$  physics from 2- $\gamma$  processes will be to require the observation of at least one hadron in the former reactions with  $x_R > 0.2$  or so, or require a calorimetric trigger with  $E_{\text{vis}} \approx E_{\text{c.m.}}$ .

Of course, the best way to study  $\gamma\gamma$  events is with an electron-hadron forward tagging detector, used not so much to measure the energy

of the electrons or hadrons, as to simply indicate that a  $\gamma\gamma$  interaction has taken place. A single tagged electron will suffice for such a trigger, although double tagging may be necessary to remove machine-related backgrounds. If one is then left with a clean enough sample of events, one can then sift them to analyze separate multi-jet contributions to the jet and single particle cross sections.

#### V. CONCLUSIONS AND DISCUSSIONS

We conclude by summarizing our predictions:

1. The cross section for producing jets at large  $p_T$  from  $\gamma\gamma$  collisions is large:  $\sigma(p_T > 3 \text{ GeV}) \simeq 0.05 \text{ nb}$  at  $\sqrt{s} = 30 \text{ GeV}$ .
2. The dominant contribution to the  $e^+e^- \rightarrow e^+e^-$  jet cross section at  $p_T > 4 \text{ GeV}/c$  is the point-like reaction  $\gamma\gamma \rightarrow q\bar{q}$ , whose magnitude is absolutely normalized to be proportional to the  $\gamma\gamma \rightarrow \mu^+\mu^-$  cross section. The constant of proportionality is  $R_{\gamma\gamma} = 3 \sum_i e_i^4$ . Observation of this reaction and its characteristic scale-invariant behavior will represent the first direct verification of the  $1/p$  shape of the quark propagator. It will also be interesting to study the logarithmic approach to scaling predicted by quantum chromodynamics and the effects of heavy  $q\bar{q}$  thresholds.
3. The single  $\pi$  cross section has a behavior  $E d\sigma/d^3p (e^+e^- \rightarrow e^+e^-X)$   $(1-x_R)^2 p_T^{-6}$  and represents a sizeable background to single photon induced hadroproduction for  $x_R < 0.2$ . The magnitude of the leading contribution is fixed by the constituent interchange model and its measurement is a check of that model.
4. A rich jet structure--two, three, and four hadronic jets, will be exposed to detectors and tagging counters aligned at small angles to

the beam. In particular, the four-jet cross section holds the possibility of seeing quark-quark scattering by vector gluon exchange. The even larger three-jet cross section allows the study of gluon jets and the QCD Compton process  $\gamma q \rightarrow gq$ .

5. Processes in which the photons vector dominate into low-mass states before scattering are essentially a negligible component of the large- $p_T$  cross section although they give the dominant contribution for small  $p_T$ .

Since these reactions are not small and since they represent such a rich laboratory for the investigation of quark and hadron dynamics, it is important that a considerable effort be made to study them. The simpler form of  $4\pi$  solid angle detectors will be able to elucidate the general features of these reactions. Detailed dynamical questions, such as the number of jets produced in a given interaction, require information which is most easily obtained from a forward tagging system used in conjunction with a central detector.

We must emphasize that this work represents only a first survey of the field of  $\gamma\gamma$ -induced jet reactions. Many interesting problems remain to be studied. On the side of phenomenology, for instance, tagging the electrons provides information on the scattering of polarized photons into jets--for which the simplest process  $\gamma\gamma \rightarrow q\bar{q}$  will give unique predictions. One may study quantum number correlations between fast hadrons in one large  $p_T$  jet and fast hadrons in the other jet, which will differ in  $\gamma\gamma \rightarrow q\bar{q}$  from the correlations seen in  $\gamma^* \rightarrow q\bar{q}$  because of the different charge structure.

The occurrence of  $\gamma\gamma$  reactions at an experimentally observable level implies that the entire range of hadronic physics which can be studied, for example, at the CERN-ISR can also be studied in parallel in  $e^+e^-$  machines. Although low  $p_T$   $\gamma\gamma$  reactions should strongly resemble meson-meson collisions, the elementary field nature of the photon implies dramatic differences at large  $p_T$ . This region can be a crucial testing ground for QCD since not only are a number of new subprocesses accessible ( $\gamma\gamma \rightarrow q\bar{q}$ ,  $\gamma q \rightarrow gq$ ,  $\gamma q \rightarrow Mq$ , etc.) essentially with no free parameters, but most important, one can make predictions for a major component of the photon structure function directly from QCD. We also note that there are open questions in hadron-hadron collisions, e.g., whether non-perturbative effects (instantons, wee parton interactions) are important for large  $p_T$  reactions.<sup>6</sup> Such effects are presumably absent for the perturbative, point-like interactions of the photon. We also note that the interplay between vector-meson-dominance and point-like contributions to the hadronic interactions of photon is not completely understood in QCD, and  $\gamma\gamma$  processes may illuminate these questions.

As a final point, we wish to remark that even though we have been mainly concerned with  $e^+e^-$  colliding beams in the  $\sqrt{s} = 30$  region, the  $\gamma\gamma$  processes become even more important as the center-of-mass energy is increased. There has already been some talk of construction of an  $e^+e^-$  storage ring with  $\sqrt{s} \sim 200$  GeV. At this energy, one unit of R of cross section is about  $2.25 \cdot 10^{-3}$  nb, whereas the integrated cross section for the process  $e^+e^- \rightarrow e^+e^-q\bar{q}$ , with  $p_T > 10$  GeV is  $\sim 0.02$  nb, or about 9 units of R. These estimates speak for themselves: very energetic  $e^+e^-$  colliding beam machines are more nearly laboratories for  $\gamma\gamma$  scattering than they are for  $e^+e^-$  annihilation.

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APPENDIX A. DERIVATION OF CONVOLUTION FORMULA

We consider the process shown in Fig. 17 where the fragments a and b of particles A and B undergo a hard scattering to produce the particles c and d. The particle c subsequently fragments to form C. In the applications discussed in this paper A and B are the incident  $e^+$  and  $e^-$ , C is an observed meson, and a and b can be either photons, rho mesons, or quarks. The relevant fragmentation functions and cross sections  $d\sigma/dt$  are given in the text.

We first derive<sup>7</sup> the inclusive cross section  $Ed\sigma/d^3p$  for the process  $A + B \rightarrow C + X$  and then specialize it to the case where the fragmentation of c is absent.

Cross sections are conveniently derived using the notation of Ref. 3. We assume throughout that all particles are ultrarelativistic ( $E_i \simeq |\vec{p}_i|$ ). We define the momentum fractions in the c.m. frame of A and B

$$x_R \equiv 1-\varepsilon = \frac{2|\vec{p}_C|}{\sqrt{s}} \quad , \quad x_F = \frac{2p_C^3}{\sqrt{s}} \quad (A1)$$

and

$$x_t = -\frac{t}{s} \equiv \frac{(p_A - p_C)^2}{(p_A + p_B)^2} = \frac{1}{2} (x_R - x_F)$$

$$x_u = -\frac{u}{s} \equiv \frac{(p_B - p_C)^2}{(p_A + p_B)^2} = \frac{1}{2} (x_R + x_F) \quad (A2)$$

Similarly for the subprocess we define

$$\hat{x}_t \equiv -\frac{\hat{t}}{\hat{s}} = \frac{(p_a - p_c)^2}{(p_a + p_b)^2}$$

$$\hat{x}_u \equiv -\frac{\hat{u}}{\hat{s}} = \frac{(p_b - p_c)^2}{(p_a + p_b)^2}$$

$$\hat{x} \equiv \frac{2|\vec{p}_c|}{\sqrt{s}} \quad (\text{A3})$$

Following Ref. 3 the probability of the whole process is given by the product of the probabilities of the subprocesses. Writing the probability of finding a constituent a in A with (light-cone/infinite momentum frame) momentum fraction between  $x_a$  and  $x_a + dx_a$  as  $G_{a/A}(x_a)dx_a$ , we have

$$d\sigma = \iiint dx_a G_{a/A}(x_a) dx_b G_{b/B}(x_b) \left[ \frac{d\sigma}{d\hat{x}_t d\hat{x}_u} d\hat{x}_t d\hat{x}_u \right] G_{C/c}(x_C) dx_C \quad (\text{A4})$$

where

$$x_C \equiv \frac{|\vec{p}_C|}{|\vec{p}_c|} = \frac{x}{\hat{x}} \quad (\text{A5})$$

Since  $m_d^2 \ll \hat{s}$ ,  $\hat{x}_u + \hat{x}_t \simeq 1$  and

$$\frac{d\sigma}{d\hat{x}_t d\hat{x}_u} = \hat{s} \frac{d\sigma}{d\hat{t}} \delta(1 - \hat{x}_t - \hat{x}_u) \quad (\text{A6})$$

Furthermore it is easy to show<sup>3</sup> that

$$\hat{x}_t = \frac{1}{x_C x_b} x_t, \quad \hat{x}_u = \frac{1}{x_C x_a} x_u \quad (\text{A7})$$

and

$$d\hat{x}_t d\hat{x}_u = \frac{1}{x_C^2 x_a x_b} dx_t dx_u = \frac{1}{x_C^2 x_a x_b} \frac{1}{\pi s} \frac{d^3 p_c}{E_c} \quad (\text{A8})$$

Inserting these in (A4) gives<sup>3</sup>

$$E_C \frac{d\sigma}{d^3 p_C} = \int_0^1 \int_0^1 dx_a dx_b G_{a/A}(x_a) G_{b/B}(x_b) \frac{\hat{s}}{\pi} \frac{d\sigma}{d\hat{t}} \\ \times \frac{G_{C/c}(x_C)}{x_C^2} dx_C \theta(1-x_C) \Big|_{x_C = \frac{x_t}{x_b} + \frac{x_u}{x_a}} \quad (A9)$$

We now assume explicit simple forms for the fragmentation functions:

$$G_{i/I}(x_i) = (1+g_i) \Lambda_{i/I} \frac{(1-x_i)^{g_i}}{x_i} \quad (A10)$$

and

$$\frac{d\sigma}{d\hat{t}} = \pi D \hat{s}^{-N} (\hat{x}_t)^{-T} (\hat{x}_u)^{-U} \quad (A11)$$

Inserting (A10) and (A11) in (A8) we obtain

$$E_C \frac{d\sigma}{d^3 p_C} = \frac{x_t^{-T} x_u^{-U}}{s^N} D \Lambda_{a/A} \Lambda_{b/B} \Lambda_{C/c} (1+g_a)(1+g_b)(1+g_c) \\ \times \int_0^1 \int_0^1 dx_a dx_b (1-x_a)^{g_a} (1-x_b)^{g_b} x_a^{U-N-1} \\ \cdot x_b^{T-N-1} x_C^{T+U-2} (1-x_C)^{g_c} \theta(1-x_C) \Big|_{x_C = \frac{x_t}{x_a} + \frac{x_u}{x_b}} \quad (A12)$$

Equation (A12) can be put into a more convenient form by changing variables so that the triangular integration region becomes the unit square:

$$x_a = \left(1 + \frac{\varepsilon z y}{x_u}\right)^{-1}, \quad x_b = \left(1 + \frac{\varepsilon z (1-y)}{x_t}\right)^{-1} \quad (A13)$$



We thus obtain

$$\begin{aligned}
 E_C \frac{d\sigma}{d^3 p_C} &= \frac{\epsilon^{g_a+g_b+g_c+2}}{\left(\frac{2}{p_T}\right)^N} D\Lambda_{a/A} \Lambda_{b/B} \Lambda_{C/c} \\
 &\times (1+g_a)(1+g_b)(1+g_c)^{\frac{g_a+g_b+T+U+2-2N}{2}} \\
 &\times \int_0^1 \int_0^1 dz dy z^{g_a+g_b+1} (1-z)^{g_c} y^{g_a} (1-y)^{g_b} \\
 &\times (1 - \epsilon(1-z))^{T+U-2} (1 + x_F - \epsilon(1-2zy))^{N-U-g_a-1} \\
 &\times (1 - x_F - \epsilon(1-2zy))^{N-T-g_b-1}
 \end{aligned} \tag{A14}$$

This is the basic formula we use in our numerical calculations. We note that for small  $\epsilon$  the final three factors in the integral are approximately unity and<sup>7</sup>

$$\begin{aligned}
 E_C \frac{d\sigma}{d^3 p_C} &\underset{\epsilon \rightarrow 0}{\sim} \frac{\epsilon^{g_a+g_b+g_c+2}}{\left(\frac{2}{p_T}\right)^N} D\Lambda_{a/A} \Lambda_{b/B} \Lambda_{C/c} \\
 &\times 2^{\frac{g_a+g_b+T+U+2-2N}{2}} \frac{\Gamma(g_a+2) \Gamma(g_b+2) \Gamma(g_c+2)}{\Gamma(3+g_a+g_b+g_c)}
 \end{aligned} \tag{A15}$$

If  $g_a$  and  $g_b$  are large while  $g_c$  is small,  $y \approx 1/2$  and  $z \approx 1$  will dominate in (A14) and the final factors are approximately

$$(1+x_F)^{N-U-g_a-1} (1-x_F)^{N-T-g_b-1}$$

but this is not the situation encountered here.

When the fragmentation of  $c$  is not present, the analog of (A12) is

$$\begin{aligned}
 E_c \frac{d\sigma}{d^3 p_c} &= \frac{x_t^{-T} x_u^{-U}}{s^N} D\Lambda_{a/A} \Lambda_{b/B} (1+g_a)(1+g_b) \\
 &\times \int_0^1 \int_0^1 dx_a dx_b (1-x_a)^{g_a} (1-x_b)^{g_b} x_a^{U-N-1} x_b^{T-N-1} \\
 &\times \delta\left(1 - \frac{x_t}{x_b} - \frac{x_u}{x_a}\right)
 \end{aligned} \tag{A16}$$

which using (A13) gives

$$\begin{aligned}
 E_c \frac{d\sigma}{d^3 p_c} &= \frac{\epsilon^{g_a+g_b+1}}{\binom{2}{p_T}^N} D\Lambda_{a/A} \Lambda_{b/B} (1+g_a)(1+g_b) \\
 &\times 2^{g_a+g_b+T+U+2-2N} \\
 &\times \int_0^1 dy y^{g_a} (1-y)^{g_b} (1+x_F - \epsilon(1-2y))^{N-U-g_a-1} \\
 &\times (1-x_F + \epsilon(1-2y))^{N-T-g_b-1} .
 \end{aligned} \tag{A17}$$

For  $\epsilon \rightarrow 0$ , we have

$$\frac{E d\sigma}{d^3 p} \sim \frac{\epsilon^{g_a+g_b+1}}{\binom{2}{p_T}^N} D\Lambda_{a/A} \Lambda_{b/B} \frac{\Gamma(2+g_a) \Gamma(2+g_b)}{\Gamma(2+g_a+g_b)} 2^{g_a+g_b+T+u+2-2N} \tag{A18}$$

This is actually exact for the dominant term in CIM process  $\gamma q \rightarrow qM$

( $a=\gamma$ ,  $b=q$ ,  $c=M$ ,  $d=q$ ) since  $N-U-g_a-1 = N-T-g_b-1 = 0$ . For small  $x_F$  and  $\epsilon$  the integrand of Eq. (A17) can be expanded. For example, this leads to corrections to (A18) of

$$\left[ 1 + 2x_F + x_F^2 + \frac{2}{3} \epsilon^2 + 0(\epsilon^3, \epsilon^2 x_F, x_F^3, \text{etc.}) \right]$$

for the QED process ( $a=b=\gamma$ ,  $c=d=q$ ).

APPENDIX B. QCD EFFECTS IN  $\gamma\gamma$  COLLISIONS

Perturbative corrections modify Born approximation calculations for  $\gamma\gamma$  collisions such as the  $q\bar{q} \rightarrow q\bar{q}$  and  $\gamma q \rightarrow gq$  subprocesses in two mutually compensating ways. First, the running coupling constant asymptotically varies with the  $p_T$  of the subprocess as<sup>22</sup>

$$\alpha_c(p_T^2) \simeq \frac{1}{b \log(p_T^2/\Lambda^2)} \quad (\text{B1})$$

where  $4\pi b = 11 - \frac{2}{3} n_f$  for color SU(3) and  $n_f$  flavors. Second the distributions of quarks in photons (and electrons) are modified, as was first discussed by Witten.<sup>4,5</sup> For our purposes it will be sufficient to retain only the "non-singlet" contributions to the photon structure functions, analogous to the valence quark contributions of the proton. The approximation is justified for our large  $p_T$  applications because the singlet or quark sea contributions (from gluon bremsstrahlung and pair production) vanish two powers faster in  $(1-x)$ . This is an especially useful approximation because it obviates the need to perform operator mixing or to redo the complicated quark charge counting of Section II.

Within the valence approximation we can use the convolution form (see also Ref. 15).

$$G_{q/\gamma}(x, Q^2) = \frac{3\alpha}{2\pi} e_q^2 \int_{\mu^2}^{Q^2} \frac{dk^2}{k^2} \int_x^1 \frac{dz}{z} \left[ z^2 + (1-z)^2 \right] G_{q/q}\left(\frac{x}{z}, Q^2, k^2\right) \quad (\text{B2})$$

where  $G_{q/q}$  is the standard non-singlet distribution for quarks in a target quark of mass  $k^2$  being probed at four-momentum squared  $Q^2$ . The factor of 3 includes the sum over color. We shall assume that the contribution from  $k^2 < \mu^2$  is accounted for by the vector-meson-dominance contributions. Taking moments, we have

$$G_{q/\gamma}(j, Q^2) = \frac{3\alpha}{2\pi} e_q^2 \int_{\mu^2}^{Q^2} \frac{dk^2}{k^2} f(j) G_{q/q}(j, Q^2, k^2) \quad (B3)$$

where

$$G(j) \equiv \int_0^1 dx x^{j-1} G(x) \quad (B4)$$

$$\begin{aligned} f(j) &= \int_0^1 dz z^{j-1} [z^2 + (1-z)^2] \\ &= \frac{1}{j} - \frac{2}{j+1} + \frac{2}{j+2} \end{aligned} \quad (B5)$$

and

$$G_{q/q}(j, Q^2, k^2) = \left[ \frac{\alpha(k^2)}{\alpha(Q^2)} \right]^{\frac{d_j}{2\pi b}} \quad (B6)$$

The  $d_j$  are the standard valence anomalous dimensions, as defined in Ref. 23. Performing the  $k^2$  integral in (B3) yields

$$G_{q/\gamma}(j, Q^2) = \frac{3}{2\pi} e_q^2 \frac{\alpha}{\alpha_c(Q^2)} \left[ \frac{2\pi f(j)}{2\pi b - d_j} \right] \quad (B7)$$

This exhibits the remarkable scaling features of the photon structure function discussed in Section III.

It is easy to invert the moment equation (B7) via the method of Yndurian.<sup>24</sup> A graph of  $xG_{q/\gamma}(x)$  calculated in valence approximation in QCD and in the parton model is given in Fig. 1(b). Good agreement is obtained with the exact (valence plus singlet) results of Llewellyn Smith<sup>5</sup> over nearly the entire range of  $x$ .

The  $x$  near 1 behavior of  $G_{q/\gamma}(x)$  can be obtained more directly from a direct integration of (B2), using the  $x \rightarrow 1$  form for the quark

structure function<sup>15</sup>

$$G_{q/q}(x, Q^2, k^2) = \exp \left[ (3-4\gamma_E) \xi C_2 \right] (1-x)^{4C_2\xi-1} / \Gamma(4C_2\xi) \quad (B8)$$

where  $\gamma_E = 0.577\dots$  is Euler's constant,  $C_2 = (N^2-1)/2N=4/3$ , and

$$\xi = \frac{1}{4\pi b} \ln \frac{\alpha(k^2)}{\alpha(Q^2)} \quad (B9)$$

We then obtain,

$$G_{q/\gamma}(x, Q^2) \underset{x \rightarrow 1}{=} \frac{3}{2\pi} e_q^2 \frac{\alpha}{\alpha_c(Q^2)} \frac{4\pi}{4\pi b - (3-4\gamma_E)C_2 + 4C_2 \ln \frac{1}{1-x}} \quad (B10)$$

This result is numerically accurate only for  $x \gtrsim 0.97$  but is off by no more than a factor of 2 for  $x > 0.1$  (see Fig. 1).

Finally, as we are interested in electron-electron scattering, we must convolute  $G_{q/\gamma}(x)$  and  $G_{\gamma/e}(x)$ . That is most easily done, once again, by taking moments and using Yndurian's method. We find

$$G_{q/\gamma/e}(x, Q^2) = 3 \left( \frac{\alpha}{2\pi} \right)^2 e_q^2 \frac{\ln \eta}{\alpha_c(Q^2)} \frac{1-x}{x} \hat{f}(x) \quad (B11)$$

where  $\hat{f}(x)$  is shown in Fig. 1(d). The  $x \rightarrow 1$  behavior of  $\hat{f}(x)$  is given in Section III.

FOOTNOTES AND REFERENCES

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Table I. Fragmentation Functions

$$xG_{a/A}(x) = \sum_i \Lambda_a^i (1+g_a^i)(1-x)^{g_a^i}$$

| A | a             | $g_a^{(1)}$ | $\Lambda_a^{(1)}$    | $g_a^{(2)}$ | $\Lambda_a^{(2)}$    |
|---|---------------|-------------|----------------------|-------------|----------------------|
| e | $\gamma$      | 0           | $2.37 \cdot 10^{-2}$ | 2           | $0.79 \cdot 10^{-2}$ |
| e | $\rho$        | 0           | $7.86 \cdot 10^{-5}$ | 2           | $2.62 \cdot 10^{-5}$ |
| e | q             | 1           | $1.1 \cdot 10^{-4}$  |             |                      |
| e | (u+d)/ $\rho$ | 2           | $2.2 \cdot 10^{-6}$  | 3           | $1.1 \cdot 10^{-6}$  |
| q | $\pi$         | 1           | 0.25                 |             |                      |

All fragmentation functions are per color and for unit charge quarks, and are evaluated at  $\sqrt{s} = 30$  GeV, with

$$\frac{f_\rho^2}{4\pi} = 2.2$$

$$\ln \eta = \ln \frac{s}{4m_e^2} = 20.4$$

Table II. QCD differential cross sections, summed over initial colors, have a generic form

$$\frac{d\sigma}{dt} (ij \rightarrow k\ell) = \frac{4\pi\alpha_c^2}{s^2} \Sigma$$

where

$$\Sigma(i=q_i \text{ or } \bar{q}_i, j=q_j \text{ or } \bar{q}_j, i \neq j) = \frac{s^2 + u^2}{t^2}$$

$$\Sigma(q_i q_i \rightarrow q_i q_i \text{ or } \bar{q}_i \bar{q}_i \rightarrow \bar{q}_i \bar{q}_i) = \frac{s^2 + u^2}{t^2} + \frac{s^2 + t^2}{u^2} - \frac{2}{3} \frac{s^2}{tu}$$

$$\Sigma(q_i \bar{q}_i \rightarrow q_i \bar{q}_i) = \frac{s^2 + u^2}{t^2} + \frac{t^2 + u^2}{s^2} - \frac{2}{3} \frac{u^2}{st}$$

$$\Sigma(q_i \bar{q}_i \rightarrow q_j \bar{q}_j, i \neq j) = \frac{t^2 + u^2}{s^2}$$

$$\Sigma(q_i \bar{q}_i \rightarrow gg) = \frac{8}{3} \left( \frac{t}{u} + \frac{u}{t} \right) - 6 \left( \frac{t^2 + u^2}{s^2} \right)$$

Table III.  $\gamma\gamma \rightarrow q\bar{q}q\bar{q}$  jet and  $\pi^+$  effective cross sections.

$$\frac{d\sigma^{\text{jet}}}{dt} (q\bar{q} + q\bar{q} + \bar{q}q + \bar{q}\bar{q} \rightarrow \text{final state quarks or antiquarks})$$

$$= \frac{4\pi\alpha_c^2}{s^2} \left\{ \left[ \frac{s^2+u^2}{t^2} + \frac{s^2+t^2}{u^2} \right] 4 \left( \sum e_i^2 \right)^2 \right. \\ \left. + \left[ \frac{t^2+u^2}{s^2} N_f - \frac{1}{3} \left( \frac{s^2}{tu} + \frac{t^2}{su} + \frac{u^2}{st} \right) \right] 4 \sum e_i^4 \right\}$$

$N_f$  = number of quark flavors

$$\frac{d\sigma^{\text{jet}}}{dt} (q\bar{q} \rightarrow \text{final state gluons})$$

$$= \frac{4\pi\alpha_c^2}{s^2} \left( \frac{8}{3} \left( \frac{t}{u} + \frac{u}{t} \right) - 6 \left( \frac{t^2+u^2}{s^2} \right) \right) 2 \sum e_i^4$$

$$\frac{d\sigma^{\pi^+}}{dt} (q\bar{q} + q\bar{q} + \bar{q}q + \bar{q}\bar{q} \rightarrow uX \text{ or } \bar{d}X)$$

$$= \frac{4\pi\alpha_c^2}{s^2} \left\{ 2 \left( \sum_{i=u,\bar{d}} e_i^2 \right) \left( \sum_{j=udsc} e_j^2 \right) \left( \frac{s^2+u^2}{t^2} + \frac{s^2+t^2}{u^2} \right) \right. \\ \left. + 4 \sum_{i=udsc} e_i^4 \left( \frac{t^2+u^2}{s^2} \right) + \sum_{i=u,d} e_i^4 \left( -\frac{2}{3} \right) \left( \frac{s^2}{tu} + \frac{t^2}{su} + \frac{u^2}{st} \right) \right\}$$

The cross sections are summed (but not averaged) over color.

FIGURE CAPTIONS

1. The structure function of the photon  $f(x) = xG_{q/\gamma}(x, Q^2) / [(\alpha/2\pi) e_q^2/\alpha_c(Q^2)]$ , as computed in perturbative QCD, (a) the Born approximation from Eq. (2.12):  $f_{\text{Born}}(x) = x[x^2 + (1-x)^2]/b$ . (b) The non-singlet (valence) structure function  $f(x)$  computed from QCD to all orders. See Eq. (2.14) and Appendix B. (c) The asymptotic form for  $f(x)$  given in Eq. (2.15). (d) The structure function  $\tilde{f}(x)$  for  $G_{q/e}$  as defined in Eq. (2.16).
2. Graphs for  $\gamma\gamma$  inclusive cross sections proportional to  $p_T^{-4}$ . A box encloses the hard scattering. (a)  $\gamma\gamma \rightarrow q\bar{q}$ ; (b)  $\gamma\gamma \rightarrow q\bar{q}q\bar{q}, q\bar{q}g\bar{g}$ ; (c)  $\gamma\rho \rightarrow q\bar{q}q\bar{q}$ ; (d)  $\rho\rho \rightarrow q\bar{q}q\bar{q}$ ; (e)  $\gamma\gamma \rightarrow q\bar{q}g$ ; and (f) a radiative correction to the three-jet topology.
3. The subprocesses which generate  $p_T^{-6}$  cross sections. The arrows on the labeled quark lines denote the direction of flow from which the quark's charge is to be read off.
4. Diagrams for  $p_T^{-6}$  inclusive cross sections (a)  $\gamma\rho \rightarrow q\bar{q}$ , (b)  $\gamma\gamma \rightarrow Mq\bar{q}$ , (c)  $\gamma\rho \rightarrow Mq\bar{q}$ .
5. Diagrams for  $p_T^{-8}$  inclusive cross sections (a)  $\rho\rho \rightarrow q\bar{q}$ , (b)  $\gamma\gamma \rightarrow q\bar{q}M\bar{M}$ , (c)  $\rho\rho \rightarrow Mq\bar{q}$ , (d)  $\gamma\rho \rightarrow Mq\bar{q}$ .
6. Inclusive jet cross sections which scale as  $p_T^{-4}$ : (a)  $\gamma\gamma \rightarrow q\bar{q}$  (b)  $\gamma\gamma \rightarrow q\bar{q}g$ , (c)  $\gamma\gamma \rightarrow q\bar{q}q\bar{q}$  and  $q\bar{q}g\bar{g}$ , (d)  $\rho\rho \rightarrow q\bar{q}q\bar{q}$ , multiplied by  $10^3$ . Curves (e) and (f) give the results of calculating (b) and (c) respectively with the full QCD perturbative structure function  $G_{q/\gamma}$ .
7. Single  $\pi^+$  inclusive cross sections which scale as  $p_T^{-4}$ : (a)  $\gamma\gamma \rightarrow q\bar{q}$  (b)  $\gamma\gamma \rightarrow q\bar{q}q\bar{q}$ .

8.  $p_T^{-6}$  jet cross sections (a)  $\gamma\rho \rightarrow q\bar{q}$ , (b)  $\gamma\gamma \rightarrow Mq\bar{q}$ , (c)  $\gamma\rho \rightarrow Mq\bar{q}$ .
9.  $p_T^{-6}$   $\pi^+$  cross sections (a)  $10 \times \gamma\rho \rightarrow q\bar{q}$ , (b)  $\gamma\gamma \rightarrow \pi^+ q\bar{q}$ ,  
(c)  $\gamma\rho \rightarrow \pi^+ q\bar{q}$ .
10.  $p_T^{-8}$  jet cross sections (a)  $\rho\rho \rightarrow q\bar{q}$ , (b)  $\gamma\gamma \rightarrow q\bar{q}M\bar{M}$ , (c)  $\rho\rho \rightarrow Mq\bar{q}$ ,  
(d)  $\gamma\rho \rightarrow Mq\bar{q}$ .
11.  $p_T^{-8}$   $\pi^+$  cross sections (a)  $10 \times \rho\rho \rightarrow q\bar{q}$ , (b)  $\gamma\gamma \rightarrow \pi Mq\bar{q}$ ,  
(c)  $10 \times \rho\rho \rightarrow \pi q\bar{q}$ , (d)  $\gamma\rho \rightarrow \pi q\bar{q}$ .
12. Combined inclusive  $\pi^+$  cross section, showing only the most important terms, which are the  $p_T^{-6}$  (a)  $\gamma\gamma \rightarrow \pi q\bar{q}$  and (b)  $\gamma\rho \rightarrow \pi q\bar{q}$  contributions. Their envelope is given by the dotted line. Also shown is (c) the  $e^+e^- \rightarrow \gamma \rightarrow \pi^+ X$  inclusive cross section for comparison. Again,  $\sqrt{s} = 30$  GeV,  $\theta_{c.m.} = 90^\circ$ .
13. Jet inclusive cross sections as they would be observed in a central detector without tagging additional forward/backward hadronic jets. The dotted line is the envelope of the most important terms  
(a)  $\gamma\gamma \rightarrow q\bar{q}$ , (b)  $\gamma\gamma \rightarrow Mq\bar{q}$ , (c)  $\gamma\gamma \rightarrow q\bar{q}g$ , (d)  $\gamma\gamma \rightarrow q\bar{q}q\bar{q}$ , (e)  $\gamma\rho \rightarrow q\bar{q}$ ,  
(f)  $\gamma\gamma \rightarrow M\bar{M}q\bar{q}$ .
14. Dominant jet inclusive cross sections with a two-jet topology  
(a)  $\gamma\gamma \rightarrow q\bar{q}g$ , (b)  $\gamma\rho \rightarrow q\bar{q}$ .
15. Jet cross sections, three-jet topology (a)  $\gamma\gamma \rightarrow q\bar{q}g$ , (b)  $\gamma\gamma \rightarrow Mq\bar{q}$ ,  
(c)  $\gamma\rho \rightarrow Mq\bar{q}$ , (d)  $\gamma\gamma \rightarrow q\bar{q}g$ , computed with the full perturbative structure function  $G_{q/\gamma}$ .
16. Jet cross sections, four-jet topology (a)  $\gamma\gamma \rightarrow q\bar{q}q\bar{q}$  (Born approximation) (b)  $\gamma\gamma \rightarrow q\bar{q}M\bar{M}$ , (c)  $\gamma\gamma \rightarrow q\bar{q}q\bar{q}$ , computed with the full perturbative structure function  $G_{q/\gamma}$ .
17. The standard hard scattering parton model diagram for  $AB \rightarrow CX$ .

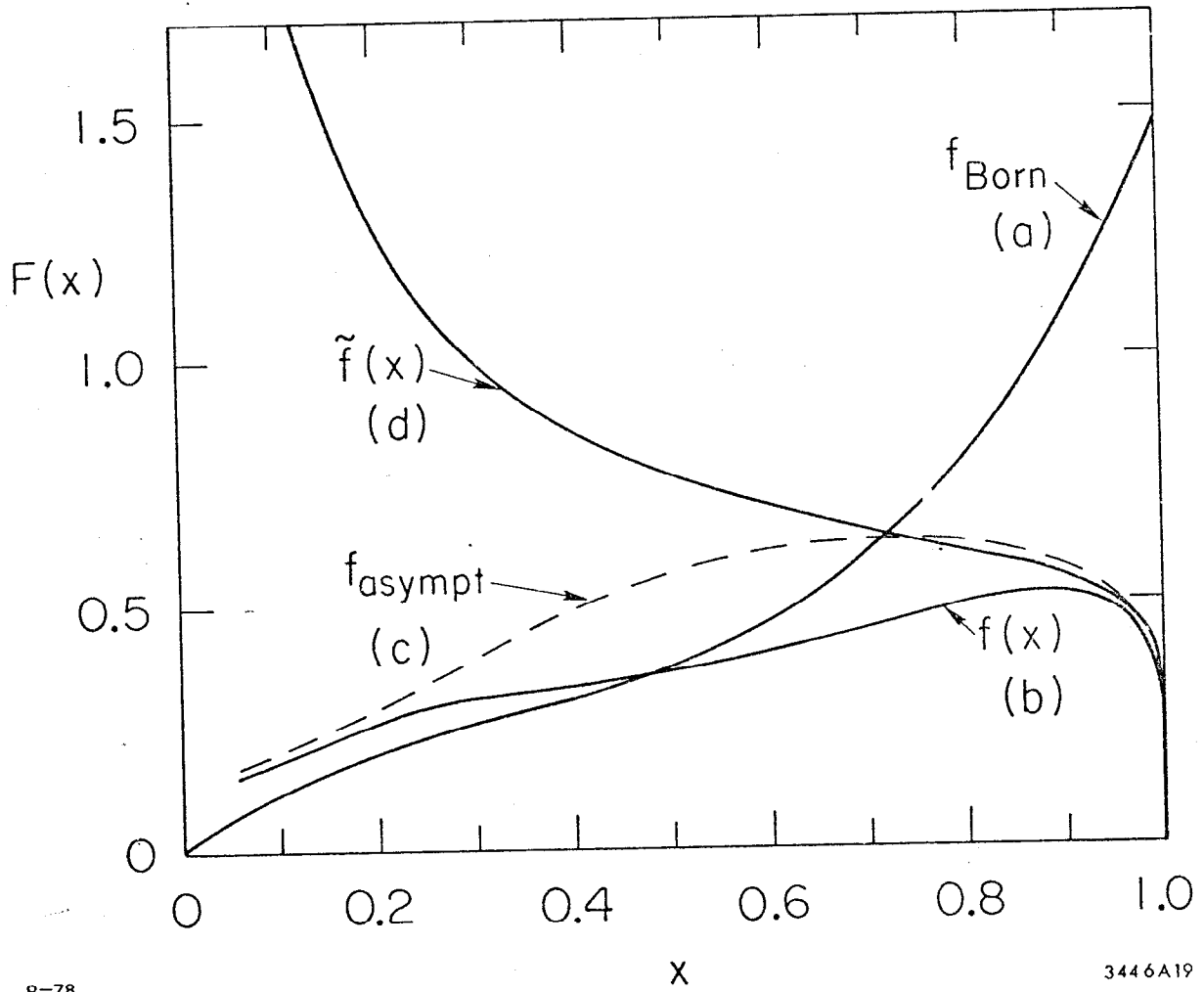
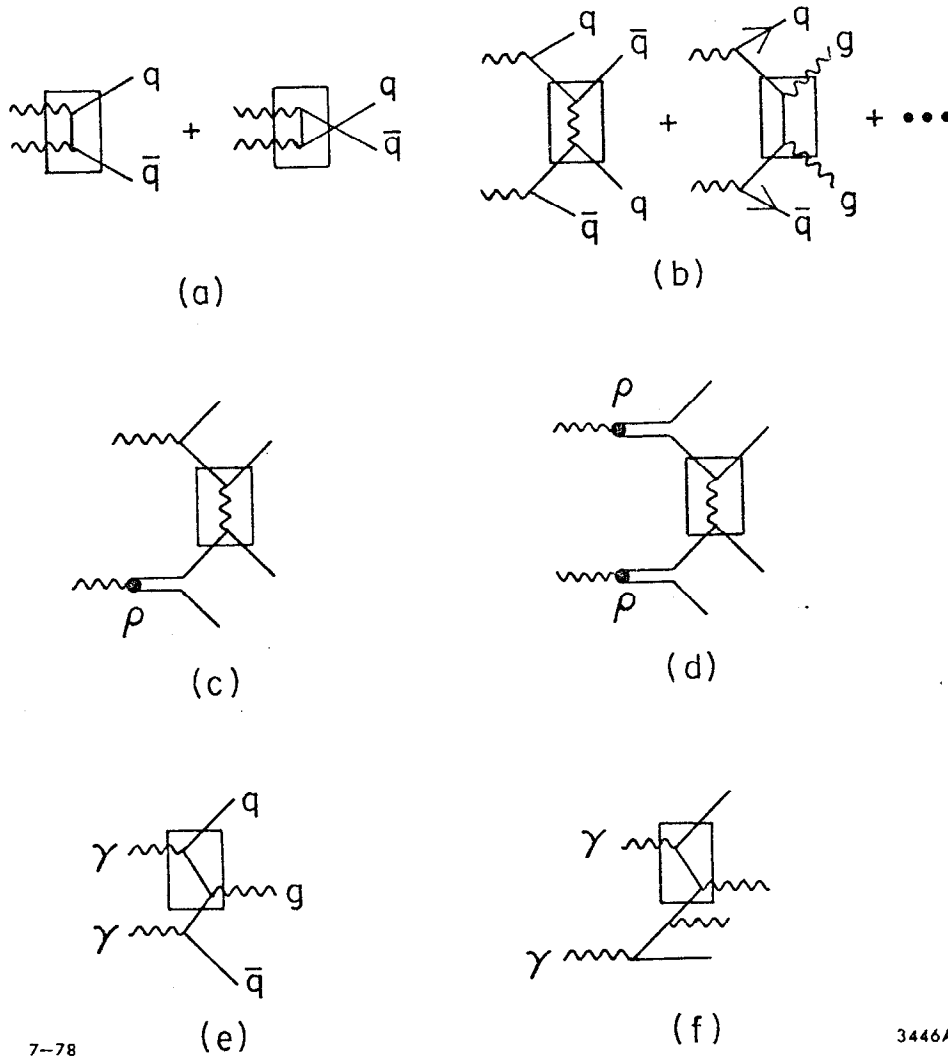


Fig. 1



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3446A2

Fig. 2



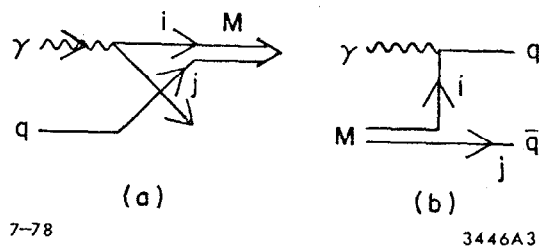
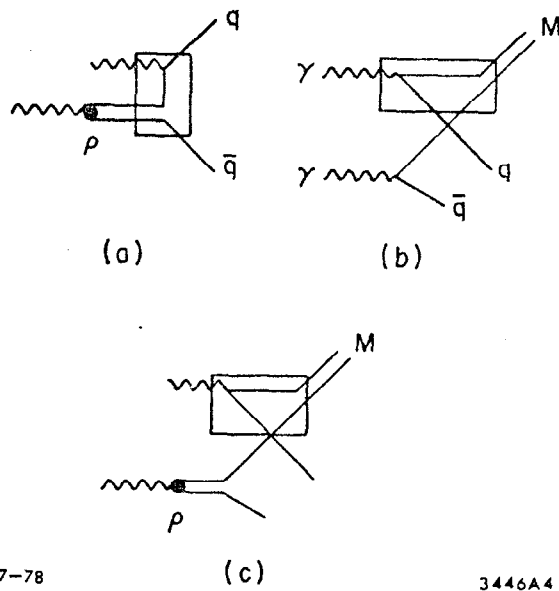


Fig. 3



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Fig. 4

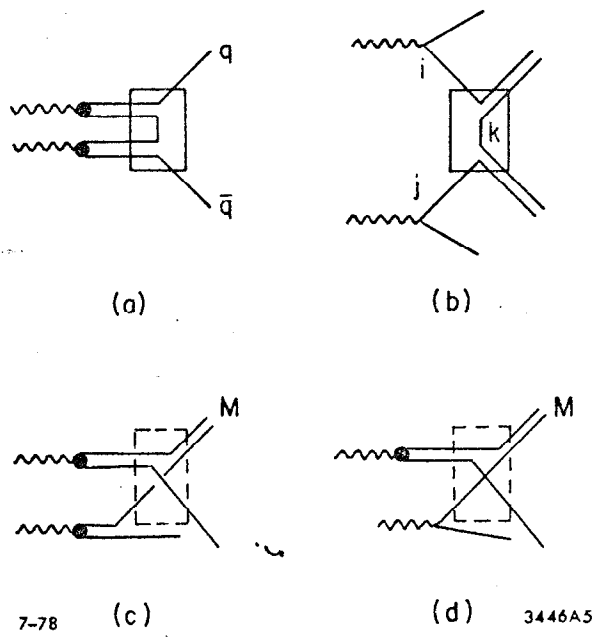


Fig. 5

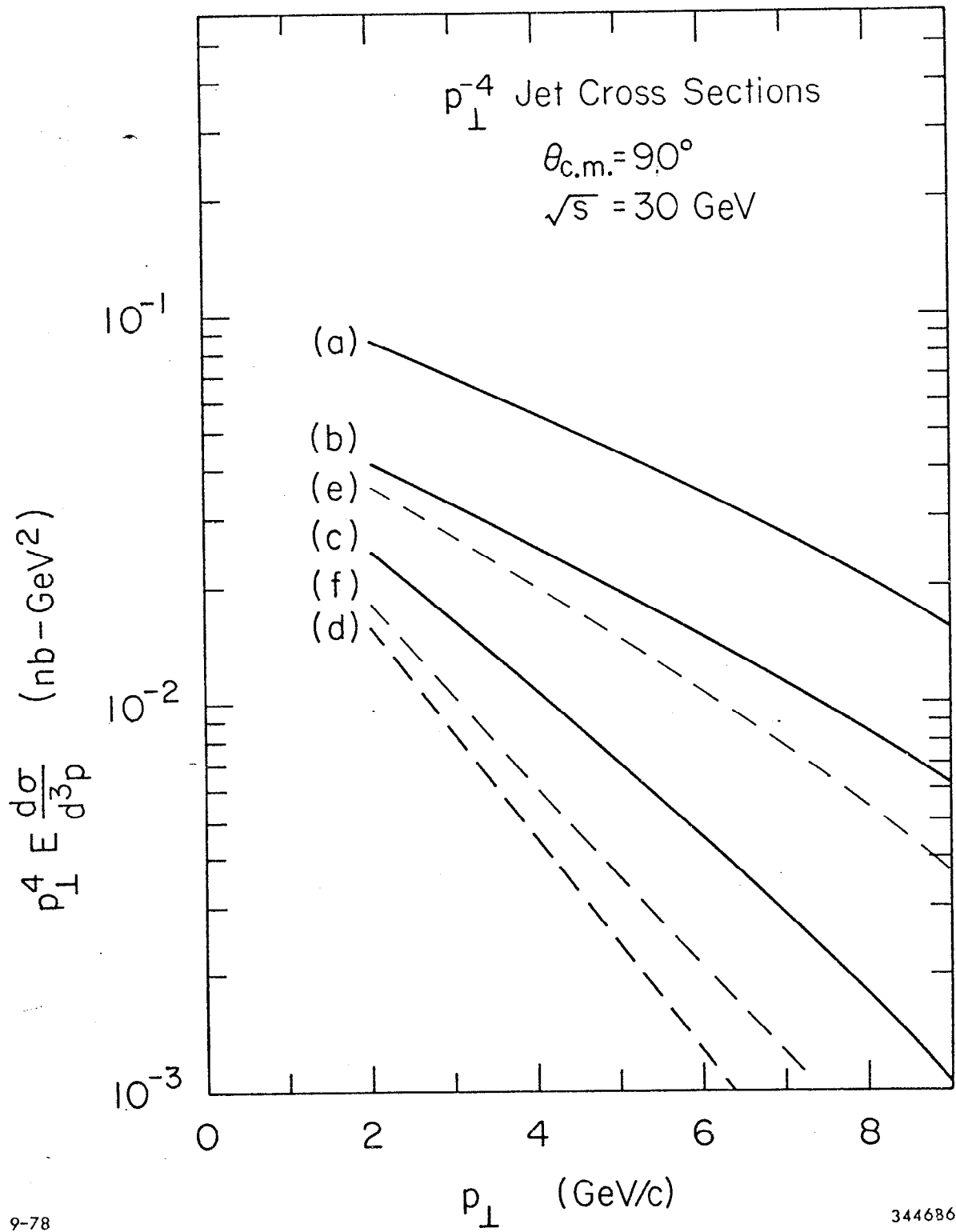


Fig. 6

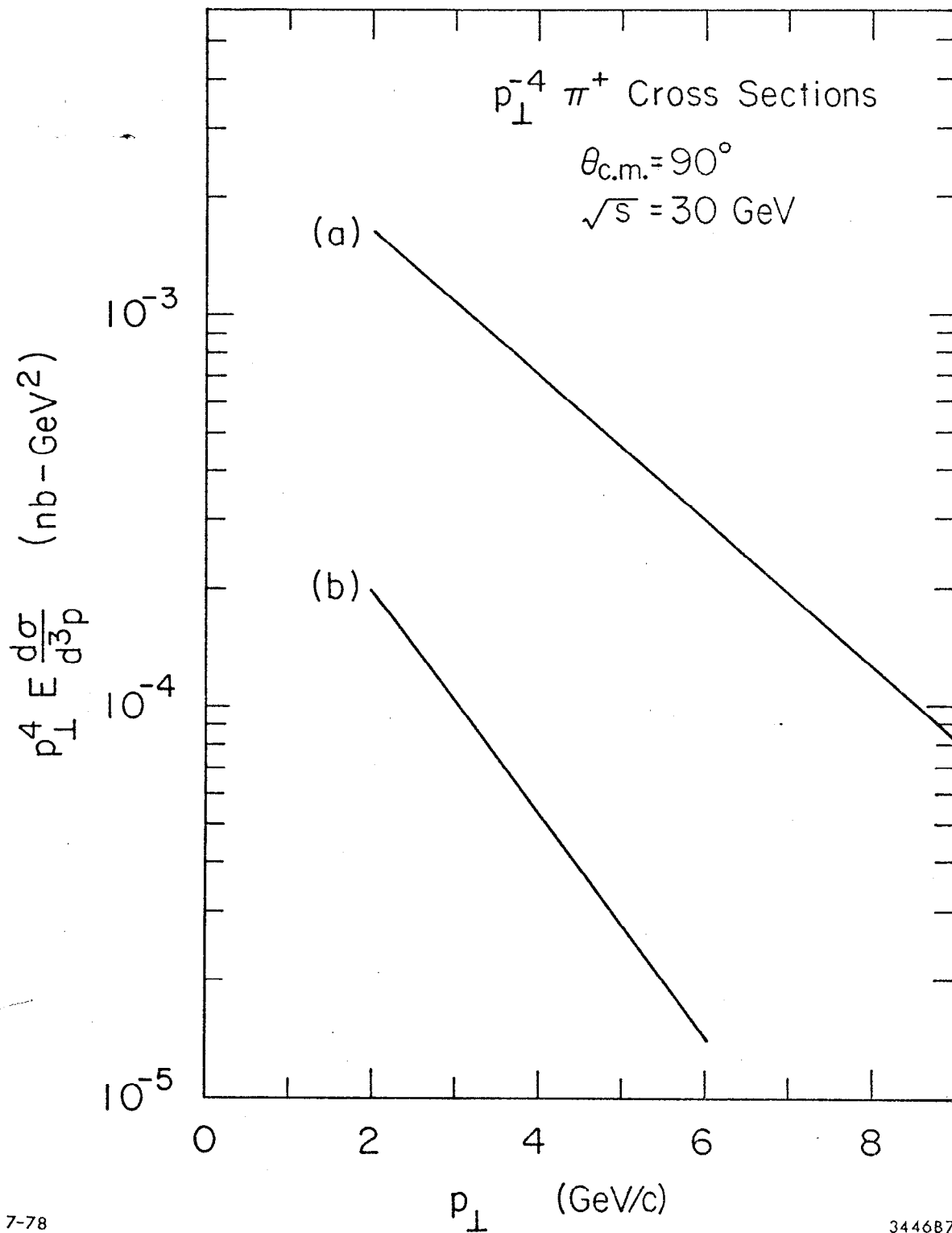


Fig. 7

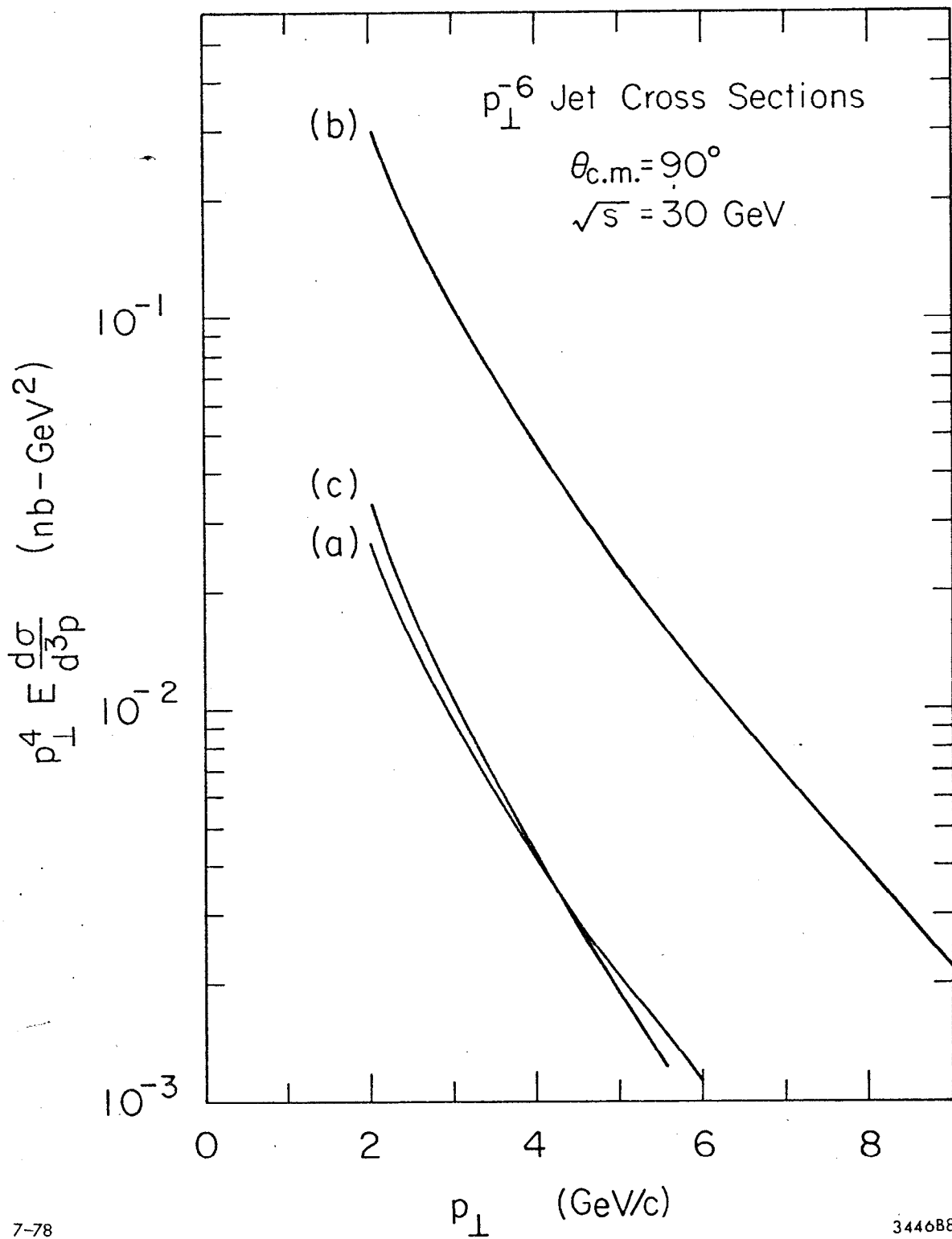


Fig. 8

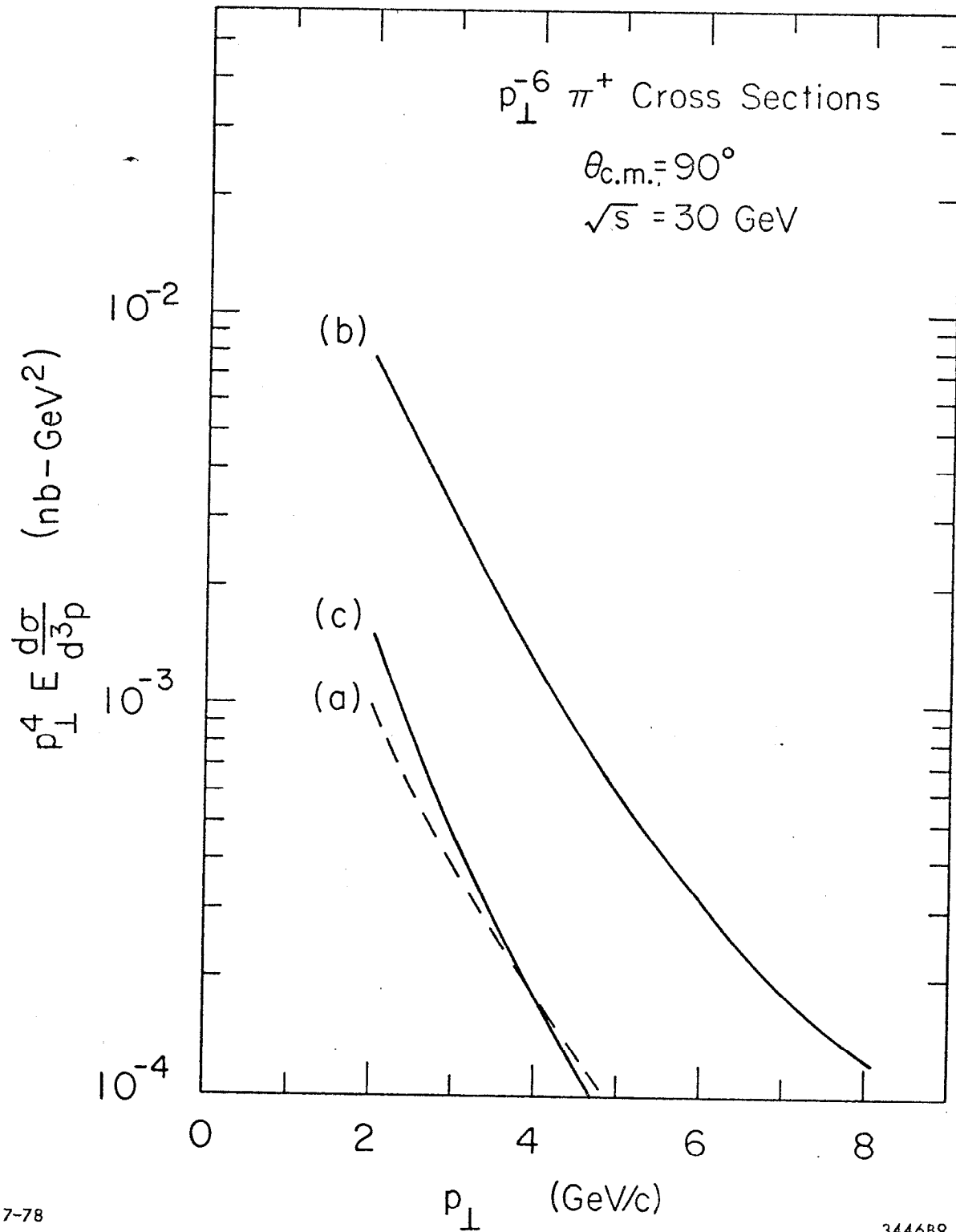


Fig. 9

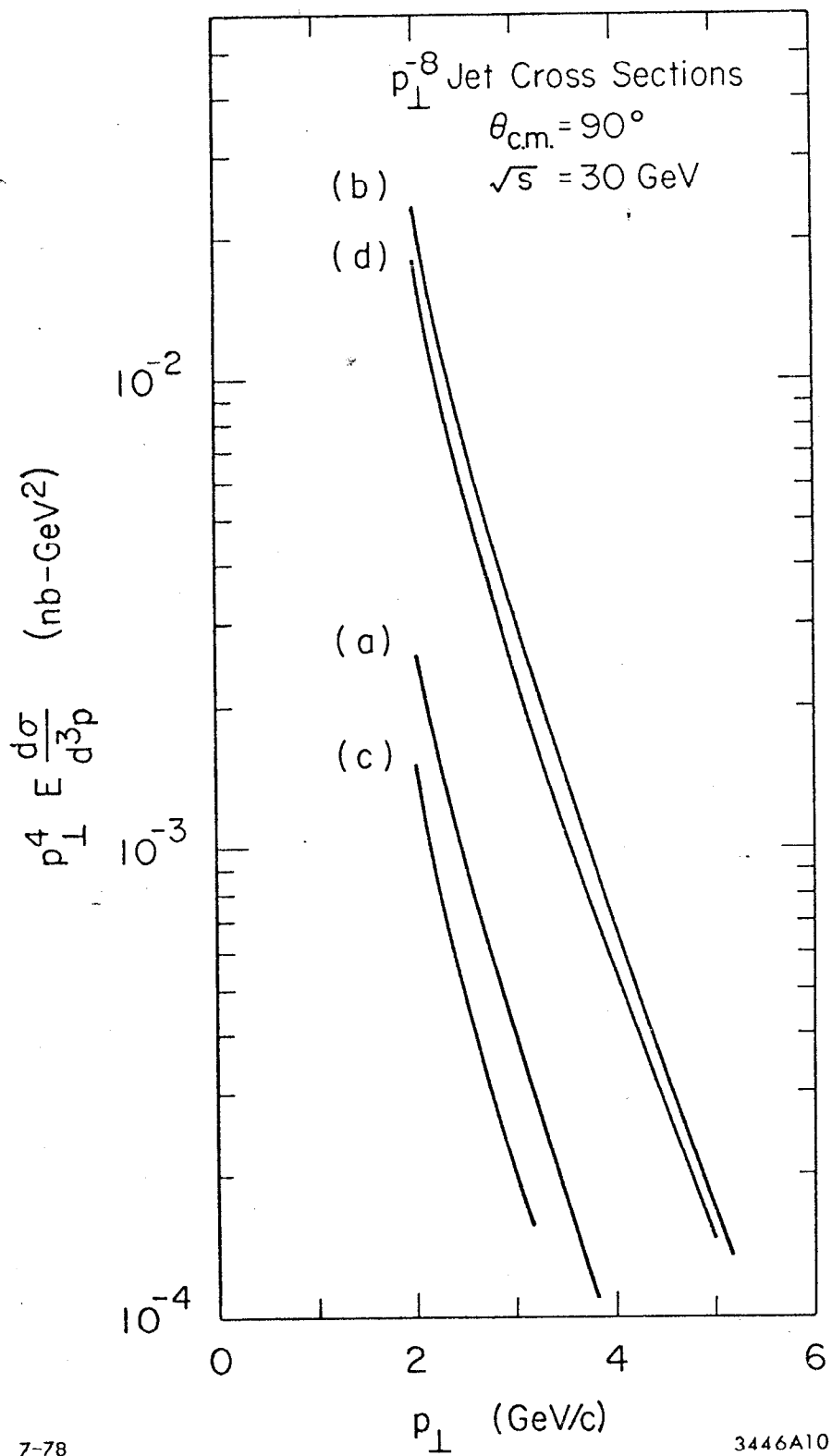


Fig. 10



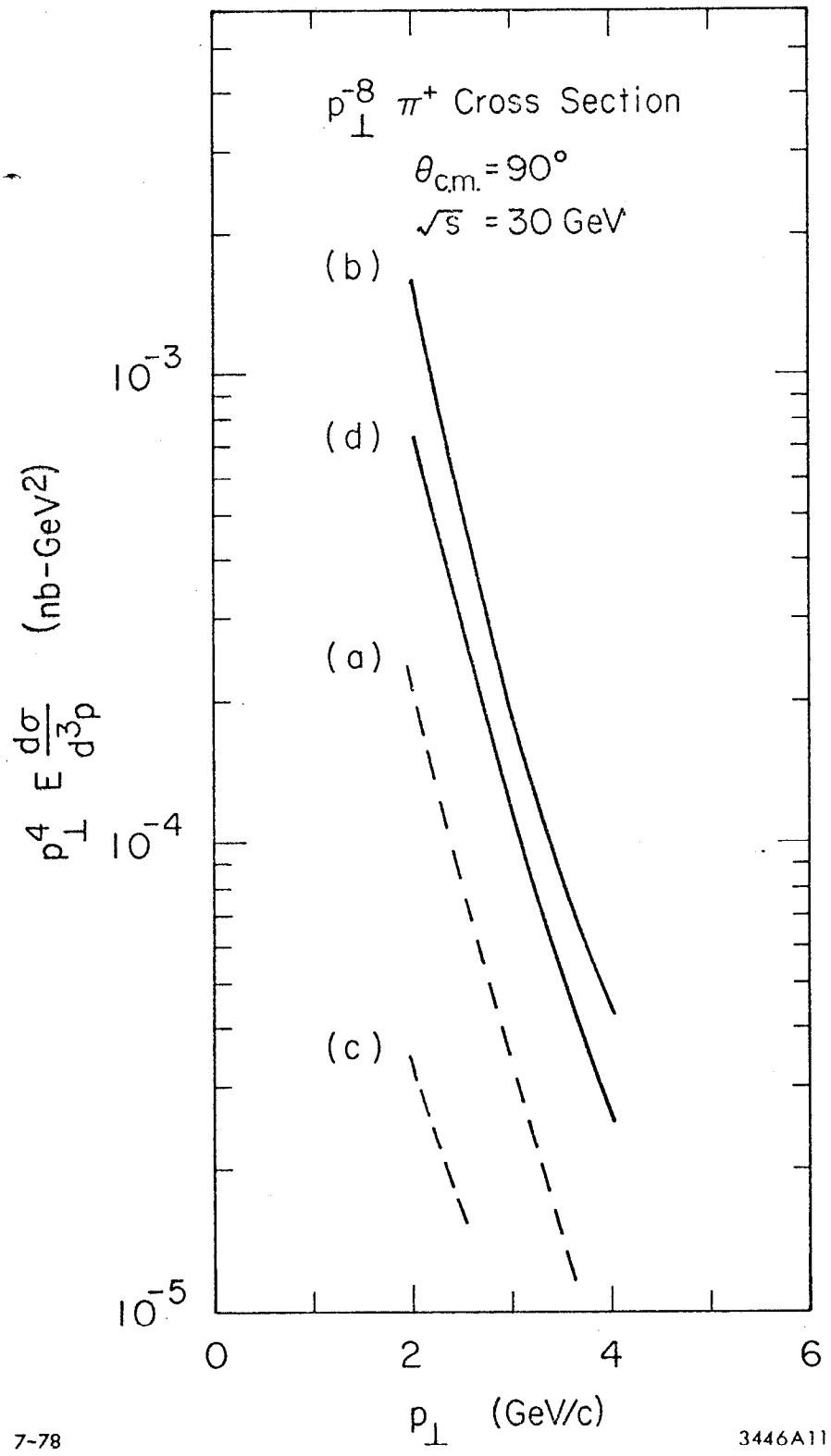


Fig. 11

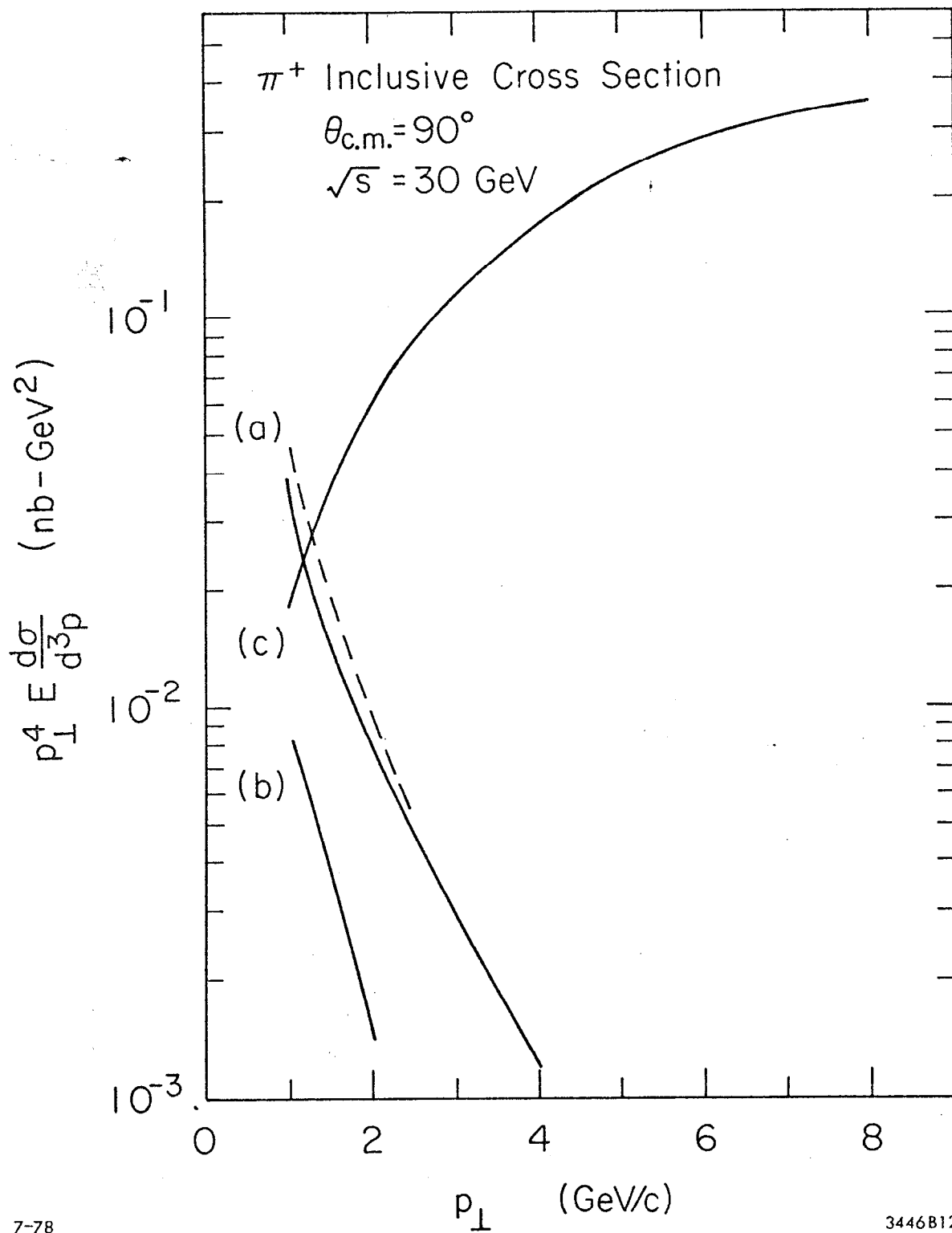


Fig. 12

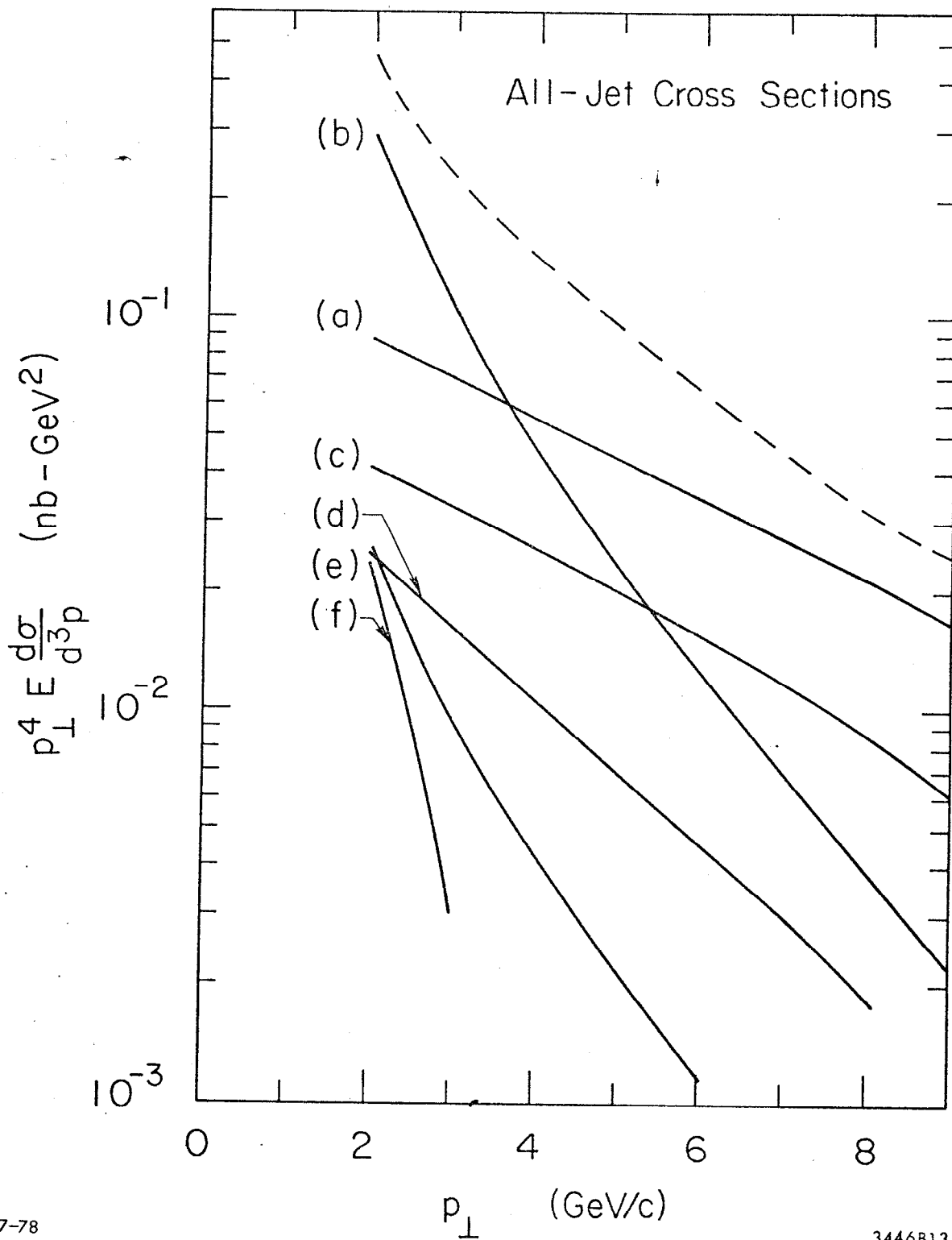


Fig. 13

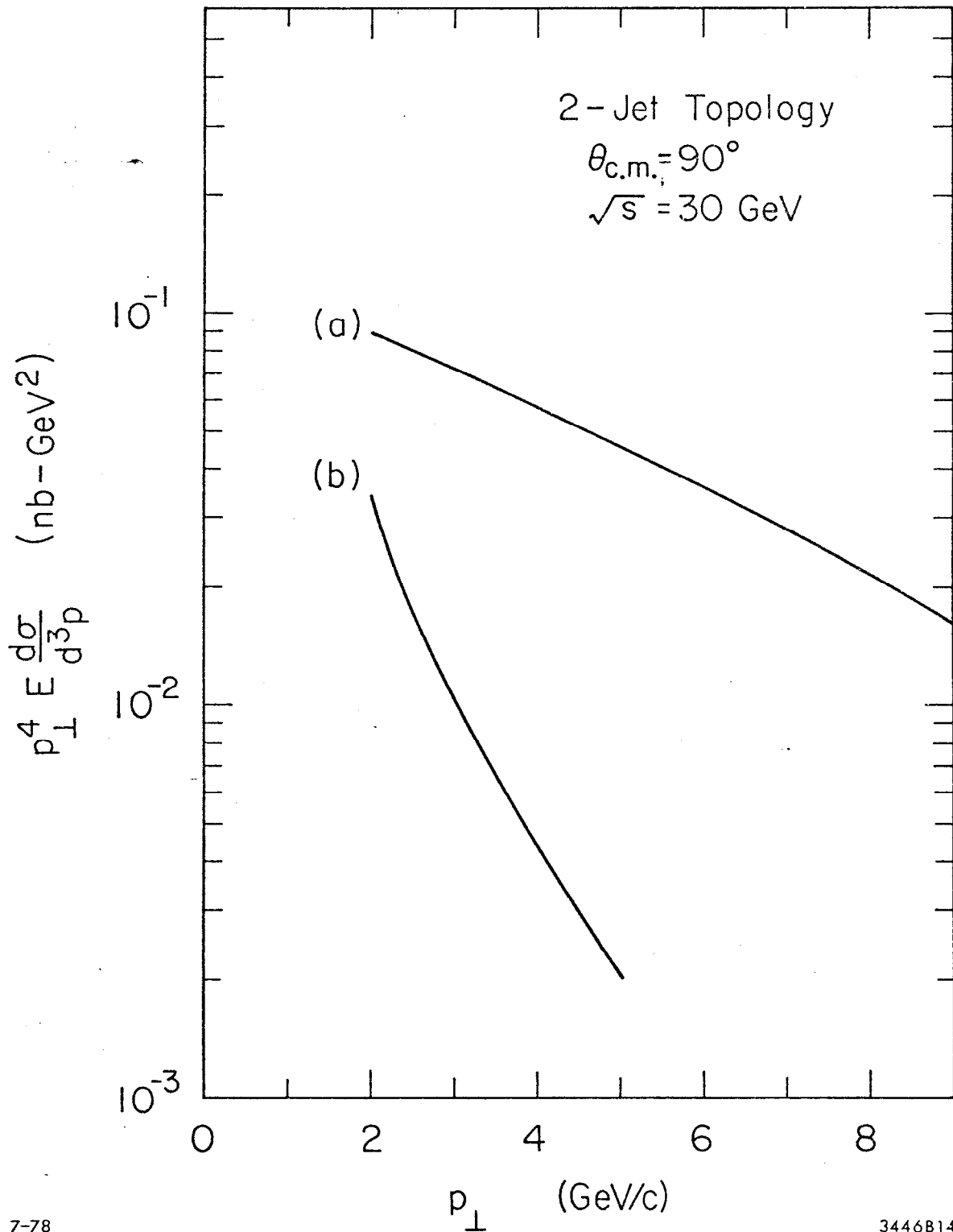


Fig. 14

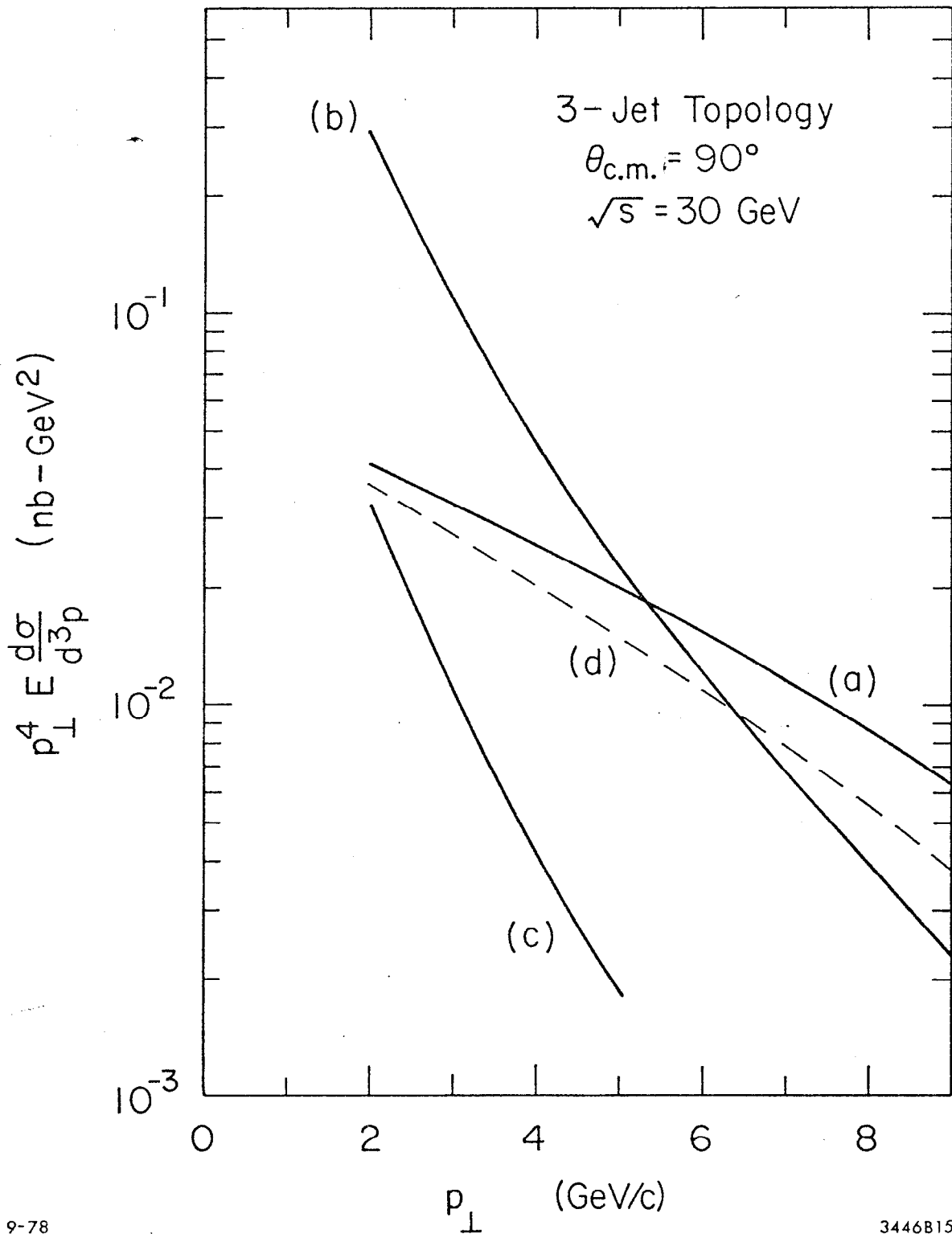


Fig. 15

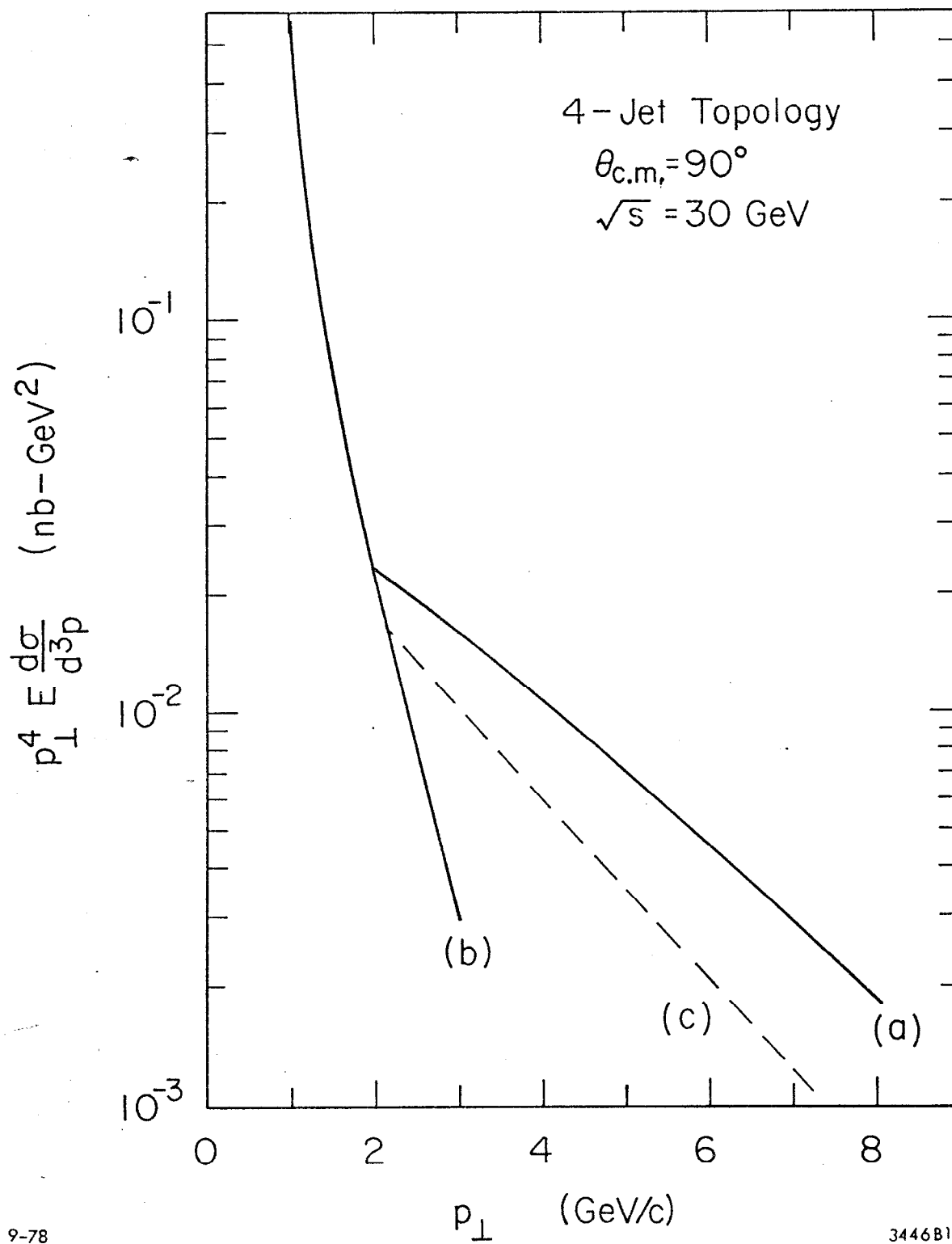


Fig. 16

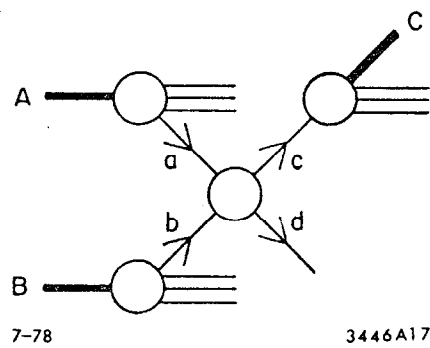


Fig. 17